GCE A LEVEL MARKING SCHEME

SUMMER 2024

A LEVEL
FURTHER MATHEMATICS
UNIT 6 FURTHER MECHANICS B
1305U60-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

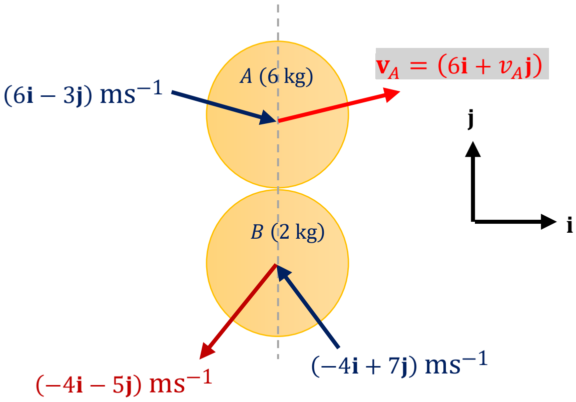
This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 6 FURTHER MECHANICS B

SUMMER 2024 MARK SCHEME

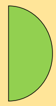
Q1	Solution	Mark	Notes
			Before collision After collision
(a)	Con. of momentum (along line of centres)	M1	Attempted
	$6(-3) + 2(7) = 6v_A + 2(-5)$	A1	All correct, oe $-4 = 6v_A - 10$
	$(v_A = 1)$		
	$\mathbf{v}_A = (6\mathbf{i} + \mathbf{j})$ $\quad$ (ms^{-1})	A1	
		[3]	
	Alternative Solution (Vector Method)		
	Conservation of momentum $6\mathbf{u}_A + 2\mathbf{u}_B = 6\mathbf{v}_A + 2\mathbf{v}_B$	(M1)	Attempted
	$6(6\mathbf{i} - 3\mathbf{j}) + 2(-4\mathbf{i} + 7\mathbf{j}) = 6\mathbf{v}_A + 2(-4\mathbf{i} - 5\mathbf{j})$	(A1)	All correct, oe
	$\mathbf{v}_A = (6\mathbf{i} + \mathbf{j})$ $\quad$ (ms^{-1})	(A1)	
		([3])	
(b)	Restitution (along line of centres)	M1	Attempted
	$(-5) - (1) = -e(7 - -3)$ OR $e = \frac{1 - -5}{7 - -3} = \frac{-5 - 1}{-3 - 7}$ OR $-e = \frac{-5 - 1}{7 - -3} = \frac{1 - -5}{-3 - 7}$	A1	All correct, oe FT component $\mathbf{j}$ of $\mathbf{v}_A$ from (a)
	$e = \frac{3}{5} = 0 \cdot 6$	A1	
		[3]	

Q1	Solution	Mark	Notes
(c)	$\cos\theta = \dfrac{(-4\mathbf{i}+7\mathbf{j})\cdot(-4\mathbf{i}-5\mathbf{j})}{\lvert -4\mathbf{i}+7\mathbf{j}\rvert\lvert -4\mathbf{i}-5\mathbf{j}\rvert}$	M1	Use of (any form) $\cos\theta = \dfrac{\mathbf{u}_B\cdot\mathbf{v}_B}{\lvert\mathbf{u}_B\rvert\lvert\mathbf{v}_B\rvert}$
	$\cos\theta = \dfrac{16-35}{\sqrt{65}\sqrt{41}}$ $(= -0\cdot368048451\ldots)$	A1	oe
	$\theta = 112°$ (nearest degree)	A1 **[3]**	or $360 - 112 = 248°$
	Alternative Solution Sight of either one from **BOTH** rows $\theta = \tan^{-1}\left(\pm\frac{7}{4}\right)$ OR $\theta = \tan^{-1}\left(\pm\frac{4}{7}\right)$ $(\theta = \pm 60\cdot 2\ldots)$ $(\theta = \pm 29\cdot 7\ldots)$ $\theta = \tan^{-1}\left(\pm\frac{5}{4}\right)$ OR $\theta = \tan^{-1}\left(\pm\frac{4}{5}\right)$ $(\theta = \pm 51\cdot 3\ldots)$ $(\theta = \pm 38\cdot 6\ldots)$	(M1)	oe
	$\theta = 60\cdot 255\ldots + 51\cdot 340\ldots$	(A1)	Correct calculation
	$\theta = 112°$ (nearest degree)	(A1) **([3])**	or $360 - 112 = 248$
(d)	Impulse, $\mathbf{I}$ = change in momentum $(-20\mathbf{i} + 18\mathbf{j}) = 2\mathbf{v}_B - 2(-4\mathbf{i} - 5\mathbf{j})$	M1	Difference in Momentum used, $(-20\mathbf{i} + 18\mathbf{j}) = -2\mathbf{v}_B + 2(-4\mathbf{i} - 5\mathbf{j})$
	$\mathbf{v}_B = (-14\mathbf{i} + 4\mathbf{j})$ (ms^{-1})	A1 **[2]**	
	Total for Question 1	11	

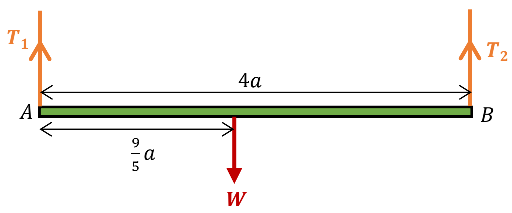
Q2	Solution	Mark	Notes
(a)	Applying N2L (mg, $0\cdot 2v^2$ opposing), $1\cdot 8g - 0\cdot 2v^2 = 1\cdot 8a$ Dividing (by $0\cdot 2, 1\cdot 8$) and using $a = \frac{dv}{dt}$ $9\frac{dv}{dt} = 9g - v^2$ $\frac{dv}{dt} = \frac{9g-v^2}{9}$	M1 A1 A1 **[3]**	Dimensionally correct equation Correct equation Convincing
(b)	$9\int \frac{1}{9g-v^2}\,dv = \int dt$ $t\ \ (+C) = \begin{cases} \frac{9}{2\times 3\sqrt{g}}\ln\left\lvert\frac{3\sqrt{g}+v}{3\sqrt{g}-v}\right\rvert \\ \frac{9}{3\sqrt{g}}\tanh^{-1}\left(\frac{v}{3\sqrt{g}}\right) \end{cases}$ When $t = 0, v = \sqrt{g}$ $C = \begin{cases} \frac{9}{2\times 3\sqrt{g}}\ln(2) \\ \frac{9}{3\sqrt{g}}\tanh^{-1}\left(\frac{1}{3}\right) \end{cases}$ Using $v = 8$, $t = \begin{cases} \frac{9}{2\times 3\sqrt{g}}\ln\left\lvert\frac{3\sqrt{g}+8}{3\sqrt{g}-8}\right\rvert - \frac{9}{2\times 3\sqrt{g}}\ln(2) \\ \frac{9}{3\sqrt{g}}\tanh^{-1}\left(\frac{8}{3\sqrt{g}}\right) - \frac{9}{3\sqrt{g}}\tanh^{-1}\left(\frac{1}{3}\right) \\ 0\cdot 878(03761\ldots) \end{cases}$ Time taken = $0\cdot 878(03761\ldots)$ (s)	M1 A1 A1 m1 A1 A1 **[6]**	Separating variables $\ln\left\lvert\frac{3\sqrt{g}+v}{3\sqrt{g}-v}\right\rvert$ or $\tanh^{-1}\left(\frac{v}{3\sqrt{g}}\right)$ Everything correct, oe Use of initial conditions Finding C Notes • These will be negative if $+C$ features on the RHS above • The numerator of 9 will not be present when the LHS is $\frac{t}{9}$ $t = \begin{cases} \frac{3}{2\sqrt{g}}\ln\left\lvert\frac{3\sqrt{g}+8}{2(3\sqrt{g}-8)}\right\rvert \\ \frac{3}{\sqrt{g}}\tanh^{-1}\left(\frac{8}{3\sqrt{g}}\right) - \frac{3}{\sqrt{g}}\tanh^{-1}\left(\frac{1}{3}\right) \end{cases}$
(c)	Using $a = v\frac{dv}{dx}$ to get $v\frac{dv}{dx} = \frac{9g-v^2}{9}$ $9\int \frac{v}{9g-v^2}\,dv = \int dx$ $\quad \frac{9}{-2}\int \frac{-2v}{9g-v^2}\,dv = \int dx$ $-\frac{9}{2}\ln\lvert 9g - v^2\rvert = x\ \ (+C)$ When $x = 0, v = \sqrt{g}$ $-\frac{9}{2}\ln\lvert 8g\rvert = C$	M1 m1 A1 A1 m1	Used with DE Separating variables $\ln\lvert 9g - v^2\rvert$ Everything correct Use of initial conditions

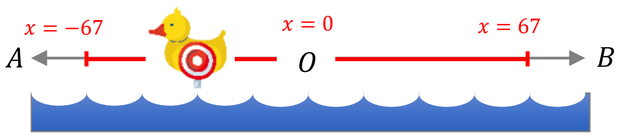
Q2	Solution	Mark	Notes
	$x = \begin{cases} -\frac{9}{2}\ln\lvert 9g - v^2\rvert + \frac{9}{2}\ln\lvert 8g\rvert \\ \frac{9}{2}\ln\left\lvert \frac{8g}{9g - v^2}\right\rvert \end{cases}$ When $v = 8$, $x = \begin{cases} -\frac{9}{2}\ln\lvert 9g - 8^2\rvert + \frac{9}{2}\ln\lvert 8g\rvert \\ \frac{9}{2}\ln\left\lvert \frac{8g}{9g - 8^2}\right\rvert \end{cases}$ $x = 5 \cdot 289(620824) \quad (\mathrm{m})$	A1 A1 **[7]**	oe, for correct expression Notes • Signs will be reversed if $+C$ features on the LHS • The numerator of 9 will not be present when the RHS is $\frac{x}{9}$
Total for Question 2		**16**	

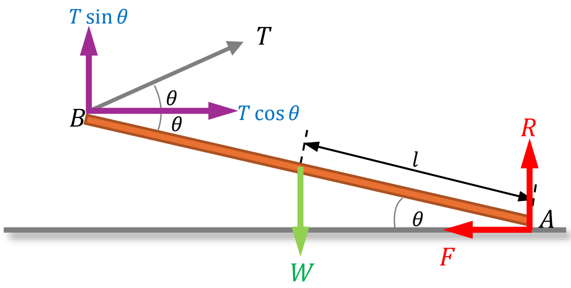
Q3	Solution	Mark	Notes
(a)			Condone omission of ρ (mass per unit area)

Shape	Area/mass	Distance from ACB	Distance from ⊥
 Big	$\frac{1}{2}\pi(2a)^2\rho$ $(= 2\pi a^2\rho)$	$\frac{4(2a)}{3\pi}$ $\left(= \frac{8a}{3\pi}\right)$	$\pm 2a$
 Small	$\frac{1}{2}\pi a^2\rho$	$\pm\frac{4(a)}{3\pi}$ $\left(= \frac{4a}{3\pi}\right)$	$\pm a$
 Lamina	$\frac{5}{2}\pi a^2\rho$	$\bar{x}$	$\bar{y}$

Solution	Mark	Notes
(table above)	B1 B1 B1	B3 6 B2 any 4 or 5, B1 any 2 or 3 correct
(Lamina row)	B1	Addition of areas/masses
(i) Moments about AB	M1	Masses and moments consistent
$\frac{5}{2}\pi a^2\bar{x} = (2\pi a^2)\left(\frac{8a}{3\pi}\right) - \left(\frac{1}{2}\pi a^2\right)\left(\frac{4a}{3\pi}\right)$	A1	
$\bar{x} = \frac{28}{15\pi}a$	A1	Convincing
(ii) Moments about ⊥ through A	M1	Masses and moments consistent
$\frac{5}{2}\pi a^2\bar{y} = (2\pi a^2)(2a) + \left(\frac{1}{2}\pi a^2\right)(a)$ OR $\frac{5}{2}\pi a^2\bar{y} = (2\pi a^2)(-2a) + \left(\frac{1}{2}\pi a^2\right)(-a)$	A1	
$\bar{y} = \pm\frac{9}{5}a = \pm 1\cdot 8a$		
Distance is $\frac{9}{5}a$ or $1\cdot 8a$	A1 **[10]**	

Q3	Solution	Mark	Notes
(b)			W = weight
	Moments about A	M1	
	$\frac{9}{5}a \times W = 4a \times T_2$	A1	
	$T_2 = \frac{9}{20}W$ (Required fraction is $\frac{9}{20}$)	A1 **[3]**	
	Total for Question 3	**13**	

Q4	Solution	Mark	Notes
(a)	Acceleration, $\frac{d^2x}{dt^2} = \pm\omega^2 x$ $1344 = \pm\omega^2(\pm 84)$ $\omega^2 = 16$ $\omega = 4$ Period, $T = \frac{2\pi}{\omega}$ $= \frac{\pi}{2}$ (s)	M1 A1 A1 **[3]**	Used with $a = 1344$, $x = \pm 84$ Convincing
(b)	$v^2 = \omega^2(a^2 - x^2)$, $\omega = 4$, $x = \pm 84$, $v = \pm 52$ $(52)^2 = 4^2(a^2 - (84)^2)$ $a = 85$ Maximum speed $= a\omega = (85)(4)$ $= 340$ (cms^{-1})	M1 A1 A1 M1 A1 **[5]**	Used FT ω from (a) Used FT ω from (a) and corresponding a
(c)			
	Using $x = \pm a\cos(\omega t)$ or $x = \pm a\sin\left(\omega t + \frac{\pi}{2}\right)$ $\pm 67 = \pm 85\cos(4t)$ (Same/opposing signs) $4t = \cos^{-1}\left(\frac{67}{85}\right)$ or $4t = \cos^{-1}\left(\frac{-67}{85}\right)$ $4t = 0 \cdot 66286 \ldots$ $\quad$ $4t = 2 \cdot 47873 \ldots$ $t = 0 \cdot 16571 \ldots$ $\quad$ $t = 0 \cdot 61968 \ldots$ Times $= T + 0 \cdot 16571 \ldots,\ T + 0 \cdot 61968 \ldots$ $= 1 \cdot 73651 \ldots,\quad 2 \cdot 190479 \ldots$ (s)	M1 m1 m1 A1 A1 **[5]**	Allow $x = \pm a\sin(\omega t)$ FT $a > 67$ 1st correct method Remaining method Corresponding time for 1st correct method.
	Total for Question 4	**13**	

Q5	Solution	Mark	Notes
			Length of rod $= 2l$
(a)	(i) Moments about A	M1	Dim. correct, no missing/extra terms
	$T \times 2l \sin 2\theta = W \times l \cos\theta$	A1	All correct
	$T \times 2l \times 2\sin\theta\cos\theta = W \times l\cos\theta$	m1	Use of $\sin 2\theta = 2\sin\theta\cos\theta$
	$T = \frac{W}{4}\operatorname{cosec}\theta$	A1	Convincing
	(ii) Resolve vertically	M1	Dim. correct equation with 3 terms
	$R + T\sin\theta = W$	A1	
	$R + \frac{W}{4}\operatorname{cosec}\theta\sin\theta = W$	m1	Elimination of T
	$R = \frac{3}{4}W$	A1	
		[8]	
(b)	Resolve horizontally	M1	Dimensionally correct equation, no missing/extra terms
	$F = T\cos\theta$	A1	
	$F = \frac{W}{4}\operatorname{cosec}\theta\cos\theta = \frac{W}{4}\cot\theta$		
	Use of $F \leq \mu R$ with $\mu = \frac{\sqrt{3}}{3}$	M1	si (or equality $F_{lim} = \mu R$)
	$\frac{W}{4}\cot\theta \leq \frac{\sqrt{3}}{3} \times \frac{3}{4}W$		
	$\cot\theta \leq \sqrt{3}$ or $\tan\theta \geq \frac{\sqrt{3}}{3}$	A1	
	$\theta \geq 30^{\circ}$	A1	
		[5]	
	Total for Question 5	13	

<table>
<tr><th>Q6</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr><td>(a)</td><td>$(V\bar{x} =)\ \pi \int_0^b xy^2\,\mathrm{d}x$
$(V\bar{x} =)\ \pi \frac{a^2}{b^2}\int_0^b x(b^2 - x^2)\,\mathrm{d}x$
$(V\bar{x} =)\ \pi \frac{a^2}{b^2}\int_0^b (b^2x - x^3)\,\mathrm{d}x$
$(V\bar{x} =)\ \pi \frac{a^2}{b^2}\left[\frac{b^2}{2}x^2 - \frac{1}{4}x^4\right]_0^b$
$(V\bar{x} =)\ \frac{1}{4}\pi a^2 b^2$
Using $V = \frac{2}{3}\pi a^2 b$ and dividing to get
$\bar{x} = \frac{3}{8}b$</td><td>M1
A1

m1
A1

A1
[5]</td><td>Used
All correct, oe

At least one term correctly integrated AND sight of correct limits

Convincing</td></tr>
<tr><td>(b)</td><td><table>
<tr><th>Shape</th><th>Mass</th><th>Distance of COM from ground</th></tr>
<tr><td></td><td>$\frac{2}{3}\pi\left(\frac{h}{4}\right)^2 h\rho \quad \left(= \frac{1}{24}h^3\pi\rho\right)$</td><td>$50 + \frac{3h}{8}$</td></tr>
<tr><td></td><td>$\pi(25)^2 \times 50 \times 20\rho \quad (= 625000\pi\rho)$</td><td>$25$</td></tr>
<tr><td></td><td>$\pi\rho\left[\frac{1}{24}h^3 + (25)^2 \times 50 \times 20\right]$</td><td>$\bar{h}$, 50</td></tr>
</table>
Moments about horizontal ground
$\pi\rho\left(\frac{1}{24}h^3\left(50 + \frac{3h}{8}\right) + (25)^2 \times 50 \times 20 \times 25\right) = \pi\rho\left(\frac{1}{24}h^3 + (25)^2 \times 50 \times 20\right) \times \bar{h}$
Using $\bar{h} = 50$ and rearranging to get
$\frac{h^4}{64} = 25^3 \times 50 \times 20$
$h = 177 \cdot 82(7941 \ldots)$</td><td>B1
B1
B1
B1
M1
A1
m1
A1
[8]</td><td>Condone omission of ρ (mass per unit area)
For Distance of COM
For Mass
Both correct
Must be addition of volumes

Notes
$h^4 = 10^9$
$h = (10^9)^{\frac{1}{4}} = 10^{\frac{9}{4}}$</td></tr>
<tr><td>(c)</td><td>One possible limitation identified.</td><td>E1
[1]</td><td>For example,
• Modelling the tree as a (uniform) solid
• Considering tree/pot to have constant density
• Shape of tree perfectly matches the region R</td></tr>
<tr><td></td><td>Total for Question 6</td><td>14</td><td></td></tr>
</table>