MathEVmatics

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A2 Applied Mathematics for WJEC

# Unit 3: The Normal Distribution

## Examples and Practice Exercises

# *Unit Learning Objectives*

- ***To understand the properties of a normal distribution curve;***
- ***To calculate probabilities, percentage points and unknown parameters for a normal distribution;***
- ***To carry out hypothesis tests for the mean of a normal distribution.***

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**Now you have completed the unit…**

| Objective | Met | Know | Mastered |
|---|---|---|---|
| *I understand the standardised normal distribution curve and can find probabilities from a table.* | | | |
| *I can find percentage points given a probability, and can standardise to the standard normal distribution.* | | | |
| *I can find unknown means and/or standard deviations for a normal distribution.* | | | |
| *I can perform one-tailed and two-tailed hypothesis tests for a normal distribution.* | | | |
| *I can apply my learning to exam-style questions.* | | | |

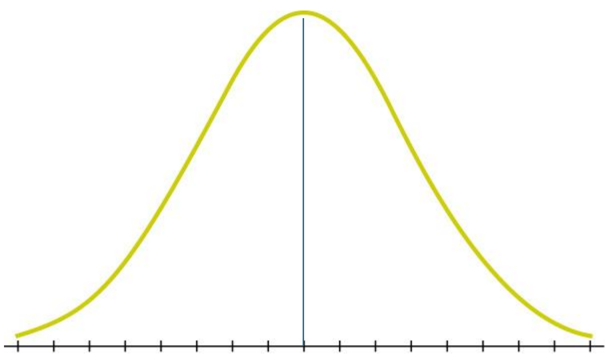
Notes/Areas to Develop:

# ***The Normal Distribution***

When we are dealing with continuous random variables (e.g. any sort of measurement such as lengths, weights etc), the number of possibilities is infinite and the probability of any one specific value is 0.

*No-one is EXACTLY 6 foot tall… or at least not for more than a fraction of a millisecond at a time before particles vibrate, water evaporates etc.!*

We use a **continuous probability distribution** to model such variables – a curve, the area under which is equal to 1. For data where values are likely to 'group' symmetrically around a central value with increasingly unlikely extreme values, we often use a **Normal Distribution curve** (sometimes called a **bell** curve)**:**



This type of spread of data is extremely common, e.g. heights of a population, intelligence metrics, error in a repeated scientific experiment, and so on!

The central value (indicated by the line) is the mean $\mu$ of the data, and we look at the other values in terms of how many standard deviations $\sigma$ they are from the mean.

A normal distribution is given by $X{\sim}N(\mu,\sigma^2)$

For the **standardised** normal curve, we use a mean of 0 and a standard deviation of 1.

- **The standard Normal Distribution is given by $\boldsymbol{Z{\sim}N(0,1^2)}$**

## ***Using Tables***

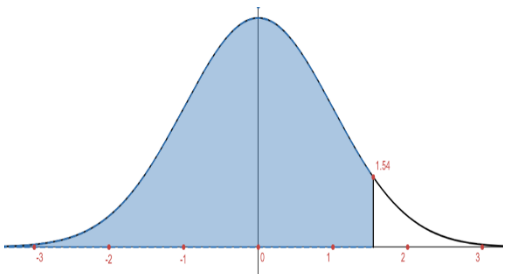
Whilst you can (and should!) use your calculator in the examination to find probabilities, it is really important that we understand how to calculate them the old-fashioned way from a table. This will give us an appreciation of the curve and what we are finding.

The standard normal tables give us the probability $P(Z \leq z)$ for some given value of $z$. It is extremely helpful to sketch the curve and shade the area you want.

**Example 1:** Given that $Z \sim N(0, 1^2)$, use the statistical tables to find:

a) $P(Z \leq 1.54)$ b) $P(Z > 0.4)$ c) $P(Z < -0.72)$

a) The area we want is:

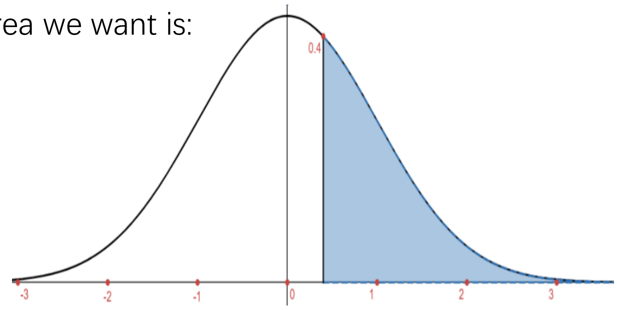



From tables, we can see:

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .50000 | .50399 | .50798 | .51197 | .51595 | .51994 | .52392 | .52790 | .53188 | .53586 |
| 0.1 | .53983 | .54380 | .54776 | .55172 | .55567 | .55962 | .56356 | .56749 | .57142 | .57535 |
| 0.2 | .57926 | .58317 | .58706 | .59095 | .59483 | .59871 | .60257 | .60642 | .61026 | .61409 |
| 0.3 | .61791 | .62172 | .62552 | .62930 | .63307 | .63683 | .64058 | .64431 | .64803 | .65173 |
| 0.4 | .65542 | .65910 | .66276 | .66640 | .67003 | .67364 | .67724 | .68082 | .68439 | .68793 |
| 0.5 | .69146 | .69497 | .69847 | .70194 | .70540 | .70884 | .71226 | .71566 | .71904 | .72240 |
| 0.6 | .72575 | .72907 | .73237 | .73565 | .73891 | .74215 | .74537 | .74857 | .75175 | .75490 |
| 0.7 | .75804 | .76115 | .76424 | .76730 | .77035 | .77337 | .77637 | .77935 | .78230 | .78524 |
| 0.8 | .78814 | .79103 | .79389 | .79673 | .79955 | .80234 | .80511 | .80785 | .81057 | .81327 |
| 0.9 | .81594 | .81859 | .82121 | .82381 | .82639 | .82894 | .83147 | .83398 | .83646 | .83891 |
| 1.0 | .84134 | .84375 | .84614 | .84849 | .85083 | .85314 | .85543 | .85769 | .85993 | .86214 |
| 1.1 | .86433 | .86650 | .86864 | .87076 | .87286 | .87493 | .87698 | .87900 | .88100 | .88298 |
| 1.2 | .88493 | .88686 | .88877 | .89065 | .89251 | .89435 | .89617 | .89796 | .89973 | .90147 |
| 1.3 | .90320 | .90490 | .90658 | .90824 | .90988 | .91149 | .91309 | .91466 | .91621 | .91774 |
| 1.4 | .91924 | .92073 | .92220 | .92364 | .92507 | .92647 | .92785 | .92922 | .93056 | .93189 |
| 1.5 | .93319 | .93448 | .93574 | .93699 | .93822 | .93943 | .94062 | .94179 | .94295 | .94408 |

So $P(Z \leq 1.54)$ = 0.9382 (4 d.p.)

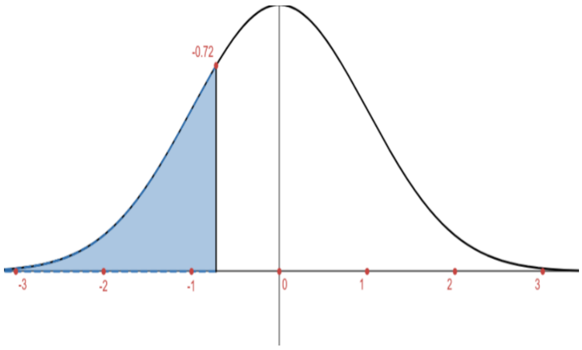
b) This time, the area we want is:




Since the table gives us $P(Z \leq z)$, we will need to find the probability we do not want, and subtract from 1.

So, $P(Z > 0.4) = 1 - 0.65542 = \ 0.3446$ to 4 decimal places.

c) This is the trickiest one from the tables. The area we want is:




Our tables only give the probability for the positive $z$-values – but luckily the curve is perfectly symmetrical. The probability we want is the same as $P(Z > 0.72)$, which we can work out as in the last example to get $0.2358$ to 4 decimal places.

**Task 1:** Use the tables to find

a) $P(Z < 1.13)$

b) $P(0.2 < Z < 1)$

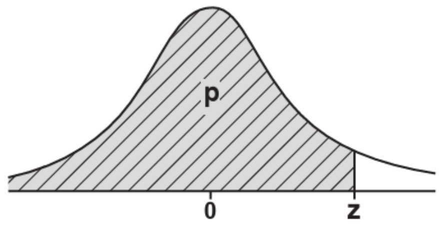
We can also find the $z$-value which corresponds to a given probability. This is another table in your formula booklet:

**TABLE 4 PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION**

**The table gives the values of z satisfying**

**P(Z ≤ z) = p**

**where Z is a normally distributed random variable with zero mean and unit variance.**




| p | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.50 | 0.000 | 0.025 | 0.050 | 0.075 | 0.100 | 0.126 | 0.151 | 0.176 | 0.202 | 0.228 |
| 0.60 | 0.253 | 0.279 | 0.305 | 0.332 | 0.358 | 0.385 | 0.412 | 0.440 | 0.468 | 0.496 |
| 0.70 | 0.524 | 0.553 | 0.583 | 0.613 | 0.643 | 0.674 | 0.706 | 0.739 | 0.772 | 0.806 |
| 0.80 | 0.842 | 0.878 | 0.915 | 0.954 | 0.994 | 1.036 | 1.080 | 1.126 | 1.175 | 1.227 |
| 0.90 | 1.282 | 1.341 | 1.405 | 1.476 | 1.555 | | | | | |

| p | .000 | .001 | .002 | .003 | .004 | .005 | .006 | .007 | .008 | .009 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.95 | 1.645 | 1.655 | 1.665 | 1.675 | 1.685 | 1.695 | 1.706 | 1.717 | 1.728 | 1.739 |
| 0.96 | 1.751 | 1.762 | 1.774 | 1.787 | 1.799 | 1.812 | 1.825 | 1.838 | 1.852 | 1.866 |
| 0.97 | 1.881 | 1.896 | 1.911 | 1.927 | 1.943 | 1.960 | 1.977 | 1.995 | 2.014 | 2.034 |
| 0.98 | 2.054 | 2.075 | 2.097 | 2.120 | 2.144 | 2.170 | 2.197 | 2.226 | 2.257 | 2.290 |
| 0.99 | 2.326 | 2.366 | 2.409 | 2.457 | 2.512 | 2.576 | 2.652 | 2.748 | 2.878 | 3.090 |

So, for example, it is useful (genuinely) to know that a probability of 0.975 corresponds to $P(Z \leq 1.96)$ (i.e. only about 2.5% of a population that is normally distributed will be 2 or more standard deviations away from the mean).

**Example 2:**

Use the table above to find the values of $z$ such that:

a) $P(Z \leq z) = 0.962$

b) $P(Z \geq z) = 0.01$

**Task 2:**

Using the percentage points table, find the $z$-values such that:

a) $P(Z \leq z) = 0.82$

b) $P(Z \leq z) = 0.05$

Most distributions that we meet will not have a mean of zero and a standard deviation of 1. In these cases, we can use the idea of **coding** the data to *transform* the data into the standard normal distribution (for which, again, we can usually use tables).

**If we have a distribution $\boldsymbol{X \sim N(\mu, \sigma^2)}$ then $\boldsymbol{Z = \frac{X-\mu}{\sigma}}$.**

**Example 3:** You are given that the continuous random variable $X$ is distributed normally with mean 20 and standard deviation 5. By standardising and using the normal tables, find:

a) $P(X < 22)$

b) $P(X \geq 25)$

**Task 3:** You are given that $Y \sim N(180, 10^2)$. By standardising and use of tables, find:

a) $P(Y \leq 182)$

b) $P(Y \geq 193.5)$

***Solving Problems using Normal Distributions***

Now we're ready to tackle some typical problems.

**Example 1:** A brand of lightbulb has a lifespan which is normally distributed with mean 1500 hours and standard deviation of 80 hours.

a) Find the probability that a randomly chosen lightbulb has a lifespan less than 1640 hours.

b) Find the probability that a randomly chosen lightbulb has a lifespan less than 1400 hours.

**Example 2:** A company's graduate training scheme tests candidates for entry. The scores on the test can be modelled normally with mean 80 and standard deviation 4.

The company only admits entry to those candidates in the top 2.5% of the cohort. Find the score required to gain entry to the graduate training scheme.

**Task 1:** A population's systolic blood pressure is modelled normally with mean 120 and standard deviation 15.

a) Find the probability that a person chosen at random from the population has a systolic blood pressure less than 132.

b) A medical researcher is looking to run a trial programme for adults with blood pressure in the highest 5%. Find the minimum blood pressure required for trial admission.

**Task 2:** A company manufactures candles in glass jars.

The weight of the glass jars is normally distributed with a mean of 122.3 grams and standard deviation of 2.6 grams.

Calculate the probability that a randomly selected glass jar will weigh:

a) less than 127g.

b) less than 121.5g.

**Task 3:** The volume of liquid in cans of a cola drink are believed to be normally distributed with a mean of 330 ml and a variance of 6.25 $ml^2$.

a) Calculate the probability that a randomly selected can from this producer contains:

i) More than 335ml

ii) Between 328ml and 330ml

b) The cans are labelled as containing 330ml. In a delivery of 800 cans, find the number of cans expected to have less than 324ml.

## Test Your Understanding 1

### Question 1

For the standardised normal distribution, and using tables, find:

a) $P(Z < 2)$   b) $P(Z \leq 0.75)$   c) $P(Z > 2.2)$   d) $P(Z \geq -1.81)$

e) $P(Z \leq -2.9)$   f) $P(Z > -0.3)$   g) $P(0.3 < Z < 1.1)$   h) $P(-0.4 \leq Z \leq 1)$

### Question 2

For each of the answers in question 1, verify the results using your calculator's Normal CD function (remember, make sure your upper/lower limit is very large where necessary).

### Question 3

You are given that $X \sim N(124, 5^2)$. By standardising and using tables, find:

a) $P(X \leq 130)$   b) $P(X < 134)$   c) $P(X \geq 127.5)$   d) $P(X > 120)$

e) $P(X < 116)$   f) $P(122 \leq X \leq 126)$

### Question 4

Verify each of the answers for Q3 using your calculator's Normal CD function.

### Question 5

An intelligence test is such that the results are normally distributed with mean 100 and standard deviation 15.

a) Show that approximately two-thirds of the population expect to score between 85 and 115.

b) Further show that approximately 2% of the population expect to score in excess of 130.

c) A group of 30 adults take the test. Given that each individual was randomly selected, find the probability that three or more exceed 130. *(HINT: what distribution could model 30 trials of an experiment?)*

**Question 6**

A random variable $X$ is such that $X \sim N(30, 4^2)$.

Find $P(X < 26 \cup X > 36)$.

**Question 7**

Glencraig produces bottles of spring water. The volume of water V in each bottle is distributed normally with mean 1000ml and standard deviation 25ml.

a) Find the probability that a randomly selected bottle contains less than 940ml.

b) A sample of three bottles is taken. Find the probability that all three bottles contain more than 1050ml.

**Question 8**

The weights $W\ kg$ of British males are such that $W \sim N(85.4, 14^2)$.

a) A man is selected at random from the population. Find the probability that his weight exceeds 110 kg.

b) A clinic is looking to run a trial for adult males whose weights exceed the $95^{th}$ percentile. Find the weight required for admission to this trial.

## *Finding an unknown mean or standard deviation*

Sometimes, we can be asked to 'work backwards' to find an unknown $\mu$ or $\sigma$.

**Example 1:** A random variable $X$ is such that $X \sim N(\mu, 4^2)$. Given that $P(X > 32) = 0.04$, find the value of $\mu$ to 3 significant figures.

**Task 1:** A random variable $Q$ is such that $Q \sim N(\mu, 3^2)$. Given that $P(Q \geq 47) = 0.015$,

a) find the value of $\mu$.

b) Hence find the 95th percentile of $Q$.

**Example 2:** A factory produces timber lengths of length $X$ cm. The timber lengths are modelled by a normal distribution with mean 180cm and standard deviation $\sigma$.

a) Given that $P(X < 170) = 0.18$, find the value of $\sigma$ to one decimal place.

b) Any timbers that fall more than 2.75 standard deviations below the mean are to be discarded. Find the length below which timbers are to be discarded.

**Task 2:**

A continuous random variable $X \sim N(\mu, \sigma^2)$.

Given that $P(X > 50) = 0.025$ and $P(X < 30) = 0.09$, find the values of $\mu$ and $\sigma$ to 1 decimal place.

## Test Your Understanding 2

**Question 1**

A random variable $X \sim N(\mu, 3^2)$. Given that $P(X > 30) = 0.15$, find $\mu$.

**Question 2**

A random variable $Y \sim N(\mu, 5^2)$. Given that $P(Y > 100) = 0.05$, find $\mu$.

**Question 3**

A random variable $W \sim N(\mu, 10^2)$. Given that $P(W \leq 230) = 0.10$, find $\mu$.

**Question 4**

A random variable $X \sim N(50, \sigma^2)$. Given that $P(X \leq 54) = 0.9$, find $\sigma$.

**Question 5**

A species of cicada is such that it's lengths $L\ cm$ are normally distributed with mean 3.5cm. Given that 80% of the population are less than 5cm long, find the standard deviation of $L$.

**Challenge 1**

The random variable $C \sim N(\mu, \sigma^2)$. Given that $P(C \leq 20) = 0.1$ and $P(C \geq 35) = 0.2$, find the values of $\mu$ and $\sigma$.

## ***Hypothesis Testing***

We can extend our knowledge of hypothesis testing from AS mathematics to test hypotheses with the normal distribution.

Some key reminders:

- The **null hypothesis** $H_0$ is the original claim or statement being tested, i.e. a manufacturer's claimed value for $\mu$.
- The **alternative hypothesis** $H_1$ could lead to a one-tailed test (e.g. if the test is to see if the original claim is too big) or two-tailed test (if we are simply testing if the actual value is *different* to the original claim).

If we have some random variable $X \sim N(\mu, \sigma^2)$, and we take some random samples of observations, the means of the samples should also be normally distributed.

**If we have a random sample of size $n$ taken from $X \sim N(\mu, \sigma^2)$, then:**

- We denote the sample mean by $\bar{X}$,
- $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

Note that this **is** given in the formula booklet!

If we have the sample mean of a normally distributed random variable $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$, then we can standardise our test statistic (our sample mean) to a normally distributed variable $Z \sim N(0, 1^2)$ using

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

This is also given in the formula booklet!

What this means, in practice, is that given a sample mean $\bar{x}$ from one particular sample of $X$, we can test whether it is statistically unlikely (significant) that this result would have occurred if $\mu$ was originally correct. Let's look at some examples of this.

**Example 1:** JoneSmells sells perfume in bottles. The amount of perfume per bottle is normally distributed with a standard deviation of 5ml.

The company claims that the mean amount of perfume per bottle is 150ml. However, a watchdog receives a number of complaints that the company is overstating the amount of perfume per bottle. The watchdog takes a random sample of 20 bottles and finds that the mean amount of perfume per bottle is 147.2ml.

Using a 5% level of significance, and stating your hypotheses clearly, test whether or not the complaint should be upheld.

**Example 2:**

VaNya manufactures metal cogs of radius $R$, where $R$ is normally distributed with mean 1.3 cm and standard deviation 0.03cm.

Following a machine repair, a random sample of 100 cogs is taken to see if the mean radius has changed.

a) Find the critical region for this test, using a 1% level of significance.

b) The mean radius of the sample of 100 cogs is found to be 1.384cm. Comment on this observation, making reference to your answer to part a).

**Task 1:**

LouisParis produces tubes of hair gel containing a mean 120ml of gel, with a standard deviation of 2ml.

Following the installation and setup of a new machine, the floor manager wishes to test whether the mean amount of gel has changed. They take a random sample of 40 tubes.

a) Write down the distribution of the test statistic and your hypotheses. Giving a reason, explain whether this is a one-tailed or a two-tailed test.

b) Find, at the 2% significance level, the critical region for this test.

c) The mean amount of gel in the sample is found to be 125ml. With reference to your answer to part b), comment on this observation.

## Test Your Understanding 3

### Question 1

In each of the following cases, a random sample of size $n$ is taken from a population that is normally distributed with mean $\mu$ and standard deviation $\sigma$. Test the following hypotheses:

a) $H_0: \mu = 40$, $H_1: \mu > 40$, $n = 20$, $\bar{x} = 44$, $\sigma = 1.5$, at a 5% level of significance.

b) $H_0: \mu = 30$, $H_1: \mu < 30$, $n = 30$, $\bar{x} = 29.5$, $\sigma = 3$, at a 5% level of significance.

c) $H_0: \mu = 120$, $H_1: \mu \neq 120$, $n = 100$, $\bar{x} = 122.5$, $\sigma = 10$, at a 1% level of significance.

### Question 2

In each of the following cases, a random sample of size $n$ is taken from a population that is normally distributed with mean $\mu$ and standard deviation $\sigma$. Find the critical region(s) for the test statistic $\bar{X}$ in the given tests.

a) $H_0: \mu = 15$, $H_1: \mu > 15$, $n = 30$, $\sigma = 2$, at a 5% level of significance.

b) $H_0: \mu = 80$, $H_1: \mu < 80$, $n = 40$, $\sigma = 4$, at a 10% level of significance.

c) $H_0: \mu = 2.3$, $H_1: \mu \neq 2.3$, $n = 50$, $\sigma = 0.2$, at a 1% level of significance.

### Question 3

The scores on a particular cognitive test are normally distributed with mean 80 and standard deviation of 10. A cognitive scientist wishes to test the theory that consuming 4 units of alcohol lowers the average test score. A random sample of 50 people are selected and they are each given the same alcoholic beverage prior to starting the cognitive test.

a) Find, at the 5% level, the critical region for this test.

b) The mean score from the sample is 77. Comment on this value with reference to your answer to part a).

**Question 4**

A machine produces screws of length $L$ cm, where $L$ is normally distributed with mean 5cm standard deviation 0.5 cm. After a repair, a sample of 30 screws is taken and their lengths measured to check that the machine is calibrated correctly.

Given that the mean of the sample was 5.1cm, test at the 5% level whether there is evidence that the machine is incorrectly calibrated.

**Question 5**

A machine fills 2 kg bags of porridge oats. The manufacturer claims that the average weight of porridge per each bag is 2025 g, with a standard deviation of 30 g.

a) Assuming a normal distribution, find the probability that, using the manufacturer's claims, the weight of porridge in a randomly selected bag is less than 2 kg.

b) Trading standards receive a number of complaints that the amount of porridge is overstated and decide to investigate the manufacturer's claims. They take a sample of 36 bags and find the mean weight of porridge to be 2010g.

i) Assuming the manufacturer's claim of $\sigma = 30$ g, test the manufacturer's claim using a 5% level of significance.

ii) Given that the manufacturer's claim is instead found to be correct for $\mu$, find the correct value of $\sigma$.

***Now: You are ready to face the Grade Enhancer™.***

## *Grade Enhancer™ - Apply your knowledge!*

*These 'Grade Enhancer' questions are designed in examination style, to test your understanding of the content learnt.*

*You should complete this task and submit full solutions within one week of the end of unit.*

**Question 1** *(WJEC 2018)*

Arwyn collects data about household expenditure on food. He records the weekly expenditure on food for 80 randomly selected households from across Wales.

| Cost, $x$ (£) | $x < 40$ | $40 \leqslant x < 50$ | $50 \leqslant x < 60$ | $60 \leqslant x < 70$ | $70 \leqslant x < 80$ | $80 \leqslant x < 90$ | $x \geqslant 90$ |
|---|---|---|---|---|---|---|---|
| Number of households | 5 | 11 | 16 | 18 | 15 | 9 | 6 |

**a)** Explain why a normal distribution may be an appropriate model for the weekly expenditure on food for this sample. [1]

Arwyn uses the distribution $N(64, 15^2)$ to model expenditure on food.

**b)** Find the number of households in the sample that this model would predict to have weekly food expenditure in the range

**i)** $60 \leqslant x < 70$,

**ii)** $x \geqslant 90$. [4]

**c)** Use your answers to part *(b)*

**i)** to comment on the suitability of this model,

**ii)** to explain how Arwyn could improve the model by changing one of its parameters. [2]

**d)** Arwyn's friend Colleen wishes to use the improved model to predict household expenditure on food in Northern Ireland. Comment on this plan. [1]

**Question 2** *(WJEC 2019)*

A company produces kettlebells whose weights are normally distributed with mean 16 kg and standard deviation 0·08 kg.

a) Find the probability that the weight of a randomly selected kettlebell is greater than 16·05 kg. [2]

The company trials a new production method. It needs to check that the mean is still 16 kg. It assumes that the standard deviation is unchanged. The company takes a random sample of 25 kettlebells and it decides to reject the new production method if the sample mean does not round to 16 kg to the nearest 100 g.

b) Find the probability that the new production method will be rejected if, in fact, the mean is still 16 kg. [4]

The company decides instead to use a 5% significance test. A random sample of 25 kettlebells is selected and the mean is found to be 16·02 kg.

c) Carry out the test to determine whether or not the new production method will be rejected. [6]

**Question 3** *(WJEC 2022)*

An interview process involves three stages of selection. The first stage involves completing an aptitude test. The scores in the aptitude test are on a continuous scale and normally distributed with mean 66 and standard deviation 14. Candidates achieving the highest 5% of scores in the aptitude test progress immediately to the third stage of the interview process. Candidates achieving the lowest 5% of scores in the aptitude test do not progress any further in the interview process.

Ffion progresses to the second stage of the interview process. Find the range of possible scores that Ffion obtained in the aptitude test. [3]

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**Question 4** *(WJEC 2022)*

In a driving simulator, the stopping distances, in metres, for cars travelling at 20 mph and 30 mph can be modelled by normal distributions with means and standard deviations shown in the table below.

| | **Stopping distance (metres)** | |
|---|---|---|
| **Travelling at** | **Mean** | **Standard deviation** |
| 20 mph | 12 | 3·5 |
| 30 mph | 23 | 3·8 |

**a)** Calculate the probability that a car travelling at 30 mph can stop within 30 metres. [2]

**b)** Suppose an obstacle suddenly appears 20 metres away. Dafydd states that you are 50 times as likely to collide with the obstacle if you are travelling at 30 mph as compared to 20 mph. Check whether Dafydd is correct. [4]

A campaigner for keeping the speed limit at 30 mph rather than reducing it to 20 mph in built-up areas claims that stopping distances are less than those stated above. He takes a random sample of 40 first-year university students and finds their mean stopping distance is 21·5 metres at 30 mph.

**c)** Test the campaigner's claim at the 1% level of significance. [6]

**d)** State a limitation to this test which calls into doubt the conclusion reached in part (c). [1]

**Question 5** *(WJEC 2023)*

A bakery produces large loaves with masses, in grams, that are normally distributed with mean $\mu$ and variance $\sigma^2$.

It is found that 11% of the large loaves weigh more than 805 g and that 20% of the large loaves weigh less than 795 g.

a) Find the values of $\mu$ and $\sigma$. [8]

The bakery also produces small loaves with masses, in grams, that are normally distributed with mean 400 and standard deviation 9.

Following a change of management at the bakery, a customer suspects that the mean mass of the small loaves has decreased. The customer weighs the next 15 small loaves that he purchases and calculates their mean mass to be 397 g.

b) Perform a hypothesis test at the 5% significance level to investigate the customer's suspicion, assuming the standard deviation, in grams, is still 9. [6]

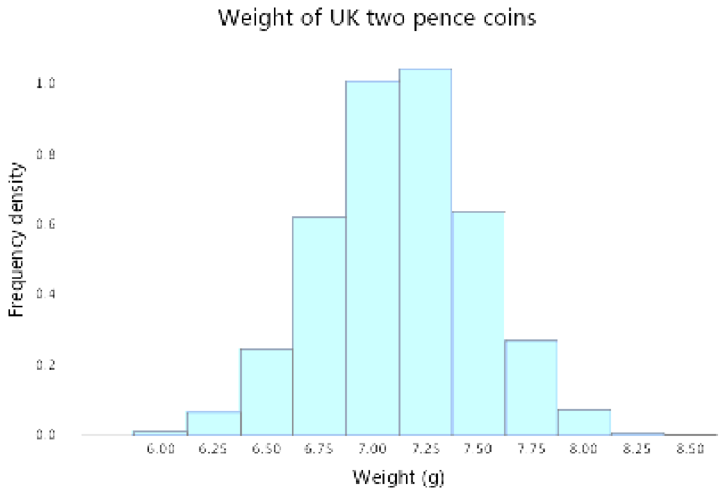
c) State another assumption you have made in part (b). [1]

**Please Turn Over For Next Question**

**Question 6** *(WJEC Sample Assessment Materials)*

Automatic coin counting machines sort, count and batch coins. A particular brand of these machines rejects 2p coins that are less than 6.12 grams or greater than 8.12 grams.

(a) The histogram represents the distribution of the weight of UK 2p coins supplied by the Royal Mint. This distribution has mean 7.12 grams and standard deviation 0.357 grams.




Explain why the weight of 2p coins can be modelled using a normal distribution. [1]

(b) Assume the distribution of the weight of 2p coins is normally distributed. Calculate the proportion of 2p coins that are rejected by this brand of coin counting machine. [2]

(c) A manager suspects that a large batch of 2p coins is counterfeit. A random sample of 30 of the suspect coins is selected. Each of the coins in the sample is weighed. The results are shown in the summary statistics table.

| Summary statistics<br>Weights (in grams) for a random sample of 30 UK 2p coins | | | | | | |
|---|---|---|---|---|---|---|
| Mean | Standard deviation | Minimum | Lower quartile | Median | Upper quartile | Maximum |
| 6.89 | 0.296 | 6.45 | 6.63 | 6.88 | 7.08 | 7.48 |

i) What assumption must be made about the weights of coins in this batch in order to conduct a test of significance on the sample mean? State, with a reason, whether you think this assumption is reasonable. [2]

ii) Assuming the population standard deviation is 0.357 grams, test at the 1% significance level whether the mean weight of the 2p coins in this batch is less than 7.12 grams. [6]

**TOTAL MARKS AVAILABLE: 62 MARKS.**