Wilson's Coaching Academy Pune

Class 10 Mathematics: High-Difficulty Real Numbers Problems

Academic Session: 2026-27 (Updated Syllabus)
Section 1: Number Theory & The Fundamental Theorem of Arithmetic

These questions focus on prime factorization, divisibility, and the properties of LCM and HCF.

1. **The Exponent Challenge:** If *n* is a natural number, prove that 12^n cannot end with the digit 0 or 5. Explain why the presence of prime factor 3 prevents it from ending in 0 even if a 2 is present.
2. **Variable HCF/LCM:** Two positive integers *a* and *b* are such that $a = x^4y^3$ and $b = x^3y^4$ where *x* and *y* are prime numbers. If $HCF(a, b) = x^my^n$ and $LCM(a, b) = x^py^q$, find the value of $(m+n)(p+q)$.
3. **Algebraic Divisibility:** Prove that for any positive integer *n*, the number $n^3 - n$ is always divisible by 6 using the Fundamental Theorem of Arithmetic.
4. **Composite Logic:** Explain whether $3 \times 12 \times 101 + 4$ is a prime or composite number without fully calculating the product.
5. **Prime Sums:** If *p* is a prime number, can $p^2 + p + 1$ ever be an even number? Justify your answer.
6. **Unknown Integers:** The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, find the other number.
7. **Co-prime Manipulation:** If *a* and *b* are co-prime numbers, what can you say about the HCF of $(a+b)$ and $(a-b)$? Provide cases for both odd and even integers.

Section 2: Advanced Word Problems

Application-based problems requiring multi-step logical reasoning.

Problem Type	Question Details
Inventory Management	A fruit vendor has 990 apples and 945 oranges. He packs them into baskets, each containing only one type of fruit and the same number of fruits. Find the **minimum** number of baskets required.
Circular Motion	Three athletes start running together on a circular track. They take 252 seconds, 308 seconds, and 198 seconds respectively to complete one round. After how many minutes and seconds will they next meet at the starting point?
Tiling & Geometry	The floor of a room is 8m 96cm long and 6m 72cm wide. Find the minimum number of square tiles of the same size needed to cover the entire floor without cutting any tiles.
Synchronization	Two electronic devices beep after every 60 seconds and 62 seconds respectively. They beeped together at 10:00 AM. Find the earliest time they will beep together again.
Morning Walk	Three people go for a morning walk. Their steps measure 80 cm, 85 cm,

Problem Type	Question Details
	and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

Section 3: Proofs of Irrationality

Rigorous proofs using the method of contradiction.

- Prove that $\sqrt{2} + \sqrt{5}$ is irrational.
- If p and q are two distinct prime numbers, prove that $\sqrt{pq}$ is irrational.
- Prove that $2\sqrt{3} / 5$ is irrational, given that $\sqrt{3}$ is an irrational number.
- Show that there is no positive integer n for which $\sqrt{(n-1)} + \sqrt{(n+1)}$ is a rational number.
- Prove that if x is a non-zero rational number and $\sqrt{y}$ is irrational, then $(x + \sqrt{y})^2$ is irrational.
- Prove that $\sqrt{13}$ is irrational using the theorem: "If p is a prime and p divides a^2, then p divides a."
- Prove that $3 + 2\sqrt{5}$ is irrational.
- Examine whether $\sqrt{2} + \sqrt{3}$ is a rational or irrational number. Justify your answer with a formal proof.