Math EV matics

Solutions - Unit 3

Test Your Understanding 1

1a) $\frac{\pi}{9}$ b) $\frac{5\pi}{6}$ c) $\frac{\pi}{10}$ d) $\frac{\pi}{90}$ e) $\frac{5\pi}{3}$

2a) $\frac{\sqrt{2}}{2}$ b) -1 c) $\frac{\sqrt{3}}{3}$ d) $-\frac{1}{2}$ e) -1

3a) $P\hat{R}Q = 0.319^c$
b) $PQ = 4.88\text{cm}$
c) $A = 12.1\text{cm}^2$

4a) $\theta = 0.775, 2.37$
b) $\theta = 1.23, 5.05$
c) $\theta = 3.45, 5.98$
d) $\theta = 1.17, 4.31$
e) $\theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$
f) $\theta = \frac{9\pi}{20}, \frac{29\pi}{20}$

5) $\theta = -2.21, 0, 2.21$

6) $\theta = 0, 0.322, \pi, 3.46, 2\pi$

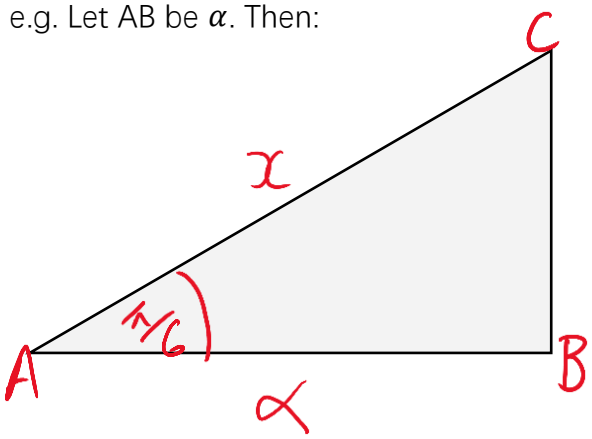
7) $\theta = 0, 0.927, \pi, 5.36, 2\pi$

8a) $\cos\beta = 2 \pm \sqrt{6}$
b) $\beta = 1.02, 2.12$

Challenge Question

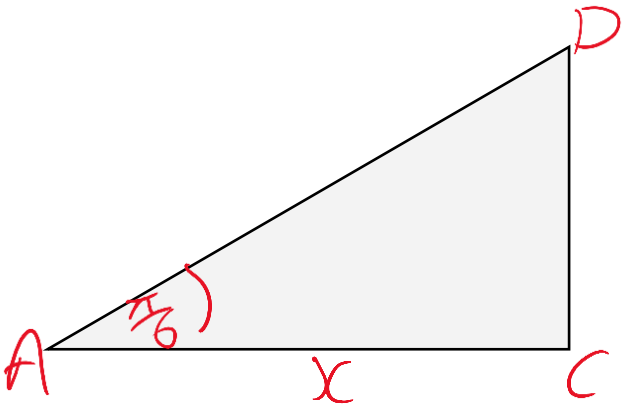
a) $CD = \frac{4}{3}, AD = \frac{8}{3}$

b) e.g. Let AB be α. Then:



$\cos\frac{\pi}{6} = \frac{\alpha}{x}$

$\therefore x = \frac{2\sqrt{3}}{3}\alpha$



$\tan\frac{\pi}{6} = \frac{CD}{x}$

$\Rightarrow CD = \frac{2\alpha}{3}$

$\cos\frac{\pi}{6} = \frac{x}{AD}$

$\Rightarrow AD = \frac{4\alpha}{3}$

$\therefore AD = 2 \times CD$

TYU 2 Solutions

Q1) $l = r\theta$ $\quad A = \frac{1}{2}r^2\theta$

a) $l = 5.24\text{cm}$, $A = 13.1\text{cm}^2$

b) $l = 10.1\text{cm}$, $A = 21.8\text{cm}^2$

c) $l = 4.8\text{cm}$, $A = 28.8\text{cm}^2$

d) $l = 14.1\text{cm}$, $A = 21.2\text{cm}^2$

Q2) Area of segment $= A_{sect} - A_{tri}$

$= \left(\frac{1}{2} \times 12^2 \times \frac{\pi}{4}\right) - \left(\frac{1}{2} \times 12^2 \times \sin\frac{\pi}{4}\right)$

$= 5.64\text{cm}^2$

Q3a) $l = r\theta$

$7.4 = x \times 0.8$

$x = 9.25\text{cm}$

b) $r\theta = 12.2$ ①

$\frac{1}{2}r^2\theta = 15$ ②

From ①, $\theta = \frac{12.2}{r}$

∴ Subbing into 2,

$\frac{1}{2}r^2\left(\frac{12.2}{r}\right) = 15$

$6.1r = 15$

$r = 2.46$

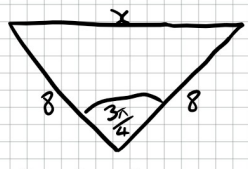
$\theta = 4.96$

Q4a) $r = 8$ ∴ $l = 6\pi$

$l = r\theta \Rightarrow 6\pi = 8\theta$

∴ $B\hat{A}C = \frac{3\pi}{4}$

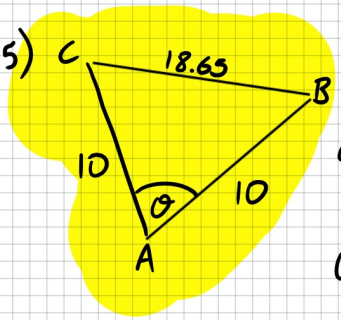
b)



$x^2 = 8^2 + 8^2 - 2(8)(8)\cos\left(\frac{3\pi}{4}\right)$

$x^2 = 218.5\ldots$ $\quad \therefore x = 14.8\text{cm}$

Q5)



$a^2 = b^2 + c^2 - 2bc\cos A$

$\cos\theta = \frac{10^2 + 10^2 - 18.65^2}{2(10)(10)}$

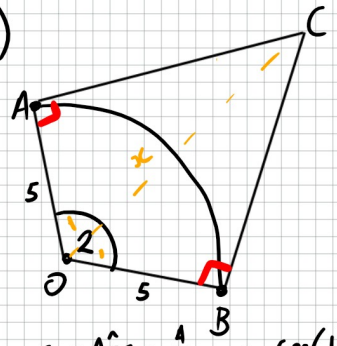
$\theta = \cos^{-1}(-0.739)$

$= 2.40$ rads

∴ Arc BC $= 10 \times 2.4$

$= 24\text{cm}$

Q6)



$\cos A\hat{O}C = \frac{A}{H}$ $\quad \therefore \cos(1) = \frac{5}{x}$

$\therefore x = 9.25\text{cm}$

Area AOBC $= 2 \times$ Area AOC

$= 2 \times \left(\frac{1}{2} \times 5 \times 9.25 \times \sin 1\right)$

$= 38.92$

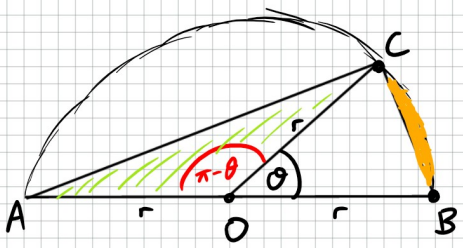
Area of sector OAB $= \frac{1}{2} \times 5^2 \times 2$

$= 25$

∴ Area ABC $= 38.92 - 25$

$= 13.92$

Q7)



Area BC $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

Area AOC $= \frac{1}{2}r^2\sin(\pi - \theta)$

$= \frac{1}{2}r^2\sin\theta$

$\therefore \frac{1}{2}r^2\sin\theta = 8 \times \left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right)$

$\frac{1}{2}r^2\sin\theta = 4r^2\theta - 4r^2\sin\theta$

$\frac{9}{2}r^2\sin\theta = 4r^2\theta \Rightarrow 8\theta - 9\sin\theta = 0$

TYU 3 Solutions

$\sin\theta \approx \theta \quad \cos\theta \approx 1 - \frac{\theta^2}{2} \quad \tan\theta \approx \theta$

1a) $\frac{1}{2}\sin\theta \approx \frac{\theta}{2}$

b) $\theta \tan 2\theta \approx \theta \cdot 2\theta$
$\approx 2\theta^2$

c) $\frac{1}{\cos 3\theta} \approx 1 \div \left(1 - \frac{(3\theta)^2}{2}\right)$
$\approx 1 \div \left(1 - \frac{9\theta^2}{2}\right)$
$\approx 1 \div \left(\frac{2 - 9\theta^2}{2}\right)$
$\approx \frac{2}{2 - 9\theta^2}$

2) $1 + \cos\theta - 3\cos^2\theta$
$\approx 1 + \left(1 - \frac{\theta^2}{2}\right) - 3\left(1 - \frac{\theta^2}{2}\right)^2$
$\approx 1 + \left(1 - \frac{\theta^2}{2}\right) - 3\left(1 - \theta^2 + \frac{\theta^4}{4}\right)$
$\approx 1 + 1 - \frac{\theta^2}{2} - 3 + 3\theta^2 - \frac{3\theta^4}{4}$ ($\frac{3\theta^4}{4} \to 0$ as θ is small)
$\approx 2.5\theta^2 - 1$ as required.

3) $\frac{\cos(\sqrt{6}\theta)}{1 - \sin\theta} = 0.5$
$\frac{1 - \frac{(\sqrt{6}\theta)^2}{2}}{1 - \theta} = 0.5$
$1 - 3\theta^2 = 0.5(1 - \theta)$
$2 - 6\theta^2 = 1 - \theta$
$6\theta^2 - \theta - 1 = 0$
$(3\theta + 1)(2\theta - 1) = 0$
$\theta = -\frac{1}{3}$ or $\theta = \frac{1}{2}$

4) $f'(x) = \frac{1}{2}(2x + \cos x)$
At $x = \alpha$, $f'(x) = 0$
$\therefore \frac{1}{2}(2\alpha + \cos\alpha) = 0$
$2\alpha + \left(1 - \frac{\alpha^2}{2}\right) = 0$
$4\alpha + 2 - \alpha^2 = 0$
$\Rightarrow \alpha^2 - 4\alpha - 2 = 0$
$\alpha = 4.45, -0.45$