MathEVmatics

Supercharge your learning

A2 Mathematics for WJEC

Unit 4: Functions and the Modulus

Examples and Practice Exercises

Unit Learning Objectives

- ***To understand what is meant mathematically by a function, by the domain of a function and the range of a function.***
- ***To be able to find composite and inverse functions;***
- ***To be able to solve problems involving functions, including equations and inequalities;***
- ***To understand the modulus function, $y = |x|$ and its graph.***
- ***To be able to solve problems involving the modulus function.***
- ***To be able to sketch transformations of graphs, including combinations of transformations.***

Prerequisite learning:

Trigonometric Graphs and Equations (AS Mathematics)

Transformation of Graphs (AS Mathematics)

Interval Notation (AS Mathematics)

Basic Skill Check:

1) Sketch the graph of $y = \sin x$ for $0 \leq x \leq 2\pi°$.

2) For a function $y = f(x)$, describe the following transformations:

(a) $y = f(x) - 1$

(b) $y = 3f(x)$

(c) $y = f(x + 2)$

(d) $y = f(4x)$

3) Write the interval $x \in (-3, 7]$ as an inequality.

When you have completed the unit…

Objective	Met	Know	Mastered
To understand and use the terms function, domain and range.			
To be able to find composite and inverse functions.			
To be able to solve problems involving functions, including equations and inequalities.			
To understand the modulus function and sketch graphs involving the modulus.			
To solve problems involving the modulus function.			
To be able to sketch transformations of graphs.			

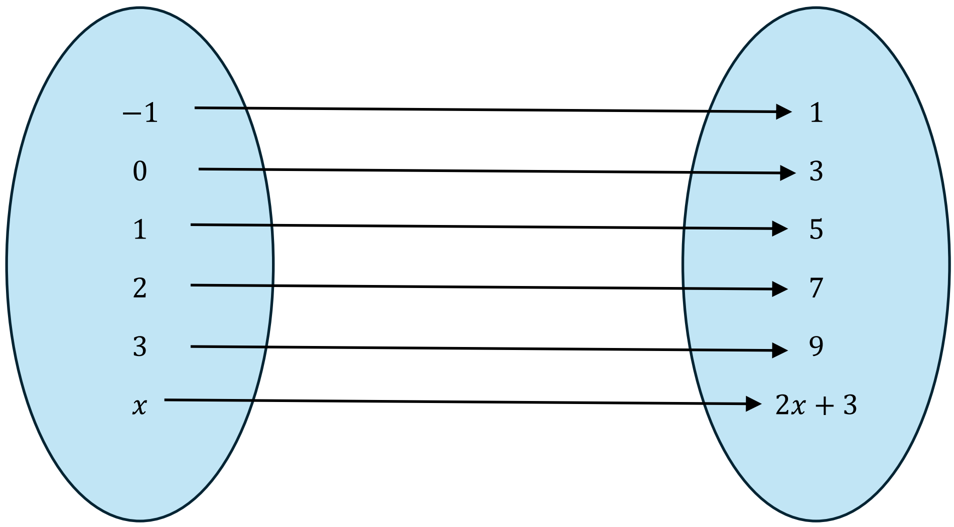
Notes/Areas to Develop:

Mappings and Functions

A mapping is a simple concept. It is a rule which takes a set of inputs (objects) and produces outputs (images).

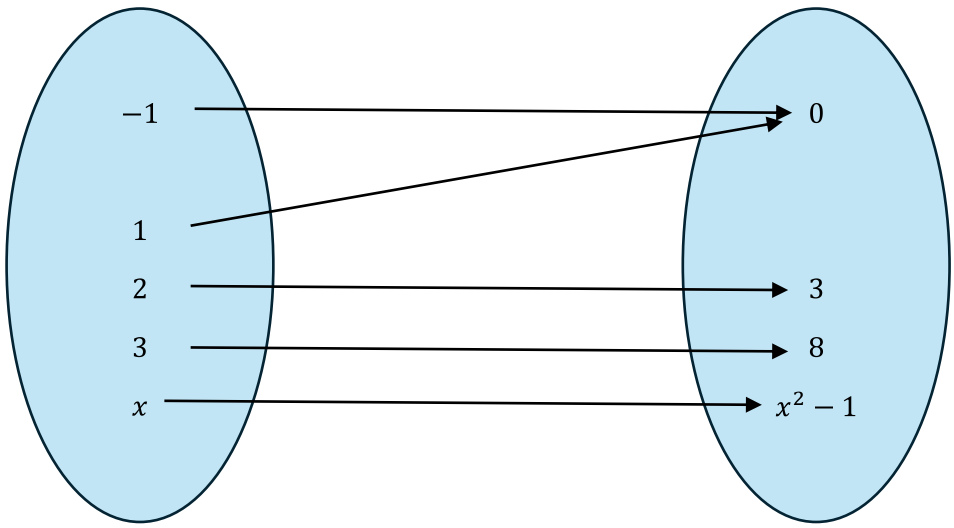
We can illustrate mappings easily with a mapping diagram:

$$x \rightarrow 2x + 3$$



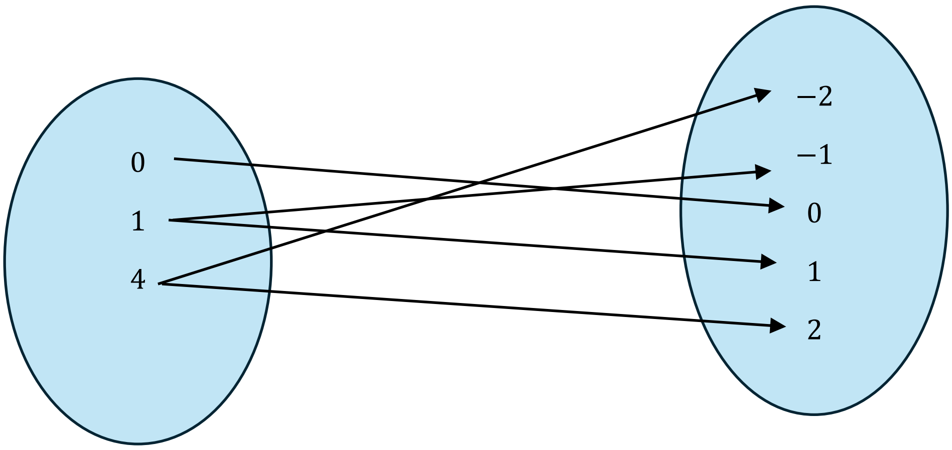
Here, each unique input corresponds to a unique output. This is known as a **one-to-one** mapping.

$$x \rightarrow x^2 - 1$$



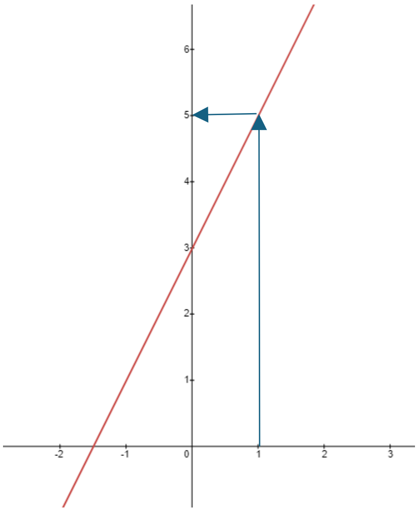
In this case, there is more than one input which gives the same output. This is called a **many-to-one** mapping.

$$x \to \pm\sqrt{2}$$

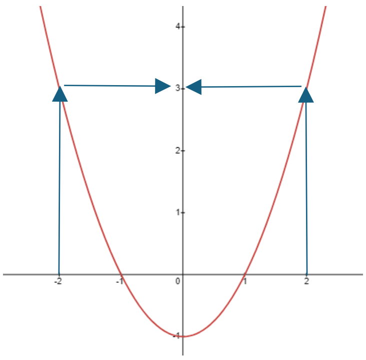


In this case, some inputs can be mapped to several different outputs. These are called **one-to-many** mappings.

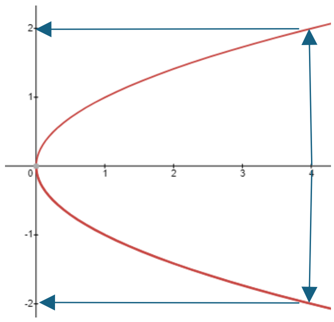
We can represent each of these graphically.



One-to-one



Many-to-one



One-to-many

There is also such a thing as a many-to-many mapping, e.g. the equation of a circle $x^2 + y^2 = r^2$*!*

We are now going to introduce some more technical language.

- The set of inputs ('x-values') is called the **domain**.
- The set of outputs ('y-values') is called the **range**.

We have often used the word 'function' loosely in mathematics up until now. However, we will now also more tightly define this word.

A **function** is a mapping in which each element of the domain is mapped to single element of the range (i.e. each input has a unique output).

This means that:

- One-to-many mappings are **not** functions because some inputs can give multiple outputs.
- It is also not a function if there are elements within the domain which have no output (e.g. mappings which have 'breaks' – vertical asymptotes – in them, such as $y = \tan x$ and $y = \frac{1}{x+1}$). **This means that just because a mapping is one-to-one or many-to-one, it is not necessarily a function.**

I like to think of functions like recipes:

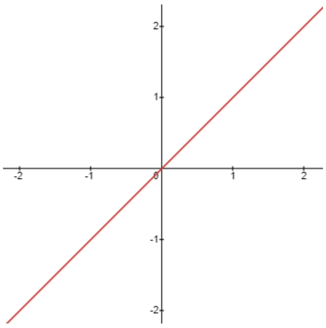
- There are several recipes which can produce the same cake (i.e. many-to-one is fine)
- However, if a recipe could produce two completely different foods through following the same instructions… that would be very strange. (So one-to-many is not a function!)
- If we start with ingredients and follow a recipe… having NO output would be even stranger! (So inputs which lead to no output mean we have no function!)

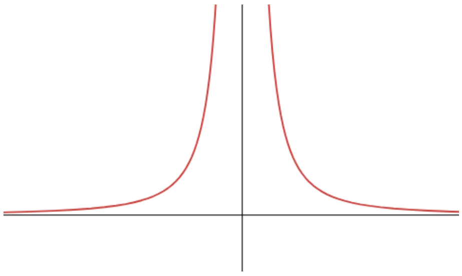
Task 1:

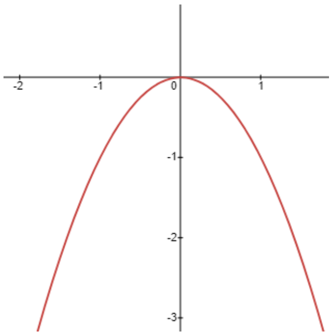
i) Circle (or otherwise indicate) the graphs which correspond to functions.

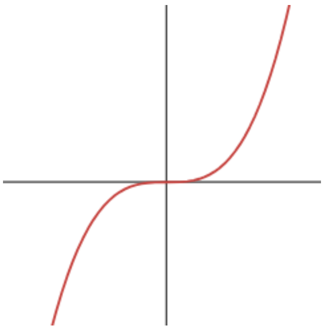
ii) For any circled functions, state whether they are one-to-one or many-to-one functions.

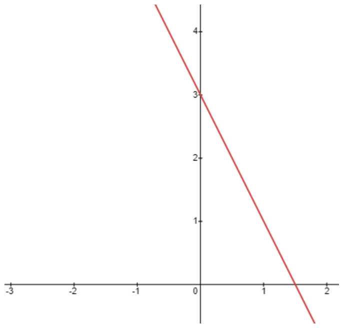
iii) For any graphs which do not represent functions, state why they are not functions.

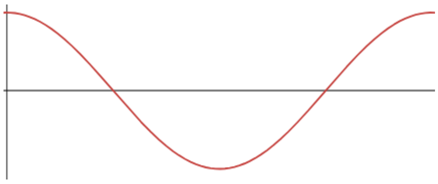


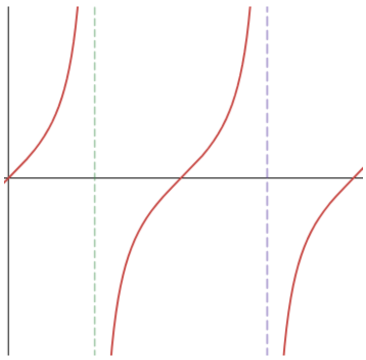






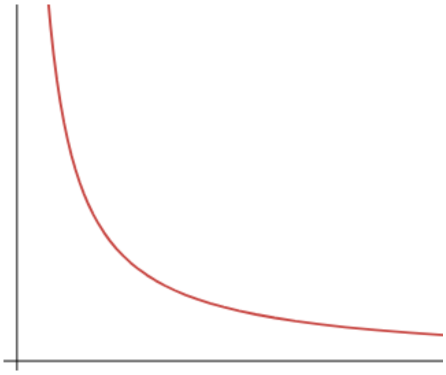






We can turn a mapping into a function by **restricting** its domain so that it removes any one-to-many elements or vertical asymptotes.

For example, if we consider $y = \frac{1}{x}, x > 0$, adding in the restriction that x must take positive values removes the vertical asymptote



Every value of x greater than zero now maps to a unique y-value, creating a one-to-one function.

One of the most important skills at A2 is to be able to determine the domain (set of possible inputs for x) and range (set of possible outputs) of a given function.

A sketch is often incredibly useful as a way of scaffolding your thinking here!

Example 1: For each of the following functions, state the range.

a) $y = 3x + 1$

b) $y = 3x + 1, 0 \leq x < 4$

c) $y = x^2 - 1$

d) $y = \frac{1}{x}, x > 0$

Example 2:

State the domain and range of the function $y = \sin x$.

Task 2:

Find the range of each of the following functions.

a) $f(x) = 3 - x, \qquad x > 0$

b) $f(x) = x^2 + 2, \quad 0 \leq x \leq 5$

c) $f: x \to x^3 + 2, \quad -2 \leq x \leq 2$

d) $y = \tan x\,, \qquad 0 \leq x < 90°$

Now: Complete TEST YOUR UNDERSTANDING 1, Page 27.

Composite and Inverse Functions

Composite Functions

$f(x)$ means that we take an input, x, and apply a function f to it.

This means that, if we see the notation $gf(x)$, the same idea must apply that we are starting with the input x, then applying f to it, and **then** applying g to this result.

Key points:

- When we combine (compose) multiple functions, we actually work right-to-left.
- We could think of this as in-to-out if we included some extra brackets, e.g. $g(f(x))$.

Example 1: Given the functions $f(x) = x^2 + 3$ and $g(x) = 3x - 1$, find:

a) $fg(2)$

b) $gf(2)$

This illustrates an incredibly important point – the order of applying operations is incredibly important, and changing the order (in general) does not give the same result. In mathematical language,

$gf(x) \neq fg(x)$.

Example 2: Given the functions $f(x) = x^2 + 3$ and $g(x) = 3x - 1$, find:

a) $fg(x)$

b) $gf(x)$

Task 1: Given the functions $p(t) = 3t - 2$ and $q(t) = \frac{5}{2t+1}$, find:

a) $pq(t)$

b) $qp(t)$

c) The value of each when $t = 3$.

We can use this skill to set up and solve equations.

Task 2: It is given that, for all real values of x, $f(x) = 2x + 1$ and $g(x) = x^2$. Find the value(s) of x that satisfy the equation

$$fg(x) = gf(x).$$

Composites and their link to domain/range

In the previous task, you were told that the two functions being composed were valid for all values of x. This meant that they could be composed without restriction.

For a composition of, let's say, $gf(x)$, it is necessary that the ***range*** of $f(x)$ (the first function being applied) is completely contained within the ***domain*** of the second function to be applied (here g). Otherwise, there would be possible outputs from f which couldn't go into g (i.e. inputs of gf with no output, which we have already seen to be not allowed in the definition of a function!).

Examiner Tip: WJEC like to ask questions testing this understanding!!

Example 3:

Two functions $f(x)$ and $g(x)$ are such that $f(x) = 2x + 1$, $-3 \leq x \leq 7$ and $g(x) = x^2 - 2, 0 \leq x \leq 3$.

a) Find $fg(x)$.

b) Explain why $gf(x)$ does not exist for the functions f and g.

Now: Complete TEST YOUR UNDERSTANDING 2, Page 29, Questions 1-7.

Inverse Functions

A function f takes elements x in the domain and maps them to corresponding elements in the range.

An inverse function f^{-1} 'undoes' this, i.e. it takes elements from the range and maps them back to elements within the original domain.

This leads to some important results about inverse functions.

- $f(x)$ and $f^{-1}(x)$ are inverses of each other. This means that $ff^{-1}(x) = f^{-1}f(x) = x$.
- The domain of $f(x)$ is the range of $f^{-1}(x)$.
- The range of $f(x)$ is the domain of $f^{-1}(x)$.
- The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are ALWAYS reflections of each other in the line $y = x$.

This last point also implies that an inverse function can only be found when $f(x)$ is one-to-one – try to understand why!

Example 1:

Sketch the graphs of $y = e^x$ and $y = \ln x$ on the same set of axes, showing the points of intersection with the axes.

Example 2:

Find the inverse of the function $f(x) = x^2 - 4, x \geq 0$, and state it's domain and range.

Make sure you understand this method – it comes up almost every year!

Task 1:

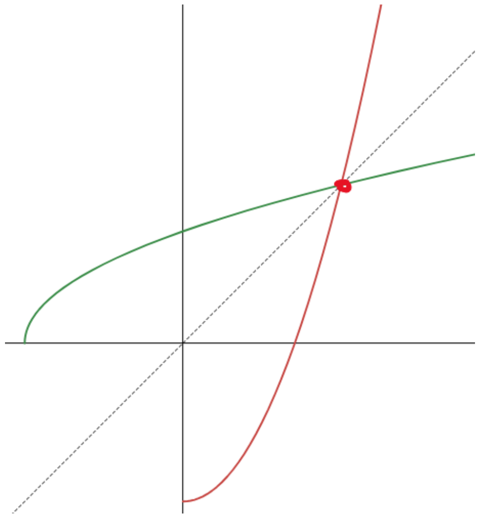
The function $f(x)$ is defined by $f(x) = x^2 + 2, x \geq 0$.

a) Find $f^{-1}(x)$ and state it's domain.

b) Sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

A really popular exam question involving inverse functions is being asked to solve the equation $f(x) = f^{-1}(x)$.

An important trick for this comes from understanding the situation graphically. The below image shows a graph $y = f(x)$ in green, the inverse $y = f^{-1}(x)$ in red, and the line of reflection $y = x$.



Notice that the point of intersection (which is where $f(x) = f^{-1}(x)$) lies on the line of reflection. This means that:

$f(x) = f^{-1}(x)$ when $f(x) = x$ (or when $f^{-1}(x) = x$ if this is easier to solve!!).

Task 2: For the function $f(x) = x^2 - 2$, where $x \geq 0$, find where $f(x) = f^{-1}(x)$.

Now: Complete TEST YOUR UNDERSTANDING 2, Page 30, Questions 8 onwards.

Space for additional notes:

The Modulus Function - Introduction

The modulus function (also known as the 'absolute value' – sometimes seen as Abs on calculators) takes an input and returns its non-negative value.

Example 1: Write down the values of the following:

a) $|-3|$ b) $|5|$ c) $|2 \times 1.4 - 6|$

Example 2: You are given that

$$f(x) = |3x - 10|.$$

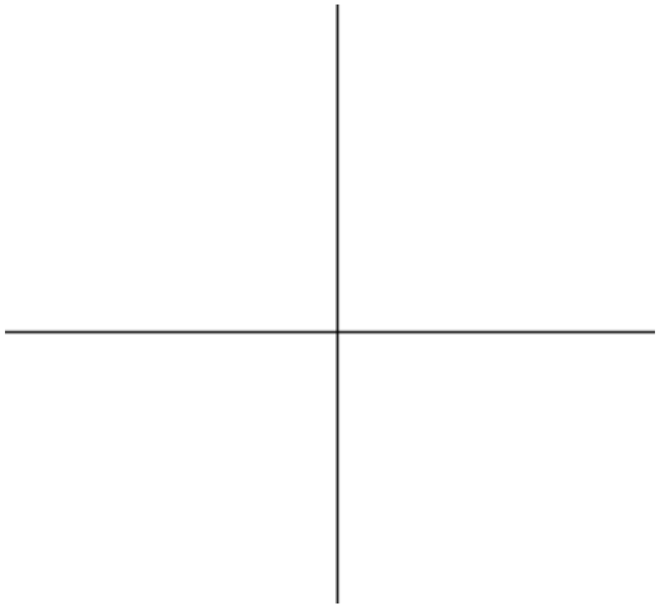
Calculate the value of:

a) $f(5)$ b) $f(2)$ c) $f(-1)$

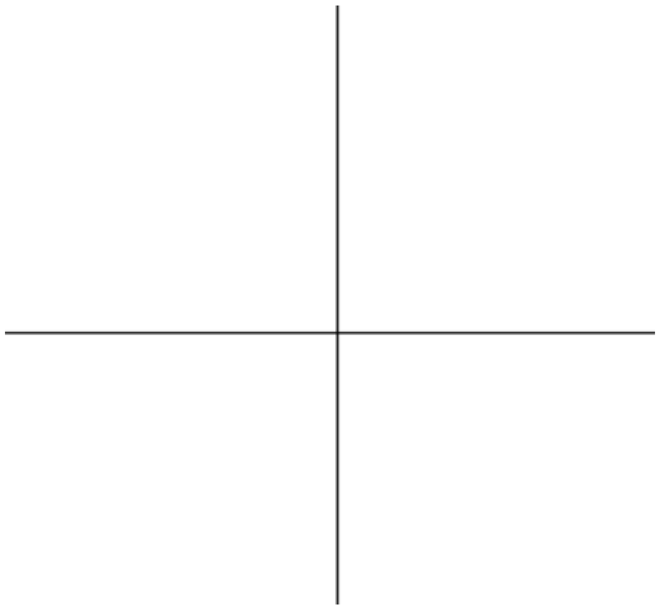
Task 1: Given that $g(x) = |5x - 2|$, find the values of:

a) $g(2)$ b) $g(-2)$ c) $g(0.2)$

We can draw graphs of modulus functions. Consider the graph of $y = x$:



Now, let's draw $y = |x|$:



As we can see, the part of the original graph that would have given negative y-values has been reflected in the x-axis.

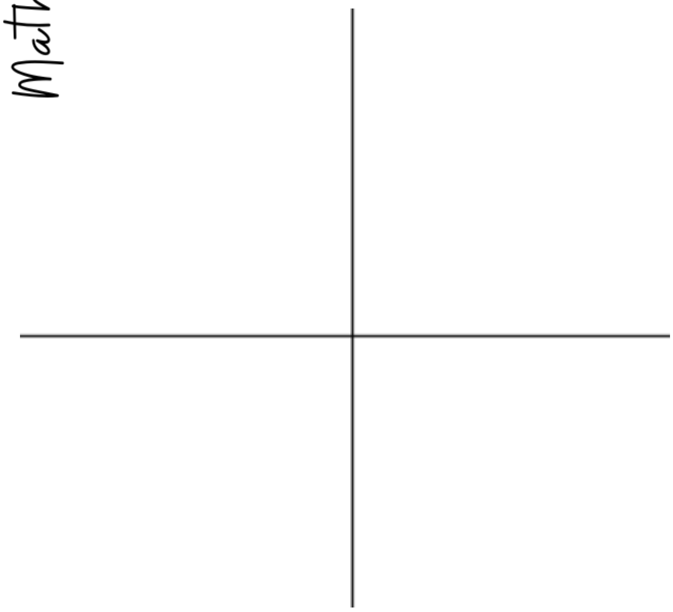
Key point: To draw the graph of a function $y = |f(x)|$:

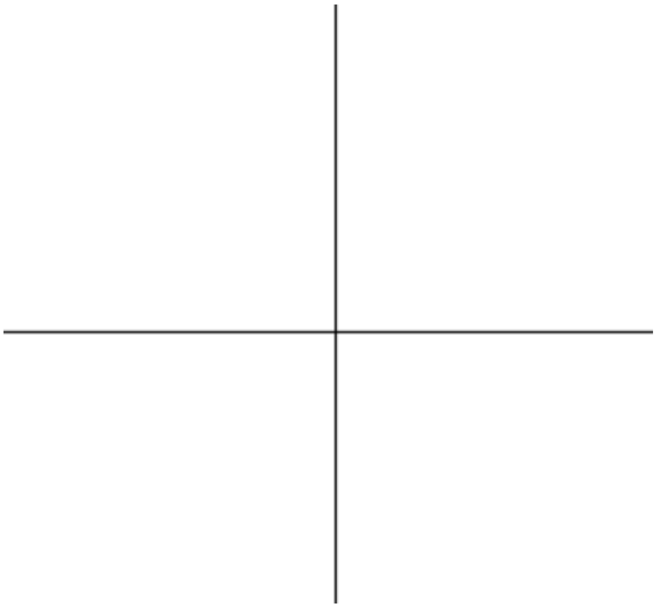
- Draw the graph of $y = f(x)$ as usual (but lightly, so you can rub out later!)
- Reflect any part of the graph below the x-axis in the x-axis.
    - (We are replacing the negative part with the graph of $y = -f(x)$).

Example 3:

Sketch the following graphs, marking any points of intersection with the axes.

a) $y = |2x - 1|$ b) $y = |x^2 - 1|$

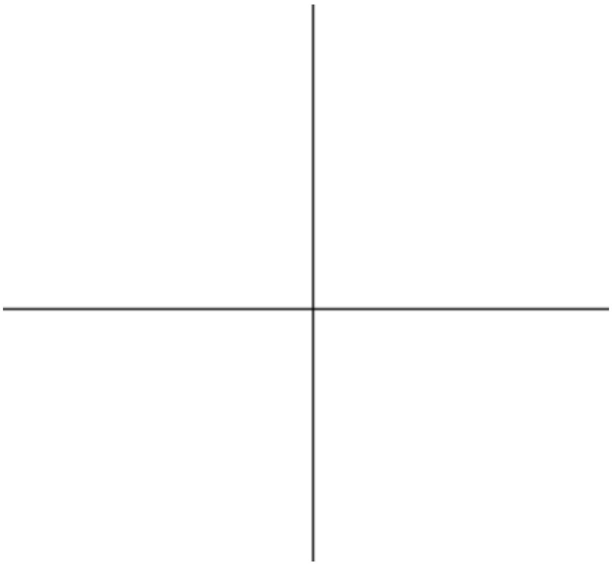




We can now start to solve equations and inequalities involving the modulus function. A sketch of the graph is *always* a useful starting point for this.

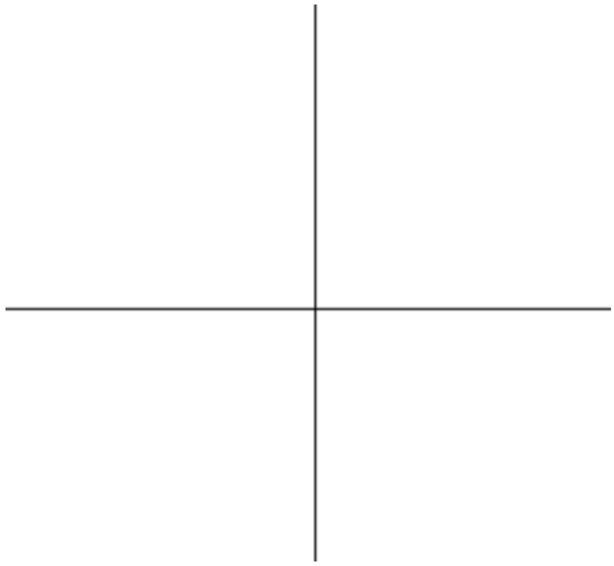
Example 4:

Solve the equation $|2x - 3| = 4$



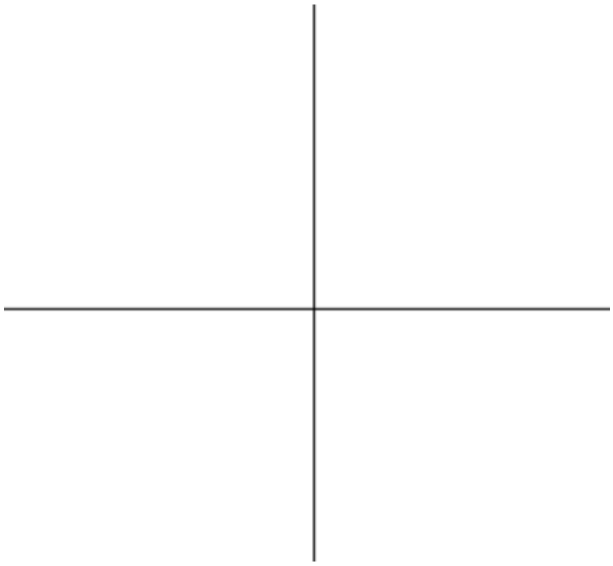
Task 2:

Solve the equation $|5x - 2| = 4 - x$



Task 3:

Solve the inequality $|4x - 2| > 2x$



Now: Complete TEST YOUR UNDERSTANDING 3, Page 33.

Transformations of Functions

We have met the six basic transformations at GCSE and more thoroughly studied these at AS level. These also form part of this module of work.

Example 1: Sketch the following graphs:

a) $y = 3|x - 2| - 1$

b) $y = -|2x + 1| + 1$

Example 2: A function $f(x)$ is given by $f(x) = 3x - 2$. On separate axes, sketch the graphs of the following, in each case giving the coordinates of any points of intersection with the axes:

a) $y = |f(x)|$

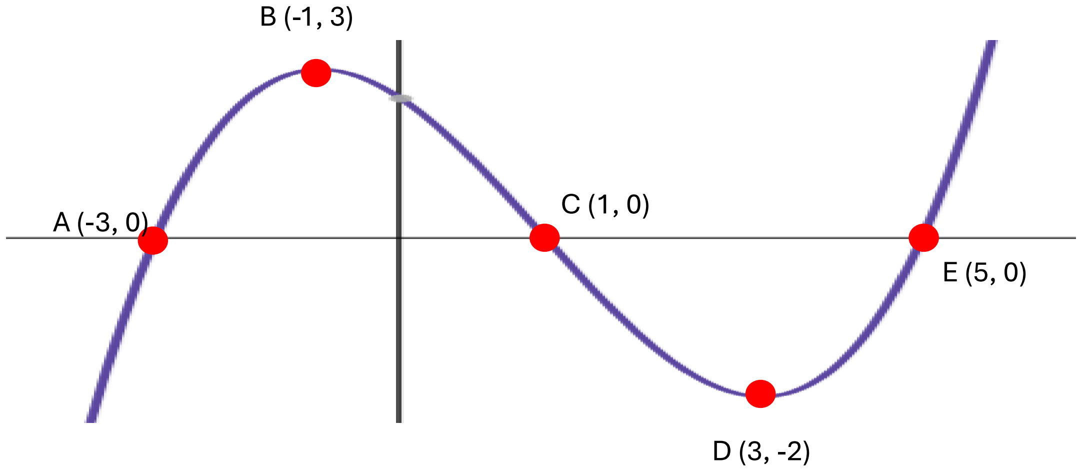
b) $y = f(|x|)$

Given the graph of $y = f(x)$,

- To draw $y = |f(x)|$, reflect any parts of the graph below the x-axis 'upwards'
- To draw $y = f(|x|)$, draw the graph for $x \geq 0$ and then reflect this in the y-axis.

Task 1:

The graph below shows the graph of the function $y = f(x)$ and the coordinates of points A, B, C, D and E.



Sketch the following graphs, indicating the coordinates of any points related to A-E.

a) $y = f(2x)$ a) $y = |f(x)|$ b) $y = f(|x|)$

Now: Complete TEST YOUR UNDERSTANDING 4, Page 35.

Putting it all together:

Solving problems

Examination questions can combine some or all of the skills learnt so far!

Example 1: It is given that $f(x) = 2|x-2| - 1$.

a) Sketch the graph of $y = f(x)$.

b) State the range of the function.

c) Explain why $f^{-1}(x)$ does not exist.

d) Solve $f(x) = 0.5x + 1$.

Space for additional notes/workings:

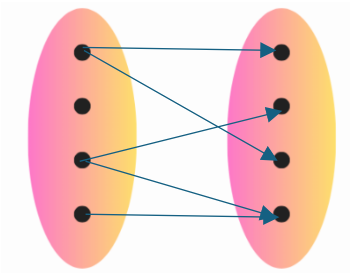
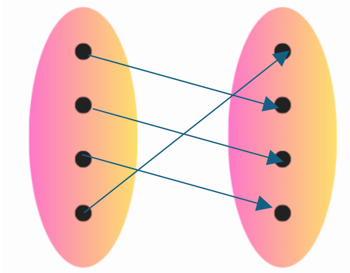
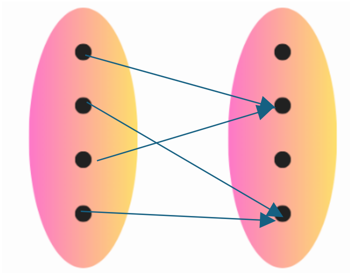
NOW – You are ready for the GRADE ENHANCER (Page 36)!

Test Your Understanding 1

Question 1

For each of the following mappings, state whether they are one-to-one, many-to-one or one-to-many.

(In each case, it can be assumed that the LHS is the domain and the RHS is the range)

a)  b)  c) 

Question 2

A function is defined by $f(x) = 3x - 2$, where $x > 0$.

a) State the range of the function.

b) Find the value of a such that $f(a) = 10$.

Question 3

A function is defined by $g(x) = 3^x$, where $x \in \mathbb{R}$.

a) State the range of the function.

b) Solve $g(x) = 12$, giving your answer to 3 significant figures.

Question 4

A function is given by $h(\theta) = 4\cos\theta\,, where\ 0 \leq \theta \leq 2\pi$.

a) State the range of the function.

b) State whether the function is one-to-one or many-to-one.

c) Sketch the graph of $y = h(\theta)$, showing any points of intersection with the axes.

Question 5

A mapping f is defined over all the real numbers by $f: x \to \frac{1}{x+2}$.

a) Explain why the above mapping is not a function.

b) Given the restriction that $x > 0$, find the range of the function $f(x)$.

c) Sketch the graph of $y = f(x),\ x > 0$.

Question 6

Two functions are given by $f(x) = x^2 + 1, x > 0$ and $g(x) = 2x + 4$, $x \in \mathbb{R}$.

a) State the range of $f(x)$.

b) Sketch $y = f(x)$ and $y = g(x)$ on the same graph, and hence state the number of solutions to the equation $f(x) = \mathrm{g}(x)$.

c) Solve $f(x) = g(x)$.

Challenge Question

i) The function f is given by $f(x) = x^2 - 8x + 10, x \geq n$. Given that the function is one-to-one, find the smallest possible value of n.

ii) A function g is such that $\mathrm{g}'(x) = 2x + 5$, and $\mathrm{g}(2) = 20$. Given that g is one-to-one and has domain $x \leq a$ where a is the largest value possible, find the function g and state the domain and range of the function.

Test Your Understanding 2

Question 1

The functions f and g are given by $f(x) = x^2$ and $g(x) = 2x - 5$.

a) Find $f(3)$

b) Find $g(3)$

c) Find $fg(3)$

d) Find $gf(3)$

e) Show that $f(x) = g(x)$ has no real roots. (CHALLENGE – Can you do this in four different ways?)

f) Find the exact roots, in surd form, of the equation $fg(x) = gf(x)$

Question 2

The functions f and g are given by $f(x) = x^3$ and $g(x) = x + 2$.

Find $fg(x)$ and $gf(x)$.

Question 3

A function f is given by $f(x) = 2x - 2$, where $x > 3$.

a) Find $f^2(x)$. (*Note: This notation means* $ff(x)$, *not squaring!)*

b) By considering the discriminant, show that there are no real solutions to the equation

$$f^2(x) = (f(x))^2$$

Question 4

The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = x + 3$.

Express each of the composite functions below in terms of f and g.

a) $\sqrt{x+3}$ b) $\sqrt{x} + 3$ c) $\sqrt{4x+12}$ d) $x + 6$

Question 5

The functions f and g are given by $f(x) = x^2 + 4$ and $g(x) = 3x + 2$.

a) Solve $fg(x) = gf(x)$, giving your answers in the form $x = a \pm \sqrt{b}$

b) Solve $fg(x) = 89$

c) Find $fgf(x)$ and $gfg(x)$

Question 6

The functions f and g are given by $f(x) = e^{2x} - 1, x \in \mathbb{R}$ and $g(x) = \ln(x + 2), x > -2$.

Find $gf(x)$, stating its range.

Question 7

Given that $f(x) = \frac{1}{x+2}$, $x \neq -2$

a) Explain why the domain $x \neq -2$ is necessary for $f(x)$ to be a function.

b) Find $f^2(x)$, giving your answer in the form $\frac{x+2}{ax+b}$.

Question 8

The functions f and g are given by $f(x) = x^2$ and $g(x) = 2x - 5$.

a) Find $g^{-1}(x)$.

b) State a reason why, currently, $f^{-1}(x)$ cannot be found.

Question 9

A function f is given by $f(x) = 2x - 2$, where $x > 3$.

Find $f^{-1}(x)$, stating its domain and range.

Question 10

Find the inverse of the function $f(x) = \dfrac{3}{5 - x}$.

Question 11

For each of the following functions $f(x)$

i) find the inverse function $f^{-1}(x)$and state its domain and range.

ii) Sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same graph, showing the line of reflection clearly.

a) $f(x) = 3x - 2. x > 1$

b) $f(x) = e^x + 1, x \in \mathbb{R}$

c) $f(x) = \sqrt{x - 1}, x \geq 5$

Question 12

a) Write $2x^2 - 4x + 1$ in the form $p(x-1)^2 + q$ where $a, b \in \mathbb{Z}$.

The function $f(x) = 2x^2 - 4x + 1$ has domain $x \in \mathbb{R}$.

b) Find the range of $f(x)$.

c) Explain why, for the given domain, $f^{-1}(x)$ does not exist.

d) Give the largest possible domain for $f(x)$, in the form $x \geq a$, such that $f^{-1}(x)$ exists. Further, for this domain, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same axes.

e) Find the exact value of the coordinates for which $f(x) = f^{-1}(x)$

Question 13

A function is self-inverse if it has the property that $f(x) = f^{-1}(x)$ for every x in the domain.

a) Show that this means that a function is self-inverse if $f^2(x) = x$

b) Show that $f(x) = \dfrac{x+5}{2x-1}$, $x \in \mathbb{R}, x \neq \frac{1}{2}$ is a self-inverse function by finding $f^2(x)$.

c) Find $f^{-1}(x)$.

Challenge Question

The functions p and q are given by $p(x) = x^2$ and $\mathrm{q}(x) = 2x - 1$, where $x \in \mathbb{R}$.

Find the range of values of n such that $p(x+n) = q^{-1}(x)$ has no solutions.

Test Your Understanding 3

Question 1

Given $f(x) = |3x - 4|$, evaluate

a) $f(1)$ b) $f(2)$ c) $f(-2)$ d) $f(-10)$

Question 2

Sketch the following graphs, showing any points of intersection with the axes:

a) $y = |x + 1|$ b) $y = |2x - 1|$ c) $y = |5x - 4|$

Question 3

a) Explain why the graph of $y = |x^2 + 2|$ is identical to the graph of $y = x^2 + 2$.

b) Sketch the graph of $y = |x^2 - 2|$, showing clearly the points of intersection with each axes.

Question 4

Solve the following equations:

a) $|2x - 1| = 4$ b) $|3x + 2| = 6$ c) $|x - 1| - 2 = 5$

d) $\left|\frac{x-5}{2}\right| = 1$ e) $|x + 2| = -1$

Question 5

Solve $|3x + 2| = 9 - x$.

Question 6

Solve each of the following inequalities:

a) $|2x - 1| < 3$ b) $|5x + 3| \geq 2$ c) $|2x + 9| > 14 - x$

Question 7

a) On the same diagram, sketch the graphs of $y = |3x - 1|$ and $y = x + 2$

b) Solve the inequality $|3x - 1| > x + 2,$ giving your answers in set notation.

Question 8

a) By way of a sketch, explain why the equation $|x + 2| = 2x + 1$ has only one solution.

b) Solve the equation $|x + 2| = 2x + 1$.

c*) State the range of values of k such that $|x + 2| < kx + 1$ has two solutions.

Question 9

a) Sketch the graph of $y = |x^2 - 6x + 8|$, showing the coordinates of the points of intersection with the axes and the coordinates of the local maximum.

b) Using your graph, or otherwise, solve the inequality $|x^2 - 6x + 8| \geq 8$.

Challenge Question

Given that the equation $|5 - 2x| = x + k$ has one solution, find the value of k.

Hint: sketch the graph of $y = |5 - 2x|$ *and think about the relative gradient of the other line.*

Further Exploration

a) Solve $|3x + 2| = x + 1$

b) Solve $(3x + 2)^2 = (x + 1)^2$

c) What do you notice about your answers to parts a) and b)?

Test Your Understanding 4

Question 1

You are given that $f(x) = \sin x, -360° \leq x \leq 360°$. Sketch the graph of:

a) $y = f(x)$ b) $y = |f(x)|$ c) $y = f(|x|)$

Question 2

You are given that $f(x) = x^2 - 2x - 3$. Sketch the following graphs, in each case giving the coordinates of all points of intersection with the axes.

a) $y = f(x)$ b) $y = |f(x)|$ c) $y = f(|x|)$

Question 3

a) Sketch the graph of $y = f(x)$, where $f(x) = |x|$.

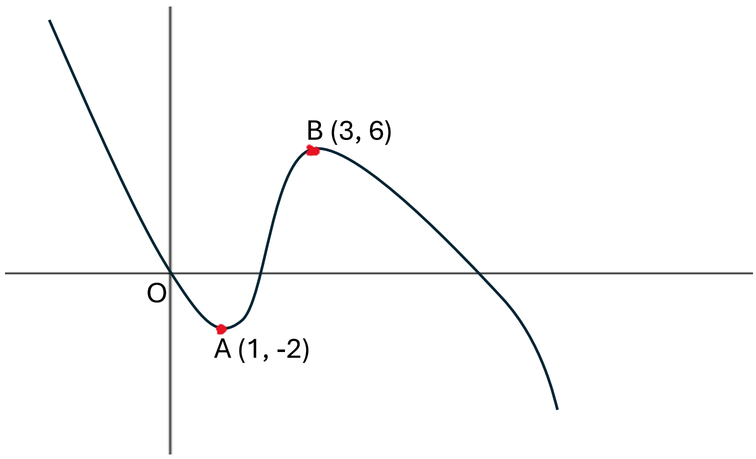
b) Sketch the following transformations:

i) $y = |x| - 1$ ii) $y = -|x|$

iii) $y = |x + 1|$ iv) $y = 2|x + 1| - 3$

In each case, give the coordinates of any points of intersection with the axes.

Question 4



The imagine above shows the graph of $y = f(x)$ along with the origin O and points A and B.

Sketch the following transformations, in each case giving the coordinates of the images of points O, A and B.

a) $y = 2f(x) - 1$ b) $y = f(x + 1) + 2$ c) $y = 0.5f(2x)$

d) $y = |f(x)|$ e) $y = -f(x) + 1$

Grade Enhancer – Apply your Knowledge!

These 'Grade Enhancer' questions are designed in examination style, to test your understanding of the content learnt.

You should complete this task and submit full solutions within one week of the end of unit.

Question 1 *(WJEC 2014)*

The function f has domain $(-\infty, 4)$ and is defined by

$$f(x) = x^2 - 8x + 7.$$

(a) Express $f(x)$ in the form

$$f(x) = (x + a)^2 + b,$$

where a, b are constants whose values are to be found. [1]

(b) Hence or otherwise, find an expression for $f^{-1}(x)$. [4]

Question 2 *(WJEC 2014)*

The function f has domain $[7, \infty)$ and is defined by

$$f(x) = 1 + \frac{2}{\sqrt{3x-5}}.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1}. [2]

Question 3 *(WJEC 2016)*

The function f has domain $(-\infty, 12]$ and is defined by

$$f(x) = e^{4-\frac{x}{3}} + 8.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1}. [2]

Question 4 *(WJEC 2023)*

a) The graphs of $y = 5x - 3$ and $y = 2x + 3$ intersect at the point A. Show that the coordinates of A are (2, 7). [2]

b) On the same set of axes, sketch the graphs of $y = |5x - 3|$ and $y = |2x + 3|$, clearly indicating the coordinates of the points of intersection of the two graphs and the points where the graphs touch the x-axis. [4]

c) Calculate the area of the region satisfying the inequalities

$$y \geqslant |5x - 3| \quad \text{and} \quad y \leqslant |2x + 3|.$$ [4]

Question 5 *(WJEC 2017)*

The function f has domain $[2, \infty)$ and is defined by

$$f(x) = 4x + k,$$

where k is a constant.

(a) Write down, in terms of k, the range of f. [1]

The function g has domain $[-3, \infty)$ and is defined by

$$g(x) = x^2 - 9.$$

(b) Find the least value of k so that the function gf can be formed. [2]

(c) (i) Write down an expression, in terms of k, for $gf(x)$.

(ii) Given that $gf(2) = 7$, find the value of k.

[5]

Question 6 *(WJEC 2023)*

Two real functions are defined as

$$f(x) = \frac{8}{x-4} \qquad \text{for} \qquad (-\infty < x < 4)\cup(4 < x < \infty),$$

$$g(x) = (x-2)^2 \qquad \text{for} \qquad -\infty < x < \infty.$$

a) **i)** Find an expression for $fg(x)$. [2]

ii) Determine the values of x for which $fg(x)$ does not exist. [3]

b) Find an expression for $f^{-1}(x)$. [3]

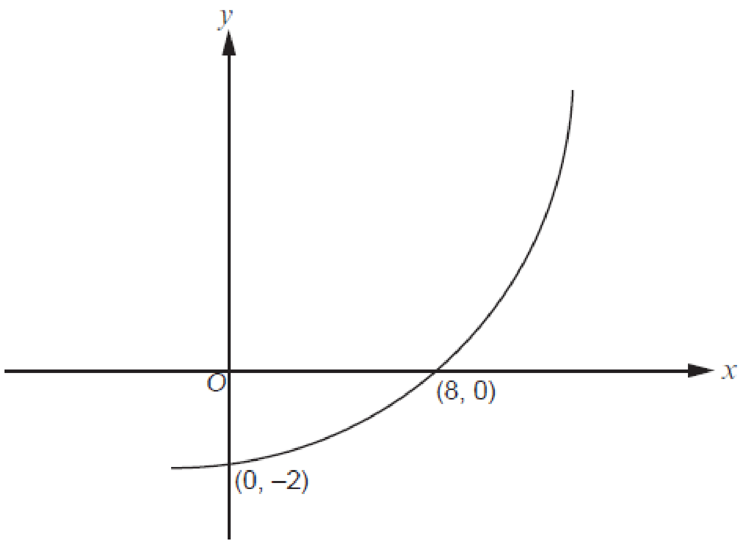
Question 7 *(WJEC 2019)*

a) Find the range of values of x for which $|1 - 3x| > 7$. [3]

b) Sketch the graph of $y = |1 - 3x| - 7$. Clearly label the minimum point and the points where the graph crosses the x-axis. [4]

Question 8 *(WJEC 2019)*

The diagram below shows a sketch of the graph of $y = f(x)$. The graph crosses the y-axis at the point (0, –2), and the x-axis at the point (8, 0).



a) Sketch the graph of $y = -4f(x + 3)$. Indicate the coordinates of the point where the graph crosses the x-axis and the y-coordinate of the point where $x = -3$. [3]

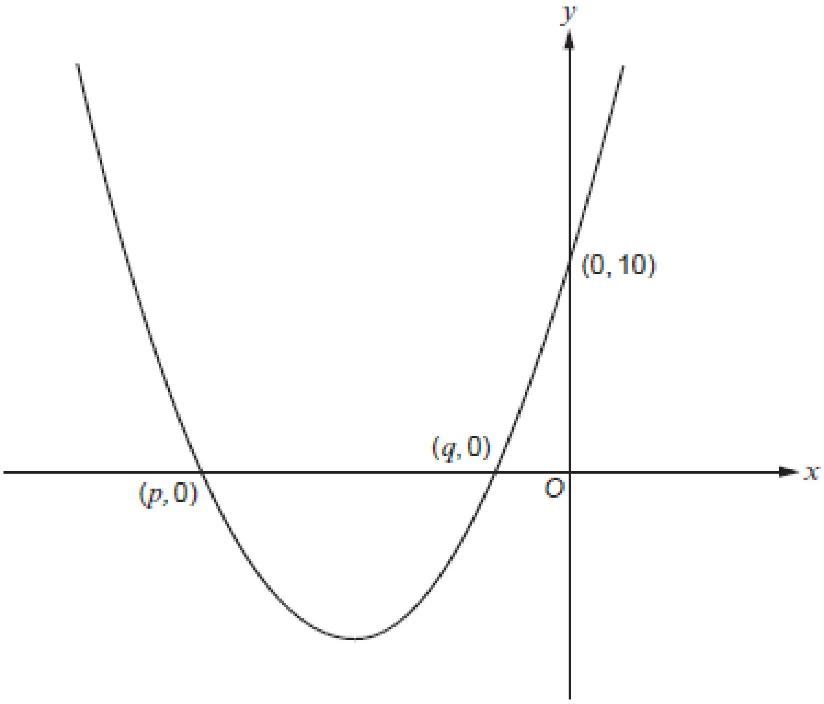
b) Sketch the graph of $y = 3 + |f(2x)$. Indicate the y-coordinate of the point where $x = 4$. [2]

Question 9 *(WJEC 2022)*

The diagram below shows a sketch of the graph of $y = f(x)$, where

$$f(x) = 2x^2 + 12x + 10.$$

The graph intersects the x-axis at the points $(p, 0)$, $(q, 0)$ and the y-axis at the point $(0, 10)$.



a) Write down the value of $ff(p)$. [1]

b) Determine the values of p and q. [2]

c) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b, c are constants whose values are to be found. Write down the coordinates of the minimum point. [3]

d) Explain why $f^{-1}(x)$ does not exist. [1]

e) The function $g(x)$ is defined as

$$g(x) = f(x) \quad \text{for} \quad -3 \leqslant x < \infty.$$

i) Find an expression for $g^{-1}(x)$. [4]

ii) Sketch the graph of $y = g^{-1}(x)$, indicating the coordinates of the points where the graph intersects the x-axis and the y-axis. [2]

A TOTAL OF 68 MARKS ARE AVAILABLE.