



GCE AS MARKING SCHEME

SUMMER 2024

**AS
MATHEMATICS
UNIT 1 PURE MATHEMATICS A
2300U10-1**

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE AS MATHEMATICS
UNIT 1 PURE MATHEMATICS A
SUMMER 2024 MARK SCHEME

Q	Solution	Mark	Notes
1	$y = 12x^{\frac{1}{2}} - 27x^{-1} + 4$	B1	index form, si
	$\frac{dy}{dx} = 12 \times \frac{1}{2} \times x^{-\frac{1}{2}} - 27 \times (-1)x^{-2}$	B1	correct 1 st term, ft fractional index.
		B1	correct 2nd term, ft -ve index.
	$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + 27x^{-2}$		
	When $x = 9$, $\frac{dy}{dx} = 2 - \left(-\frac{1}{3}\right) = \frac{7}{3}$	B1	cao, accept decimal answers correctly derived from $\frac{7}{3}$, isw

Q	Solution	Mark	Notes
2	$2\sin 2\theta = 1$ $\sin 2\theta = \frac{1}{2}$ $2\theta = 30^\circ, 150^\circ$ $\theta = 15^\circ,$ $\theta = 75^\circ$	 B1 B1 B1	 either angle, si

Notes:

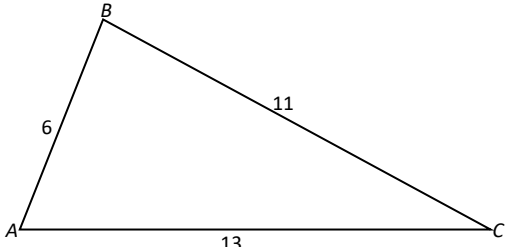
Ignore answers not in the range $0^\circ < \theta < 180^\circ$.

For an incorrect 3rd answer, -1 mark from the last 2 marks.

For an incorrect 4th answer, -1 mark from the last 2 marks.

Q	Solution	Mark	Notes
3	$\int (5x^{\frac{1}{4}} + 3x^{-2} - 2) \, dx$ $= \frac{5}{5/4} \times x^{\frac{5}{4}} + \frac{3x^{-1}}{-1} - 2x + C$	B1	$\frac{5}{5/4} \times x^{\frac{5}{4}}$, oe, isw $+ \frac{3}{-1} x^{-1}$, oe, isw $-2x$, isw -1 no C
	$= 4x^{\frac{5}{4}} - 3x^{-1} - 2x + C$		

Q	Solution		Mark	Notes
4	n	$n^2 - 2$		
	1	-1		
	2	2		
	3	7		
	4	14		
	5	23		
	6	34	M1	evaluating $n^2 - 2$ (for 1,...,6) at least twice
			A1	at least 4 values correct. Implied by correct conclusion.
	$n^2 - 2$ is not divisible by 3 for $1 \leq n \leq 6$		A1	all values correct, and conclusion.

Q	Solution	Mark	Notes
5			
	<p>Cosine rule attempted</p> $11^2 = 6^2 + 13^2 - 2 \times 6 \times 13 \cos \alpha$ $13^2 = 6^2 + 11^2 - 2 \times 6 \times 11 \cos \beta$ $6^2 = 13^2 + 11^2 - 2 \times 13 \times 11 \cos \gamma$ $\cos \alpha = \frac{7}{13}, \cos \beta = -\frac{1}{11}, \cos \gamma = \frac{127}{143},$ $(\alpha = 57.421^\circ, \beta = 95.216^\circ, \gamma = 27.363^\circ)$ $\text{Area} = \frac{1}{2} \times 6 \times 13 \sin \left(\cos^{-1} \left(\frac{7}{13} \right) \right)$ $\text{Area} = \frac{1}{2} \times 6 \times 11 \sin \left(\cos^{-1} \left(-\frac{1}{11} \right) \right)$ $\text{Area} = \frac{1}{2} \times 11 \times 13 \sin \left(\cos^{-1} \left(\frac{127}{143} \right) \right)$ $\text{Area} = 32.86 \dots (\text{cm}^2)$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>allow one error</p> <p>Must have 2 and cos</p> <p>any one</p> <p>si</p> <p>any one, FT their angle</p> <p>Accept 32.7 to 33.0, cao</p>

Q	Solution	Mark	Notes
6(a)	$7x^{\frac{3}{4}} = \sqrt{147}$ $7x^{\frac{3}{4}} = 7\sqrt{3}$ $x^{\frac{3}{4}} = 3^{\frac{1}{2}}$ $x = \left(3^{\frac{1}{2}}\right)^{\frac{4}{3}} \quad \text{or} \quad x^3 = 9$ $x = 3^{\frac{2}{3}}$	B1	collect x term, $x^{\frac{3}{4}}$.
		B1	collect constant term, $3^{\frac{1}{2}}$.
		B1	si by correct answer
		B1	cao, oe $\sqrt[3]{3^2} = \sqrt[3]{9}$
	OR		
	$7x^{\frac{5}{4}} = \sqrt{147}x^{\frac{1}{2}}$ $49x^{\frac{5}{2}} = 147x$ $49x^{\frac{3}{2}} = 147$ $x^{\frac{3}{2}} = 3$ $x = 3^{\frac{2}{3}}$	(B1)	collect x term, $x^{\frac{3}{2}}$.
		(B1)	collect constant term, 3.
		(B1)(B1)	

Note

Sight of 2.08... without working 0 marks

Sight of 2.08... from correct working B1 B1 B1 B0

Q	Solution	Mark	Notes
6(b)	$\frac{(8x-18)}{(2\sqrt{x}-3)} = \frac{(8x-18)(2\sqrt{x}+3)}{(2\sqrt{x}-3)(2\sqrt{x}+3)}$	M1	
	$= \frac{(8x-18)(2\sqrt{x}+3)}{(4x-9)}$	A1	correct denominator simplified
	$= \frac{2(4x-9)(2\sqrt{x}+3)}{(4x-9)}$		
	$= 2(2\sqrt{x}+3)$	A1	convincing
	OR		
	$\frac{(8x-18)}{(2\sqrt{x}-3)} = \frac{2(4x-9)}{(2\sqrt{x}-3)}$	(B1)	
	$= \frac{2(2\sqrt{x}+3)(2\sqrt{x}-3)}{(2\sqrt{x}-3)}$	(M1)	
	$= 2(2\sqrt{x}+3)$	(A1)	convincing

Q	Solution	Mark	Notes
7(a)	Gradient $L_1 = \frac{4-0}{1-(-3)} (= 1)$	B1	si
	Correct method for finding equation of line	M1	
	Equation of L_1 is $y - 0 = 1(x - (-3))$	A1	$y - 4 = 1(x - 1)$ isw
	$y = x + 3$		
7(b)(i)	$x + 3 = 3x - 3$	M1	FT part (a) for M1 A1 A1
	$x = 3$	A1	
	$y = 6$	A1	
7(b)(ii)	At D , $y = 0$, $0 = 3x - 3$		
	$x = 1$		
	D is the point $(1, 0)$	B1	allow verification
7(c)	area of $ACD = \frac{1}{2} \times 4 \times 6$	M1	oe, FT their coord of C
	$= 12$	A1	cao

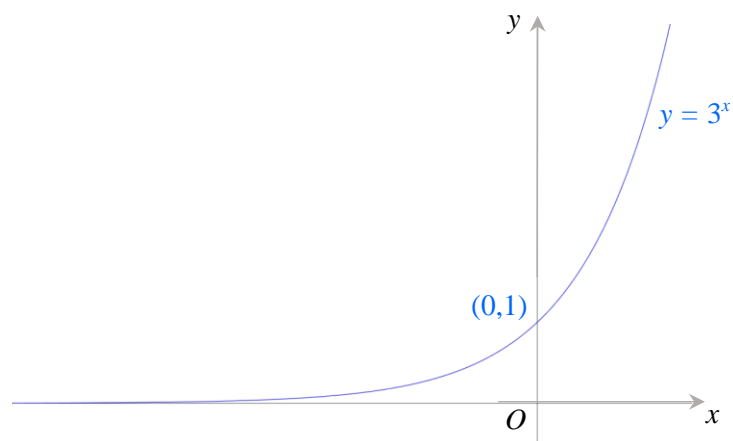
Q	Solution	Mark	Notes
7(d)	$\text{angle } ACD = \tan^{-1}(6/6) - \tan^{-1}(2/6)$ $= 45^\circ - 18.4349^\circ$ $= 26.57^\circ$	M1 A1	oe, FT their coord of C cao Accept 26.6°
OR	$\text{angle } ACD = \tan^{-1}(3) - \tan^{-1}(1)$ $= 71.5651^\circ - 45^\circ$ $= 26.57^\circ$	(M1) (A1)	oe, FT their coord of C cao Accept 26.6°
OR	$AC = \sqrt{(6-0)^2 + (3-(-3))^2} = \sqrt{72} = 6\sqrt{2}$ $CD = \sqrt{(6-0)^2 + (3-1)^2} = \sqrt{40} = 2\sqrt{10}$ $\text{Area} = \frac{1}{2} \times \sqrt{72} \times \sqrt{40} \sin ACD = 12$ $\sin ACD = \frac{1}{\sqrt{5}}$ $\text{angle } ACD = 26.57^\circ$	 (M1) (A1)	FT their coord of C cao Accept 26.6°

Q	Solution	Mark	Notes
8	$x - 10 < x^2 - 5x$		
	$x^2 - 6x + 10 > 0$	M1	For collecting terms on to one side
	$(x - 3)^2 + 1 > 0$	M1	oe
			or Showing minimum > 0
			or Discriminant < 0 (and a point > 0)
			or Discriminant < 0 (and +ve quadratic)
			or correct sketch
		A1	all correct
	Valid explanation, e.g. minimum ≥ 1	A1	Convincing
	This is true for all real values of x .		

Q	Solution	Mark	Notes
9(a)	$(2 - x)^6$ $= 2^6 + 6 \times 2^5 \times (-x) + \frac{6 \times 5}{2 \times 1} \times 2^4 \times (-x)^2 + \dots$ $= 64 - 192x + 240x^2 - \dots$	B3	B1 for each term, isw
9(b)	$(1 + ax)(2 - x)^6$ $= (1 + ax)(64 - 192x + 240x^2 - \dots)$ $= 64 + 64ax - 192x + 240x^2 - 192ax^2 + \dots$	M1 A1	replace $(2 - x)^6$ by answer in (a) provided 3 terms. allow one slip, ignore extra terms FT (a)
	Therefore $64 + (64a - 192)x + (240 - 192a)x^2$ $\equiv 64 + bx + 336x^2 + \dots$		
	$64a - 192 = b$ $240 - 192a = 336$ $a = -\frac{1}{2}$ $b = -224$	m1 A1 A1 A1	FT (a) equating coefficients both equations correct cao cao
	OR $(1 + ax)(2 - x)^6$ $= (1 + ax)(64 - 192x + 240x^2 - \dots)$ $(1 + ax)(64 - 192x + 240x^2 - \dots) \equiv$ $64 + bx + 336x^2 + \dots$	M1 A1	FT their (a), implied by next line si
	$64a - 192 = b$ $240 - 192a = 336$ $a = -\frac{1}{2}$ $b = -224$	m1 A1 A1 A1	FT (a) equating coefficients both equations correct cao cao

Q	Solution	Mark	Notes
10(a)	$t^2 - 14t + 49 = 25$ $t^2 - 14t + 24 = 0$ $(t - 2)(t - 12) = 0$ $t = 2, 12$ Required value is $t = 2$, (since $t \leq 7$).	M1 A1 A1	 method to solve must be seen $t = 12$ rejected
10(b)	$\frac{dy}{dt} = 2t - 14$ $t = 3, \frac{dy}{dt} = 2 \times 3 - 14$ $\frac{dy}{dt} = -8$ Rate of decrease is 8 (cms^{-1})	M1 A1 A1	at least 1 correct term. substitute $t = 3$, si cao

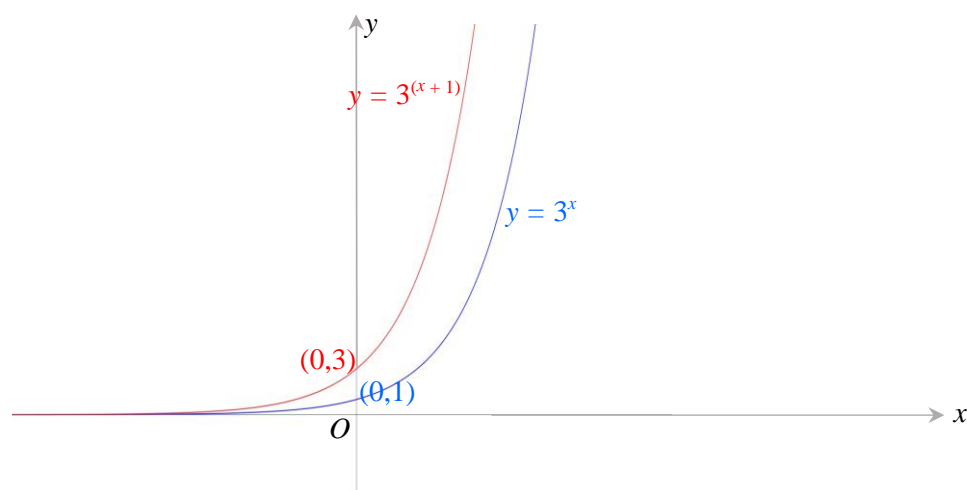
11(a)



G1 graph of $y = 3^x$, -ve domain required. x -axis not crossed.

B1 $(0, 1)$ accept all correct methods.

11(b)

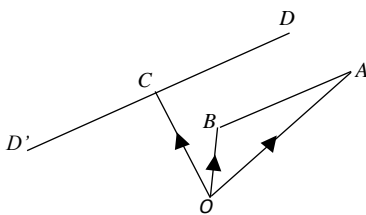


B1 graph of $y = 3^{x+1}$, same shape as (a), graphs do not intersect, ft (a) provided G1 awarded.

B1 $(0, 3)$ accept all correct methods.

Q	Solution	Mark	Notes
12(a)	$\frac{dy}{dx} = -3x^2 + 12$	B1	
	$\frac{dy}{dx} = -3x^2 + 12 = 0$	M1	si
	$x = 2, x = -2$	A1	cao any pair of correct values
	$y = -4, y = -36$	A1	cao all 4 values correct, no extra.
	$\frac{d^2y}{dx^2} = -6x$	M1	oe, ft quadratic $\frac{dy}{dx}$, si Eg -ve cubic has min before max. Correct sketch of negative cubic.
	FT only if x -coordinate of min $<$ x -coordinate of max.		
	$(x = 2, \frac{d^2y}{dx^2} = -12 < 0.)$		
	$(2, -4)$ is a maximum point	A1	ft their x value except $x = 0$.
	$(x = -2, \frac{d^2y}{dx^2} = 12 > 0.)$		
	$(-2, -36)$ is a minimum point	A1	ft their x value provided different conclusion except $x = 0$.

Q	Solution	Mark	Notes
12(b)	Curve is decreasing when		
	$\frac{dy}{dx} < 0$		FT their $\frac{dy}{dx}$
	$-3x^2 + 12 < 0$	M1	allow \geq or \leq throughout, oe
	$x^2 > 4$		
	$x < -2$ or $x > 2$	A1	A0 if 'and' instead of 'or'
	$(x \in) (-\infty, -2) \cup (2, \infty)$	B1	cao Allow $-2] \cup [2$
	OR		
	$\{x : x < -2\} \cup \{x : x > 2\}$	(B1)	cao
	<u>Alternative Solution</u>		
	$x < -2$	B1	ft their -2, if used.
	or $x > 2$	B1	ft their 2, if used.
	$(x \in) (-\infty, -2) \cup (2, \infty)$	B1	cao Allow $-2] \cup [2$
	OR		
	$\{x : x < -2\} \cup \{x : x > 2\}$	(B1)	cao

Q	Solution	Mark	Notes
13(a)	$\mathbf{AB} = (\mathbf{i} + 3\mathbf{j}) - (4\mathbf{i} + 7\mathbf{j})$ $\mathbf{AB} = -3\mathbf{i} - 4\mathbf{j}$	M1 A1	allow $(4\mathbf{i} + 7\mathbf{j}) - (\mathbf{i} + 3\mathbf{j})$ cao
13(b)	Distance = $\sqrt{(-3)^2 + (-4)^2}$ Distance = 5	M1 A1	correct method for $ a\mathbf{i} + b\mathbf{j} $, $a, b \neq 0$, si ft (a)
13(c)	 $\mathbf{d} = \mathbf{c} - \mathbf{BA} = \mathbf{c} + \mathbf{AB}$ $\mathbf{d} = (-2\mathbf{i} + 5\mathbf{j}) + (-3\mathbf{i} - 4\mathbf{j})$ $\mathbf{d} = -5\mathbf{i} + \mathbf{j}$ or $\mathbf{d} = \mathbf{c} + \mathbf{BA} = \mathbf{c} - \mathbf{AB}$ $\mathbf{d} = (-2\mathbf{i} + 5\mathbf{j}) - (-3\mathbf{i} - 4\mathbf{j})$ $\mathbf{d} = \mathbf{i} + 9\mathbf{j}$	M1 A1 M1 A1	si ft \mathbf{AB} si $\mathbf{a} + \mathbf{BC}$, $\mathbf{BC} = -3\mathbf{i} + 2\mathbf{j}$ $4\mathbf{i} + 7\mathbf{j} - 3\mathbf{i} + 2\mathbf{j}$ ft \mathbf{AB}

Q	Solution	Mark	Notes
14(a)	A is the point $(-2, 0)$	B1	
	B is the point $(0, 2)$	B1	
14(b)	$I = \int_{-2}^0 (2 - 3x - 2x^2) dx$	M1	attempt to integrate y wrt x Limits not required. At least one power of x increased
	$= \left[2x - \frac{3}{2}x^2 - \frac{2}{3}x^3 \right]_{-2}^0$	A1	correct integration
	$= \left[0 - \left(-4 - 6 + \frac{16}{3} \right) \right]$	m1	correct use of candidate's limits,
	$= \frac{14}{3}$	A1	cao, allow one decimal place correctly derived from $\frac{14}{3}$.
	Area of triangle $\left(= \frac{1}{2} \times 2 \times 2 \right) = 2$	B1	oe, ft (a)
	Required area $\left(= \frac{14}{3} - 2 \right) = \frac{8}{3}$	A1	cao, allow one decimal place correctly derived from $\frac{8}{3}$.

Note:

Must be supported by workings.

If M0, award SC1 for sight of $\frac{14}{3}$, OR SC2 for $\frac{8}{3}$

Q Solution**Mark Notes**Alternative solution

$$14(b) \quad I = \int_{-2}^0 (2 - 3x - 2x^2 - x - 2) \, dx$$

(M1) attempt to integrate y wrt x
Limits not required.
At least one power of x increased

(A1) attempt to subtract integrand

$$= \int_{-2}^0 (-4x - 2x^2) \, dx$$

$$= \left[-2x^2 - \frac{2x^3}{3} \right]_{-2}^0$$

(A1) correct integration

$$= \left[0 - \left(-8 + \frac{16}{3} \right) \right]$$

(m1) correct use of candidate's limits,

$$= \frac{8}{3}$$

(A2) cao

Note:

Must be supported by workings.

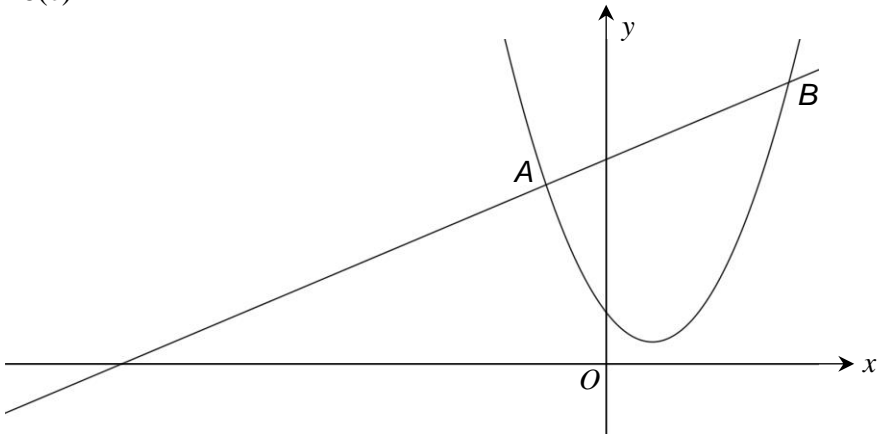
If M0, SC2 for $\frac{8}{3}$

Q	Solution	Mark	Notes
15	$2\sin x + 3\cos^2 x - 3 = 0$	M1	
	$2\sin x + 3(1 - \sin^2 x) - 3 = 0$	M1	$\sin^2 x + \cos^2 x = 1$
	$3\sin^2 x - 2\sin x = 0$	A1	si by $\sin x = 0$ AND $\sin x = \frac{2}{3}$
	$\sin x(3\sin x - 2) = 0$		
	$\sin x = \frac{2}{3}$ or $\sin x = 0$	A1	cao both roots (no extra roots)
			FT their $\sin x = k$ for $0 < k \leq 1$.
	(At A,) $x = 41.81^\circ$	B1	
	(At B,) $x = 138.19^\circ$	B1	
	(At C,) $x = 180^\circ$	B1	

Note:

Do not follow through for trig functions other than sine.

Ignore angles greater than 180°

Q	Solution	Mark	Notes
16(a)	$\text{Discriminant} = (-k)^2 - 4 \times 1 \times 4$ $= k^2 - 16$ If no real roots, discriminant < 0 $k^2 - 16 < 0$ Critical values $-4, 4$ $-4 < k < 4$	B1 M1 B1 A1	si condone \leq , si by correct answer. si cao, oe, e.g. $k > -4$ and $k < 4$ Mark final answer
16(b)	$x^2 - 3x + 4 = x + 16$ $x^2 - 4x - 12 = 0$ $(x + 2)(x - 6) = 0$ $x = -2, x = 6$ $y = 14, y = 22$ Points of intersection are $(-2, 14)$ and $(6, 22)$.	M1 m1 A1 A1	 write as quadratic equation cao one correct pair cao all correct
16(c)		G1 G1	+ve quadratic, above x -axis straight line, +ve gradient 1 point of intersection in 1 st quadrant, 1 point of intersection in 2 nd quadrant

Q	Solution	Mark	Notes
17(a)	$1 = \log_{10}(2 - c)$ $2 - c = 10$ $c = -8$	M1 A1	si
17(b)	$\log_{10}(5 - \alpha) - \log_{10}(2 - \alpha) = 1.2$ $\log_{10}\left(\frac{5 - \alpha}{2 - \alpha}\right) = 1.2$ $\frac{5 - \alpha}{2 - \alpha} = 10^{1.2} (= 15.8489\dots)$ $5 - \alpha = 10^{1.2}(2 - \alpha)$ $5 - \alpha = 2 \times 10^{1.2} - 10^{1.2} \times \alpha$ $\alpha(10^{1.2} - 1) = 2 \times 10^{1.2} - 5$ $\alpha = 1.798$	M1 B1 m1 A1 A1	Condone $\log_{10}(2 - \alpha) - \log_{10}(5 - \alpha) = 1.2$ subtraction law logs removed correctly removal of denominator cao

Q	Solution	Mark	Notes
18(a)	$(x - -3)^2 + (y - -1)^2 = (\sqrt{5})^2$ $(x + 3)^2 + (y + 1)^2 = 5$ $x^2 + 6x + 9 + y^2 + 2y + 1 = 5$ $x^2 + y^2 + 6x + 2y + 5 = 0$	M1	condone $\sqrt{5}$ on RHS
	OR		
	Equation of circle radius r is		
	$x^2 + y^2 + 2fx + 2gy + c = 0$		
	Centre $(-f, -g)$, $c = f^2 + g^2 - r^2$	(M1)	used
	$f = 3, g = 1, c = 3^2 + 1^2 - \sqrt{5}^2 = 5$	(A1)	
	OR		
	$x^2 + y^2 + 6x + 2y + 5 = 0$		
	$(x + 3)^2 + (y + 1)^2 - 9 - 1 + 5 = 0$	(M1)	M0 if no working shown.
	$(x + 3)^2 + (y + 1)^2 = 5$		
	Hence centre = $(-3, -1)$, radius = $\sqrt{5}$	(A1)	

Q	Solution	Mark	Notes
18(b)(i)	Tangents have equations $y = mx$	B1	
	Touches circle when		
	$x^2 + (mx)^2 + 6x + 2mx + 5 = 0$	M1	
	$(1 + m^2)x^2 + 2(3 + m)x + 5 = 0$	A1	si
	Discriminant $= 4(3 + m)^2 - 4(1 + m^2) \times 5$	B1	ft 1 slip in quadratic
	If tangent, discriminant $= 0$	M1	used
	$9 + 6m + m^2 - 5 - 5m^2 = 0$		
	$2m^2 - 3m - 2 = 0$		
	$(2m + 1)(m - 2) = 0$		
	$m = -\frac{1}{2} \quad m = 2$	A1	cao, both values
	$y = -\frac{1}{2}x \quad y = 2x$		
	<u>Special case 1</u>		
	Candidates who substitute $y = mx + c$ can only earn method marks, B0 M1 A0 B0 M1 A0		
	<u>Special case 2</u>		
	Candidates who obtain the correct answer using any method, award as follows:		
	$y = -\frac{1}{2}x \quad y = 2x$	SC1	

Q	Solution	Mark	Notes
18(b)(ii)	$m = -\frac{1}{2}, \quad \frac{5}{4}x^2 + 5x + 5 = 0$ $x^2 + 4x + 4 = 0$ $(x + 2)^2 = 0$ $x = -2, y = 1 \quad (-2, 1)$	M1	FT their derived m provided $y = mx$
		m1	must be a perfect square, si
		A1	cao
	$m = 2, \quad 5x^2 + 10x + 5 = 0$ $x^2 + 2x + 1 = 0$ $(x + 1)^2 = 0$ $x = -1, y = -2 \quad (-1, -2)$	(M1)	award if previous M1 not awarded
		(m1)	must be a perfect square, si
		A1	cao
	OR		
	$x = \frac{-b}{2a} = \frac{-2(3+m)}{2(1+m^2)} = \frac{-(3+m)}{(1+m^2)}$	(M1)	
	At $m = -\frac{1}{2}, x = \frac{-(3-\frac{1}{2})}{(1+\frac{1}{4})} = -2$	(m1)	
	$y = -\frac{1}{2} \times (-2) = 1 \quad (-2, 1)$	(A1)	
	At $m = 2, x = \frac{-(3+2)}{(1+4)} = -1$		
	$y = 2 \times (-1) = -2 \quad (-1, -2)$	(A1)	

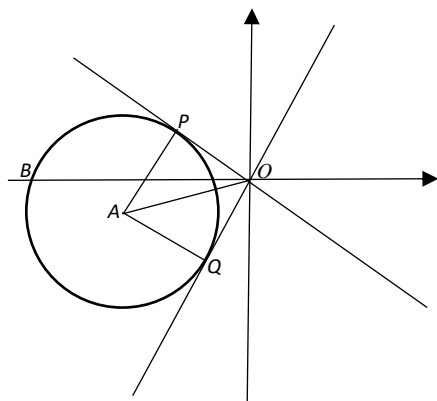
Special case

Candidates who obtained the correct answer using any method, award as follows:

$(-2, 1) \quad (-1, -2)$ SC1

Alternative solution

18(b)(i)



$$\tan(\angle BOA) = \frac{1}{3}$$

$$\angle BOA = 18.43494882^\circ$$

$$\text{B1} \quad \angle BOP = 26.56505118^\circ$$

$$OA = \sqrt{(3)^2 + (1)^2} = \sqrt{10}$$

B1

$$OP = OQ = \sqrt{(\sqrt{10})^2 - (\sqrt{5})^2} = \sqrt{5}$$

B1

Triangle POA and QOA are isosceles right angled

$$\text{Angles } \angle POA = 45^\circ \text{ and } \angle QOA = 45^\circ$$

B1

$$\text{Gradient } OQ = \tan(45 + 18.43494882) = 2$$

B1

$$\tan(243.43494882) = 2$$

$$y = 2x$$

$$\text{Gradient } OP = -\frac{1}{2}$$

B1

$$\tan(26.56505118) = \frac{1}{2}$$

$$\tan(116.56505118) = -\frac{1}{2}$$

$$y = -\frac{1}{2}x$$