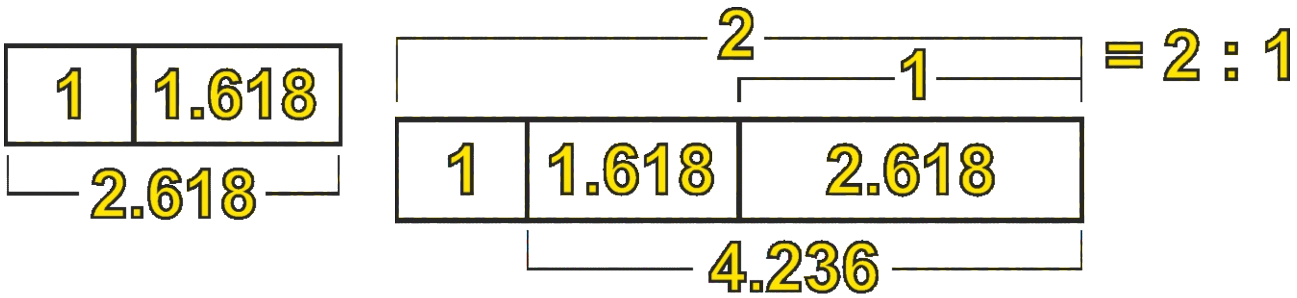
Phionic Intonation

By Cree M-J.

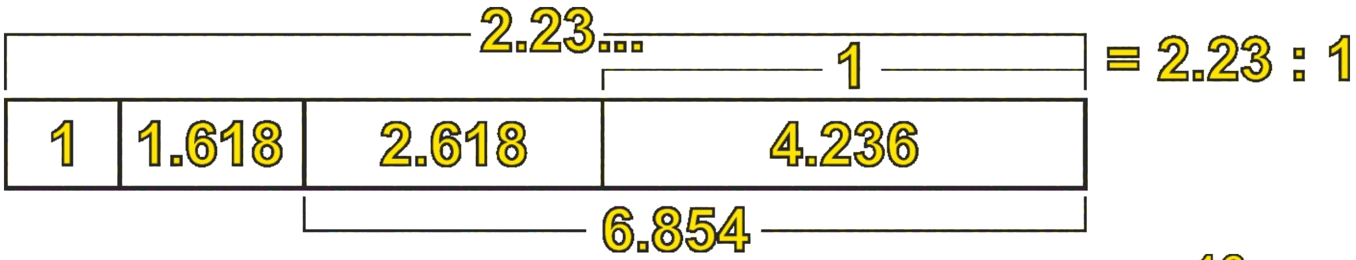
Phionic Geometry

Fundemental Structure

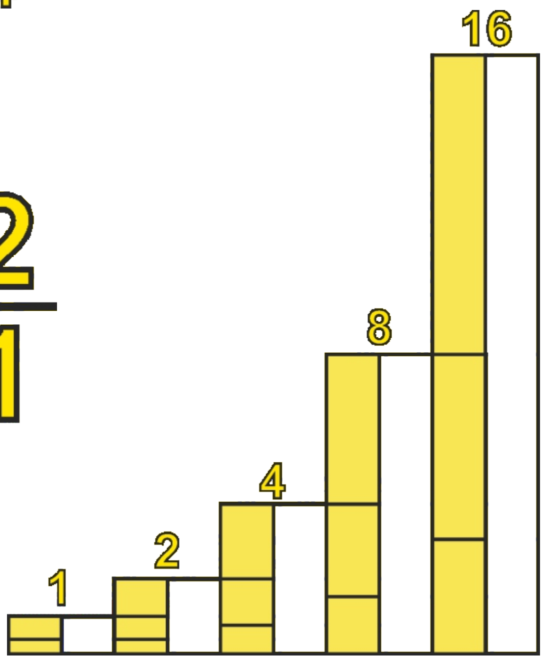
By definition Phi is, the lesser is to the greater, as the greater is to the whole. which also implies the whole of this equation being the next value in our sequence of Phi multiples. In This new case of Phi, The greater is to the whole in the perfect ratio of 2 : 1, the ratio of the octave.



This is the only case in Phi that we find a perfect whole ratio, between Phi^3 to its cumulative whole. If we were to continue the sequence of comparing the progressively greater to its whole, the values would converge at 1 : 2.618 (phi Squared) or inversely at 1 : 0.381... (square root of Phi).



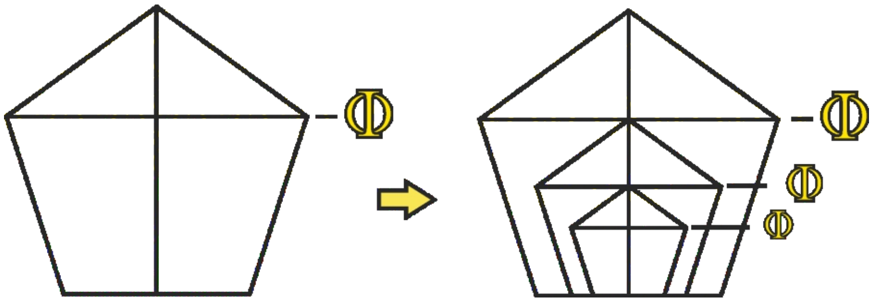
$$\frac{\Phi + \Phi^2 + \Phi^3}{\Phi^3} = \frac{2}{1}$$



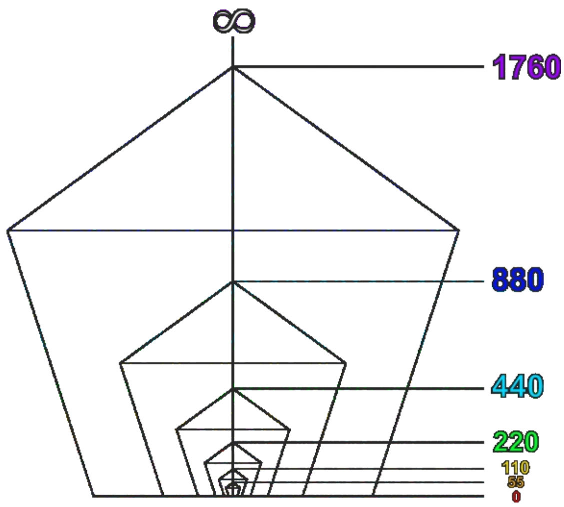
With this in mind we can hypothesize that if we impose Phi ratios over a 2 : 1 octave framework, we will still be creating intervals congruent with Phi and its exponential expressions, namely Phi^3 / Phi^1 + Phi^2 + Phi^3 or vice versa.

Fundemental Structure

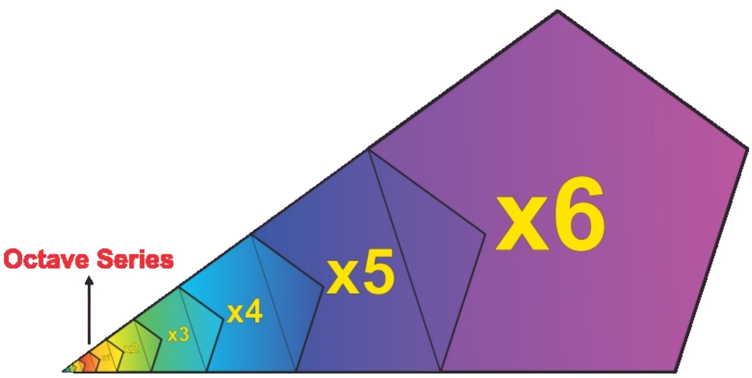
The inbuilt Phi geometries of the pentagon will be a fun and useful guide in visually laying out phi related ratios with the structure as we progress, namely the distance between the apex, the bottom and the point between the outer vertices, which is in the 1 : 1.618 Phi ratio, so we can draw a line there and use this shape as the visual reference.

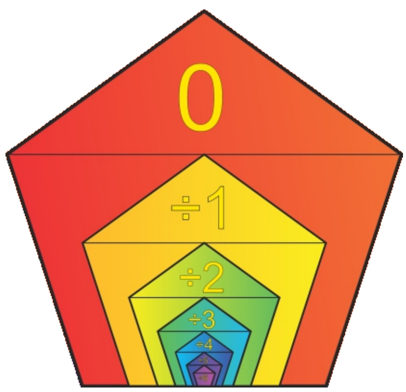


Lets start by establishing our first series of octaves, the exact values are arbitrary but for the sake of comparison we will assign the corresponding octave values of 12 Tone Equal Temperament with A = 440hz being 55, 110, 220,440, 880, 1760 etc..



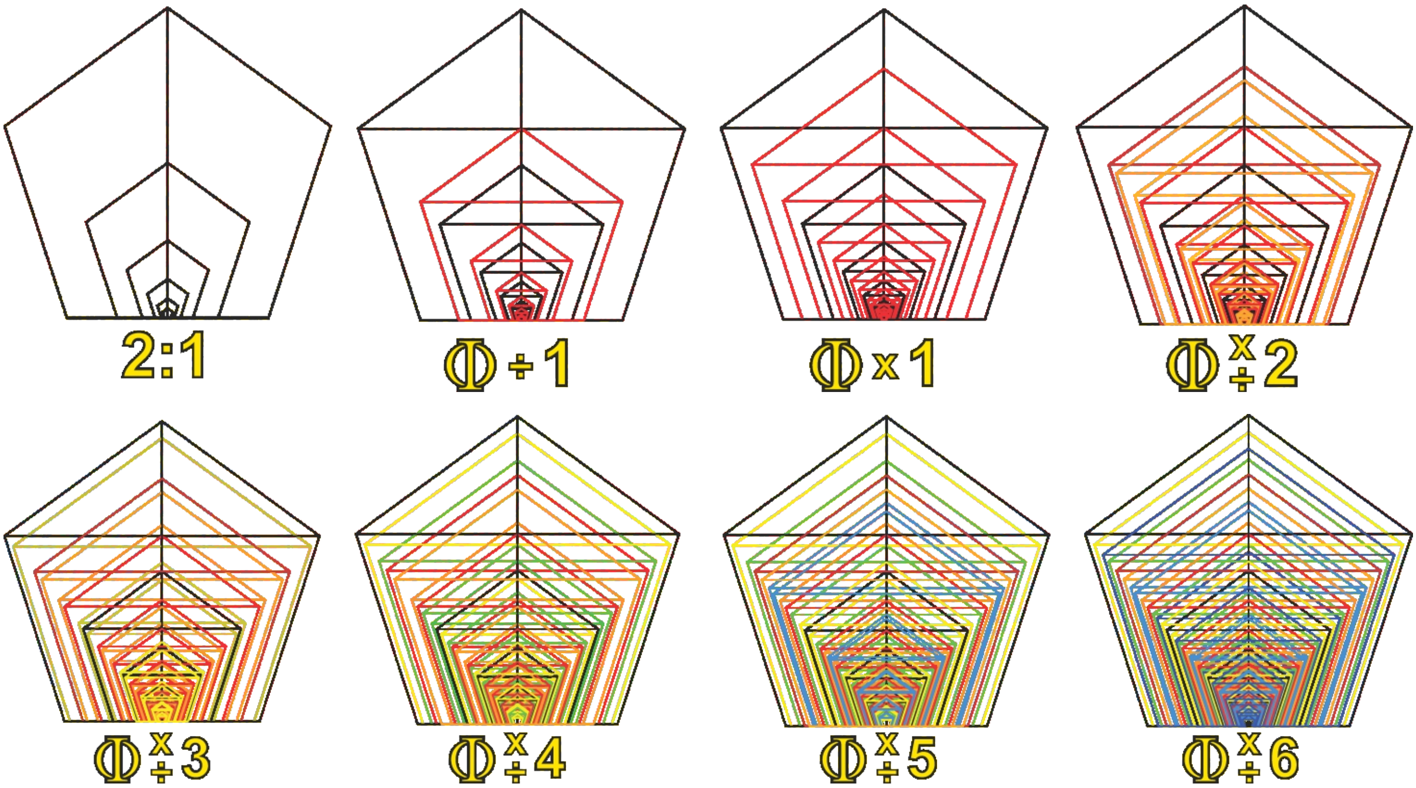
With the foundation established, Next we will simultaneously divide and multiply each of our individual octave positions by Phi . To complete our first basic structure, all we are going to do is continue along this sequence of multiplying and dividing by Phi, off of the original octave and labelling them either Phi times or divided, to indicate its position above or below the starting point.



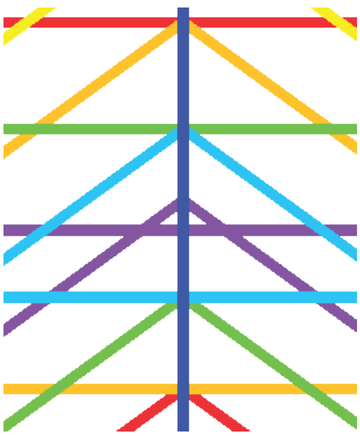


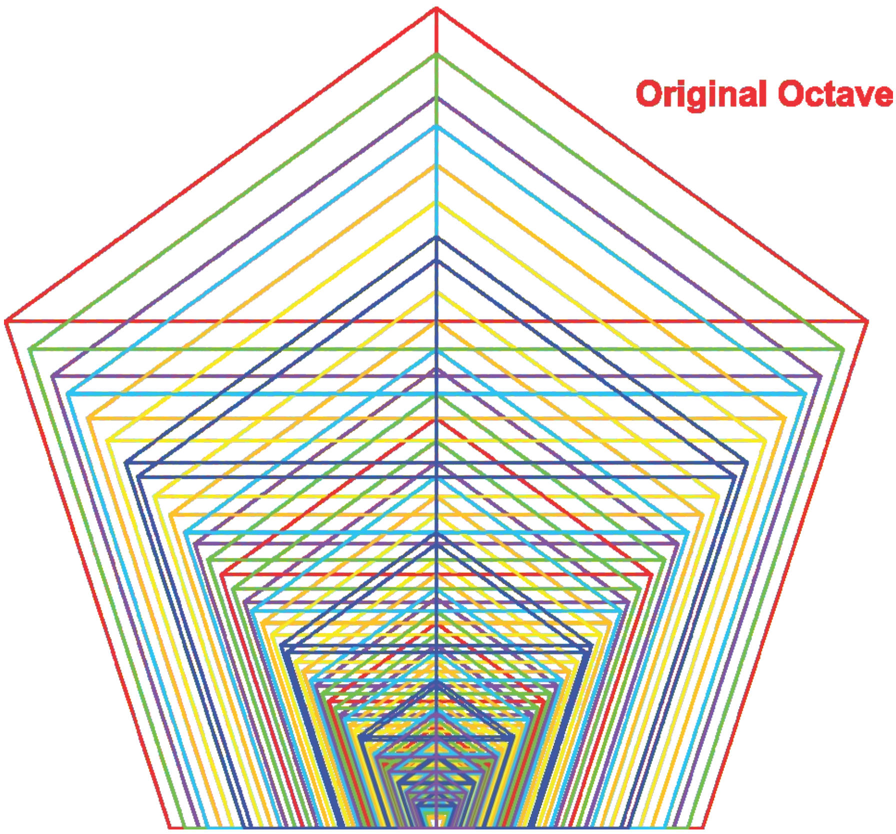
Fundemental Structure

So we create Phi x/÷ 1, Phi x/÷ 2, Phi x/÷ 3, Phi x/÷ 4, Phi x/÷ 5, and then Phi x/÷ 6 off of each of our octave positions.



There are multiple things to point out at this point. Firstly we can see that our octave has been regularly yet unequally divided into 13 intervals, secondly if we observe the colour coding of the different series of Phi, we can see that the overlapping intervals have created a regular pattern even though many of the individual Phi values have extended over from adjacent octaves. This is not our finished structure but lets unpack exactly what is going on here.

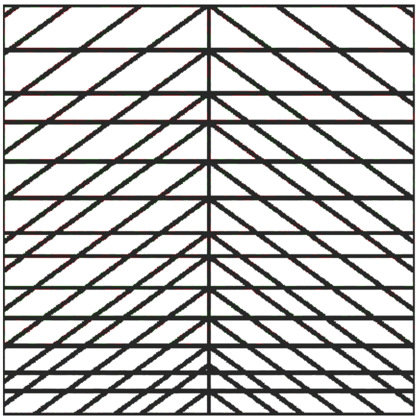




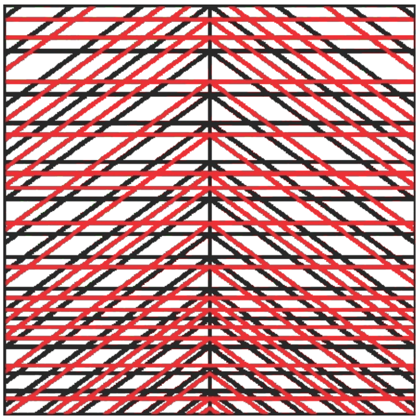
Fundemental Structure

If you pay close attention you will see that all of the lines of geometry are aligned, except the last series of Phi x/÷ 6 Which almost come full circle to meet each other Phi x6 tip to Phi line of Phi÷6 or Phi÷7. If we trace back the sources of these Phi intervals we will find that this meeting point is a convergence spanning 10 octaves and 13 individual steps which are 7 steps of Phi divisions and 6 steps of Phi multiplications, note that the 7th phi division is visibly implied by the pentagonal Phi line of Phi ÷6, we have not made that step yet.

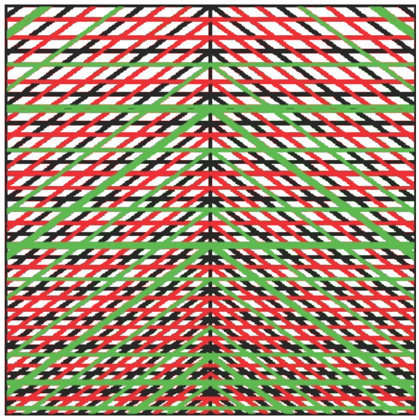
If we were to create another pentagon at this (Phi÷6) Phi line and compare its overall ratio with (Phi+6) we come to a ratio of 1 : 1.01758...etc, which we could also express as Phi^13/512. 512 being related to our octaves as 1 doubled 10 times. While Phi x/÷ 6 or 13 steps could seem like an arbitrary number in which to define the structure, if we were to continue our Phi series beyond 13 or Phi x/÷ 6, we would no longer be creating semi regular intervals, but intervals in the ratio of 1 : 1.01758 off of our original intervals in the same ratio as between Phi x6 and Phi÷7.



Φ x/÷ 6
+ octave



Φ x/÷ 6



Φ x/÷ 7

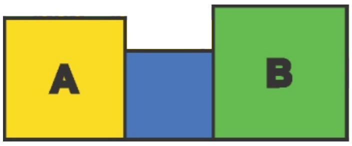
This would continue for another 13 steps, in after which the intervals would be created in the 1 : 1.01758 ratio in the other opposing direction from the original set, giving a 1 : 1.0758 ratio either side. From this we can hypothesize that in this context of the 2:1 structure of simultaneous octave divisions or multiplications, phi has revolved through different cycles of 13 and 3 , and as such is not a completely arbitrary point of reference from a mathematical perspective. Beyond the 3 cycles of 13, the structure would begin to breakdown accordingly into an even smaller series of divisions.

Intervals and Iterations

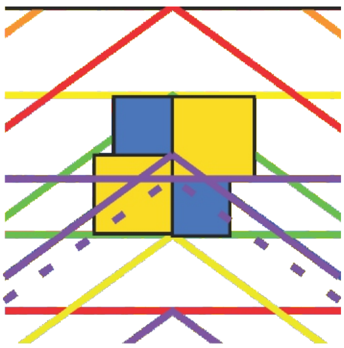
If we now highlight these thirteen intervals, a rather interesting pattern emerges. A block of 4 intervals bridge the octave, and inside we see two blocks of three separated by 3 smaller intervals. The ratio of these larger intervals is 1 : 1.05901 , which is extremely close to the 1 : 1.059463 ratio intervals of equal temperament. The smaller intervals are in the ratio of 1 : 1.0407.

$1 : 1.059 = \Phi^{1/9}$

$1 : 1.0407 = \Phi^{1/12}$

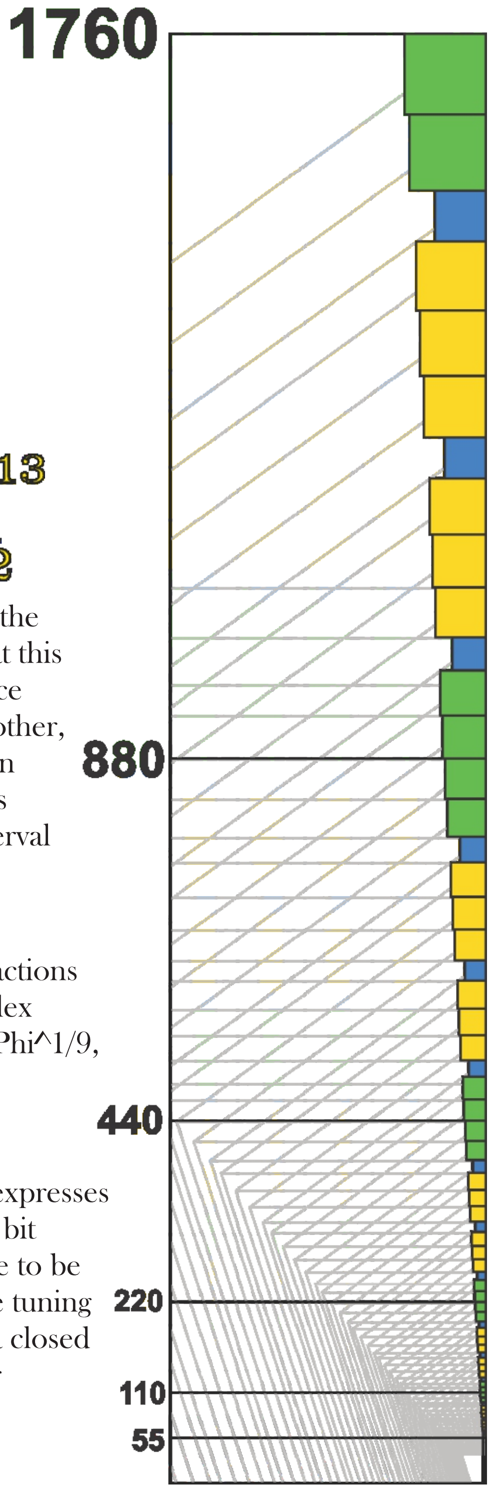


$1 : 1.102 = \Phi^{1/5}$



$1 : 1.01758 = \Phi^{1/28} = \frac{\Phi^{13}}{512}$

By manipulating the intervals between the Phi ÷7 and Phi x6 point we can see that this 1 : 1.01758 ratio is exactly the difference between these ratios being one or the other, almost as if the ratio is functioning as an intermediary overlap for other intervals hiding within the structure and this interval is a bridge in between.

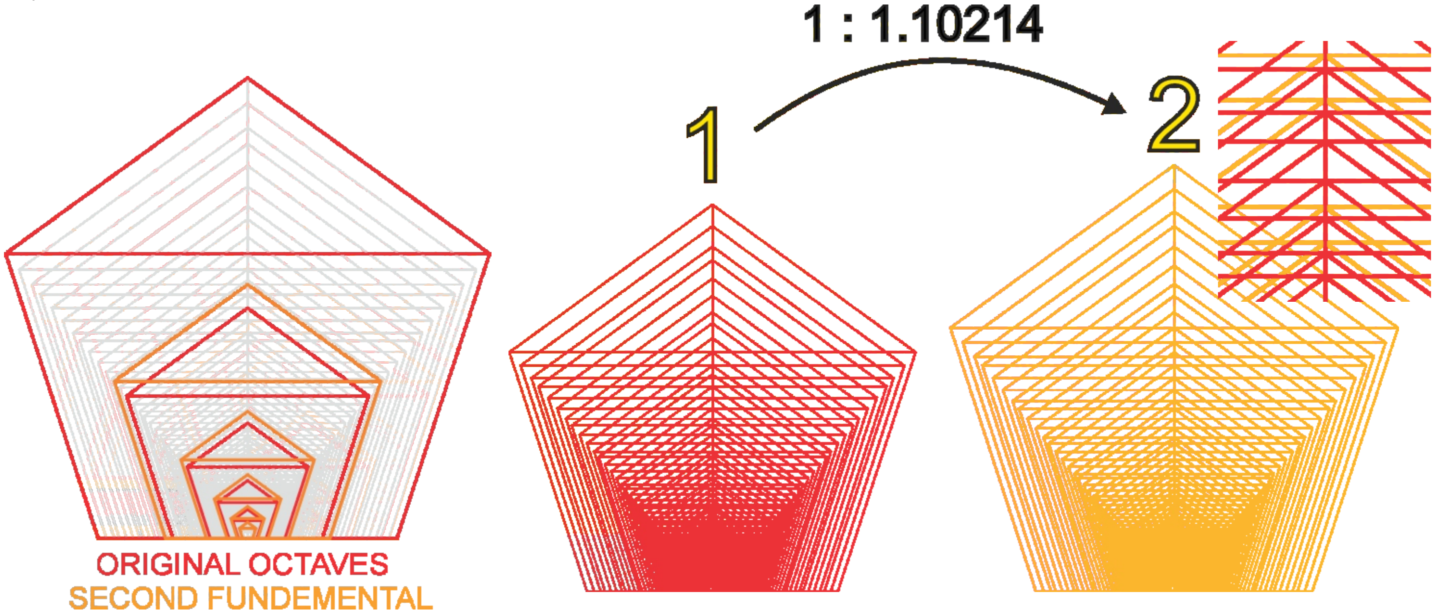


These intervals could also be thought of as corresponding to rough fractions of Phi, although not precisely due to them being the product of complex overlapping. 1 : 1.059 would roughly correspond with approximately Phi^1/9, and 1 : 1.0407 would roughly equal Phi^1/12.

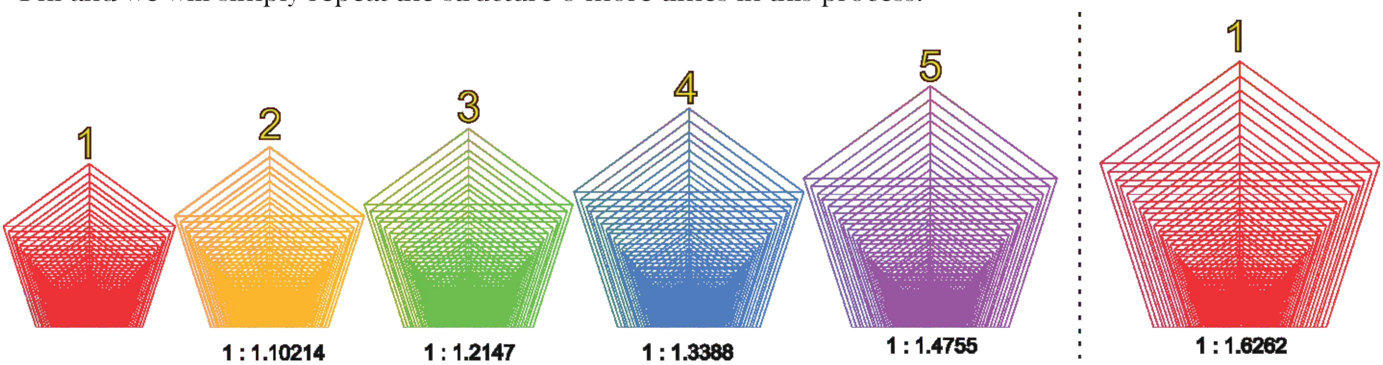
Assigning these values to Hz and playing them will reveal a scale that expresses some harmony and pleasant chords but enough dissonance to seem a bit rough around the edges. To get beyond this point we are going to have to be able to "tune" our scale, and the next step in the process of refining the tuning of the frequencies and understanding Phi, is that Phi does not create a closed system, and rather it spirals infinitely through the spectrum of number relationships.

Intervals and Iterations

To begin we return to the Phi x 6 mark, at the 1 : 1.01758 ratio point, and where we can see the Phi ÷7 mark we create that corresponding interval. Not as an extension to our current structure, but as an entirely new one and we will assign that intervals value and all of its octave counterparts as our second fundamental value. Then we will complete exactly the same process as described to create the first iteration, by multiplying and dividing by Phi off of the octave values until we reach the 6th.



Once that is complete we can see how the new intervals created, relative to the first iteration have created a repeating 2 overlapping, 2 adjacent pattern. The ratio between iteration 1 and iteration 2 is approximately 1 : 1.102141 which corresponds to the ratio between the original octave and Phi ÷7 or also between the two intervals adjacent the smaller interval on our highlighted scale. While this extra set alone can provide a series of alternative values we can use to tune to musically, in this state it seems rather arbitrary and the exact relationships are still not apparent. We can get the hint of how many times we repeat this process by looking to the particular ratio of these scales, 1 : 1.102141 or approximately 1/5 of Phi. So we want to create 5/5 of Phi and we will simply repeat the structure 3 more times in this process.



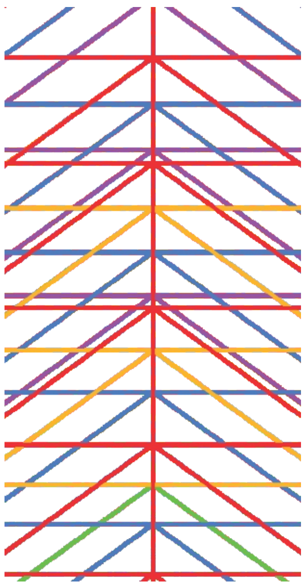
The iterations end before we reach the approximate 1.6262 Phi interval, as we have to include the first set in the equation. 1.6262 which is 1.618 x 1.00507, is a ratio that will make sense later. This 1.6262 interval is actually the beginning of yet another series of 5 iterations or Phi-Cycle. This goes both ways as the beginning of our set would be the rough Phi ratio of a preceding Phi-Cycle. It is worth mentioning the iterations and Phi-Cycles are distinct and do not necessarily bleed into one another arbitrarily, in the sense that if one Phi-Cycle is clearly defined, then the others will fit into place..

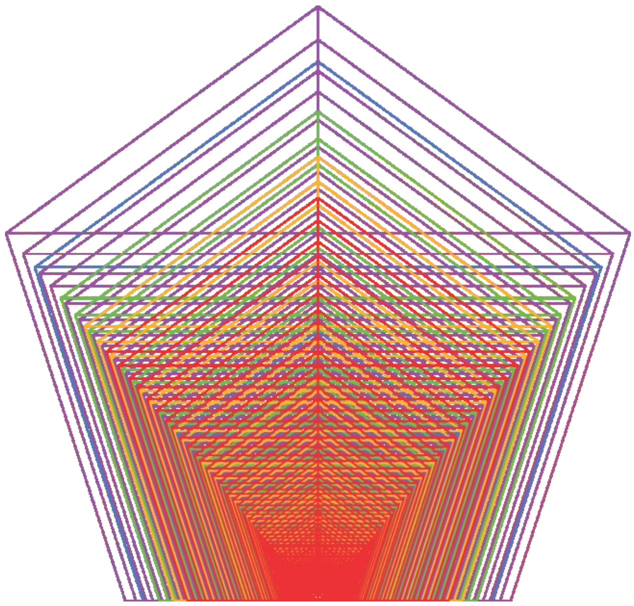
Intervals and Iterations

After we have created the 5 iterations, we now have the basic structure from which we can begin to derive our musical structure, you may consider this Phi-Cycle of 5 iterations the basic "skeleton" of our structure. A few simple supporting proofs at this point with which we can consider this structure mathematically notable and not just abstract interpretation is:

By analysing the close up of the intervals, we can see that, for the same reason as why we stopped our fundamental structure at 6 Phi steps, only through the combination of the first and last iterations between the purple (first) and red (last) lines, have we started to create the first irregular intervals, as Phi seems to "put its foot in the door" of the next series of values, instead of rounding itself up. In fact we have created exactly the same intervals as if we had simply continued multiplying and dividing by phi from our original octaves in 3 cycles of 13 as noted on the "Fundamental structure" page.

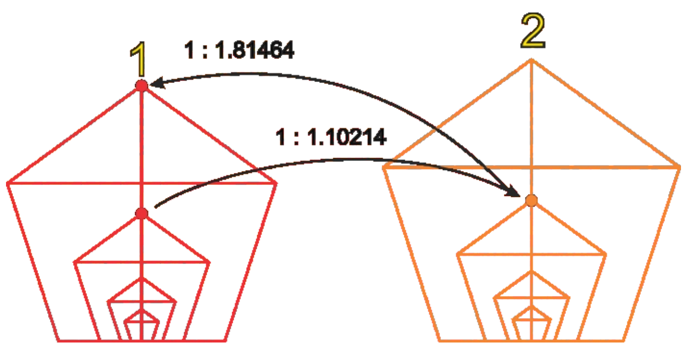
It simply took more iterations to get there by this route because remember only half of each iterations values are unique and half overlap with the previous iteration. One might interpret this as the iterations being woven and interconnected in unique ways, because as we will see there is a distinct advantage to seeing the structure in this context.





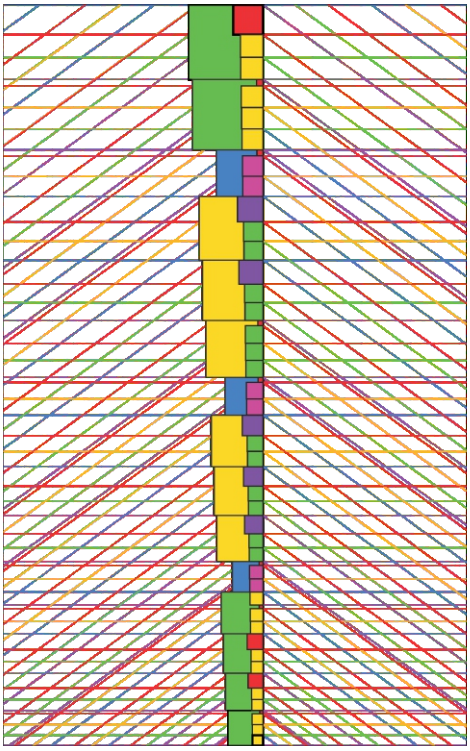
Another point is that if we look at how we build the iterations, we are creating off of the Phi ÷7 mark as the creation point of the next iterations fundamental octave values, approximately 1 : 1.102141 from the original octave. If we switch to the Phi x 7 mark instead and create our fundamental octaves from that point, the resulting octaves will be in the ratio of 1 : 1.81464 to our original octaves.

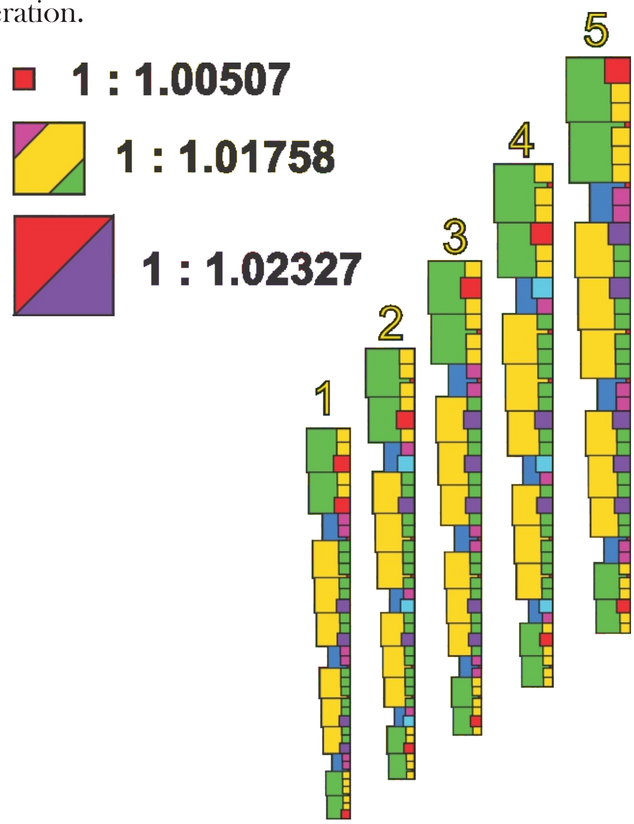
We would find the iteration that results from that, exactly equal to the values of what would be the previous iteration but up one octave. We can identify at what point in the overlapping of phi we get these particular points. The number 1.102141 is approximately equal to 32 / Phi^(7) and the number 1.81464 is approximately equal to Phi^(7) / 16. We can see the correlation of 16 and 32 between the octave series as 5 and 6 octaves. Thus, we can see the balance with Phi ÷7 defining the iterations in one direction and Phi x7 defining the iterations in the other direction.



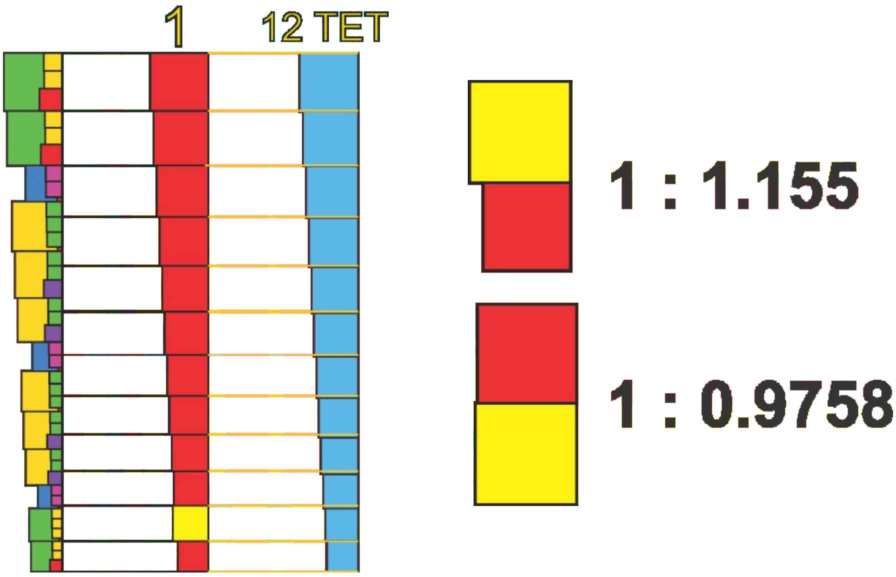
The Connected Whole

By now we have created 5 iterations of our first basic structure, which together are equal to the value that we would get to if we had simply continued multiplying and dividing off of our octaves by phi in 3 cycles of 13. In the following process we can demonstrate how the 5 iterations could be likened to particular Phi-harmonic nodes and points of inter-relation within the total structure. The next step is to overlay the 5 iterations, and highlight the intervals naturally created by each, relative to each iteration. By viewing the base intervals of the iteration relative to the inter-penetrating intervals of the other four iterations over the top, we create a series of sub-intervals from the perspective of each unique iteration.





Now we can see a complex relationship of intervals across the 5 iterations. Though the literal values of each interval remains constant , relative to each set, it has a different framework to work with. Humans have studied music for a long time and come up with many different approaches to music theory, though the 12 tone system has become dominant. The initial aim of this project was to create a marriage of the Phi ratio, musical harmony and consonance without any other preconceived notions, we will see how the Phionic structure naturally resolves into 12 tones as the most apparent configuration. We will solve the puzzle of these new intervals by dividing the octaves into a familiar 12 tone system by lining up intervals in the approximate 1 : 1.059 ratio which we see in the system naturally and we will be constricted to the closest available interval from the structure.

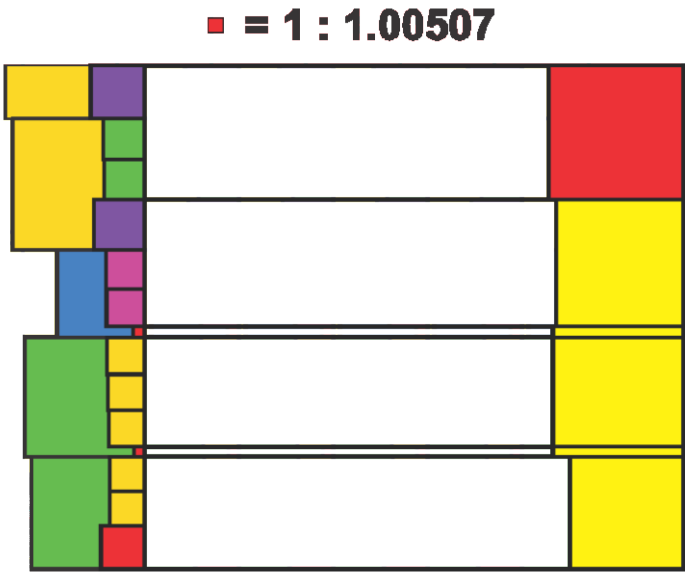


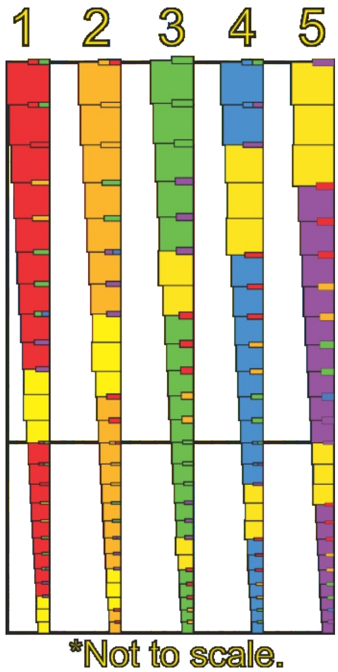
The Connected Whole

We can now see that we have a scale with two different intervals, As our intervals are smaller than the intervals of equal temperament, it does not complete the 12 divisions of the octave, so there is a slightly larger interval. As we will see, this larger interval in yellow is not a unfortunate inconsistency but has mathematical inter-relations and it may be referred to as the "bridge" interval.

If we sweep through the scale and see where we can place this Bridge interval what we will find is that only certain locations on the scale permit it. The particular locations are wherever a new created interval that would be in our scale is adjacent the smallest sub interval 1 : 1.00507. This interval is the result of the first and last iterations overlapping and completing the Phi-Cycle and "spilling over" into the next series of divisions.

After going about this process, we will find that iteration 1 and its sub-intervals have 3 possible locations for the Bridge, all of which are at the bottom of the octave. This effectively extends the possible notes from this scale from 12 to 14. Understanding this process, we can go through the 5 iterations and map out the particular scales that are created.



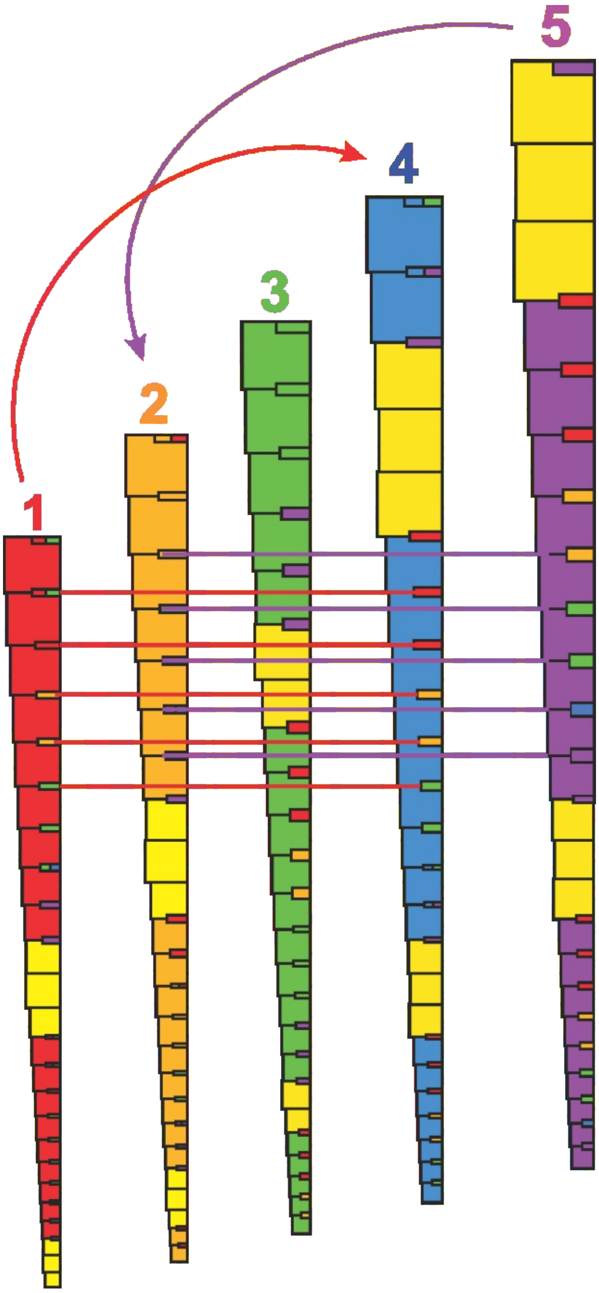


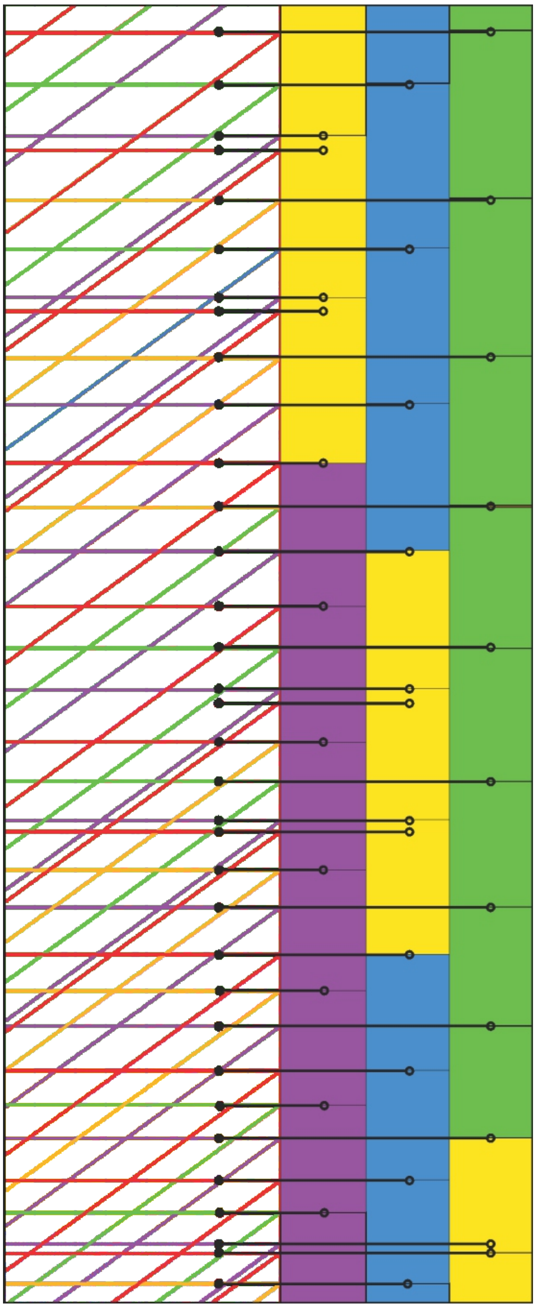
As we can see, the "Bridge" interval clearly makes a sweep across the 5 iterations. By showing which iteration that any given interval was derived from as coloured, we can see that there is even a coherent gradient system through which these intervals themselves are pulled from the iterations and put into the scale coherently, We can observe how the "Bridge" is always defined by values between the first (purple) on top and last (red) iterations on the bottom where the Phi-Cycle comes full circle, and a even gradient through the iterations in between, defining the intervals.

Though one note is that the values within the iterations do overlap, so some intervals would be representative of multiple iterations, hence why there seems to not be as much of some colours, once again the interwoven nature becomes apparent.

The Connected Whole

When we put the scales into their correct ratio and we analyse the resulting relationships, and we can see that there are only 3 truly unique scales, as when in their proper ratios, the first is equal to the fourth and the second is equal to the fifth, giving us a uniquely woven structure. This leaves the central Phionic scale as the most unique and apparently "stable" with only two bridge locations and the cherry on top for this mathematical model, is that if we compare the 3 unique scales that are created, to the five iterations from which they were derived, we can see that every single interval is accounted for and used without overlap in an efficient and wholistic fashion.

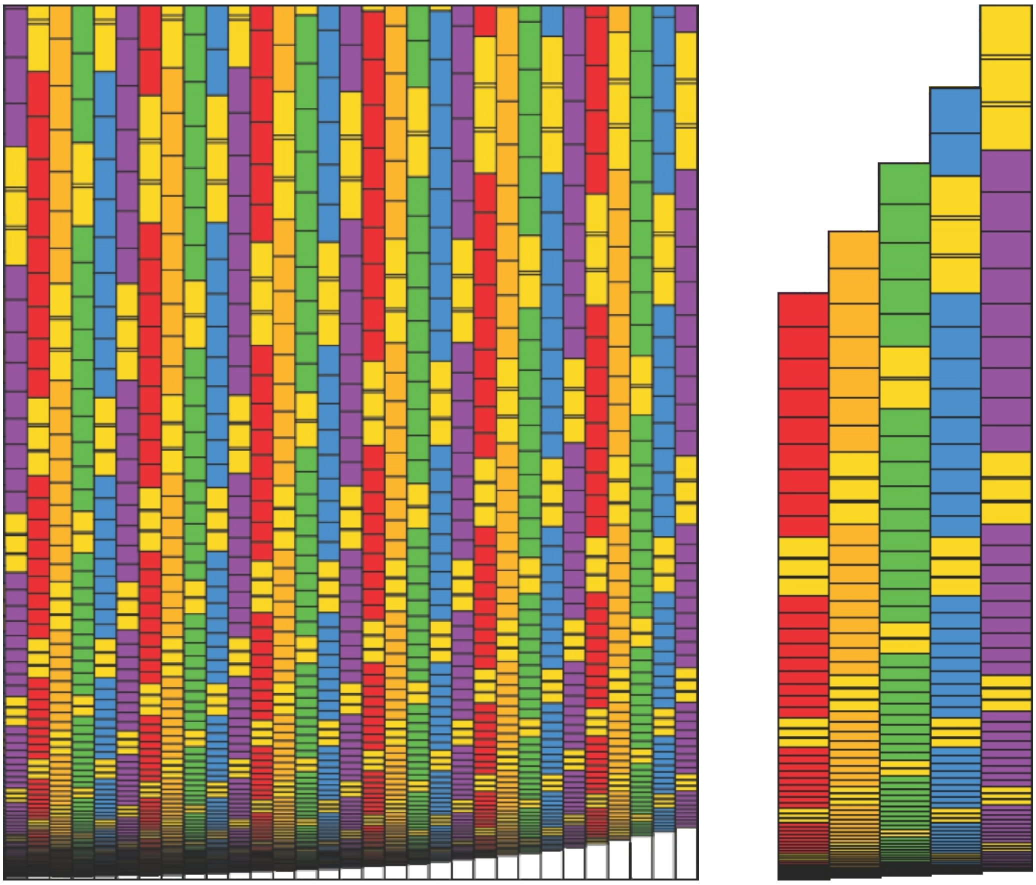




Now that we have identified the basic core of the Phionic series, we can expand our scale and reveal an endless chart of possible Phi-Octave relationships. Intended for musicians so inclined, to explore tuning to the golden ratio with this series as a geometric, poly-scale, microtonal series of intervals.

The Phionic Series

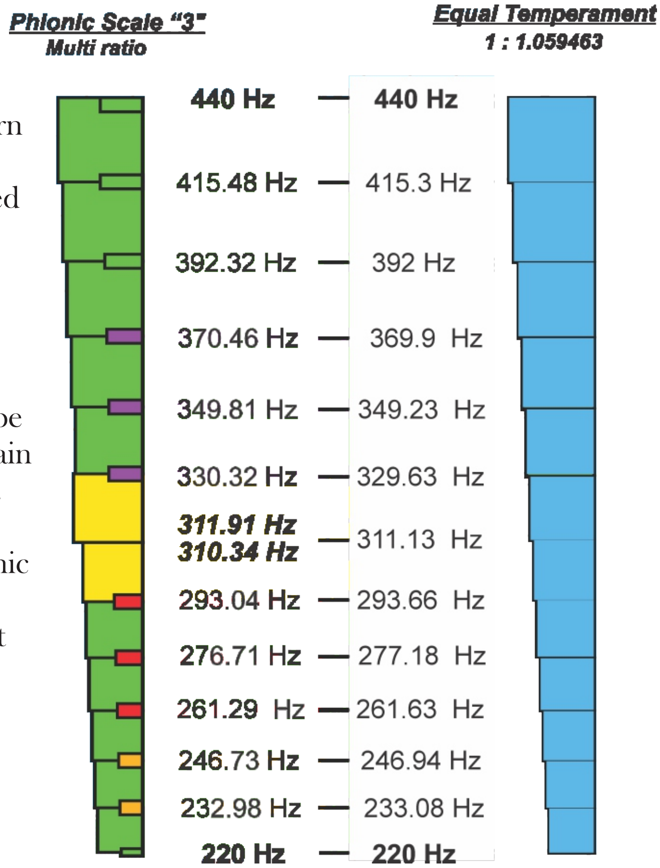
Phionic Intervals



Phionic Intonation and the Phionic series is a natural convergence of the golden ratio and the layout of modern 12 tone temperament. It is a chart of Phi relationships that can be used as the basis for a mathematically aligned tonal tuning system that can satisfy our current concepts of musical harmony and intervals.

The Phionic scales, in comparison to western 12 tone equal temperament, is extremely similar as equivalent intervals will only differ by a small amount and will not be harmonically unfamiliar. Current music theory will remain applicable, though may need to be adjusted accordingly.

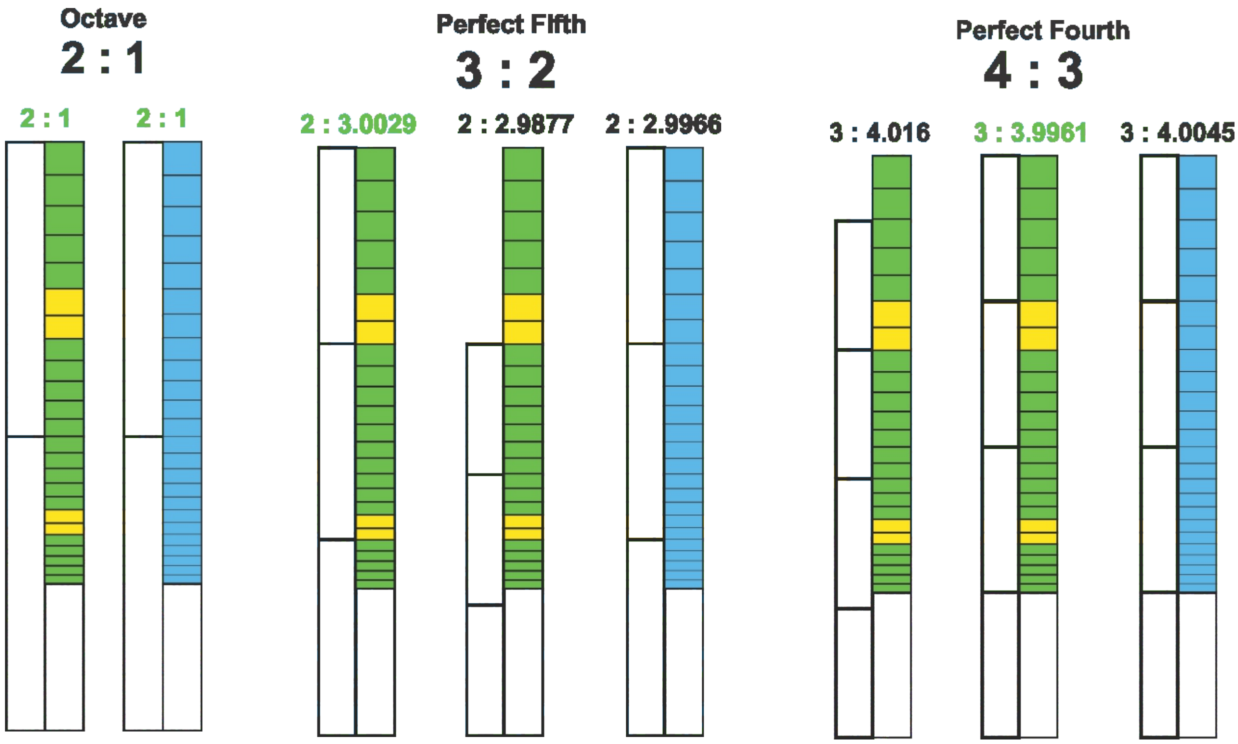
Unlike Equal Temperaments uniform ratios, The Phionic scales are comprised of two different ratios. One ratio is slightly smaller that equal temperaments 1 : 1.059463, at approximately 1 : 1.0591. The Other ratio which is highlighted is approximately 1 : 1.155 and is called the Bridge interval.



The Phionic Series

Phionic Intervals

This Bridge interval is one of the Phionic scales main quirks and charms, it both bridges the underlying geometry of the scales as well as bridging between the various scales of the Phionic Series. Because there are two different ratios, there are two different conditions that determines the exact harmonic ratios of an interval. The condition is whether or not the interval crosses the "Bridge". By laying out the various musical intervals and comparing the Phionic series with Equal Temperament, we can see how the Phionic series has both slightly more and slightly less consonant intervals in all standard cases.



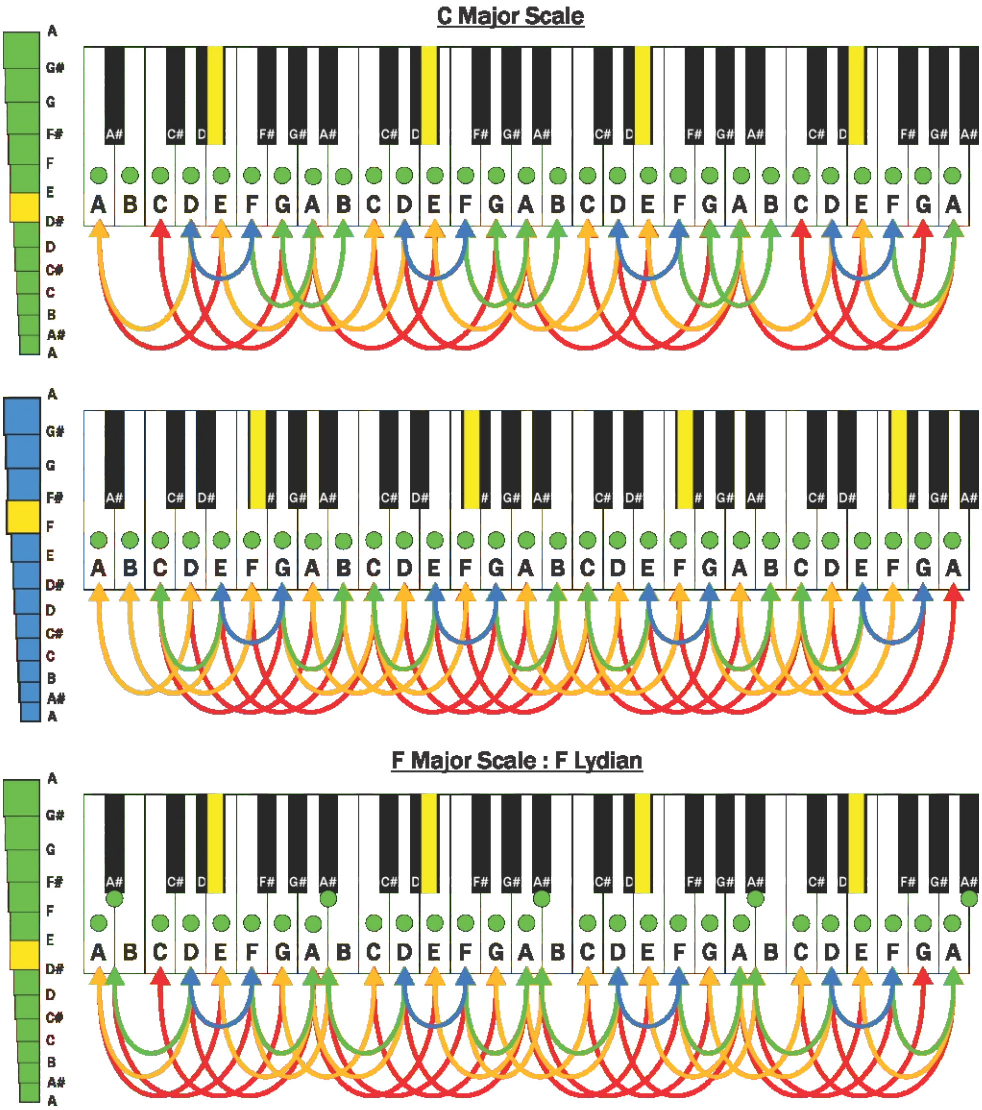
We can interpret the Phionic intervals as having a lot of potential character musically as certain intervals drop slightly more out and slightly more into tune when compared to equal temperament. One could imagine using this creatively, as well as selecting one of the five different Phionic scales as a base would alter the tonal palette of a song, or too optimise the intervals for any given musical key or scale of notes. The position of the "Bridge" interval varies between the scales of one Phi-cycle and we can observe how in one Phi-Cycle, this bridge makes a full sweep of the octave , and relative to the static ratios of Equal Temperament, the Phionic ratios appear to undulate like a wave.

Exactly how one might utilise the scales is up to interpretation and imagination. The regions in yellow designate potential positions for the bridge, and we can decide on either choosing just one for a 12 tone structure, or including all of the bridge positions available for up to 14 notes as a method of a transient bridge interval that can either be crossed or not depending on what is desired. We can also decide to utilise multiple scales at once, but we will discuss this on the next page as it is a broad topic. Choosing a bridge position within the proper regions wont affect the values of the surrounding intervals because it is "baked" into the structure of any given scale.

Phionic Intervals

"Refined" Intervals

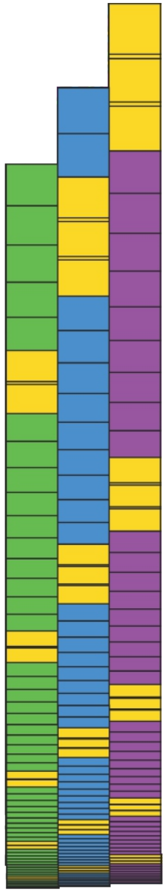
Perfect Fifth	2 : 3.0029	7 Semitones	Bridge
Perfect Fourth	3 : 3.9961	5 Semitones	
Major Third	4 : 5.031	4 Semitones	
Minor Third	5 : 5.968	3 Semitones	Bridge



From the diagrams above we can see how different bridge positions across the octaves of a 12 tone system will have an affect on the intervals of different scales. As far as quantities of refined intervals for a C major scale are concerned, we can see that the bridge between F and F# is more efficient than between D# and E though the same Bridge at D# and E will have improved results for an F Major scale. I would suggest that these refined intervals be structured around the dominant scale of any song or tuning.

The overall differences between the ratios of equal temperament and the Phionic Series are rather small so overall the increased wavering between consonance and dissonance will not be overly noticeable in an obvious sense overall, but if we can increase the consonance for the key scales, notes and moments of a song then we may be able to derive more of a musical and psychological satisfaction. As we tend to play certain scales more often than simply all of the notes, it makes sense for them to have a consonance bias in certain circumstances.

3	4	5
440 Hz	**484.942 Hz**	**534.475 Hz**
415.48 Hz	457.912 Hz	***504.68 Hz***
392.32 Hz	432.402 Hz	***502.141 Hz***
370.46 Hz	***408.305 Hz***	***476.571 Hz***
349.81 Hz	***406.248 Hz***	***474.149 Hz***
330.32 Hz	***385.55 Hz***	447.729 Hz
311.91 Hz	***383.6 Hz***	422.778 Hz
310.34 Hz	362.226 Hz	399.218 Hz
293.04 Hz	342.038 Hz	376.971 Hz
276.71 Hz	322.976 Hz	355.963 Hz
261.29 Hz	304.977 Hz	336.125 Hz
246.73 Hz	287.98 Hz	317.395 Hz
232.98 Hz	271.931 Hz	299.706 Hz
220 Hz	256.775 Hz	283.012 Hz
	242.464 Hz	**267.237 Hz**

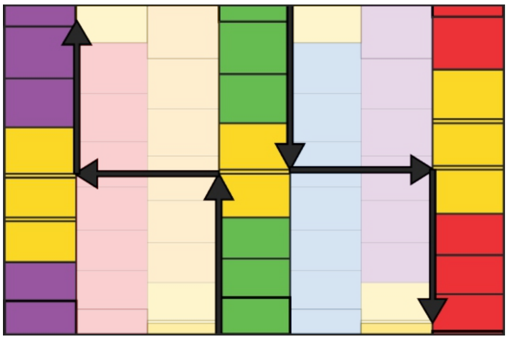


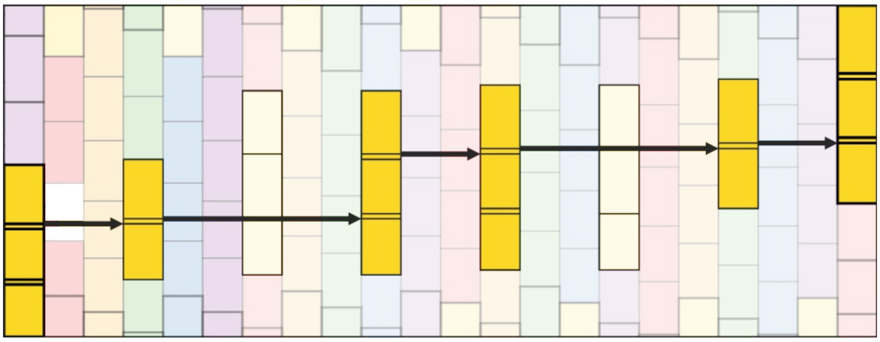
3	5	4
432 Hz	**524.757 Hz**	**476.125 Hz**
407.918 Hz	***495.507 Hz***	449.586 Hz
385.189 Hz	***493.011 Hz***	424.545 Hz
363.723 Hz	***467.906 Hz***	***400.881 Hz***
343.454 Hz	***465.528 Hz***	***398.862 Hz***
324.314 Hz	439.589 Hz	***378.54 Hz***
306.244 Hz	415.091 Hz	***376.63 Hz***
304.694 Hz	391.959 Hz	355.64 Hz
287.714 Hz	370.117 Hz	335.819 Hz
271.681 Hz	349.491 Hz	317.104 Hz
256.544 Hz	330.014 Hz	299.432 Hz
242.234 Hz	311.624 Hz	282.744 Hz
228.744 Hz	294.249 Hz	266.987 Hz
216 Hz	277.867 Hz	252.107 Hz
	262.378 Hz	**238.056 Hz**

Phionic Polytonality & Chorusing

The extended chart of the Phionic series beyond the first "Phi-Cycle" of 5 scales, reveals an interesting pattern of sweeping bridge intervals. We may question exactly what its significance is and how can it be utilised as a polytonal model. In the case of microtonal music, we could experiment utilising two or more different scales to access unique combinations of intervals that may not be accessible with just one scale and even intervals that may be more in tune harmonically across the scales.

Given that all of the scales are geometrically fixed, we can identify between which scales does a specific interval most accurately lie and the conditions that align with it e.g. across or between the bridge. We can also observe that if for some reason we were not to cross the bridge interval in our scale and just keep increasing our ratio by 1 : 1.059, at the point of the bridge we would technically jump across to another scale.





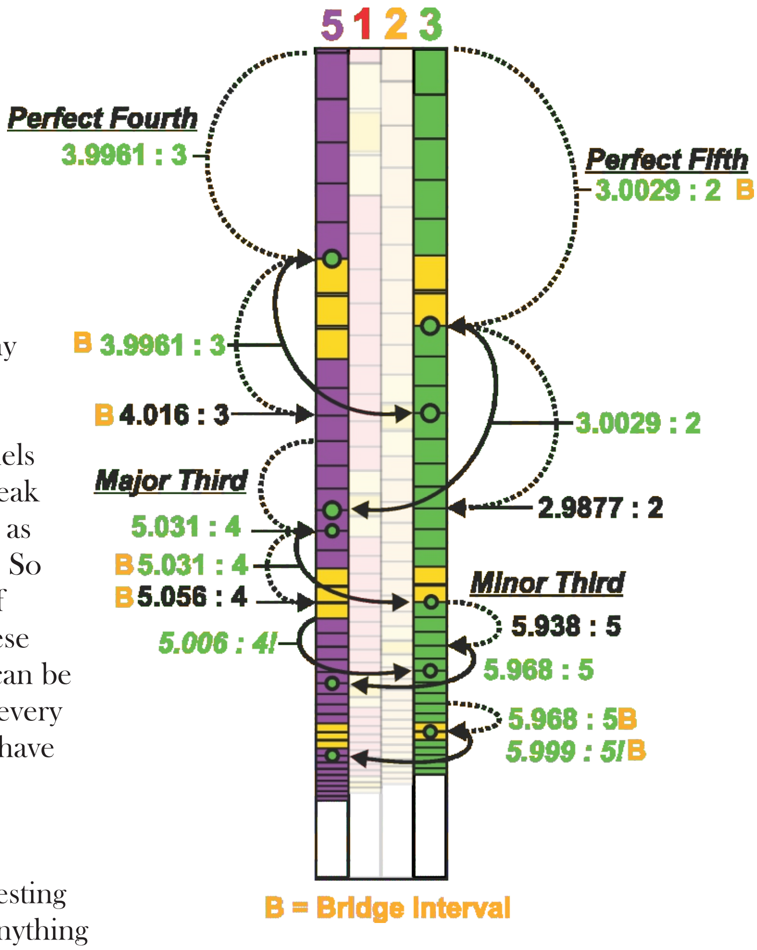
For example if we start on scale three, and as we climb up our scale note by note toward the bridge, as we don't cross the bridge interval we then end up on the scale 5 of the previous Phi-cycle, the inverse is true were if we are going down and do not cross the bridge interval we will end up on scale 1 of the next Phi-cycle.If we analyse the positions of the bridge, we can see that there is also a horizontal ascending pattern as well as the veritcal sweeping curve pattern. We can see how the consecutive bridges in this pattern are aligning to the other bridges along the series. We can see that a bridge occurs every 3 steps across, with 15 steps between the middle scales, crossing the bridge of each different scale before returning back.

One potentially interesting way to utilise the multiple scales of the Phionic series is to take advantage of the bridge interval over multiple scales. Relative to the middle scale 3, the scales 3 steps up or down that start the next Phi cycles are in the overall ratio of 4.016 : 3, close to a perfect fourth between the fundamental octaves. By aligning the closest intervals or observing the bridge alignments we can see that these scales closest intervals are also precisely in the 1 : 1.00507 ratio that is the difference between the bridge interval and the standard interval. By some creative means of utilising two scales or being able to transiently shift between them, we could take advantage of the favourable intervals of the Phionic series. This would only imply a small shift in key and basically a shift of the whole system by a fixed ratio of 1 : 1.00507.

These scales could also be played simultaneously in what would be described as a "chorus" effect, a rich shimmering caused by slightly out of phase frequencies such as found naturally when many voices are singing in unison, due to the slight differences between multiple singer's pitches and overtones. The effect could be described as ethereal, dreamy and nostalgic and contains frequencies and overtones that shimmer. This creates a unique opportunity for acoustic instruments that have multiple strings or courses that are played together such as twelve string guitar, mandolin, lute, dulcimer etc. where if we were to tune the individual strings within the courses of these instruments to the two different scales, we can create a transient tonal centre which implies a situation where we simultaneously have an omnipresent bridge interval and standard interval.

This creates the situation where at any given note we are playing at least one note as in tune as possible within the bridge interval system, as both scales are simultaneously played, exactly which scale is leading harmonically becomes vague and ambiguous. To the effect of creating a situation where either scale takes turns in leading the harmony as the most consonantly resolved interval moves from either both scales simultaneously to one scale at a time

Phionic Polytonality & Chorusing



The frequency differences between the scales are rather small, for example middle c at 261.63hz to its bridge equivalent is 262.95hz, a difference of 1.32Hz.Practical tests acoustically suggest that the harmonics of the fundamental note begin to spread further and further apart, thus we we do hear more of this beating effect in the harmonics, in fact a very pleasant harmonic excitation and complex symphony of beating overtones could be heard in tests.

The phenomenon of acoustic beating has had parallels drawn to the idea of dissonance, though when the peak of a beating note is in tune, we start to call it vibrato, as simply both ideas are just a fluctuation in frequency. So what we get with this overlapping system is a field of tones effectively beating into tune.The pattern of these 1 : 1.00507 scales occurs in an orderly fashion that can be mapped, starting at scale 3 the bridged scales occur every 3 steps, keeping in mind that these other two scales have twin values so if we ignore them then the pattern is (scale 3 - 3 steps- 6 steps -6 steps- back at scale 3.)

Thank you for reading about this project, It is interesting how by applying Phi to the octave, without forcing anything we come to a complex and interlaced structure that fundamentally resolves to a system so close to our current model of equal temperament and very human concept of musicality, but with the unique weave of this "bridge interval".

Through the weaving of this interval expressed through multiple scales at once, it is also very compelling how other scales along the series can complement one another and achieve quite precise musical intervals.

There are definitely more interesting relationships to be found in the series, as any system based on phi will have an abundance of emergent properties especially in an infinite one such as this. It is worth further practical experimentation and exploration.

Other Resolved intervals:

Major Sixth:
5.026 : 3 B
5.0009 : 3

Minor Sixth:
7.99 : 5 B
7.95 : 5

Minor Seventh:
8.96 : 5 B
8.91 : 5

Major Seventh:
15.032 : 8 B
15.032 : 8

Major Second:
9.017 : 8

Minor Second:
15.966 : 15

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