



# Math<sup>EV</sup>matICS

Supercharge your learning

*A2 Mathematics for WJEC*

## Unit 1 – Partial Fractions

Examples and Practice Exercises

## *Unit Learning Objectives*

- *To recall techniques for working with algebraic fractions;*
- *To be able to split a fraction into two or more partial fractions;*
- *To understand how to modify the technique in cases where there are repeated factors.*

### ***Prerequisite atoms:***

*The Four Operations with Fractions (KS3 Mathematics)*

*Basic Algebra (GCSE Mathematics)*

***Atom Check:*** See section 1.

*When you have completed the unit...*

Objective	Met	Know	Mastered
<i>I can add, subtract, multiply and divide algebraic fractions.</i>			
<i>I understand what is meant by the term 'Partial Fractions' and can split a fraction into two or three Partial Fractions.</i>			
<i>I can extend the technique to working with repeated factors (extension: and improper fractions).</i>			

Notes/Areas to Develop:

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

### Atom Recap – Algebraic Fractions

*"Cause tonight is the night when two become one" – The Spice Girls*

We met the techniques for the four operations with algebraic fractions, and their further simplification, at GCSE. This section serves as a recap of these techniques.

**Example 1:** Simplify the following fully:

a)  $\frac{3}{x} + \frac{2}{x-1}$

b)  $\frac{4q}{q^2 + 3q} - \frac{3}{4q + 12}$

c)  $\frac{4x^2}{3x-6} \times \frac{8x-16}{2x}$

$$\text{d) } \frac{10x - 10}{5x + 15} \div \frac{4 - 3x - x^2}{x^2 + 7x + 12}$$

[illegible]

**Task 1:** For each of the following, simplify fully.

a)  $\frac{3}{x} + \frac{1}{x+1}$

$$\text{b) } \frac{4q}{q^2 + 4q} - \frac{3}{3q + 12}$$

c)  $\frac{2x^3}{3x+9} \times \frac{8x+24}{2x}$

$$\text{d) } \frac{x-1}{3x+9} \div \frac{4-3x-x^2}{x^2+7x+12}$$

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

**Task 2:** Simplify the fraction  $\frac{x(x^2 - 4)}{x^3 - 5x^2 + 6x}$  fully

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***Now:***

- If fully confident, move onto the next section.
- If not fully confident, ask for additional practice questions.

### Introducing Partial Fractions

We have seen how to combine two fractions by addition/subtraction.

However, as A-level mathematicians this should beg the question – given one single fraction, is there some way to ‘split’ it into two (or more) constituent parts?

*Spoiler alert: Yes.*

**Example 1:** Show that  $\frac{3x+1}{(x+2)(x-3)}$  can be written in the form  $\frac{A}{x+2} + \frac{B}{x-3}$ .

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There are approximately 20 lines visible. The paper has a slightly textured appearance and is set against a dark background.

**Task 1:** Express the following in partial fractions:

a)  $\frac{17 - x}{(x - 2)(x + 3)}$

b)  $\frac{7x - 13}{x^2 - 5x + 4}$

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

We can extend this method to any cubic denominator that we can express as a product of three linear factors!

**Example 2:** Express  $\frac{x^2 + 9x + 26}{(x + 3)(x - 1)(x + 2)}$  in the form  $\frac{A}{x + 3} + \frac{B}{x - 1} + \frac{C}{x + 2}$ .

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**NOW:** Complete Test Your Understanding 1, Page 11.

Further Partial Fractions:Repeated Factors

Sometimes we may have to deal with repeated factors as per the following example.

**Example:** Express  $\frac{2x + 3}{(x + 3)^2}$  using partial fractions.

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**Key Points:**

- If our expression contains a factor in the denominator of the form  $(x + a)^2$  then we will need two fractions to account for the two possible fractions that could be added with that denominator, i.e.  $\frac{1}{x + a}$  and  $\frac{1}{(x + a)^2}$ .
- When we reach the 'substituting values for  $x$ ' stage, we will be one value 'short' – however we can substitute any value for  $x$  at this stage to find the remaining numerator.



**Task:** Express the following using partial fractions.

a)  $\frac{3x - 7}{(x - 2)^2}$

b)  $\frac{19 - 13x}{(x + 1)(x - 3)^2}$

**NOW:** Complete *Test Your Understanding 2*, Page 12.

*According to the specification, this should be beyond the scope of the course. However, I:*

- Since we need no new learning to tackle this problem, and it provides a helpful revision on a skill learnt at AS-level... let's take a look.*

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There are approximately 20 lines visible. The paper has a slight shadow on the right side, suggesting it's resting on a surface. There is no handwriting or other markings on the paper.



**Question 1**

Show that  $\frac{2}{(x+1)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+3}$

**Question 2**

Express the following in partial fractions:

a)  $\frac{x+1}{(x-5)(x-3)}$

b)  $\frac{15}{(x+2)(3-x)}$

c)  $\frac{x-4}{x(x-2)}$

**Question 3**

Show that  $\frac{8x+14}{(x-2)(x+3)(x+1)} \equiv \frac{A}{x-2} + \frac{B}{x+3} + \frac{C}{x+1}$

**Question 4**

Express the following in partial fractions:

a)  $\frac{7-5x}{(x-1)(x+1)(x-2)}$

b)  $\frac{16x-2}{(x+1)(x-2)(x+3)}$

c)  $\frac{4x-48}{x(x^2-16)}$

**Question 5**

Express the following in partial fractions:

a)  $\frac{1-3x}{(3x+4)(2x+1)}$

b)  $\frac{12x+12}{(x+3)(x-1)(x+5)}$

c)  $\frac{2x-10}{8x^2+10x-3}$

d)  $\frac{4x-1}{x^2+x-2}$

e\*)  $\frac{2x^2+4}{x(x-1)(x-4)}$

**Challenge Question**

Express in partial fractions

$$\frac{2x-10}{x^3+6x^2+11x+6}$$

**Question 1**

Show that  $\frac{5x-4}{(x-1)^2} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2}$

**Question 2**

Express the following as partial fractions:

a)  $\frac{3x+8}{(x+3)^2}$

b)  $\frac{2x-5}{(x-3)^2}$

c)  $\frac{x+4}{(x+1)^2}$

**Question 3**

You are given that

$$\frac{-25}{(x+3)(x-2)^2} \equiv \frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

where  $A, B$  and  $C$  are integers. Find the values of  $A, B$  and  $C$ .

**Question 4**

Express the following as partial fractions:

a)  $\frac{3}{x(x+1)^2}$

b)  $\frac{3x-2}{(x-1)(x-2)^2}$

c)  $\frac{9}{(x-2)(x+1)^2}$

**Question 5**

Express the following in partial fractions, simplifying your coefficients where possible.

$$\frac{9}{x^2(x-2)}$$

**Challenge Question**

Express in partial fractions

$$\frac{3x^3 - 20x^2 + 47x - 38}{(x-1)^2(x-3)^2}$$

**Beyond The Boundaries™**

*This section contains questions at/beyond the specification or at the very top end of the A\* grade range. These questions are optional!*

**BtB1.**

a) Find the quotient and remainder when  $3x^2 - 4x - 13$  is divided by  $x^2 - x - 2$ .

b) Hence, show that

$$\frac{3x^2 - 4x - 13}{x^2 - x - 2} \equiv A + \frac{B}{x + p} + \frac{C}{x - q}$$

**BtB2.**

Show that

$$\frac{4x^3 - 15x^2 - 32x + 14}{(x + 2)(x - 5)} \equiv Ax + B + \frac{C}{x + 2} + \frac{D}{x - 5}$$

**BtB3.**

a) Show that

$$\frac{2x^3 - 2x^2 - 2}{x^2 - 1} \equiv Ax + B + \frac{C}{x + 1} + \frac{D}{x - 1}$$

b) Hence, or otherwise, solve

$$\frac{2x^3 - 2x^2 - 2}{x^2 - 1} = \frac{2x - 4}{(x + 1)(x - 1)}$$

**Grade Enhancer™ - Apply your Knowledge!**

These 'Grade Enhancer' questions are designed in examination style, to test your understanding of the content learnt.

You should complete this task and submit full solutions within one week of the end of unit.

**Note:** Many of these questions are (no pun intended, honestly) 'partial', as this topic is often co-examined as part of another topic.

**Question 1 (WJEC 2015)**

Express

$$\frac{2}{n(n+2)}$$

in partial fractions.

[3]

**Question 2 (WJEC 2016)**

The function  $f$  is defined by

$$f(x) = \frac{17 + 4x - x^2}{(2x-1)(x-3)^2}.$$

Express  $f(x)$  in terms of partial fractions.

[4]

**Question 3 (WJEC 2017)**

The function  $f$  is defined by

$$f(x) = \frac{24x^2 + 31x + 9}{(x+1)(2x+1)(3x+1)}.$$

Express  $f(x)$  in partial fractions.

[4]

**Question 4 (WJEC 2018)**

Show that

$$\frac{3x}{(x-1)(x-4)^2} \equiv \frac{A}{(x-1)} + \frac{B}{(x-4)} + \frac{C}{(x-4)^2},$$

where  $A$ ,  $B$  and  $C$  are constants to be found.

[3]

Question 5 (WJEC 2019)

Express  $\frac{9}{(x-1)(x+2)^2}$  in terms of partial fractions. [4]

Question 6 (WJEC 2014)

(a) Express  $\frac{5x^2 + 7x + 17}{(x+1)^2(x-4)}$  in terms of partial fractions. [4]

(b) Use your answer to part (a) to express  $\frac{5x^2 + 9x + 9}{(x+1)^2(x-4)}$  in terms of partial fractions. [2]

Question 7 (WJEC 2015)

Given that  $f(x) = \frac{2x^2 + 5x + 25}{(x+3)^2(x-1)}$  express  $f(x)$  in terms of partial fractions [4]

Question 8 (WJEC 2017)

(a) Express  $\frac{8x^2 + 7x - 25}{(x-1)^2(x+4)}$  in terms of partial fractions. [4]

(b) Use your result to part (a) to express  $\frac{9x^2 + 5x - 24}{(x-1)^2(x+4)}$  in terms of partial fractions. [3]

Total Mark Available is 35.