SIG718

Real World Analytics

Final Term Assessment

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Contents

Question 1

A garment factory produces shirts and pants for Kmart chain. The contract is such that quality control is done before shipping and all products supplied to Kmart satisfying the quality requirements would be accepted by the chain. The factory employs 20 workers in the cutting department, 50 workers in the sewing department, and 14 workers in the packaging department. The garment factory works 8 productive hours a day (no idle time during these 8 hours). There is a daily demand for at most 180 shirts. The demand for pants is unlimited. Each worker can participate only in one activity- the activity to which they are assigned. The table below gives the time requirements (in minutes) and profit per unit for the two garments.

	Amount (minutes) per operation			
	Cutting	Sewing	Packaging	Profit per unit($)
Shirts	40	40	20	$ 10.00
Pants	20	100	20	$ 8.00

a) Explain why a Linear Programming (LP) model would be suitable for this case study.

Objective: The garment factory's main objective is to maximize its profits. This goal can be quantified as a linear function of the quantity of shirts and pants produced, taking into account their respective unit profits.

Limitations The factory is constrained by its resources, specifically labor (the workforce in each department) and time (8 productive hours per day). These limitations can be represented as linear inequalities. Each department (cutting, sewing, packaging) has a certain capacity based on the workforce and the time needed for each garment. These capacities form additional linear constraints. The market demand also imposes constraints: there is a daily maximum demand for shirts, while the demand for pants is unlimited. These market constraints can be expressed as linear inequalities as well.

Linearity The problems relationships are linear. For example, if the production of shirts is doubled, both the profit from shirts and the resources required will double, assuming no changes in efficiency or costs. The total time and profit are the sum of the individual times and profits for shirts and pants. The resources consumed and profits made are directly proportional to the number of garments produced.

Divisibility Although the actual garments cannot be divided, the production quantities (number of shirts and pants) can be considered as continuous variables in the model. This allows for fractional solutions, which can be rounded to the nearest whole number for practical purposes.

Ease of Use and Solvability Linear programming (LP) models are relatively simple to formulate and can be efficiently solved using established methods like the Simplex algorithm. This makes them perfect for optimizing production in sectors like garment manufacturing.

Decision Making LP offers a clear structure for decision-making. By solving the LP model, the factory can identify the optimal quantity of shirts and pants to produce to maximize profits while complying with labor, time, and demand constraints.

Conclusion In conclusion, an LP model is appropriate for this case study because it can effectively manage the linear relationships, constraints, and profit maximization objective in the garment factory's production process. It offers a clear, numerical method for making optimal production decisions.

b) Formulate a LP model to help the factory management to determine the optimal daily production schedule that maximises the profit while satisfying all constraints.

Decision Variables:

Let x be the number of shirts produced per day.
Let y be the number of pants produced per day.

Objective Function:

The objective is to maximize profit. The profit per unit is $10 for shirts and $8 for pants. Therefore, the objective function is:

Maximize Z = 10x + 8y

Constraints:
Cutting Department Constraint:

Each shirt requires 40 minutes of cutting, and each pair of pants requires 20 minutes. The cutting department has 20 workers, each working 480 minutes a day.

40x + 20y <= 20×480
40x + 20y <= 9600

Set y = 0 then	set x = 0 then
40x + 20(0) <= 9600 x = 9600 / 40 x = 240, y =0	40(0) + 20y <= 9600 y = 9600 / 20 x = 0, y = 480

Sewing Department Constraint:

Each shirt requires 40 minutes of sewing, and each pair of pants requires 100 minutes. The sewing department has 50 workers, each working 480 minutes a day.

40x + 100y <= 50×480
40x + 100y <= 24000

Set y = 0 then	set x = 0 then
40x + 100(0) <= 24000 x = 24000 / 40 x = 600, y =0	40(0) + 100y <= 24000 y = 24000 / 100 x = 0, y = 240

Packaging Department Constraint:

Each shirt and each pair of pants require 20 minutes of packaging. The packaging department has 14 workers, each working 480 minutes a day.

20x + 20y <= 14×480
20x + 20y <= 6720

Set y = 0 then	set x = 0 then
20x + 20(0) <= 6720 x = 6720 / 20 x = 336, y =0	20(0) + 20y <= 6720 y = 6720 / 20 x = 0, y = 336

Demand Constraint for Shirts:

The daily demand for shirts is at most 180.
x <= 180

Non-Negativity Constraint:
The number of shirts and pants produced cannot be negative.
x >= 0, y >= 0

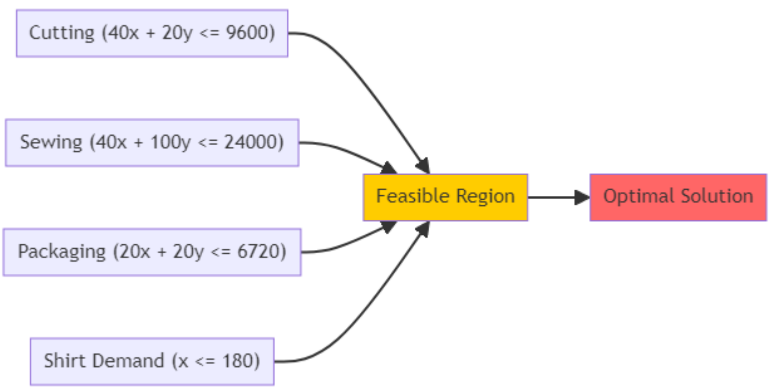
LP Model:		
40x + 20y <= 9600	(Cutting Dept.)	Graph points (x = 240, y =0), (x = 0, y = 480)
40x + 100y <= 24000	(Sewing Dept.)	Graph points (x = 600, y =0), (x = 0, y = 240)
20x + 20y <= 6720	(Packaging Dept.)	Graph points (x = 336, y =0), (x = 0, y = 336)
x <= 180	(Shirt Demand)	Graph points (x <= 180)
x >= 0, y >= 0	(non-negativity)	Graph points (x>=0, y>=0)
Maximize Z = 10x + 8y		
While plotting in graph **shirts (x)** would be plotted in **X-Axis** and **pants (y)** would be plotted in **Y-Axis**		

This LP model can be solved using graph, simplex methods to find the values of Z.

c) Use the graphical method to find the optimal solution. Show the feasible region and the optimal solution on the graph. Annotate all lines on your graph. What is the optimal daily profit for the factory?

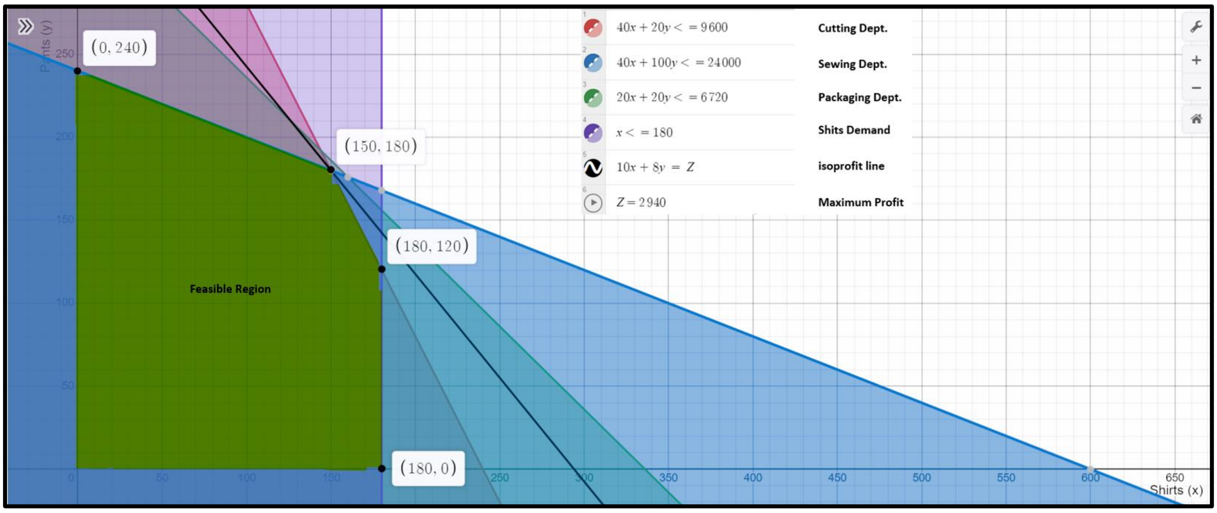
As stated earlier, shirts (x) will be represented on the X-Axis and pants (y) on the Y-Axis. The graphical method for plotting will be carried out using an online https://www.desmos.com/calculator.

Flowchart:



The Optimal Solution is the point within this feasible region that maximizes the profit function is **Maximize Z = 10x + 8y**

Graphical Method: desmos.com



In the graph, shirts (x) are represented on the X-axis and pants (y) on the Y-axis. The feasible region, which is the intersection of all constraints, illustrates all the potential combinations of shirts and pants that the factory can manufacturer within its resource limits. The lines for each constraint are plotted based on the given endpoints, and the feasible region is the overlapping area of all these constraints.

Optimal Solution: Once these constraints are graphically represented, the optimal point (where the objective function reaches its maximum) is identified at (150 shirts, 180 pants). At this juncture, the maximum profit is computed using the objective function: **Z=10×150+8×180=2940**

Regions:

We have totally 8 regions out of which 1 feasible region with 4 feasible points. The same has been indicated in the above graph.

Hence, under these constraints, the factory's optimal daily profit is **$2940**. This solution efficiently balances the constraints imposed by the factory's resources and the market demands.

d) Find the range for the profit ($) per shirt (if any) that can be obtained without affecting the optimal point of part (c).

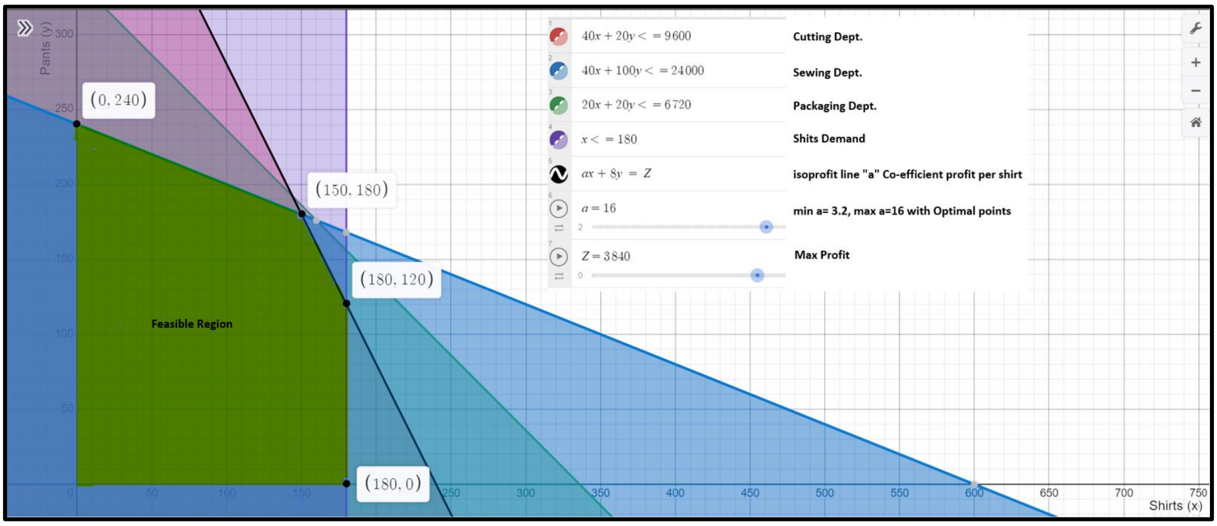
Original Objective Function:

Maximize Z=10x + 8y
Optimal solution: (150 shirts, 180 pants) with a maximum profit of $2940.

Sensitivity analysis in linear programming involves exploring how changes in the coefficients of the objective function like the profit per unit affect the optimal solution. In this case, varying the profit per shirt **originally $10 per shirt** in the objective function **Z=10x+8y** and observing max profit can be obtained without changing the optimal solution.

Modified Objective Function:

Altered the profit per shirt to "**a**" in the objective function, making it Z=**a**x + 8y, where "**a**" represents the new profit per shirt.
Maximize Z=**a**x + 8y



Experimented on the graph model by adjusting the 'a' value from 2 to 20 using a slider for sensitivity analysis in linear programming. The smallest adjustment that can be made to 'a' is 3.2, and the largest is up to 16, which resulted in the maximum profit of $3,840 without impacting the optimal value. This has been corroborated in the Excel solver, and the proof is provided below.

Observations from Desmos.com:

For a=3.2: new objective function is Z=3.2x + 8y & with optimal point (150, 180), the total profit is Z=3.2×150+8×180=1920. → Minimum expected shirt profit value with Least Profit
For a=14: new objective function is Z=14x + 8y & with optimal point (150, 180), the total profit is Z=14×150+8×180=3540.
For a=16: new objective function is Z=16x + 8y & with optimal point (150, 180), the total profit is Z=16×150+8×180=3840. → Maximum expected shirt profit value with maximum Profit

Even with the profit per shirt rising between $10 to $16 with the optimal point (150 shirts, 180 pants). This constant optimal point suggests that within this profit range per shirt ($10 to $16), the production quantities continue to be the most profitable option while meeting all constraints. The solution is robust in the profit per shirt within this range. Choosing a profit of $16 per shirt results in the maximum profit of $3840, without changing the optimal production quantities and still complying with all the factory's production constraints.

Conclusion: The sensitivity analysis shows that the factory can raise the profit per shirt to $16 without changing its production plan of 150 shirts and 180 pants. This analysis is vital for decision-making as it offers a range in which the price of shirts can be varied without impacting the production strategy and while maximizing profit.

Appendix using excel Solver for both optimal point and sensitivity analysis & executive summary:

In addition, the Excel Solver Add-In was utilized to address the problem, and the following conclusions were drawn. The results align with the previous findings regarding both the optimal point and the permissible increase and decrease per shirt.

Solution using xl solver:

	x	y			
Input	150	180			
Constraints			**Used**		**Available**
C1	40	20	9600	<=	9600
C2	40	100	24000	<=	24000
C3	20	20	6600	<=	6720
C4	1	0	150	<=	180
			Total		
profit	10	8	2940		

Sensitivity Analysis:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
J10	Input x	150	0	10	6	6.8
K10	Input y	180	0	8	17	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
M14	C1 Used	9600	0.2125	9600	320	4800
M15	C2 Used	24000	0.0375	24000	960	4800
M16	C3 Used	6600	0	6720	1E+30	120
M17	C4 Used	150	0	180	1E+30	30

Permissible Increase/Decrease: These figures represent the extent to which the objective coefficient can rise or fall before the existing optimal solution alters. For the input shirts (x), the permissible increase is 6, and the permissible decrease is 6.8. This implies that the coefficient can be raised from **10 to 16** or lowered to **3.2** without modifying the structure of the optimal solution.

Question 2

A factory makes three products called Bloom, Amber, and Leaf, from three materials containing Cotton, Wool and Nylon. The following table provides details on the sales price, production cost and purchase cost per ton of products and materials respectively.

	Sales price	Production cost		Purchase price
Bloom	\$60	\$5	Cotton	\$40
Amber	\$55	\$4	Wool	\$45
Leaf	\$60	\$5	Nylon	\$30

The maximal demand (in tons) for each product, the minimum cotton and wool proportion in each product are as follows:

	Demand	min Cotton proportion	min Wool proportion
Bloom	4200	50%	40%
Amber	3200	60%	40%
Leaf	3500	50%	30%

a) Formulate an LP model for the factory that maximises the profit, while satisfying the demand and the cotton and wool proportion constraints.

Hints:

1. Let xij ≥ 0 be a decision variable that denotes the number of tons of products j for j ∈ {1 = Bloom, 2 = Amber, 3 = Leaf to be produced from Materials i ∈ {C=Cotton, W=Wool, N=Nylon}.
2. The proportion of a particular type of Material in a particular type of Product can be calculated as: e.g., the proportion of Cotton in product Bloom is given by: xC1 / (xC1 + xW1 + xN1).

Decision variable:

x(ij) represents

the number of tons of product "j" j ∈ {1 = Bloom, 2 = Amber, 3 = Leaf}
produced from material "i" i ∈ {C = Cotton, W = Wool, N = Nylon}

There are 3 materials and 3 products, so we have a total of 3×3=9 decision variables

x(C1), x(W1), x(N1), x(C2), x(W2), x(N2), x(C3), x(W3), x(N3)

Objective Function:

The objective is to maximize profit which is calculated as the difference between sales revenue and the sum of production and material costs.

Profit = Total Sales Revenue - Total Production Cost - Total Material Cost

Total Sales Revenue:

It is calculated by multiplying the sales price of each product by the total quantity produced from all materials for each product:
For Bloom: 60×(x(C1)+x(W1)+x(N1) + For Amber: 55×(x(C2)+x(W2)+x(N2) + For Leaf: 60×(x(C3)+x(W3)+x(N3)

Total Material Cost:

It is the sum of the costs of all materials used in the production of all products:
For Cotton: 40×(x(C1)+x(C2)+x(C3) + For Wool: 45×(x(W1)+x(W2)+x(W3))+ For Nylon: 30×(x(N1)+x(N2)+x(N3)

Total Production Cost:

It is the sum of the production costs for all products:
For Bloom: 5×(x(C1)+x(W1)+x(N1) + For Amber: 4×(x(C2)+x(W2)+x(N2) + For Leaf: 5×(x(C3)+x(W3)+x(N3)

Hence:
(60×(x(C1)+x(W1)+x(N1) + 55×(x(C2)+x(W2)+x(N2) + 60×(x(C3)+x(W3)+x(N3)) – (40×(x(C1)+x(C2)+x(C3) + 45×(x(W1)+x(W2)+x(W3)) + 30×(x(N1)+x(N2)+x(N3)) – (5×(x(C1)+x(W1)+x(N1) + 4×(x(C2)+x(W2)+x(N2) + 5×(x(C3)+x(W3)+x(N3))

Group like Terms & Calculations:

For x(C1): 60x(C1) - 40x(C1) - 5x(C1) *then* 15x(C1)
For x(W1): 60x(W1) - 45x(W1) - 5x(W1) *then* 10x(W1)
For x(N1): 60x(N1) - 30x(N1) - 5x(N1) *then* 25x(N1)
For x(C2): 55x(C2) - 40x(C2) - 4x(C2) *then* 11x(C2)
For x(W2): 55x(W2) - 45x(W2) - 4x(W2) *then* 6x(W2)
For x(N2): 55x(N2) - 30x(N2) - 4x(N2) *then* 21x(N2)
For x(C3): 60x(C3) - 40x(C3) - 5x(C3) *then* 15x(C3)
For x(W3): 60x(W3) - 45x(W3) - 5x(W3) *then* 10x(W3)
For x(N3): 60x(N3) - 30x(N3) - 5x(N3) *then* 25x(N3)

Final Profit Objective Function:

Maximize Z = 15x(C1) + 10x(W1) + 25x(N1) + 11x(C2) + 6x(W2) + 21x(N2) + 15x(C3) + 10x(W3) + 25x(N3)

Demand Constraint:

Bloom: x(C1) + x(W1) + x(N1) <= 4200
Amber: x(C2) + x(W2) + x(N2) <= 3200
Leaf: x(C3) + x(W3) + x(N3) <= 3500

Material Constraints:

Constraints for Bloom
Cotton Proportion in Bloom: X(C1) / (x(C1) + x(W1) + x(N1)) >= 0.50
Constraint: x(C1) >= 0.50 * (x(C1) + x(W1) + x(N1))
then **0.50x(C1) - 0.50x(W1) - 0.50x(N1) >= 0**

Wool Proportion in Bloom: x(W1) / (x(C1) + x(W1) + x(N1)) >= 0.40
Constraint: x(W1) >= 0.40 * (x(C1) + x(W1) + x(N1))
then **-0.40x(C1) + 0.60x(W1) - 0.40x(N1) >= 0**

Constraints for Amber
Cotton Proportion in Amber: x(C2) / (x(C2) + x(W2) + x(N2)) >= 0.60
Constraint: x(C2) >= 0.60 * (x(C2) + x(W2) + x(N2))
then **0.40x(C2) - 0.60x(W2) - 0.60x(N2) >= 0**

Wool Proportion in Amber: x(W2) / (x(C2) + x(W2) + x(N2)) >= 0.40
Constraint: x(W2) >= 0.40 * (x(C2) + x(W2) + x(N2))
then **-0.40x(C2) + 0.60x(W2) - 0.40x(N2) >= 0**

Constraints for Leaf
Cotton Proportion in Leaf: x(C3) / (x(C3) + x(W3) + x(N3)) >= 0.50
Constraint: x(C3) >= 0.50 * (x(C3) + x(W3) + x(N3))
then **0.50x(C3) - 0.50x(W3) - 0.50x(N3) >= 0**

Wool Proportion in Leaf: x(W3) / (x(C3) + x(W3) + x(N3)) >= 0.30
Constraint: x(W3) >= 0.30 * (x(C3) + x(W3) + x(N3))
then **- 0.30x(C3) + 0.70x(W3) - 0.30x(N3)>= 0**

Non-Negativity Constraints: All x(ij) must be >= 0

Total Constraints Count Demand Constraints: There are 3, one for each product. Material Proportion Constraints: There are 6, two for each product. Non-Negativity Constraints: There are 9, one for each decision variable.

We have totally 18 constraints

b) Solve the model using R/R Studio. Find the optimal profit and optimal values of the decision variables.

Developed a code to address a linear programming (LP) problem that optimizes the production mix of textile products. The LP model, built using the lpSolveAPI library, includes 9 constraints and 9 decision variables. These represent the production quantities of three products (Bloom, Amber, Leaf) made from three materials (Cotton, Wool, Nylon). The goal is to maximize profit while considering demand and material proportion constraints by setting up the objective function, constraints, and bounds for variables, the model is solved to determine the optimal production quantities and profit. This approach ensures an efficient and profitable production process.
R-Code available in ArunkumarBalaraman-Code.R

Model Snapshot from R-Studio

```
> textile.products.production.mix
Model name:
                              C1    W1    N1    C2    W2    N2    C3    W3    N3
Maximize                      15    10    25    11     6    21    15    10    25
Demand Constraints - Bloom     1     1     1     0     0     0     0     0     0  <=  4200
Demand Constraints - Amber     0     0     0     1     1     1     0     0     0  <=  3200
Demand Constraints - Leaf      0     0     0     0     0     0     1     1     1  <=  3500
Cotton Proportion in Bloom   0.5  -0.5  -0.5     0     0     0     0     0     0  >=     0
Wool Proportion in Bloom    -0.4   0.6  -0.4     0     0     0     0     0     0  >=     0
Cotton Proportion in Amber     0     0     0   0.4  -0.6  -0.6     0     0     0  >=     0
Wool Proportion in Amber       0     0     0  -0.4   0.6  -0.4     0     0     0  >=     0
Cotton Proportion in Leaf      0     0     0     0     0     0   0.5  -0.5  -0.5  >=     0
Wool Proportion in Leaf        0     0     0     0     0     0  -0.3   0.7  -0.3  >=     0
Kind                         Std   Std   Std   Std   Std   Std   Std   Std   Std
Type                        Real  Real  Real  Real  Real  Real  Real  Real  Real
Upper                        Inf   Inf   Inf   Inf   Inf   Inf   Inf   Inf   Inf
Lower                          0     0     0     0     0     0     0     0     0
```

Using excel Solver:

Decision Variables	C1	W1	N1	C2	W2	N2	C3	W3	N3			
Input	2100	1680	420	1920	1280	0	1750	1050	700			
Maximize										Used	Sign	Available
Demand Constraints - Bloom	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	4200	<=	4200
Demand Constraints - Amber	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0	3200	<=	3200
Demand Constraints - Leaf	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	3500	<=	3500
Cotton Proportion in Bloom	0.5	-0.5	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0	>=	0
Wool Proportion in Bloom	-0.4	0.6	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0	>=	0
Cotton Proportion in Amber	0.0	0.0	0.0	0.4	-0.6	-0.6	0.0	0.0	0.0	0	>=	0
Wool Proportion in Amber	0.0	0.0	0.0	-0.4	0.6	-0.4	0.0	0.0	0.0	0	>=	0
Cotton Proportion in Leaf	0.0	0.0	0.0	0.0	0.0	0.0	0.5	-0.5	-0.5	0	>=	0
Wool Proportion in Leaf	0.0	0.0	0.0	0.0	0.0	0.0	-0.3	0.7	-0.3	0	>=	0
Maximum Profit	15	10	25	11	6	21	15	10	25	$1,41,850.00		

Sensitivity report

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
J10	Input C1	2100	0	15	10	28
K10	Input W1	1680	0	10	15	35
L10	Input N1	420	0	25	1E+30	10
M10	Input C2	1920	0	11	10	15
N10	Input W2	1280	0	6	15	22.5
O10	Input N2	0	0	21	1E+30	10
P10	Input C3	1750	0	15	10	31
Q10	Input W3	1050	0	10	15	51.66666667
R10	Input N3	700	0	25	1E+30	10

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
S14	C1 Used	4200	14	4200	1E+30	4200
S15	C2 Used	3200	9	3200	1E+30	3200
S16	C3 Used	3500	15.5	3500	1E+30	3500
S17	C4 Used	5.68434E-14	-10	0	420	2100
S18	C5 Used	2.84217E-14	-15	0	420	1680
S19	C6 Used	-1.13687E-13	-10	0	0	1920
S20	C7 Used	1.13687E-13	-15	0	0	1280
S21	C8 Used	0	-10	0	700	1750
S22	C9 Used	0	-15	0	700	1050

Executive Summary:

The optimal profit is **$141,850**. The factory should produce:

Bloom: 4200 tons, using **2100** tons of **Cotton**, **1680** tons of **Wool**, and **420** tons of **Nylon**.
Amber: 3200 tons, using **1920** tons of **Cotton** and **1280** tons of **Wool**.
Leaf: 3500 tons, using **1750** tons of **Cotton**, **1050** tons of **Wool**, and **700** tons of **Nylon**.
This production plan maximizes the factory's profit while meeting the demand and material proportion constraints for each product.

Sensitivity:

I have also used Excel solver and it provides exactly same result.

Decision Variables: Current production plan is at its best. Any alterations within the permissible increase or decrease range won't impact the excellence of our existing solution.

Constraints: The shadow prices serve as indicators of how much our profit (the objective function) would rise if we were to ease the constraints, such as demand limits. For those constraints with a shadow price of zero, making them more or less stringent won't have any effect on the profit.

Question 3

Two construction companies, Giant and Sky, bid for the right to build in a field. The possible bids are $ 10 Million, $ 20 Million, $ 30 Million, $ 35 Million and $ 40 Million. The winner is the company with the higher bid.

The two companies decide that in the case of a tie (equal bids), Giant is the winner and will get the field.

Giant has ordered a survey and, based on the report from the survey, concludes that getting the field for more than $ 35 Million is as bad as not getting it (assume loss), except in case of a tie (assume win). Sky is not aware of this survey.

(a) State reasons why/how this game can be described as a two-players-zero-sum game

Binary Outcomes: The outcome of each bid is binary; either one company wins the bid or the other does. There are no shared outcomes or partial wins.
Opposing Interests: The interests of Giant and Sky are strictly opposed. If Giant wins the bid, Sky loses, and vice versa.
Fixed Sum: The "sum" of the outcome is fixed in that if Giant wins the bid, they gain the field, and Sky gains nothing (and potentially loses their bid amount), and the reverse is true if Sky wins.
Tie Situation: Even in the event of a tie, the rules dictate that Giant wins, so the gain for one is a loss for the other, maintaining the zero-sum nature.
Mutual Exclusivity: Only one company can win the bid. There is no scenario in which both companies can win simultaneously

In a two-player zero-sum game, we have two players. In this case, the construction companies Giant and Sky. Their decisions directly impact each other, and the total gain or loss is always zero. If one gains, the other loses. Here only one company can win the field. Both companies make strategic decisions to maximize their payoff. They decide on their bid amounts, knowing the highest bid wins. However, Giant has an edge - they won't bid over $35 Million unless there's a tie due to their survey information.

Generally, all players have complete information about the game, but here, **Sky doesn't know about Giant's survey results**, adding complexity to the game. The rules are clear – the highest bid wins, and if there's a tie, Giant wins. This creates a predictable payoff structure. It's assumed that both players are rational and aim to maximize their payoff. For example, Giant believes that winning the bid at more than $35 Million is as bad as losing, unless there's a tie.

So, this bidding can be seen as a two-player zero-sum game with opposing interests, strategic decision-making, and a clear payoff structure. The twist is the partial asymmetry of information, as Sky isn't aware of Giant's survey. The outcome of each bid is binary; either one company wins the bid or the other does. There are no shared outcomes or partial wins. The "sum" of the outcome is fixed - if Giant wins the bid, they gain the field, and Sky gains nothing.

(b) Considering all possible combinations of bids, formulate the payoff matrix for the game.

Lets

1 → represents the Giants win
-1→ represents the Giants losing

Conditions:

- Highest bid wins
- In the event of a tie, the rules dictate that Giant win
- Giant has an edge - they won't bid over $35 Million unless there's a tie due to their survey information

Sky / Giant	$10	$20	$30	$35	$40	R Min
$10	1	-1	-1	-1	-1	-1
$20	1	1	-1	-1	-1	-1
$30	1	1	1	-1	-1	-1
$35	1	1	1	1	-1	-1
$40	-1	-1	-1	-1	1	-1
C Max	1	1	1	1	1	

Row Max(Min) Lower Bound	-1
Column Min(Max) Upper Bound	1
LB = -1 < UB = 1, Game Value is between -1 < 1	
Pure Strategy does not exist & No Saddle point	

Formulated the matrix based on the problem statement by meeting all below conditions.

- Highest bid wins
- In the event of a tie, the rules dictate that Giant win
- Giant has an edge - they won't bid over $35 Million unless there's a tie due to their survey information

The Lower Bound value was found by performing Max(Min) on the rows, resulting in -1. Similarly, the Upper Bound value was determined by performing Min(Max) on the columns, resulting 1. Therefore, the game value ranges from -1 to 1, and no saddle point is reached.

(c) Explain what is a saddle point. Verify: does the game have a saddle point?

Saddle Point: If there is a pair of strategies in which either player can do no better, then we have a saddle point As I already derived the same in manual method in Matrix, I have also developed the code to check the saddle point in this context and ***code is available in ArunkumarBalaraman.R***

Definition Saddle Point:

A saddle point is a situation where neither player can benefit from changing their strategy while the other player keeps theirs unchanged. This point is also known as an equilibrium point.
In terms of the payoff matrix, the saddle point is the element that is both the smallest in its row (the best of the worst outcomes for the row player) and the largest in its column (the worst of the best outcomes for the column player).
If a saddle point exists, it represents the value of the game, and both players have a clear, optimal strategy. However, not all games have a saddle point, in which case players may need to use mixed strategies.

Snippet of the Code Outcome for Saddle Point:

```
> lowerbound <- calculate_max_secured_payoff(payoff_matrix)
> upperbound <- calculate_min_secured_payoff(payoff_matrix)
> cat('Lower Bound Value:', lowerbound, '<', 'Upper Bound Value:', upperbound, '\n')
Lower Bound Value: -1 < Upper Bound Value: 1
>
> # Condition to check saddle point
> if (lowerbound < upperbound) {
+   cat("Saddle point did not exist.\n")
+ } else if (lowerbound == upperbound) {
+   cat("Saddle point exists.\n")
+ } else {
+   cat("Unexpected case: min_secured_payoff is greater than max_secured_payoff.\n")
+ }
Saddle point did not exist.
```

Saddle Point in the context of this problem:

Pure Strategy **does not exist** & **No Saddle point** for this game, as there's no single strategy (pure strategy) for either player that guarantees the best possible outcome irrespective of what the other player does that is, there's no dominant strategy for either player and no stable outcome "saddle point" where both players would not benefit from deviating unilaterally.

From our analysis using both the R- code and manual calculations in Excel, its clear that there's no saddle point in this bidding problem between Giant and Sky. Neither player has a dominant strategy, and there's no stable outcome (saddle point).

(d) Construct a linear programming model for Company Sky in this game

Mixed Strategy:

A mixed strategy is a tactical approach where a player doesn't stick to one fixed action (known as a pure strategy), but instead selects from a range of possible actions based on a certain probability distribution. In other words, the player chooses to play different strategies at random, assigning specific probabilities to each.

Giant's

max z = v
s.t. v - (+1x1+1x2+1x3+1x4-1x5) <= 0
v - (-1x1+1x2+1x3+1x4-1x5) <= 0
v - (-1x1-1x2+1x3+1x4-1x5) <= 0
v - (-1x1-1x2-1x3+1x4-1x5) <= 0
v - (-1x1-1x2-1x3-1x4+1x5) <= 0
x1+x2+x3+x4+x5=1
xi >= 0, → i = 1,2,3,4,5

then
-x1-x2-x3-x4+x5+v <= 0
x1-x2-x3-x4+x5+v <= 0
x1+x2-x3-x4+x5+v <= 0
x1+x2+x3-x4+x5+v <= 0
x1+x2+x3+x4-x5+v <= 0
x1+x2+x3+x4+x5=1
xi >= 0, → i = 1,2,3,4,5
v u.r.s (-Inf)

Sky's

min z = v
s.t. v-(+1y1-1y2-1y3-1y4-1y5) >=0
v-(+1y1+1y2-1y3-1y4-1y5) >=0
v-(+1y1+1y2+1y3-1y4-1y5) >=0
v-(+1y1+1y2+1y3+1y4-1y5) >=0
v-(-1y1-1y2-1y3-1y4+1y5) >=0
y1+y2+y3+y4+y5=1
yi >= 0, → i = 1,2,3,4,5

then
-y1+y2+y3+y4+y5+v >=0
-y1-y2+y3+y4+y5+v >=0
-y1-y2-y3+y4+y5+v >=0
-y1-y2-y3-y4+y5+v >=0
+y1+y2+y3+y4-y5+v >=0
y1+y2+y3+y4+y5=1
yi >= 0, → i = 1,2,3,4,5
v u.r.s (-Inf)

I have created the Linear programming model for both Player 1 (Giants) and Player 2 (Sky).

For Giant:

- Giant's strategy is to maximize its minimum guaranteed payoff 'v' by choosing a mix of bids.
- The strategy selects probabilities for each bid to optimize Giant's expected payoff, regardless of Sky's actions.
- Probabilities must be non-negative, and 'v' can represent any payoff outcome.

For Sky: (Highlighted in blue)

- Sky aims to minimize its potential maximum loss 'v' by selecting probabilities for each bid.
- Constraints ensure 'v' is at least the loss against any fixed bid by Giant.
- The sum of Sky's bid probabilities must be one.

(e) Produce an appropriate code to solve the linear programming model in part (d).

Developed the R-Code in R-studio and ***available in ArunkumarBalaraman.R***

Developed to solve a linear programming (LP) problem with the aim of minimizing ‘v’, which signifies Sky’s maximum potential loss in the bidding game against Giant. The LP model, constructed using the lpSolveAPI library, comprises 6 constraints and 6 decision variables. These correspond to the 6 strategies as constraints ($10M, $20M, $30M, $35M, $40M, and Probability) and decision variables for y1, y2, y3, y4, y5, and v. The objective is to minimize ‘v’, representing Sky’s maximum possible loss. The model will be solved to find the probability values for each strategy

Model Build for Sky:

```
Model name:
              y1    y2    y3    y4    y5     v
Minimize       0     0     0     0     0     1
$10           -1     1     1     1     1     1  >=  0
$20           -1    -1     1     1     1     1  >=  0
$30           -1    -1    -1     1     1     1  >=  0
$35           -1    -1    -1    -1     1     1  >=  0
$40            1     1     1     1    -1     1  >=  0
Prob           1     1     1     1     1     0   =  1
Kind         Std   Std   Std   Std   Std   Std
Type        Real  Real  Real  Real  Real  Real
Upper        Inf   Inf   Inf   Inf   Inf   Inf
Lower          0     0     0     0     0  -Inf
```

Solution:

```
> # Solves the model
> solve(bidding.problem.model)
[1] 0
> # Get the solution
> solution <- get.variables(bidding.problem.model)
> objective_value <- get.objective(bidding.problem.model)
> # Display the solution
> cat("Solution:\n")
Solution:
> print(solution)
[1] 0.5 0.0 0.0 0.0 0.5 0.0
> cat("\nObjective Value (v):\n")

Objective Value (v):
> print(objective_value)
[1] 0
```

Basic Explanation:

Sky should alternate between bidding the lowest ($10M) and the highest ($40M) amounts with equal probability (50%) to avoid any expected loss.

(f) Solve the game for Sky using the linear programming model and the code you constructed in parts (d) and (e). Interpret your solution.

Executive Summary:

Decision Variables: Sky's decision variables are y1 to y5, each representing the probability of bidding a specific amount ($10M to $40M).

Objective: Sky aims to minimize 'v', which signifies its maximum potential loss in the game.

Constraints: The constraints ensure that Sky's loss 'v' is not negative, regardless of Giant's bid.

Solution: Sky's optimal strategy is to bid either $10M or $40M, each with a 50% probability. This is shown by the solution values of 0.5 for y1 and y5.

Objective Value (v): The optimal value of 'v' is 0, indicating that Sky's expected loss is minimized to zero when following the optimal strategy.

In summary, the linear programming model suggests a mixed strategy for Sky that safeguards it from losses, irrespective of Giant's actions. Sky should alternate between bidding the lowest ($10M) and the highest ($40M) amounts with equal probability to avoid any expected loss. Bidding $20M, $30M, or $35M is not recommended for construction company Sky.

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