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# **GCE A LEVEL MARKING SCHEME**

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**SUMMER 2024**

**A LEVEL  
MATHEMATICS  
UNIT 3 PURE MATHEMATICS B  
1300U30-1**

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## About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

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**WJEC GCE A LEVEL MATHEMATICS**  
**UNIT 3 PURE MATHEMATICS B**  
**SUMMER 2024 MARK SCHEME**

| Q    | Solution  | Mark | Notes  |
|------|---|------|--|
| 1(a) | $\frac{25x + 32}{(2x-5)(x+1)(x+2)} = \frac{A}{(2x-5)} + \frac{B}{(x+1)} + \frac{C}{(x+2)}$ $25x + 32 = A(x+1)(x+2) + B(2x-5)(x+2) + C(2x-5)(x+1)$ <p>Put <math>x = -1</math>, <math>-25 + 32 = B(-7)(1)</math></p> $B = -1$ <p>Put <math>x = -2</math>, <math>-50 + 32 = C(-9)(-1)</math></p> $C = -2$ <p>Coefficient <math>x^2</math> (or put <math>x = 2.5</math>),</p> $0 = A + 2B + 2C = A - 2 - 4$ $A = 6$ $f(x) = \frac{6}{(2x-5)} - \frac{1}{(x+1)} - \frac{2}{(x+2)}$ | M1   |  |
|      |   | m1   | correct removal of denom, oe, si                                 |
|      |   | A1   | one constant correct   |
|      |   |      |  |
|      |   |      | oe   |
|      |   | A1   | all three constants correct                                      |
| 1(b) | $\int_1^2 \left( \frac{6}{(2x-5)} - \frac{1}{(x+1)} - \frac{2}{(x+2)} \right) dx$ $= \left[ \frac{6}{2} \ln 2x-5  - \ln x+1  - 2 \ln x+2  \right]_1^2$ $= [3 \ln -1  - \ln 3 - 2 \ln 4] -$ $[3 \ln -3  - \ln 2 - 2 \ln 3]$ $= -2 \ln 3 - 3 \ln 2$ $= -\ln 72$   | M1   | FT at least 2 of their non-zero A, B, C, limits not required, si |
|      |   | A1   | for one correct term, FT A, B, C.                                |
|      |   | A1   | all 3 correct, FT A, B, C<br>condone brackets instead of   .     |
|      |   | m1   | correct use of limits, si  |
|      |   | A1   | cao, $P = 72$  |

| Q    | Solution   | Mark | Notes  |
|------|--|------|--|
| 1(c) | $f(2) = -\frac{82}{12} < 0$<br>$f(3) = \frac{107}{20} > 0$ | B1   |  |
|      | Denominator = 0 when $x = 2.5$ .                           | E1   | oe<br>Eg (vertical) asymptote at $x = 2.5$ .<br>Function discontinuous<br>when $x = 2.5$ . |

| Q    | Solution   | Mark | Notes  |
|------|--|------|--|
| 2(a) | $3\cot\theta + 4(1 + \cot^2\theta) = 5$<br>$4\cot^2\theta + 3\cot\theta - 1 = 0$<br>$(4\cot\theta - 1)(\cot\theta + 1) = 0$<br>$\cot\theta = -1, \frac{1}{4}$<br>$\tan\theta = -1$<br>$x = 135^\circ, 315^\circ$ | M1   | Use of $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$   |
|      |  | m1   | coeff $\cot\theta$ multiply to 4,<br>constant terms multiply to $-1$   |
|      |  | A1   | cao, both values   |
|      |  | B1   | ft only if $\tan\theta$ is correct from $\cot\theta$ .<br>or allow one correct value from<br>each branch<br>$2.326^\circ, 5.498^\circ$ |
|      |  |      |  |
|      |  | B1   | ft if sign different<br>$1.326^\circ, 4.467^\circ$   |

### Note

Ignore all roots outside range.

For each branch, award B0 if extra root(s) in range present.

2 +ve roots ft for B1, 2 -ve roots ft for B1.

Only award for  $\tan\theta$ .

If radians used, units are required.

| Q    | Solution                                | Mark | Notes  |
|------|---|------|--|
| 2(a) | OR                                      |      |  |
|      | $3\cot\theta + 4(1 + \cot^2\theta) = 5$ | (M1) | Use of $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$                           |
|      | $4\cot^2\theta + 3\cot\theta - 1 = 0$   |      |  |
|      | multiplying by $\tan^2\theta$           |      |  |
|      | $4 + 3\tan\theta - \tan^2\theta = 0,$   |      |  |
|      | $\tan^2\theta - 3\tan\theta - 4 = 0$    |      |  |
|      | $(\tan\theta - 4)(\tan\theta + 1) = 0$  | (m1) | coeff $\tan\theta$ multiply to 1,<br>constant terms multiply to $-4$               |
|      | $\tan\theta = -1, 4$                    | (A1) | cao  |
|      | $\tan\theta = -1,$                      |      |  |
|      | $\theta = 135^\circ, 315^\circ$         | (B1) | ft<br>or allow one correct value from<br>each branch<br>$2.326^\circ, 5.498^\circ$ |
|      | $\tan\theta = 4,$                       |      |  |
|      | $\theta = 75.96^\circ, 255.96^\circ$    | (B1) | ft if sign different, $1.326^\circ, 4.467^\circ$                                   |

### Note

Ignore all roots outside range.

For each branch, award B0 if extra root(s) in range present.

2 +ve roots ft for B1, 2 -ve roots ft for B1.

Only award for  $\tan\theta$ .

If radians used, units are required.

| Q    | Solution  | Mark | Notes   |
|------|---|------|---|
| 2(b) | $24\cos x - 7\sin x = R\cos(x + \alpha)$<br>$= R\cos x \cos \alpha - R\sin x \sin \alpha$<br>$R\cos \alpha = 24, R\sin \alpha = 7$<br>$R = \sqrt{24^2 + 7^2} = 25$<br>$\alpha = \tan^{-1}\left(\frac{7}{24}\right)$<br>$\alpha = 16.26^\circ$<br>$25\cos(x + 16.26^\circ) = 16$<br>$\cos(x + 16.26^\circ) = \frac{16}{25} = 0.64$<br>$x + 16.26^\circ = 50.21^\circ, 309.79^\circ$<br>$x = 33.95^\circ, 293.53^\circ$ |      |   |
|      |   | M1   | implied by $\alpha = 16.26^\circ$   |
|      |   | B1   | no working required   |
|      |   |      | $\sin^{-1}\left(\frac{7}{25}\right), \cos^{-1}\left(\frac{24}{25}\right)$ |
|      |   | A1   | cao   |
|      |   | M1   | ft $R, \alpha$  |
|      |   | A1   | any one value, ft $R, \alpha$ , si  |
|      |   | A1   | ft $R, \alpha$ , both values.   |
|      |   |      | A0 if 3 or more values between $0^\circ < x < 360^\circ$ .                |
|      |   |      | Accept whole numbers.   |
|      |   |      | Ignore values outside the range.  |

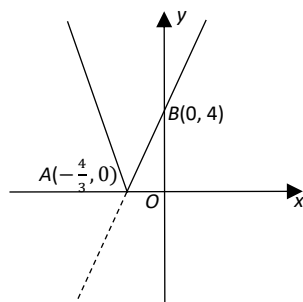
| Q    | Solution  | Mark | Notes  |
|------|---|------|--|
| 3(a) | Area of sector = $\frac{1}{2}r^2\theta$   | B1   | used si                                      |
|      | $\frac{1}{2}(5+r)^2 \times \frac{\pi}{5} - \frac{1}{2}r^2 \times \frac{\pi}{5} = \frac{13\pi}{2}$ | M1   | difference between two sectors               |
|      |   | A1   | Condone wrong way round.<br>correct equation |
|      | $r^2 + 10r + 25 - r^2 = 65$   |      |  |
|      | $10r + 25 = 65$   |      |  |
|      | $2r + 5 = 13$   |      |  |
|      | $r = 4$   | A1   | cao  |
|      |   |      | FT their part (a)                            |
| 3(b) | length $CD = \frac{(5+r)\pi}{5} = \frac{9\pi}{5}$   |      |  |
|      | length $AB = \frac{r\pi}{5} = \frac{4\pi}{5}$   | B1   | si, either correct with $r$ substituted      |
|      | Perimeter = $5 + 5 + CD + AB$   | M1   |  |
|      | Perimeter = $5 + 5 + \frac{4\pi}{5} + \frac{9\pi}{5}$   |      |  |
|      | Perimeter = $10 + \frac{13\pi}{5}$ (= 18.168...)  | A1   | cao, isw                                     |



**Q Solution**

**Mark Notes**

4(a)

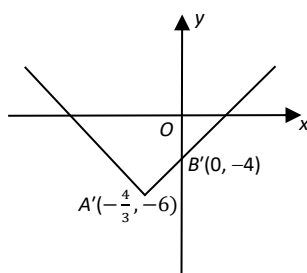


G1 shape, above the  $x$ -axis  
and vertex on  $x$ -axis

B1  $(-\frac{4}{3}, 0)$ , any correct method

B1  $(0, 4)$ , any correct method

4(b)



G1 move downwards, ft (a)

B1  $(-\frac{4}{3}, -6)$ , ft their  $(-\frac{4}{3}, 0)$ ,  
any correct method

B1  $(0, -4)$ , any correct method

| Q | Solution   | Mark                                    | Notes   |
|---|--|---|---|
| 5 | <p>(Assume that there is a real and positive value of <math>x</math> such that)</p> $x + \frac{81}{x} < 18.$ <p>Then</p> $x^2 + 81 < 18x$ $x^2 - 18x + 81 < 0$ $(x - 9)^2 < 0$ <p>which is impossible, hence contradiction</p> <p>Therefore <math>x + \frac{81}{x} \geq 18.</math></p> | <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> | <p>condone <math>\leq</math></p> <p></p> <p></p> <p>cso</p> |

| Q    | Solution   | Mark | Notes  |
|------|--|------|--|
| 6(a) | $y = \cos x$   |      |  |
|      | $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h}$  | M1   |  |
|      | $= \lim_{h \rightarrow 0} \frac{\cos x \cosh - \sin x \sinh - \cos x}{h}$                                    | B1   | $\frac{\cos x \cosh - \sin x \sinh - \cos x}{h}$ |
|      | As $h \rightarrow 0$ , $\sinh \cong h$ and $\cosh \cong 1 - \frac{h^2}{2}$                                   |      |  |
|      | $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{\cos x \left(1 - \frac{h^2}{2}\right) - h \sin x - \cos x}{h}$ | M1   | substitution                                     |
|      | $\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{\cos x \left(-\frac{h^2}{2}\right) - h \sin x}{h}$             | A1   |  |
|      | $\frac{dy}{dx} = \lim_{h \rightarrow 0} \left(-\frac{h}{2} \cos x - \sin x\right)$                           |      |  |
|      | $\frac{dy}{dx} = -\sin x$  | A1   | Everything correct & convincing                  |

| Q    | Solution   | Mark | Notes   |
|------|--|------|---|
|      | OR   |      |   |
| 6(a) | $y = \cos x$   |      |   |
|      | $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\cos(x + \delta x) - \cos x}{\delta x}$   | (M1) |   |
|      | $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\cos x \cos \delta x - \sin x \sin \delta x - \cos x}{\delta x}$                              | (B1) | $\frac{\cos x \cos \delta x - \sin x \sin \delta x - \cos x}{\delta x}$ |
|      | $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \left[ \frac{\cos x (\cos \delta x - 1)}{\delta x} - \frac{\sin x \sin \delta x}{\delta x} \right]$ | (M1) | factorisation   |
|      | $\frac{dy}{dx} = \cos x \lim_{\delta x \rightarrow 0} \left[ \frac{\cos \delta x - 1}{\delta x} \right]$   |      |   |
|      | $- \sin x \lim_{\delta x \rightarrow 0} \left[ \frac{\sin \delta x}{\delta x} \right]$   |      |   |
|      | But $\lim_{\delta x \rightarrow 0} \left[ \frac{\cos \delta x - 1}{\delta x} \right] = 0$  |      |   |
|      | and $\lim_{\delta x \rightarrow 0} \left[ \frac{\sin \delta x}{\delta x} \right] = 1$  | (A1) | both limits used  |
|      | Hence $\frac{dy}{dx} = \cos x \times 0 - \sin x \times 1$  |      |   |
|      | $\frac{dy}{dx} = -\sin x$  | (A1) | Everything correct & convincing   |

| Q    | Solution  | Mark | Notes   |
|------|---|------|---|
| 6(b) | $y = e^{3x}\sin 4x$   |      |   |
|      | $\frac{dy}{dx} = e^{3x} \times 4\cos 4x + 3e^{3x} \times \sin 4x$ | M1   | $e^{3x}f(x) + g(x)\sin 4x, f(x), g(x) \neq 0, 1.$ |
|      |   | A1   | $f(x) = 4\cos 4x, \text{ isw}$                    |
|      |   | A1   | $g(x) = 3e^{3x}, \text{ isw}$                     |
|      | $\frac{dy}{dx} = e^{3x}(4\cos 4x + 3\sin 4x)$                     |      |   |

| Q    | Solution  | Mark | Notes  |
|------|---|------|--|
| 6(c) | $\int x^2 \sin 2x \, dx$  |      |  |
|      | $= \left[ -\frac{1}{2} \cos 2x \times x^2 \right] - \int -\frac{1}{2} \cos 2x \times 2x \, dx$                                      | M1   | $[f(x)x^2] - \int f(x)g(x)dx,$<br><br>$g(x) \neq 0,1$ , condone a sign error.<br>$f(x) \neq \sin 2x$ |
|      |   | A1   | $f(x) = -\frac{1}{2} \cos 2x$  |
|      |   | A1   | $g(x) = 2x$  |
|      | $= \left[ -\frac{1}{2} x^2 \cos 2x \right] + \left[ \frac{1}{2} \sin 2x \times x \right] - \int \frac{1}{2} \sin 2x \times 1 \, dx$ |      |  |
|      |   | A1   | correct second and third terms   |
|      | $= \left[ -\frac{1}{2} x^2 \cos 2x \right] + \left[ \frac{1}{2} x \sin 2x \right]$  |      |  |
|      | $+ \left[ \frac{1}{4} \cos 2x \right] + C$  | A1   | third term correct and + C. isw  |

| Q    | Solution   | Mark | Notes                                       |
|------|--|------|---|
| 7(a) | $\sum_{r=3}^{50} (4r + 5)$ $= 17 + 21 + 25 + \dots (+ 205)$ $= \frac{48}{2} [2 \times 17 + (48 - 1) \times 4]$ | M1   | oe AP recognised                            |
|      |  | m1   | use of formula, condone 1 error.            |
|      |  |      | $\frac{48}{2} [17 + 205]$                   |
|      |  | A1   | all correct                                 |
|      |  |      | Accept $a = 13, d = 4, n = 49$ , Sum = 5341 |
|      |  |      | Accept $a = 9, d = 4, n = 50$ , Sum = 5350  |
|      |  |      | Accept $a = 5, d = 4, n = 51$ , Sum = 5355  |
|      | $= 5328$   | A1   | cao, from correct working                   |
|      | <u>Note:</u> No workings, M0   |      |   |

| Q    | Solution   | Mark | Notes   |
|------|--|------|---|
| 7(b) | $\sum_{r=2}^{\infty} \left( 540 \times \left( \frac{1}{3} \right)^r \right)$ $= 540 \left( \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \dots \right)$ $= 60 + 20 + \frac{20}{3} + \dots$ $= \frac{540 \times \frac{1}{9}}{1 - \frac{1}{3}} = \frac{60}{1 - \frac{1}{3}}$ | M1   | GP recognised   |
|      |  | m1   | correct use of formula, ratio = $\frac{1}{3}$   |
|      |  |      | Accept $\frac{540 \times \frac{1}{3}}{1 - \frac{1}{3}} = \frac{180}{1 - \frac{1}{3}}$ , Sum = 270 |
|      |  |      | Accept $\frac{540}{1 - \frac{1}{3}}$ , Sum = 810  |
|      | = 90   | A1   | cao   |

Note: No workings, M0



| Q | Solution | Mark | Notes |
|---|----------|------|-------|
|---|----------|------|-------|

8(a)  $f(0) = -1 < 0$

$$f(1) = 1 > 0$$

$f(x)$  changes sign in the interval  $[0, 1]$ ,

(and  $f$  is continuous /  $f$  is a cubic curve.) B1

Hence, there is a root in the interval  $[0, 1]$

8(b)(i)  $f'(x) = 3x^2 + 8x - 3$  B1 si

$$x_{n+1} = x_n - \frac{x_n^3 + 4x_n^2 - 3x_n - 1}{3x_n^2 + 8x_n - 3}$$

M1  $x_{n+1} = \frac{2x_n^3 + 4x_n^2 + 1}{3x_n^2 + 8x_n - 3}$ , si

$$x_0 = 0.8$$

$x_1 = 0.861\ 654\ 135\ 3\dots$  A1 cao

8(b)(ii)  $x_2 = 0.857\ 641\ 074\ 1\dots$

$$x_3 = 0.857\ 623\ 607\ 4\dots$$

$$x_4 = 0.857\ 623\ 607\dots$$

Root = 0.857 624 (correct to 6 dp) A1 cao

| Q    | Solution  | Mark | Notes   |
|------|---|------|---|
| 8(c) | $f'(x) = 3x^2 + 8x - 3$<br>$f'\left(\frac{1}{3}\right) = 3 \times \frac{1}{9} + 8 \times \frac{1}{3} - 3 = 0$ | B1   | $f'(x) = 3x^2 + 8x - 3 = (3x - 1)(x + 3)$   |
|      | Therefore N-R requires division by 0.   | E1   | <p>When <math>x = \frac{1}{3}</math>, <math>f'(x) = 0</math></p> <p>oe eg tangent at <math>x = \frac{1}{3}</math> is horizontal, hence not intersecting the <math>x</math>-axis for the next iteration.</p> |

| Q | Solution   | Mark | Notes   |
|---|--|------|---|
| 9 | Area under $C_1 = \int_0^{\pi} (-x^2 + \pi x + 1) dx$                              | M1   | Attempt to integrate, index increased in at least one term. |
|   | $= \left[ -\frac{x^3}{3} + \frac{\pi x^2}{2} + x \right]_0^{\pi}$                  | B1   | correct integration, limits not required                    |
|   | $= -\frac{\pi^3}{3} + \frac{\pi^3}{2} + \pi$                                       | m1   | correct use of limits, si                                   |
|   | $= \pi + \frac{\pi^3}{6}$  | A1   | cao, si, allow $-\frac{\pi^3}{3} + \frac{\pi^3}{2} + \pi$   |
|   | Area under $C_2$   |      |   |
|   | $= \int_0^{\frac{\pi}{4}} (\cos 2x) dx + \int_{\frac{3\pi}{4}}^{\pi} (\cos 2x) dx$ | M1   | oe, either integral   |
|   | $= 2 \left[ \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{4}}$                         | B1   | correct integration of $\cos 2x$ , limits not required      |
|   | $= 2 \left( \frac{1}{2} - 0 \right)$   | (m1) | si correct use of limits                                    |
|   | $= 1$  | A1   | cao, si or each integral $= \frac{1}{2}$ , both required.   |

Note: Award the m1 once for correct substitution of limits seen in either integration.

Required area

|   |    |      |
|---|----|------|
| $= \text{Area under } C_1 - \text{Both Areas under } C_2$ | m1 | used |
|---|----|------|

|   |    |                           |
|---|----|---------------------------|
| Required area $= \pi + \frac{\pi^3}{6} - 1$ | A1 | cao, exact value required |
|---|----|---------------------------|

Note: SC3 for answer only of 7.3093.

SC2 for answer only of 8.3093 AND  $\frac{1}{2}$  or 1.

SC1 for answer only of 8.3093 OR  $\frac{1}{2}$  or 1



| Q     | Solution  | Mark | Notes   |
|-------|---|------|---|
| 10(a) | $\frac{2(3x+1)}{x^2-2x-3} + \frac{x}{x+1} = \frac{2(3x+1)}{(x-3)(x+1)} + \frac{x}{x+1}$ $= \frac{2(3x+1) + x(x-3)}{(x-3)(x+1)}$ $= \frac{x^2 + 3x + 2}{(x-3)(x+1)}$ $= \frac{(x+1)(x+2)}{(x-3)(x+1)} = \frac{x+2}{x-3}$ | B1   | factorise denominator   |
|       |   | B1   | common denominator  |
|       |   | B1   | simplify numerator  |
|       |   | B1   | factorise and cancel ( $x \neq -1$ ). AG  |
| 10(b) | $f(x) = \frac{x+2}{x-3} = 1 + \frac{5}{x-3}$ <p>Range is (1, 6]</p>   | B1   | (1, oe  |
|       |   | B1   | 6] oe   |
|       |   | SC1  | [6, 1)  |
| 10(c) | $y = \frac{x+2}{x-3}$ $xy - 3y = x + 2$ $xy - x = 3y + 2$ $x(y - 1) = 3y + 2$ $x = \frac{3y+2}{y-1}$ $f^{-1}(x) = \frac{3x+2}{x-1}$ <p>Domain (1, 6], range [4, <math>\infty</math>)</p>                                | M1   | x and y may be interchanged at start  |
|       |   | m1   | attempt to isolate x (or y)   |
|       |   | A1   | oe  |
|       |   | B1   | FT part (b) only for domain, oe   |
| 10(d) | $\frac{x+2}{x-3} = \frac{3x+2}{x-1}$ $2x^2 - 8x - 4 = 0$ $x = 2 \pm \sqrt{6}$ <p>Since <math>x \geq 4</math>, <math>x = 2 + \sqrt{6} = 4.449\dots</math></p> <p><u>Note:</u> Answer only M1 A0 m0 A0</p>                | M1   | $x = \frac{3x+2}{x-1}; x = \frac{x+2}{x-3}; \frac{\frac{x+2}{x-3}+2}{\frac{x+2}{x-3}-3} = x$ ft (c) |
|       |   | A1   | $x^2 - 4x - 2 = 0$ , ft (c) only if similar expression for $f^{-1}(x)$                              |
|       |   | A1   | cao   |
|       |   | A1   | cao   |

| Q     | Solution  | Mark | Notes   |
|-------|---|------|---|
| 11(a) | $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$                             | M1   | used, si  |
|       | $\frac{dx}{d\theta} = 2 + 2\cos 2\theta$  | B1   |   |
|       | $\frac{dy}{d\theta} = -2\sin 2\theta$   | B1   |   |
|       | $\frac{dy}{dx} = \frac{-2\sin 2\theta}{2(1 + \cos 2\theta)}$                                |      |   |
|       | $\frac{dy}{dx} = \frac{-2\sin \theta \cos \theta}{1 + 2\cos^2 \theta - 1}$                  | m1   | $\sin 2\theta = 2\sin \theta \cos \theta$         |
|       |   | m1   | $\cos 2\theta = 2\cos^2 \theta - 1$               |
|       | $\frac{dy}{dx} = -\frac{\sin \theta}{\cos \theta} = -\tan \theta$                           | A1   | convincing, AG                                    |
| 11(b) | Where $\theta = \frac{\pi}{4}$ , point is $P\left(\left(\frac{\pi}{2} + 1\right), 1\right)$ | B1   | si, $P(2.57, 1)$                                  |
|       | Gradient of tangent $= -\tan \frac{\pi}{4} = -1$  | B1   | si  |
|       | Equation of tangent is  |      |   |
|       | $y - (1 + \cos 2\theta) = -\tan \theta (x - (2\theta + \sin 2\theta))$                      | M1   | oe, method for equation, si<br>e.g. $y = -1x + c$ |
|       | $y - 1 = -1\left(x - \left(\frac{\pi}{2} + 1\right)\right)$                                 | A1   | oe, all correct, isw                              |
|       | $y + x = 2 + \frac{\pi}{2} (= 3.57)$  |      |   |

| Q     | Solution   | Mark | Notes                                     |
|-------|--|------|---|
| 12(a) | For small $\theta$ , $\sin\theta \approx \theta$ ; $\cos\theta \approx 1 - \frac{\theta^2}{2}$ .   | M1   | used                                      |
|       | $2\cos\theta + \sin\theta - 1$ $= 2\left(1 - \frac{\theta^2}{2}\right) + \theta - 1$ $= 2 - \theta^2 + \theta - 1$ $= 1 + \theta - \theta^2$ | A1   | AG  |
| 12(b) | $[1 + \theta - \theta^2]^{-1}$   | M1   |   |
|       | $= [1 + (\theta - \theta^2)]^{-1}$ $= 1 + (-1)(\theta - \theta^2) + \frac{(-1)(-2)}{2}(\theta - \theta^2)^2 + \dots$                         | A1   | $1 + (-1)(\theta - \theta^2)$             |
|       | $= 1 - \theta + \theta^2 + \theta^2 + \dots$ $= 1 - \theta + 2\theta^2 + \dots$  | A1   | $\frac{(-1)(-2)}{2}(\theta - \theta^2)^2$ |
|       | $(a = -1, b = 2)$  | A1   | cao                                       |

| Q     | Solution   | Mark         | Notes                             |
|-------|--|--------------|-----------------------------------|
| 13(a) | $f(x)$ is decreasing, so $f'(x) < 0$<br>$f(x)$ is convex, so $f''(x) > 0$ ,<br>$f'(x)$ is increasing.<br>Required interval is $(x_3, 0)$       | E1<br><br>B1 | oe<br><br>condone $[x_3, 0]$ , oe |
| 13(b) | (Points of inflection occur when there is<br>a change of concavity,<br>ie change of sign in $f''(x)$ .)<br>$x$ -coordinates are $x_3$ or $x_4$ | B1           | either                            |



**Q Solution****Mark Notes**

14(a)  $y = \frac{1+\ln x}{x}$

$$\frac{dy}{dx} = \frac{xf(x) - (1+\ln x)g(x)}{x^2}$$

$$= \frac{x\left(\frac{1}{x}\right) - (1+\ln x)1}{x^2}$$

$$\frac{dy}{dx} = \frac{-\ln x}{x^2}$$

M1 Allow omission of  $g(x)$ 

$$f(x) = \frac{1}{x}, (g(x) = 1)$$

A1 convincing, AG

Alternative solution 1

$$y = \frac{1}{x} + \frac{\ln x}{x}$$

$$\frac{dy}{dx} = -\frac{1}{x^2} - \frac{1}{x^2} \ln x + \frac{1}{x} \times \frac{1}{x}$$

(M1) one correct term

$$\frac{dy}{dx} = \frac{-\ln x}{x^2}$$

(A1) convincing, AG

Alternative solution 2

$$y = (1 + \ln x)x^{-1}$$

$$\frac{dy}{dx} = x^{-1}f(x) + (1 + \ln x)g(x)$$

(M1)

$$= x^{-1}\frac{1}{x} + (1 + \ln x) \times -1x^{-2}$$

$$f(x) = \frac{1}{x}, g(x) = -1x^{-2}$$

$$\frac{dy}{dx} = \frac{-\ln x}{x^2}$$

(A1) convincing, AG

Alternative solution 3

$$xy = 1 + \ln x$$

$$y + x \frac{dy}{dx} = \frac{1}{x}$$

(M1)

$$x \frac{dy}{dx} = \frac{1}{x} - y = \frac{1}{x} - \frac{1+\ln x}{x} = -\frac{\ln x}{x}$$

$$\frac{dy}{dx} = \frac{-\ln x}{x^2}$$

(A1) convincing, AG

| Q     | Solution   | Mark                   | Notes  |
|-------|--|------------------------|--|
| 14(b) | $\int \frac{\ln x}{x^2} dx = \int t \, dt$ $-\frac{1 + \ln x}{x} = \frac{t^2}{2} (+ C)$ <p>When <math>x = 1, t = 3</math></p> $-1 = \frac{9}{2} + C$ $C = -\frac{11}{2}$ $t^2 = 11 - \frac{2(1 + \ln x)}{x}$ | M1<br><br>A1<br><br>A1 | variables separated, 1 error only.<br><br>$\frac{1 + \ln x}{x}$<br><br>$\frac{t^2}{2}$<br><br>–1 for incorrect signs if A1A1 |
|       |  | m1                     | use of conditions  |
|       |  | A1                     | cao, oe, isw   |

| Q        | Solution  | Mark                                     | Notes                                    |
|----------|---|--|--|
| 15       | For £ $P$ invested, the return after $n$ years is $S$ .                     |  |  |
|          | Use of GP formula for $k$ th term   | M1                                       |  |
|          | $S_A = P(1.01)^n$   | B1                                       | Allow $P = 1$                            |
|          | $S_B = P(1.05)(1.006)^{n-1}$  | B1                                       | Allow $P = 1$                            |
|          | We want minimum $n$ st $S_A > S_B$  |  |  |
|          | $P(1.01)^n > P(1.05)(1.006)^{n-1}$  | m1                                       | Allow $P = 1$ . Condone $\geq$ or $=$    |
|          | $(1.006)(1.01)^n > (1.05)(1.006)^n$   |  |  |
|          | $\left(\frac{1.01}{1.006}\right)^n > \left(\frac{1.05}{1.006}\right)$       |  |  |
|          | $n \ln\left(\frac{1.01}{1.006}\right) > \ln\left(\frac{1.05}{1.006}\right)$ |  |  |
|          | $n > 10.78(762515)$   |  |  |
|          | min $n = 11$  | A1                                       | cao Mark final answer                    |
|          | <u>Note:</u> Accept any valid method (eg trial and error).                  |  |  |
|          | Use of GP formula for $k$ th term   | M1                                       | at least two iterations                  |
|          | $S_A = P(1.01)^n$   | B1                                       | Allow $P = 1$ , used for 2 values of $n$ |
|          | $S_B = P(1.05)(1.006)^{n-1}$  | B1                                       | Allow $P = 1$ , used for 2 values of $n$ |
|          | Two iterations for $n = 9, 10, 11, 12, 13$                                  | m1                                       |  |
|          | min $n = 11$  | A1                                       | cao Mark final answer                    |
|          | <b>Bank B</b>   | <b>Bank A</b>                            |  |
|          | $\frac{1.05 \times 1.006^{n-1}}{1.01^n}$                                    | $\frac{1.01^n}{1.05 \times 1.006^{n-1}}$ |  |
| $n = 9$  | 1.101 471 196...  | 1.093 685 272...                         | $B > A$                                  |
| $n = 10$ | 1.108 080 023...  | 1.104 622 125...                         | $B > A$                                  |
| $n = 11$ | 1.114 728 503...  | 1.115 668 346...                         | $B < A$                                  |
| $n = 12$ | 1.121 416 874...  | 1.126 825 030...                         | $B < A$                                  |
| $n = 13$ | 1.128 145 376...  | 1.138 093 280...                         | $B < A$                                  |
|          | Therefore $n = 11$  |  |  |