

Test Your Understanding 1

Question 1

a) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots$

b) $1 - 3x + 6x^2 - \dots$

c) $1 + \frac{3}{4}x - \frac{3}{32}x^2 + \dots$

Question 2

a) $1 + x - \frac{1}{2}x^2 + \dots$

b) $1 + 6x + 27x^2 + \dots$

c) $1 + \frac{1}{6}x - \frac{1}{36}x^2 + \dots$

Valid for $|x| < \frac{1}{2}$

Valid for $|x| < \frac{1}{3}$

Valid for $|x| < 2$

Question 3

$$1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \dots$$

Question 4

$$1 + \frac{2}{3}x + \frac{8}{9}x^2$$

Question 5

a) $(3 - 12x + 48x^2 - \dots) - (2 + 10x + 50x^2 + \dots)$

$$= 1 - 22x - 2x^2 - \dots$$

b) $f(0.01) = \frac{385}{494}$

$$f(0.01) \approx \frac{3899}{5000}$$

Error: $\frac{\left| \frac{3899}{5000} - \frac{385}{494} \right|}{\frac{385}{494}} \times 100 = 0.0575\%$

c) The range of validity for the expression is $|x| < \frac{1}{5}$ (the narrower of the two intervals).

Challenge

a) $\frac{2 - 4x}{(1 + x)(1 - x)} \equiv \frac{3}{1 + x} - \frac{1}{1 - x}$

Test Your Understanding 2

Question 1

a) $\frac{1}{3} - \frac{1}{9}x + \frac{1}{27}x^2 - \dots$ b) $2 - \frac{1}{4}x - \frac{1}{64}x^2 - \dots$ c) $\frac{1}{4} - \frac{3}{4}x + \frac{27}{16}x^2 - \dots$

Question 2

a) $\frac{1}{8} - \frac{3}{16}x + \frac{3}{16}x^2 - \dots$ b) $2 - \frac{4}{3}x - \frac{8}{9}x^2 - \dots$ c) $\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \dots$

Valid for $|x| < 2$

Valid for $|x| < \frac{1}{2}$

Valid for $|x| < 4$

Question 3

$$3 + \frac{2}{3}x - \frac{2}{27}x^2 + \frac{4}{243}x^3 - \dots$$

Question 4

$$\sqrt{2} - \frac{1}{2\sqrt{2}}x + \frac{3}{16\sqrt{2}}x^2 - \dots$$

Question 5

$$\frac{1}{\sqrt{a + bx}} = \frac{1}{\sqrt{a}} \left(1 - \frac{b}{2a}x + \frac{3b^2}{8a^2}x^2 + \dots \right)$$

Comparing terms, $a = 25$ and $b = -4$

Hence, coefficient of x^3 term is $\frac{4}{15625}$.

Super Challenge Question

A fully worked solution to this problem is available at

<https://pmt.physicsandmathstutor.com/download/Admissions/STEP/Solutions-and-Reports/2012%20Solutions.pdf>