GCE A LEVEL MARKING SCHEME

SUMMER 2024

A LEVEL
MATHEMATICS
UNIT 4 APPLIED MATHEMATICS B
1300U40-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE A LEVEL MATHEMATICS

UNIT 4 APPLIED MATHEMATICS B

SUMMER 2024 MARK SCHEME

SECTION A – Statistics

Qu	Solution	Mark	Notes
1	$\text{P(Both girls}\vert\text{1st in school or college)} = \frac{80}{162} \times \frac{87}{179}$ $\frac{40}{81}$	M2	M2 for correct method. M1 for either correct fraction in a product of 2 fractions.
	$= \frac{1160}{4833} = 0.2400\ldots$ oe	A1	Cao A1 for 3sf or greater if using decimals. 0.24 with no working scores 0 marks.
	Alt Method $\text{P(Both girls}\vert\text{1st Education)} = \frac{\text{P(girleducation} \cap \text{girl)}}{\text{P(1st Education)}}$ $= \frac{\frac{80}{180} \times \frac{87}{179}}{\frac{162}{180}}$	(M1) (M1)	M1 for sight of $\frac{80}{180} \times \frac{87}{179}$ as a numerator M1 for sight of $\frac{162}{180} (= 0.9)$ as a denominator
	$= \frac{1160}{4833} = 0.2400\ldots$ oe	(A1) **[3]**	cao A1 is 3sf or greater if using decimals. 0.24 with no working scores 0 marks.
	Total for Question 1	**3**	

Qu	Solution	Mark	Notes
2(a)	$\theta \sim \mathrm{U}[0,45]$	B1	Allow $\theta \sim \mathrm{U}(0,45)$ si by correct values, diagram, or calculation of mean (22.5), variance (675/4=168.75) or SD ($15\sqrt{3}/2$=12.99). Condone mislabelling of θ.
	(mean $=\frac{0+45}{2}=$) 22.5 oe (e.g. $\frac{45}{2}$)	B1	FT their $\mathrm{U}[a,b]$ provided $0 \leq a < 90$, $a < b$ and $0 < b \leq 90$ if stated, implied by a diagram or implied by calculations.
	(SD $= \sqrt{\frac{(45-0)^2}{12}} =) \frac{15\sqrt{3}}{2}$ oe (e.g. $\frac{45}{2\sqrt{3}}, 12.99$)	B1	FT their $\mathrm{U}[a,b]$ provided $0 \leq a < 90$, $a < b$ and $0 < b \leq 90$ if stated or implied by a diagram or implied by calculations. 3sf or better for SD
		[3]	
(b)	$X = 8\sin\theta$	B1	si
	$\mathrm{P}(X > 5) = \mathrm{P}(8\sin\theta > 5)$ $= \mathrm{P}(\sin\theta > \frac{5}{8})$ $= \mathrm{P}(\theta > 38.68\ldots)$ OR $1 - \mathrm{P}(\theta < 38.68\ldots)$	M1 A1	oe, e.g. $1 - \mathrm{P}(8\sin\theta \leq 5)$
	$= \frac{45-38.68\ldots}{45}$ OR $= 1 - \frac{38.68\ldots-0}{45}$	m1	FT their Uniform distribution for θ from (a) where possible (even if mislabelled). FT their 38.68 provided $38.68 < \text{their } b$. Do not award M1 if a uniform distribution used for the length X instead of the angle θ e.g.
	$= 0.1404$	A1	Cao (condone 0.14 from use of 38.7) Do not condone 0.133… from using 39
		[5]	
	Total for Question 2	**8**	

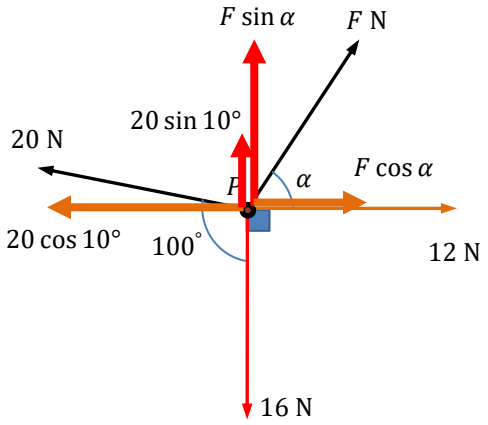
Qu	Solution	Mark	Notes
3(a)	Valid reason, e.g., • It is obvious that, in the general population, people with bigger biceps would have bigger forearms. • It is very unreasonable to think that people with bigger forearms have smaller biceps in general. • No need to check for negative correlation. • She knows there will be some kind of positive correlation.	E1 **[1]**	Condone "negative correlation is impossible." E0 for the diagram shows positive correlation. E0 for "it should be a two-tailed test." E0 for "specifically looking for a positive or negative correlation" (both positive and negative stated). E0 for implying a causal relationship, e.g. "increase in forearm girth will cause an increase in bicep girth".
(b)	(Let ρ denote the population correlation coefficient between forearm girth and bicep girth.) H_0: $\rho = 0$ $\quad$ H_1: $\rho > 0$ TS $\left(= \sqrt{0.9412}\right) = 0.97015\ldots$ CV = 0.7348 Since 0.970 > 0.7348 (there is sufficient evidence to) Reject H_0. Sufficient evidence to suggest there is a **positive** correlation between bicep girth and forearm girth.	B1 B1 B1 B1 E1 **[5]**	Allow other letters if defined. Allow worded hypotheses B0 for H_0: correlation = 0. Population must be stated or implied. B0 for omission of ρ or use of r B0 for a non-strict inequality in H_1 Labelled as TS or used in correct comparison with a critical value (not with significance level). B0 for $r = 0.97015$ without label as TS or used in comparison. B0 for $TS = \pm 0.97015$ unless the positive value correctly used later. B0 for 0.7545 from Spearman's rank. Condone 0.7348 stated as the CV even if using a two-tailed test. FT for using 0.9412 and their CV. (Comparison may be implied by a diagram.) cso Do not allow categorical statements E0 for omission of the word positive (unless positive implied by contextualised comment, e.g. E1 for sufficient evidence to suggest that the bigger the bicep girth the bigger the forearm girth.)
(c) (i)	Her n is not in the table.	E1	oe Condone "likely not in the table".
(ii)	Valid comment, e.g. • Use a software package to carry out the test. • Use a software package to find the CV. • Use a software package to find the p-value. • Realise that the CVs are lower than those for $n = 100$ in the tables. • Realise that there is no need to carry out a test for correlation when the scatter diagram clearly shows very strong correlation (and is bivariate normal).	E1 **[2]**	Condone "Use an online table that goes to 507." Allow "Use a calculator to find the CVs." E0 for "use the calculator" without reference to a possible use of the calculator (e.g. find the CV). E0 for "use a smaller sample size." E0 for "take a sample so that you can use tables." Allow "work out critical values using a t-distribution."
	Total for Question 3	**8**	

Qu	Solution	Mark	Notes
4 (a)	Let the random variable X be the mass in kg of a parcel. $X \sim \mathrm{N}(2.2, 0.3^2)$ $\mathrm{P}(X < 1.8) = 0.091211$	M1A1 **[2]**	3sf or better M1A1 for correct answer from calculator M1 for correctly standardising $z = \frac{1.8-2.2}{0.3}\left(= -\frac{4}{3} = -1.3333..\right)$ M1A1 for 0.09176 from tables using -1.33
(b)	$\mathrm{P}(X < m) = 0.8$ $m = 2.452$ Mass of parcel is 2.452 kg.	M1 A1 **[2]**	M1 implied by correct answer from calculator or for correctly standardising $0.842 = \frac{m-2.2}{0.3}$ (note: 0.8 on the LHS earns M0, allow 0.84 or better) A1 for 2.45 or better (A0 for 2.5) M1A1 for 2.4526 from tables.
(c)	$\mathrm{P}(X < 3\|\mathrm{car}) = \frac{\mathrm{P}(m<X<3)}{\mathrm{P}(\mathrm{car})}$ $= \frac{0.196(1697275)}{0.2}$ $= 0.981$	M1 M1 m1 A1 **[4]**	Recognising conditional probability leading to fraction M1 for $\mathrm{P}(m < X < 3)$ (not needed in a fraction), FT their (b) if $0 < m < 3$. Dependent on first M1 only. Correct denominator in a fraction cao 3sf or better A1 for use of 2.452, which gives 0.983. A0 for use of 2.45, which gives 0.993. Use of tables gives 0.983(3) from 0.19666/0.2.

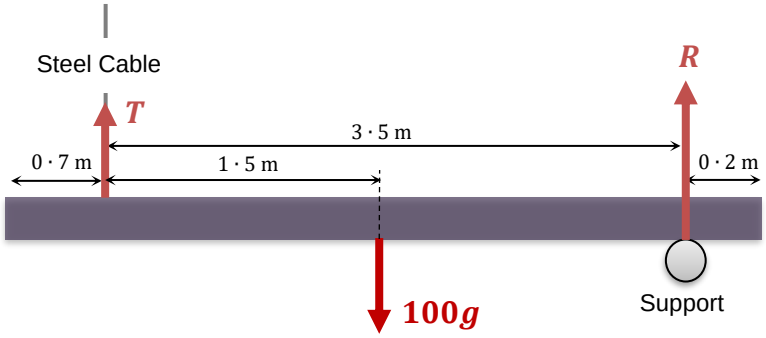
Qu	Solution	Mark	Notes
4 (d)	(Let μ be the mean mass of a parcel in kg) $H_0: \mu = 2.2$ $H_1: \mu > 2.2$	B1	Allow other letters if defined. Allow worded hypotheses (must refer to population). B0 for $H_0: \text{mean} = 2.2$, must imply/refer to population. B0 for omission of μ, or $\bar{x}$, or m, in place of μ. B0 for a non-strict inequality in H_1.
	$\bar{X} \sim \text{N}\left(2.2, \frac{0.3^2}{20}\right)$ under H_0	B1	Distribution of $\bar{X}$ si.
	Use of $\bar{x} = \frac{46}{20} = 2.3$.	B1	B1 for appropriate **use** of 2.3, not just calculating.
	METHOD 1 (*p*-value) $\text{P}(\bar{X} > 2.3 \mid H_0)$	M1	Must use $\sigma_{\bar{X}} = 0.3/\sqrt{20}$ for M1. M1 for $\text{P}\left(Z > \frac{2.3-2.2}{\frac{0.3}{\sqrt{20}}}\right) = \text{P}(Z > 1.49)$ 1.49 earns M1 only in a *p*-value method provided $\text{P}(Z > 1.49)$ considered. M0 for $\text{P}(Z < 1.49)$.
	$= 0.0680(18\ldots)$	A1	0.06811 from tables.
	Since $0.0680 > 0.05$ there is insufficient evidence to reject H_0.	m1	Dep on previous M1.
	METHOD 2 (Critical value) CV = 2.31 (CR is $\bar{X} > 2.31$)	(M1A1)	M1 implied by correct answer from calculator or for correctly standardising and equating to 1.645: $\frac{\text{CV}-2.2}{\frac{0.3}{\sqrt{20}}} = 1.645$. Must use $\sigma_{\bar{X}} = 0.3/\sqrt{20}$ for M1.
	Since $2.3 < 2.31$ there is insufficient evidence to reject H_0.	(m1)	Dep on previous M1.
	METHOD 3 (critical value with standardising): $\text{TS} = \frac{2.3-2.2}{\frac{0.3}{\sqrt{20}}}$	(M1)	Must use $\sigma_{\bar{X}} = 0.3/\sqrt{20}$ for M1.
	$= 1.49$ Since $1.49 < 1.645$ there is insufficient evidence to reject H_0.	(A1) (m1)	1.49 earns M1A1 if used as a TS. Dependent on previous M1. FT their TS. Correct comparison with 1.645 required.
	There is insufficient evidence to suggest that parcels are heavier in the run up to Christmas.	A1	cso Do not allow categorical statements
		[7]	

Qu	Solution	Mark	Notes
4(e)	Probability of ABC is $\frac{1}{2}\times\frac{1}{6}\times\frac{1}{3}\left(=\frac{1}{36}\right)$	M1	M1 for sight of correct product or correct value (1/36 or $0.02\dot{7}$)
	In any order is $\times 6 \therefore \frac{1}{6}$	A1	CAO (3sf or better if working in decimals)
		[2]	
(f)	$\mathrm{P}(AA)=\frac{1}{2}\times\frac{1}{2}(=\frac{1}{4}=0.25)$		
	$\mathrm{P}(BB)=\frac{1}{6}\times\frac{1}{6}(=\frac{1}{36}=0.02\dot{7})$		
	$\mathrm{P}(CC)=\frac{1}{3}\times\frac{1}{3}(=\frac{1}{9}=0.\dot{1})$	B2	B2 for all three products correct and no incorrect or additional products.
	$\mathrm{P}(AA \text{ or } BB \text{ or } CC)=\frac{1}{4}+\frac{1}{36}+\frac{1}{9}=\frac{14}{36}=\frac{7}{18}$		B1 for at least one correct product for either $\mathrm{P}(AA)$ or $\mathrm{P}(BB)$ or $\mathrm{P}(CC)$
	$\mathrm{P}(\text{More than one area})$ $=1-\mathrm{P}(\text{only one area})=1-\frac{7}{18}$	M1	M1 for $1-$ P(their only one area from sum of products for at least two correct terms).
	$\mathrm{P}(\text{More than one area})=\frac{11}{18}(=0.6\dot{1})$	A1	CAO 3sf or better if working in decimals
	Alternative solution 1		
	$\mathrm{P}(AA')=\frac{1}{2}\times\frac{1}{2}\left(=\frac{1}{4}\right)$ $\mathrm{P}(BB')=\frac{1}{6}\times\frac{5}{6}\left(=\frac{5}{36}\right)$ $\mathrm{P}(CC')=\frac{1}{3}\times\frac{2}{3}\left(=\frac{2}{9}\right)$	(B2)	B2 for all three products correct and no incorrect or additional products. B1 for at least one correct product for either $\mathrm{P}(AA')$, $\mathrm{P}(BB')$ or $\mathrm{P}(CC')$.
	$\mathrm{P}(\text{More than one area})=\frac{1}{4}+\frac{2}{9}+\frac{5}{36}$	(M1)	M1 for P(their >1 area from sum of products for at least two correct terms)
	$\mathrm{P}(\text{More than one area})=\frac{11}{18}(=0.6\dot{1})$	(A1)	CAO 3sf or better if working in decimals.
	Alternative solution 2		
	$\mathrm{P}(AB)=\frac{1}{2}\times\frac{1}{6}\left(=\frac{1}{12}\right)$ $\mathrm{P}(AC)=\frac{1}{2}\times\frac{1}{3}\left(=\frac{1}{6}\right)$ $\mathrm{P}(BC)=\frac{1}{6}\times\frac{1}{3}\left(=\frac{1}{18}\right)$	(B2)	B2 for all three products correct and no incorrect or additional products. $\frac{11}{36}$ earns B2. B1 for at least one correct product.
	$\mathrm{P}(\text{More than one area})=\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{18}\right)\times 2$	(M1)	M1 for sum of products for at least two correct terms and multiplying by 2 oe.
	$\mathrm{P}(\text{More than one area})=\frac{11}{18}(=0.6\dot{1})$	(A1)	CAO 3sf if working in decimals.
		[4]	
	Total for Question 4	**21**	

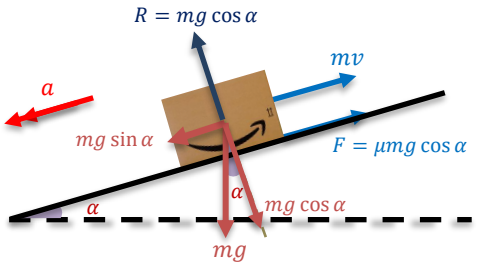
SECTION B – Differential Equations and Mechanics

Q5	Solution	Mark	Notes
			$20 \sin 10° = 3 \cdot 47296 \ldots$ $20 \cos 10° = 19 \cdot 69615 \ldots$ $20 \sin 100° = 19 \cdot 69615 \ldots$ $20 \cos 100° = -3 \cdot 47296 \ldots$
	Resolving horizontally **OR** vertically Parallel to 12 N (↔) $F \cos \alpha + 12 = 20 \cos 10$ $F \cos \alpha = 20 \cos 10 - 12 \quad (= 7 \cdot 696 \ldots)$ Parallel to 16 N (↕) $F \sin \alpha + 20 \sin 10 = 16$ $F \sin \alpha = 16 - 20 \sin 10 \quad (= 12 \cdot 527 \ldots)$ $\tan \alpha = \frac{16 - 20 \sin 10}{20 \cos 10 - 12} \quad (= 1 \cdot 6277 \ldots)$ $\alpha = 58 \cdot 4 (349\ldots) \quad (°)$ Correct method for calculating F $F = \begin{cases} \sqrt{(20 \cos 10 - 12)^2 + (16 - 20 \sin 10)^2} \\ \frac{16 - 20 \sin 10}{\sin \alpha} \\ \frac{20 \cos 10 - 12}{\cos \alpha} \end{cases}$ $= 14 \cdot 7 (0229386\ldots)$	M1 A1 A1 m1 A1 m1 A1 **[7]**	No missing or extra forces Correct equation, oe Correct equation, oe Attempt to eliminate F
	Total for Question 5	7	

Q6	Solution	Mark	Notes
a)	Working horizontally (→) $4w = \frac{60}{5}$ $w = 3$ Working vertically using $s = ut + \frac{1}{2}at^2$, with $u = \pm 21\ (\pm 7w), t = 5, a = \pm 9 \cdot 8$ $s = (\pm 21)\ (5) + \frac{1}{2}(\mp 9 \cdot 8)(5)^2$ $s = \mp 17 \cdot 5$ Height of tower is $17 \cdot 5$ (m) <u>Alternative solution for final 3 marks</u> $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2 + (\mathbf{s_0})$, with $\mathbf{u} = (12\mathbf{i} + 21\mathbf{j}), \mathbf{a} = \pm g\mathbf{j}, t = 5,$ $(\mathbf{s} = 60\mathbf{i})$ $\mathbf{s} = (12\mathbf{i} + 21\mathbf{j})(5) + \frac{1}{2}(-g\mathbf{j})(5)^2$ $+ (\mathbf{s_0} = h\mathbf{j})$ $\mathbf{s} = 60\mathbf{i} - 17 \cdot 5\mathbf{j}$ Height of tower is $17 \cdot 5$ (m)	M1 A1 M1 A1 A1 **[5]** (M1) (A1) (A1)	Using speed $= \frac{\text{distance}}{\text{time}}$ Numerically or in terms of w FT their '$7w$' 21 $(7w)$ opposing g Numerically or in terms of w FT their '$7w$' 21 $(7w)$ opposing g
b)	Working vertically using $v = u + at$, with $u = \pm 21\ (\pm 7w), v = 0,\ a = \pm g = \pm 9 \cdot 8,$ $0 = \mp 21 \pm 9 \cdot 8t$ or $0 = \mp 21 \pm gt$ $t = \frac{15}{7}$ (s) (Time to reach maximum height) Proportion of journey on way down $= \frac{5 - \frac{15}{7}}{5} = \frac{4}{7} = 0 \cdot 57(14 \ldots)$	M1 A1 A1 **[3]**	FT their '$7w$' 21 $(7w)$ opposing g
	Total for Question 6	**8**	

Q7	Solution	Mark	Notes
			
(a)	(i) Moments about base of cable $100g \times 1 \cdot 5 = R \times 3 \cdot 5$ $980 \times 1 \cdot 5 = R \times 3 \cdot 5$ $1470 = R \times 3 \cdot 5$ $R = \frac{300}{7}g = 420 \quad (\text{N})$ $420 \times 4 = 1680 < 2000 \quad (\text{N})$ OR $\frac{2000}{420} = 4 \cdot 7619 \ldots (> 4)$ Therefore, safety requirement satisfied.	M1 A1 A1	Dim. correct equation Allow incorrect distances with $R, 100g$ Correct equation, oe Convincing
	(ii) Resolve vertically $T + R = 100g$ $T = \frac{400}{7}g = 560 \quad (\text{N})$ Safety requirement satisfied: $560 \times 4 = 2240 \quad (\text{N})$ OR $\frac{2500/3000}{560} = 4 \cdot 46 \ldots / 5 \cdot 3 \ldots (> 4)$ Therefore, **Categories A** and/or **B**.	M1 A1 A1 **[6]**	Dim. correct eqn, allow 1 sign error (or 2nd moment equation) Correct equation, oe FT R Convincing
(b)	Examples, • **Weight** acts at the **centre** of the rod. • **Mass** is at the **centre** of the rod.	E1 **[1]**	
	Total for Question 7	**7**	

Q8	Solution	Mark	Notes
a)	$\mathbf{F_1}+\mathbf{F_2}+\mathbf{F_3}=(15c-12)\mathbf{i}+(9c)\mathbf{j}+(6c-12)\mathbf{k}$ $\mathbf{a}=\dfrac{\mathbf{F_1}+\mathbf{F_2}+\mathbf{F_3}}{3}=(5c-4)\mathbf{i}+(3c)\mathbf{j}+(2c-4)\mathbf{k}$ $\mathbf{a}$ is parallel to $(\mathbf{i}+\mathbf{j})$ $\Rightarrow$ $\mathbf{i}=\mathbf{j}$ component or $\mathbf{k}$ component $=\mathbf{0}$ $5c-4=3c$ or $2c-4=0$ $c=2$ Substitute $c=2$ into expression for $\mathbf{a}$ $\mathbf{a}=6\mathbf{i}+6\mathbf{j}$ (ms^{-2})	M1 m1 A1 A1 **[4]**	Sum attempted Convincing
b)	Use of $\mathbf{r}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ $(+\mathbf{r_0})$ with $\mathbf{u}=(-17\mathbf{i}+8\mathbf{j})$, $t=4$, $\mathbf{a}=6\mathbf{i}+6\mathbf{j}$ $\mathbf{r}=(-17\mathbf{i}+8\mathbf{j})(4)+\frac{1}{2}(6\mathbf{i}+6\mathbf{j})(4)^2$ $(+\mathbf{r_0})$ $\mathbf{r}=-20\mathbf{i}+80\mathbf{j}$ $(+\mathbf{r_0})$ $-13\mathbf{i}+84\mathbf{j}=-20\mathbf{i}+80\mathbf{j}+\mathbf{r_0}$ $\mathbf{r_0}=7\mathbf{i}+4\mathbf{j}$ (m)	M1 A1 A1 **[3]**	
	Total for Question 8	**7**	

Q9	Solution	Mark	Notes
			$\sin\alpha = \frac{7}{25} = 0\cdot 28$ $\cos\alpha = \frac{24}{25} = 0\cdot 96$ $\mu = \frac{1}{12}$
(a)	$R = mg\cos\alpha \qquad \left(= \frac{24}{25}mg\right)$	B1	si
	$F = \frac{1}{12}\times R \qquad \left(F = \frac{1}{12}\times mg\cos\alpha = \frac{1}{12}\times\frac{24}{25}mg = \frac{2}{25}mg\right)$	B1	si
	Apply N2L to parcel, downwards positive	M1	Dim. correct equation, all forces/terms
	$mg\sin\alpha - F - mv = ma$	A1	
	$\frac{7}{25}mg - \frac{2}{25}mg - mv = ma$		
	$5\frac{dv}{dt} = g - 5v$	A1	Convincing
		[5]	
(b)	$\int \frac{5}{g-5v}dv = \int dt \qquad \frac{5}{-5}\int \frac{1}{g-5v}dv = \int dt$	M1	Separating variables
	$-\frac{5}{5}\ln\lvert g-5v\rvert = t\ (+C)$	A1	Correct integration
	When $t = 0$, $v = 0$ $C = -\ln(g)$	m1	Used
	$-t = \ln\left\lvert\frac{g-5v}{g}\right\rvert$	m1	Attempted inversion,
	$e^{-t} = 1 - \frac{5}{g}v$		
	$v = \frac{g}{5}(1 - e^{-t})$	A1	oe (in terms of g) Examples, $v = \frac{g}{5}\left(1 - \frac{1}{e^t}\right) = \frac{ge^t - g}{5e^t}$
		[5]	
(c)	Parcel will **not exceed 2** with reason, e.g. • Limiting speed is $\frac{g}{5} = 1\cdot 96\ (< 2)$ • DE Valid for $v < \frac{g}{5} = 1\cdot 96\ \ (< 2)$	B1	Convincing
		[1]	
	Total for Question 9	**11**	