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# **GCE AS MARKING SCHEME**

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**SUMMER 2024**

**AS  
FURTHER MATHEMATICS  
UNIT 1 FURTHER PURE MATHEMATICS A  
2305U10-1**

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## About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

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**WJEC GCE AS FURTHER MATHEMATICS**

**UNIT 1 FURTHER PURE MATHEMATICS A**

**SUMMER 2024 MARK SCHEME**

Qu	Solution	Mark	Notes
1.	<p><b>METHOD 1:</b></p> $\frac{v}{w} = \frac{-16 + 11i}{5 + 2i} \times \frac{5 - 2i}{5 - 2i}$ $\frac{v}{w} = \frac{-80 + 32i + 55i - 22i^2}{25 - 10i + 10i - 4i^2}$ $\frac{v}{w} = \frac{-58 + 87i}{29} = -2 + 3i = z$ $ z  = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$ $\arg z = \tan^{-1}\left(\frac{3}{-2}\right) + 180^\circ = 123.69^\circ$ $z = \sqrt{13}(\cos 123.69^\circ + i \sin 123.69^\circ)$ <p><b>METHOD 2:</b></p> $ v  = \sqrt{(-16)^2 + 11^2} = \sqrt{377}$ $ w  = \sqrt{5^2 + 2^2} = \sqrt{29}$ $\arg v = \tan^{-1}\left(\frac{11}{-16}\right) + 180^\circ = 145.49^\circ$ $\arg w = \tan^{-1}\left(\frac{2}{5}\right) = 21.80^\circ$ $ z  = \frac{\sqrt{377}}{\sqrt{29}} = \sqrt{13}$ $\arg z = 145.49^\circ - 21.80^\circ = 123.69^\circ$ $z = \sqrt{13}(\cos 123.69^\circ + i \sin 123.69^\circ)$	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>(B1)</p> <p>(B1)</p> <p>(M1)</p> <p>(A1)</p> <p>(B1)</p> <p><b>Total [5]</b></p>	<p>Multiplying by conjugate</p> <p>No workings M0A0</p> <p>FT <math>z</math></p> <p>FT <math>z</math> 2.16 radian Accept <math>124^\circ</math></p> <p>FT arg</p> <p>Both <math>v</math> and <math>w</math></p> <p>Both <math>v</math> and <math>w</math> 2.54 and 0.38 rad</p> <p>FT using both mod and arg</p> <p>FT both <math> z </math> and <math>\arg(z)</math> correct 2.16 radian Accept <math>124^\circ</math> FT arg</p>

Qu	Solution	Mark	Notes
2.	$x^3 + x^2 - 7x - 15 = (x - 3)(x^2 + 4x + 5)$ When $x - 3 = 0$ , $x = 3$  When $x^2 + 4x + 5 = 0$ , Solving, e.g. $x = \frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times 5)}}{2}$  or $(x + 2)^2 + 1 = 0$ $(x + 2)^2 = -1$ $x + 2 = \pm i$  $x = -2 \pm i$	B1   M1      A1  <b>Total</b> <b>[3]</b>	May be awarded at any point   Complete method M0 no workings      Both, cao

Qu	Solution	Mark	Notes
3.	<p>METHOD 1:</p> <p>Original quadratic with roots <math>\alpha</math></p> $\alpha + \alpha = -p$ $\alpha \times \alpha = q$ <p>EITHER:</p> <p>New quadratic with roots <math>\frac{1}{\alpha}</math></p> $\frac{1}{\alpha} + \frac{1}{\alpha} = -m$ $\frac{1}{\alpha} \times \frac{1}{\alpha} = m$ <p>OR:</p> <p>New quadratic with roots <math>\frac{1}{\alpha}</math></p> $\left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\alpha}\right) = 0$ $x^2 - \frac{2}{\alpha}x + \frac{1}{\alpha^2} = 0$ <p>THEN:</p> $-\left(\frac{1}{\alpha} + \frac{1}{\alpha}\right) = \frac{1}{\alpha} \times \frac{1}{\alpha}$ $\frac{-2}{\alpha} = \frac{1}{\alpha^2}$ $-2\alpha^2 = \alpha$ $2\alpha^2 + \alpha = 0$ $\alpha(2\alpha + 1) = 0$ $\therefore \alpha = 0 \quad \text{or} \quad \alpha = -\frac{1}{2}$ <p>As <math>\alpha \neq 0</math>, <math>\alpha = -\frac{1}{2}</math>.</p> <p>Therefore,</p> $-\frac{1}{2} + -\frac{1}{2} = -p \rightarrow p = 1$ $-\frac{1}{2} \times -\frac{1}{2} = q \rightarrow q = \frac{1}{4}$ <p>METHOD 2:</p> <p>Let <math>w = \frac{1}{x}</math>, then <math>x = \frac{1}{w}</math></p> <p>leading to <math>\left(\frac{1}{w}\right)^2 + p\left(\frac{1}{w}\right) + q = 0</math></p> $qw^2 + pw + 1 = 0.$ $w^2 + \frac{p}{q}w + \frac{1}{q} = 0$ <p>Hence, <math>\frac{p}{q} = m</math> and <math>\frac{1}{q} = m</math></p> <p>therefore <math>\frac{p/q}{1/q} = \frac{m}{m}</math></p> <p>such that <math>p = 1</math>.</p> <p>Given that <math>\frac{1}{\alpha} + \frac{1}{\alpha} = -m</math>, <math>-\frac{2}{\alpha} = m \rightarrow \alpha = -\frac{2}{m}</math></p> <p>and <math>p = -2\alpha = \frac{4}{m} \rightarrow m = 4</math></p> <p>Therefore, <math>\frac{1}{q} = 4 \rightarrow q = \frac{1}{4}</math></p>	<p>B1</p> <p>B1</p> <p>(B1)</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>(M1)</p> <p>(A1)</p> <p>(A1)</p> <p>(A1)</p> <p>(B1)</p> <p>(B1)</p>	<p>Both</p> <p>Allow use of other notation for <math>p, q</math> but must be <math>\alpha</math></p> <p>Both si</p> <p>Allow use of other notation for <math>m</math></p> <p>Either value</p> <p>Must reject <math>\alpha = 0</math></p> <p>FT their <math>\alpha</math></p>

Qu	Solution	Mark	Notes
3.	<p>METHOD 3:</p> <p>Given repeated root, <math>b^2 - 4ac = 0</math> therefore <math>m^2 - 4m = 0</math></p> <p>Solving, e.g. <math>m(m - 4) = 0</math> <math>m = 0</math> or <math>m = 4</math></p> <p>When <math>m = 0</math>, no possible roots so new equation is <math>x^2 + 4x + 4 = 0</math></p> <p>Solving, e.g. <math>(x + 2)^2 = 0</math> <math>x = -2</math></p> <p>Therefore, <math>-2 = \frac{1}{\alpha} \rightarrow \alpha = -\frac{1}{2}</math></p> <p>Original equation: <math>\left(x + \frac{1}{2}\right)^2 = 0</math> <math>x^2 + x + \frac{1}{4} = 0</math> such that <math>p = 1, q = \frac{1}{4}</math>.</p>	<p>(M1)</p> <p>(A1)</p> <p>(A1)</p> <p>(A1)</p> <p>(B1)</p> <p>(B1)</p> <p><b>Total</b> <b>[6]</b></p>	<p>Must reject <math>m = 0</math></p>

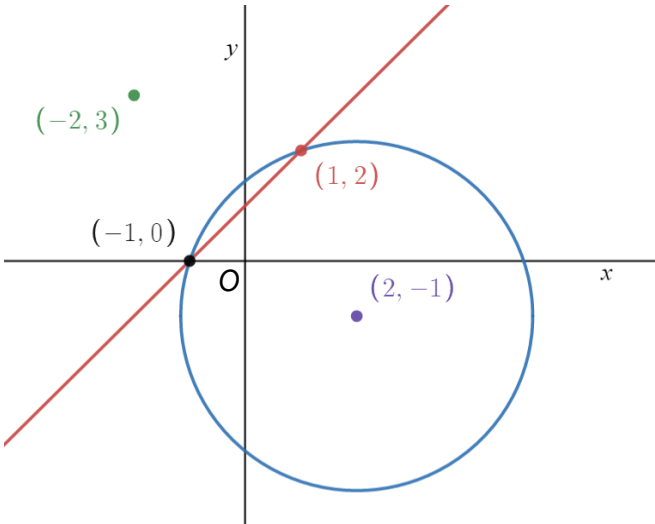
Qu	Solution	Mark	Notes
4. a)	$u + iv = \frac{x + iy}{1 - x - iy}$ $u + iv = \frac{x + iy}{(1 - x) - iy} \times \frac{(1 - x) + iy}{(1 - x) + iy}$ $u + iv = \frac{(x + iy)(1 - x) + (x + iy)(iy)}{(1 - x)^2 - (iy)^2}$ $u + iv = \frac{x - x^2 + iy - ixy + ixy - y^2}{(1 - x)^2 + y^2}$ $u + iv = \frac{(x - x^2 - y^2) + iy}{(1 - x)^2 + y^2}$	M1  m1  A1	
	<p>Comparing coefficients</p> <p>Imaginary parts: <math>v = \frac{y}{(1-x)^2 + y^2}</math> (given)</p> <p>Real parts: <math>u = \frac{x - x^2 - y^2}{(1-x)^2 + y^2}</math></p>	m1  A1 [5]	<p>Dependent on M1</p> <p>Both correct If expanded, denominator must be correct, otherwise A0</p>
b)	<p>Putting <math>y = 1 - x</math></p> $v = \frac{y}{y^2 + y^2} \left( = \frac{y}{2y^2} = \frac{1}{2y} \right)$ $u = \frac{(1 - y)y - y^2}{y^2 + y^2} = \frac{y - 2y^2}{2y^2}$ $u = \frac{y}{2y^2} - 1$ <p>Eliminating <math>y</math>, the equation of the locus <math>Q</math> is</p> $u = v - 1$ <p>OR</p> <p>Putting <math>y = 1 - x</math></p> $v = \frac{1 - x}{(1 - x)^2 + (1 - x)^2} \left( = \frac{1 - x}{2(1 - x)^2} = \frac{1}{2(1 - x)} \right)$ $u = \frac{x - x^2 - (1 - x)^2}{(1 - x)^2 + (1 - x)^2}$ $u = \frac{x - x^2 - 1 + 2x - x^2}{2(1 - x)^2} = \frac{-2x^2 + 3x - 1}{2(1 - x)^2}$ $u = \frac{(1 - x)(2x - 1)}{2(1 - x)^2} = \frac{2x - 1}{2(1 - x)} = \frac{1 - 2(1 - x)}{2(1 - x)}$ $u = \frac{1}{2(1 - x)} - 1$ <p>Eliminating <math>x</math>, the equation of the locus <math>Q</math> is</p> $u = v - 1$	M1  A1  A1   m1 A1  (M1) (A1)  (A1)   (m1) (A1)  [5]  <b>Total</b> <b>[10]</b>	<p>FT their <math>u</math> for method marks only</p> <p>cao oe</p> <p>FT their <math>u</math> for method marks only</p> <p>cao oe</p>

Qu	Solution	Mark	Notes
5.	<p><b>METHOD 1:</b></p> $\sum_{r=k}^{76}(r-31) = \sum_{r=1}^{76}(r-31) - \sum_{r=1}^{k-1}(r-31)$ $= \left[ \left( \frac{1}{2} \times 76 \times 77 \right) - (31 \times 76) \right]$ $- \left[ \left( \frac{1}{2} \times (k-1) \times k \right) - (31 \times (k-1)) \right]$ <p><b>METHOD 2:</b></p> $\sum_{r=k}^{76}(r-31) = \sum_{r=1}^{76}(r-31) - \sum_{r=1}^{k-1}(r-31)$ $\sum_{r=1}^n(r-31) = \sum_{r=1}^n r - \sum_{r=1}^n 31$ $= \frac{n(n+1)}{2} - 31n$ $= \frac{n(n-61)}{2}$ $\sum_{r=k}^{76}(r-31) = \frac{76 \times 15}{2} - \frac{(k-1)(k-62)}{2}$ <p><b>THEN</b></p> <p>If <math>\sum_{r=k}^{76}(r-31) = 980</math>,</p> $570 - \left[ \left( \frac{1}{2} \times (k-1) \times k \right) - (31 \times (k-1)) \right] = 980$ $570 - \frac{k^2}{2} + \frac{63k}{2} - 31 = 980$ $\frac{k^2}{2} - \frac{63k}{2} + 441 = 0$ <p>Solving,</p> $k^2 - 63k + 882 = 0$ $(k-21)(k-42) = 0$ $k = 21 \text{ or } k = 42$ <p><math>\therefore k</math> has two possible values.</p> <p>If m0A0, then SC1 for:</p> <p>Discriminant = <math>(-63)^2 - (4 \times 1 \times 882)</math></p> <p>Discriminant = <math>441 &gt; 0</math></p> <p><math>\therefore k</math> has two possible values</p> <p>Note: Trial and improvement, 0 marks</p>	<p>M1</p> <p>m1</p> <p>A1</p> <p>(M1)</p> <p>(m1)</p> <p>(A1)</p> <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p><b>Total</b> <b>[7]</b></p>	<p>Condone <math>\sum_{r=1}^{76} \square - \sum_{r=1}^k \square</math></p> <p>Use of <math>\sum \square</math> formulae All correct</p> <p>Condone <math>\sum_{r=1}^{76} \square - \sum_{r=1}^k \square</math></p> <p>Use of <math>\sum \square</math> formulae</p> <p>All correct</p> <p>FT their expression provided at least quadratic</p> <p>cao, oe</p> <p>m0 no working FT their quadratic for m1</p> <p>cao</p>



Qu	Solution	Mark	Notes
6. a)	<p>METHOD 1:</p> <p><math>L_1</math></p> $ z - 2 + i  =  z + 2 - 3i $ $ x + iy - 2 + i  =  x + iy + 2 - 3i $ $ (x - 2) + i(y + 1)  =  (x + 2) + i(y - 3) $ $(x - 2)^2 + (y + 1)^2 = (x + 2)^2 + (y - 3)^2$ $x^2 - 4x + 4 + y^2 + 2y + 1 = x^2 + 4x + 4 + y^2 - 6y + 9$ $-8x + 8y - 8 = 0$ $y = x + 1$ <p><math>L_2</math></p> $ z - 2 + i  = \sqrt{10}$ $ x + iy - 2 + i  = \sqrt{10}$ $ (x - 2) + i(y + 1)  = \sqrt{10}$ $(x - 2)^2 + (y + 1)^2 = 10$ $x^2 - 4x + 4 + y^2 + 2y + 1 = 10$ $x^2 + y^2 - 4x + 2y - 5 = 0$ <p>Substituting from <math>L_1</math> into <math>L_2</math>:</p> $x^2 + (x + 1)^2 - 4x + 2(x + 1) - 5 = 0$ $2x^2 - 2 = 0$ $x^2 = 1$ $\therefore x = 1 \quad \text{or} \quad x = -1$ <p>OR</p> $(y - 1)^2 + y^2 - 4(y - 1) + 2y - 5 = 0$ $2y^2 - 4y = 0$ $2y(y - 2) = 0$ $\therefore y = 0 \quad \text{or} \quad y = 2$ <p>Therefore points of intersection are (1,2) and (-1,0)</p>	<p>M1 A1 m1  A1    A1  m1  m1 A1   B1</p>	<p>si si  oe   Award M1A1m1 here if not awarded for <math>L_1</math>  FT their equation of a line Solving equation cao  Award m1A1 for <math>y</math> if not awarded for <math>x</math>  Must be paired correctly FT their <math>x</math> or <math>y</math> into their equation of a line provided real</p>

Qu	Solution	Mark	Notes
6. a)	<p>METHOD 2:</p> <p><math>L_2</math></p> $ z - 2 + i  = \sqrt{10}$ $ x + iy - 2 + i  = \sqrt{10}$ $ (x - 2) + i(y + 1)  = \sqrt{10}$ $(x - 2)^2 + (y + 1)^2 = 10$ $x^2 - 4x + 4 + y^2 + 2y + 1 = 10$ $x^2 + y^2 - 4x + 2y - 5 = 0$ <p>Equating <math>L_1</math> to <math>\sqrt{10}</math>:</p> $ z + 2 - 3i  = \sqrt{10}$ $ x + iy + 2 - 3i  = \sqrt{10}$ $ (x + 2) + i(y - 3)  = \sqrt{10}$ $(x + 2)^2 + (y - 3)^2 = 10$ $x^2 + 4x + 4 + y^2 - 6y + 9 = 10$ $x^2 + y^2 + 4x - 6y + 3 = 0$ <p>Finding the equation of <math>L_1</math> and substituting into equation of circle:</p> $(x^2 + y^2 + 4x - 6y + 3) - (x^2 + y^2 - 4x + 2y - 5) = 0 - 0$ $8x - 8y + 8 = 0$ $y = x + 1$ $x^2 + (x + 1)^2 - 4x + 2(x + 1) - 5 = 0$ $2x^2 - 2 = 0$ $\therefore x = 1 \quad \text{or} \quad x = -1$ <p>OR</p> $(y - 1)^2 + y^2 - 4(y - 1) + 2y - 5 = 0$ $2y^2 - 4y = 0$ $2y(y - 2) = 0$ $\therefore y = 0 \quad \text{or} \quad y = 2$ <p>OR</p> $x^2 + (x + 1)^2 + 4x - 6(x + 1) + 3 = 0$ $2x^2 - 2 = 0$ $\therefore x = 1 \quad \text{or} \quad x = -1$ <p>OR</p> $(y - 1)^2 + y^2 + 4(y - 1) - 6y + 3 = 0$ $2y^2 - 4y = 0$ $2y(y - 2) = 0$ $\therefore y = 0 \quad \text{or} \quad y = 2$ <p>Therefore points of intersection are (1,2) and (-1,0)</p>	<p>(M1)</p> <p>(A1)</p> <p>(m1)</p> <p>(A1)</p> <p>(A1)</p> <p>(m1)</p> <p>(m1)</p> <p>(A1)</p> <p>(B1)</p>	<p>si</p> <p>si</p> <p>Award M1A1m1 here if not awarded for <math>L_2</math></p> <p>oe</p> <p>Solving to find <math>x</math> or <math>y</math></p> <p>Award m1A1 for any of these 4 options of solutions</p> <p>Must be paired correctly FT their <math>x</math> or <math>y</math> into their equation of a line provided real</p>

Qu	Solution	Mark	Notes
6. a)	<p>METHOD 3: (Geometric argument)</p> <p><math>L_1</math> is perp bisector of line of (2, -1) and (-2, 3)</p> $m = \frac{3-(-1)}{-2-2} = \frac{4}{-4} = -1 \rightarrow m(\text{perp}) = 1$ <p>Midpoint: (0,1)</p> <p>Equation of <math>L_1</math>: <math>y = x + 1</math></p> <p><math>L_2</math> is a circle centred at (2, -1) with radius <math>\sqrt{10}</math>, such that <math>(x - 2)^2 + (y + 1)^2 = 10</math> leading to <math>(x - 2)^2 + (x + 2)^2 = 10</math> <math>2x^2 + 8 = 10</math> <math>x^2 = 1</math> <math>\therefore x = \pm 1</math></p> <p>When <math>x = 1, y = 2</math> and when <math>x = -1, y = 0</math></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>[9]</p>	<p>oe</p> <p>or <math>y = 2, 0</math></p> <p>FT their <math>x</math> provided real If B0B0, award SC1 for incorrectly paired coordinates</p>
b)		<p>G1</p> <p>G1</p> <p>G1</p> <p>[3]</p> <p><b>Total</b> <b>[12]</b></p>	<p>Circle, centred in 4th quadrant, extending into all 3 other quads</p> <p>Line, positive gradient, intersecting circle twice FT their line</p> <p>Fully correct, with points of intersection clearly labelled</p>

Qu	Solution	Mark	Notes
7.	<p>When <math>n = 1</math>, <math>13^1 + 8 = 21</math> which is a multiple of 7. Therefore, proposition is true for <math>n = 1</math>.</p> <p>Assume the proposition is true for <math>n = k</math></p> <p>i.e. <math>13^{2k-1} + 8</math> is a multiple of 7 or <math>13^{2k-1} + 8 = 7N</math></p> <p>Consider <math>n = k + 1</math></p> $  \begin{aligned}  13^{2(k+1)-1} + 8 &= 13^{2k-1+2} + 8 \\  &= 13^2(13^{2k-1}) + 8 \\  &= 169(7N - 8) + 8 \\  &= 1183N - 1344 \\  &= 7(169N - 192)  \end{aligned}  $ <p>Since this is a multiple of 7, <math>13^{2(k+1)-1} + 8</math> is also a multiple of 7.</p> <p>So, if the proposition is true for <math>n = k</math>, it is also true for <math>n = k + 1</math>.</p> <p>Since we have shown it is true for <math>n = 1</math>, by mathematical induction, it is true for all positive integers <math>n</math>.</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p><b>Total</b> <b>[7]</b></p>	<p><math>169(13^{2k-1} + 8) - 1344</math></p> <p>cs0</p>

Qu	Solution	Mark	Notes
8. a)	<p>METHOD 1:</p> <p>When <math>\cos \theta = 0.8</math> and <math>\theta</math> acute, <math>\sin \theta = 0.6</math></p> <p>Rotation matrix: <math>\begin{bmatrix} 0.8 &amp; -0.6 \\ 0.6 &amp; 0.8 \end{bmatrix}</math></p> <p><math>\begin{bmatrix} 0.8 &amp; -0.6 \\ 0.6 &amp; 0.8 \end{bmatrix} [\text{Reflection}] = \frac{1}{85} \begin{bmatrix} -84 &amp; -13 \\ -13 &amp; 84 \end{bmatrix}</math></p> <p><math>[\text{Reflection}] = \frac{1}{85} \begin{bmatrix} 0.8 &amp; -0.6 \\ 0.6 &amp; 0.8 \end{bmatrix}^{-1} \begin{bmatrix} -84 &amp; -13 \\ -13 &amp; 84 \end{bmatrix}</math></p> <p><math>\begin{bmatrix} 0.8 &amp; -0.6 \\ 0.6 &amp; 0.8 \end{bmatrix}^{-1} = \frac{1}{1} \begin{bmatrix} 0.8 &amp; 0.6 \\ -0.6 &amp; 0.8 \end{bmatrix}</math></p> <p><math>[\text{Reflection}] = \frac{1}{85} \begin{bmatrix} 0.8 &amp; 0.6 \\ -0.6 &amp; 0.8 \end{bmatrix} \begin{bmatrix} -84 &amp; -13 \\ -13 &amp; 84 \end{bmatrix}</math></p> <p><math>[\text{Reflection}] = \begin{bmatrix} -\frac{15}{17} &amp; \frac{8}{17} \\ \frac{8}{17} &amp; \frac{15}{17} \end{bmatrix}</math></p> <p>As <math>[\text{Reflection}] = \begin{bmatrix} \cos 2\alpha &amp; \sin 2\alpha \\ \sin 2\alpha &amp; -\cos 2\alpha \end{bmatrix}</math></p> <p><math>\cos 2\alpha = -\frac{15}{17}</math> AND <math>\sin 2\alpha = \frac{8}{17}</math> (<math>2\alpha</math> is in the second quadrant)</p> <p><math>2\alpha = 151.9(275131 \dots)</math> <math>\alpha = 75.9(6375653 \dots)</math></p> <p><math>y = (\tan 75.96375653 \dots)x = 4x</math> Therefore, <math>k = 4</math></p>	<p>B1</p> <p>M1</p> <p>m1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>B1</p>	<p>si</p> <p>Attempt to use rotation matrix in equation</p> <p>Use of inverse</p> <p>Multiplication</p> <p>cao</p> <p>FT their reflection matrix provided of correct format</p>

Qu	Solution	Mark	Notes
8. a)	<p>METHOD 2: When <math>\cos \theta = 0.8</math> and <math>\theta</math> acute, <math>\sin \theta = 0.6</math></p> <p>Rotation matrix: <math>\begin{bmatrix} 0.8 &amp; -0.6 \\ 0.6 &amp; 0.8 \end{bmatrix}</math></p> <p>Let reflection matrix be <math>\begin{bmatrix} a &amp; b \\ c &amp; d \end{bmatrix}</math></p> $\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{85} \begin{bmatrix} -84 & -13 \\ -13 & 84 \end{bmatrix}$ $0.8a - 0.6c = -\frac{84}{85} \quad \text{and} \quad 0.6a + 0.8c = -\frac{13}{85}$ $0.8b - 0.6d = -\frac{13}{85} \quad \text{and} \quad 0.6b + 0.8d = \frac{84}{85}$ <p>Solving simultaneous equations</p> $a = -\frac{15}{17} \quad b = \frac{8}{17} \quad c = \frac{8}{17} \quad d = \frac{15}{17}$ $[\text{Reflection}] = \begin{bmatrix} -\frac{15}{17} & \frac{8}{17} \\ \frac{8}{17} & \frac{15}{17} \end{bmatrix}$ <p>As <math>[\text{Reflection}] = \begin{bmatrix} \cos 2\alpha &amp; \sin 2\alpha \\ \sin 2\alpha &amp; -\cos 2\alpha \end{bmatrix}</math></p> $\cos 2\alpha = -\frac{15}{17} \quad \text{AND} \quad \sin 2\alpha = \frac{8}{17}$ <p>(<math>2\alpha</math> is in the second quadrant)</p> $2\alpha = 151.9(275131 \dots)$ $\alpha = 75.9(6375653 \dots)$ $y = (\tan 75.96375653 \dots)x = 4x$ <p>Therefore, <math>k = 4</math></p>	<p>(B1)</p> <p>(M1)</p> <p>(m1)</p> <p>(A1)</p> <p>(m1)</p> <p>(A1)</p> <p>(B1)</p> <p>(B1)</p> <p>(B1)</p>	<p>si</p> <p>Attempt to form simultaneous eqn All correct</p> <p>m0A0 no working</p> <p>cao</p> <p>FT their reflection matrix provided of correct format</p>

Qu	Solution	Mark	Notes
8. a)	<p>METHOD 3:</p> <p>When <math>\cos \theta = 0.8</math> and <math>\theta</math> acute, <math>\sin \theta = 0.6</math></p> <p>Rotation matrix: <math>\begin{bmatrix} 0.8 &amp; -0.6 \\ 0.6 &amp; 0.8 \end{bmatrix}</math></p> <p>Let reflection matrix be <math>\begin{bmatrix} \cos 2\alpha &amp; \sin 2\alpha \\ \sin 2\alpha &amp; -\cos 2\alpha \end{bmatrix}</math></p> $\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix} = \frac{1}{85} \begin{bmatrix} -84 & -13 \\ -13 & 84 \end{bmatrix}$ <p><math>0.8 \cos 2\alpha - 0.6 \sin 2\alpha = -\frac{84}{85}</math> and <math>0.6 \cos 2\alpha + 0.8 \sin 2\alpha = -\frac{13}{85}</math></p> <p>Solving</p> $\cos 2\alpha = -\frac{15}{17}$ <p>As [Reflection] = <math>\begin{bmatrix} \cos 2\alpha &amp; \sin 2\alpha \\ \sin 2\alpha &amp; -\cos 2\alpha \end{bmatrix}</math>  <math>\cos 2\alpha = -\frac{15}{17}</math> AND <math>\sin 2\alpha = \frac{8}{17}</math>  <math>(2\alpha \text{ is in the second quadrant})</math></p> <p><math>2\alpha = 151.9(275131 \dots)</math>  <math>\alpha = 75.9(6375653 \dots)</math></p> <p><math>y = (\tan 75.96375653 \dots)x = 4x</math>  Therefore, <math>k = 4</math></p>	<p>(B1)</p> <p>(M1)</p> <p>(m1)</p> <p>(A1)</p> <p>(m1)</p> <p>(A1)</p> <p>(B1)</p> <p>(B1)</p> <p>(B1)</p> <p>[9]</p>	<p>si</p> <p>Attempt to form simultaneous eqn All correct</p> <p>m0A0 no working cao Accept <math>\sin 2\alpha = \frac{8}{17}</math></p> <p>FT their <math>\cos 2\alpha</math> or <math>\sin 2\alpha</math></p>
8. b)	<p><math>T = \frac{1}{85} \begin{pmatrix} -84 &amp; -13 \\ -13 &amp; 84 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}</math></p> <p>Giving,  <math>\frac{1}{85}(-84x - 13y) = x</math> or <math>\frac{1}{85}(-13x + 84y) = y</math>  Leading to  <math>169x + 13y = 0</math> or <math>13x + y = 0</math>  Therefore the line of fixed points is <math>13x + y = 0</math>.</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p> <p><b>Total</b> <b>[12]</b></p>	

Qu	Solution	Mark	Notes
Qu	Solution	Mark	Notes
9. a)	$\Pi_1: \mathbf{r} \cdot (4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = 5$ , let $\mathbf{n}_1 = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ $\Pi_2: \mathbf{r} \cdot (6\mathbf{i} + \mathbf{j} + \mathbf{k}) = 9$ , let $\mathbf{n}_2 = 6\mathbf{i} + \mathbf{j} + \mathbf{k}$  $\mathbf{n}_1 \cdot \mathbf{n}_2 =  \mathbf{n}_1  \mathbf{n}_2 \cos\theta$  $\cos\theta = \frac{(4 \times 6) + (-3 \times 1) + (2 \times 1)}{\sqrt{4^2 + (-3)^2 + 2^2}\sqrt{6^2 + 1^2 + 1^2}}$ $\cos\theta = \frac{23}{\sqrt{29}\sqrt{38}}$ $\theta = 46.1^\circ$	B1  M1  A1  A1 [4]	Both $\mathbf{n}_1, \mathbf{n}_2$ si  Use of   Mark final answer Accept 0.805 rad
b)	$D = \frac{ (4 \times 5) + (-3 \times -2) + (2 \times -6) - 5 }{\sqrt{4^2 + (-3)^2 + 2^2}}$ $D = \frac{9}{\sqrt{29}} \quad (1.67 \dots)$	M1  A1 [2]	
c) i)	Point $B$ : $(4 \times 5) + (-3 \times 5) + (2 \times 0) = 5$ Point $C$ : $(6 \times 1) + (1 \times 3) + (1 \times 0) = 9$	B1	Both shown convincingly
ii)	Any valid plane equation e.g. $z = 0$ (because $z$ -coordinate in $B, C$ is 0)  $-x + 2y + kz = 5 \quad (\text{where } k \in R)$	B1  [2]  <b>Total</b> <b>[8]</b>	