Faculty of Science & Technology

First Semester B.Tech. (CSE/CE/IT/CT/CSE(Data Science)/CSE(Cyber Security)/CSE(AIML)) (CBCS) (NEP) Examination

APPLIED ALGEBRA

Time : Three Hours] [Maximum Marks : 70

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Solve Question **1** OR Question No. **2**.

(3) Solve Question **3** OR Question No. **4**.

(4) Solve Question **5** OR Question No. **6**.

(5) Solve Question **7** OR Question No. **8**.

(6) Solve Question **9** OR Question No. **10**.

(7) Use of non-programmable calculator is permitted.

1. (a) Investigate the linear dependence of vectors if possible, find the relation between them where

$X_1 = (2,-1,3,2), X_2 = (1,3,4,2), X_3 = (3,-5,2,2)$. 7

(b) Find the Eigenvalue and Eigenvectors of Matrix 7

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}.$$

OR

2. (a) Find the largest eigenvalues and corresponding eigenvectors of the matrix 7

$$\begin{bmatrix} -4 & -5 \\ 1 & 2 \end{bmatrix}.$$

(b) Solve by Gauss elimination method : 7

$2x + 3y + z = 13$

$x - y - 2z = -1$

$3x + y + 4z = 15$

3. (a) Determine the dimension and basis for the solution space of the system 7

$x + y + z = 0$

$3x + 2y - 2z = 0$

$4x + 3y - z = 0$

$6x + 5y + z = 0$.

(b) $T: R_3 \to R_3$ defined by $T(x, y, z) = (x + y, x - y, 2x + z)$. Find the range space, null space, rank, and nullity of T and verify rank of T + nullity of T = dim (R_3). 7

OR

4. (a) Find the singular value decomposition of the matrix 7

$$A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}.$$

(b) Using Gram-Schmidt orthogonalization process, orthonormalize the linearly independent subset {(1,0,0), (1,1,1), (1,2,3)}. 7

5. (a) If $y = \sin(m \sin^{-1} x)$ then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0.$$ 7

(b) Evaluate :

(i) $\lim_{x \to 0} \sin x \log x$ 3

(ii) $\lim_{x \to 1} (2-x)^{\tan \frac{\pi x}{2}}$. 4

OR

6. (a) Using Taylor's series, find the value of cos 64° correct up to fourth decimal place. 7

(b) Find the maximum and minimum values of f(x) where 7

$f(x) = 8x^5 - 15x^4 + 10x^2$.

7. (a) Evaluate :

(i) $\int_0^\infty e^{-4x} x^{3/2}\, dx$ 3

(ii) $\int_0^{\pi/2} \sqrt{\tan \theta}\, d\theta$. 4

(Contd.)

(b) Trace the curve: 7

$ay^2 = x^2(a-x)$.

OR

8. (a) Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the X-axis. 7

(b) Trace the curve astroid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ and find the perimeter of the astroid. 7

9. (a) Discuss the convergence of the sequence $\{a_n\}$ where $a_n=\frac{n+1}{n}$. If $\{a_n\}$ is a sequence in R, where $a_n = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{n}$. Evaluate $\lim_{n\to\infty}|a_{n+1}-a_n|$ and verify whether this sequence satisfies the Cauchy condition. 7

(b) Prove that $\left\{\frac{2n-7}{3n+2}\right\}$ is a monotonic increasing sequence and further show that it is bounded and tends to limit $\frac{2}{3}$. 7

OR

10. (a) Test the convergence of the series $\frac{1}{1^p}+\frac{1}{3^p}+\frac{1}{5^p}+\frac{1}{7^p}+\cdots$ 7

(b) Test the absolute convergence of the series $\frac{2}{1^2}-\frac{3}{2^2}+\frac{4}{3^2}-\frac{5}{4^2}\cdots$ 7

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