# Relative Price of Equipment, Investment Shocks and Oil-Food Prices

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August 1, 2025

**Abstract**

This paper reexamines the role of IST shocks in macroeconomic fluctuations by accounting for movements in oil and food prices. Using a structural VAR and a DSGE model with a commodity sector, I show that standard IST shock estimates embed information from food and oil price fluctuations, distorting their macroeconomic impact. After adjusting for these effects, IST shocks lead to declining price levels, immediate increases in GDP and investment, and a lagged rise in consumption. Additionally, their contribution to GDP and consumption variance decreases. These findings highlight the importance of refining IST shock identification to avoid overstating their role in business cycle dynamics.

***Keywords***— Relative Price of Equipment, Investment shocks, Oil prices.

# 1 Introduction

The relative price of equipment (RPE), defined as the price of equipment and durable consumption goods relative to non-durable consumption goods (Ben Zeev & Khan, 2015), has been a central variable in macroeconomic research since the foundational work of Greenwood et al. (2000). The long-term decline in the RPE in the U.S. economy since the mid-1950s (Figure 1) is widely interpreted as evidence of sustained technological progress in the investment goods sector. This interpretation forms the basis of the investment-specific technological change (IST) hypothesis, which posits that such trends reflect improvements in the productivity of investment goods production. Within this framework, unexpected innovations to the underlying technology, so-called IST shocks, are often treated as exogenous forces driving both long-run growth and business cycle fluctuations.

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However, standard empirical methods for identifying IST shocks, which rely in the information contained in the RPE, may overlook important sources of relative price variation. The RPE is an imperfect measure of investment technology, and it identifies IST shocks only under restrictive assumptions. In particular, I show that fluctuations in oil and food prices can disproportionately affect the price of non-durable consumption goods, thereby affecting the RPE independently of technological change. Because these components are often excluded from empirical specifications, the estimated impact of IST shocks may be overstated. In this paper, I show that standard empirical estimates of IST shocks exhibit a significant correlation with oil shocks identified in previous studies. The underlying intuition is straightforward: an increase in oil prices raises the cost of non-durable consumption goods (the denominator of the relative price of equipment) causing the RPE to decline. After adjusting the estimation for oil and food price fluctuations, I find that the share of forecast error variance (FEV) attributed to IST shocks decreases, particularly for GDP, investment, and consumption from zero to 10 quarters. This suggests that the empirical significance often ascribed to IST shocks may, in part, reflect the influence of commodity markets.

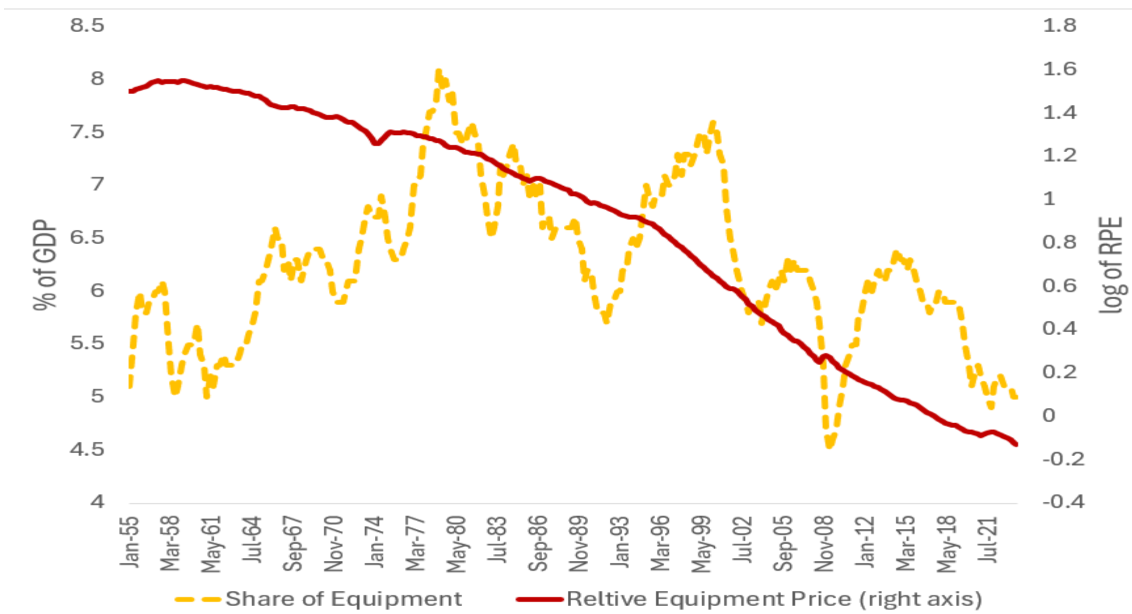



Figure 1: Relative price of equipment and share of equipment.

Furthermore, IST shocks adjusted for oil and food price movements generate impulse response functions (IRFs) that align more closely with standard macroeconomic theory. When oil price fluctuations are ignored, IST shocks produce a counterintuitive decline in real wages and an increase in consumer price indexes. However, after accounting for oil price movements, I find that real wages rise following the shock, while consumer prices decline, resolving these anomalies.

To further investigate these findings, I develop a DSGE model that explicitly incorporates oil and food price shocks. The goal is to examine whether accounting for these commodity prices within a structural framework helps reconcile the conflicting views in the literature regarding the importance of IST shocks. A key tension exists between DSGE studies that assign a central role to IST shocks in driving business cycles (Chen & Wemy, 2015; Fisher, 2006; Galí & Gambetti, 2009), and empirical estimations that include the RPE as an observable to discipline the DSGE model and find IST shocks to be far less influential (Ben Zeev & Khan, 2015; Justiniano et al., 2011; Schmitt-Grohé & Uribe, 2012). By embedding oil and food price dynamics into the DSGE model, I assess whether the model's internal consistency and empirical fit improve, particularly with respect to the behavior of macroeconomic variables following IST shocks. While the inclusion of these prices enhances the model's ability to replicate observed impulse responses (especially for wages and consumer prices) the overall contribution of IST shocks to macroeconomic fluctuations remains limited. Thus, although the model performs better in some dimensions, it does not fully resolve the empirical disconnect identified in the literature.

I estimate a structural DSGE model that incorporates both IST shocks and oil and food price shocks. The model includes a commodity-producing sector whose output serves as an input for intermediate goods producers, which in turn supply both consumption and investment goods producers, as well as the final consumption goods sector. This structure allows commodity prices to directly affect final goods production costs and influence the dynamics of the relative price of investment. The model is estimated using Bayesian methods, with the RPE included as an observable to discipline its behavior over time.

The estimated model produces three key findings: (i) IST plays a small role in explaining the variance of GDP, investment and consumption; (ii) the theoretical IRFs of macroeconomic variables in response to IST shocks closely align with those observed in the empirical analysis; and (iii) applying the empirical strategy from Section 2 to the simulated data yields similar results, highlighting that using a measure of the relative price of investment that still contains commodity prices into the denominator can lead to divergent conclusions in the IRFs.

*Related Literature* - The role of IST in business cycle fluctuations has been explored through both empirical analysis and structural modeling. Empirical studies consistently highlight the significance of IST shocks, often relying on a VAR framework to extract these shocks and using forecast error variance decomposition (FEVD) to assess their macroeconomic impact (Ramey, 2016). For example, Fisher (2006), Galí and Gambetti (2009), and Chen and Wemy (2015) employ long-run and medium-run restrictions to identify IST shocks, providing evidence of their substantial influence on macroeconomic dynamics. Moreover, Ben Zeev and Khan (2015) examines news shocks to IST (anticipated exogenous changes in the technology variable), showing that they explain a considerable share of the forecast error variance (FEV) in consumption, hours worked, investment, and GDP.

Structural models calibrated to the U.S. economy also suggest that IST plays a major role in macroeconomic fluctuations. Studies by Greenwood et al. (2000), Christensen and Dib (2008), Jaimovich and Rebelo (2009), and Justiniano et al. (2010) indicate that IST accounts for a significant share of the variance in consumption, investment, and GDP at business cycle frequencies. Furthermore, research by Jaimovich and Rebelo (2009), Choi (2020), and Liao and Chen (2023) examines the role of news shocks to IST and finds that these shocks substantially contribute to the variance of key macroeconomic indicators.

Despite substantial evidence supporting the role of IST shocks, another strand of the literature challenges their significance in explaining macroeconomic behavior. This research employs a DSGE framework with a rich stochastic structure, following the methodology of Smets and Wouters (2007). These studies estimate the model using Bayesian techniques and incorporate the RPE as a key disciplining variable. Notable contributions include Justiniano et al. (2011), which examines non-news shocks, as well as Schmitt-Grohé and Uribe (2012) and Ben Zeev and Khan (2015), which focus on news shocks. Understanding the factors driving these contrasting perspectives is essential for accurately assessing the macroeconomic impact of IST. This paper seeks to reconcile part of this discrepancy by examining the role of oil and food prices in the estimation of IST shocks through the RPE.

The paper is structured as follows: Section 2.2 presents the empirical methodology, explores the relationship between IST and oil price shocks, and analyzes the responses of real and price variables after adjusting for oil and food prices. Section 2.3 conducts robustness checks to validate the main findings. Section 2.4 introduces a medium-scale DSGE model, and Section 2.5 concludes.

# 2 RPE, IST shocks and oil-food prices

In this section, I follow the empirical literature by using the RPE to identify IST shocks. I then apply local projections, following Jordà (2005), to examine the impulse responses of several macroeconomic variables to these shocks. The results indicate that IST shocks lead to increases in consumer prices, which can explain the observed decline in real wages. Additionally, I show that the estimated IST shocks exhibit a significant correlation with oil price shocks documented in the literature. However, once oil and food prices are incorporated into the analysis, both the correlation with oil shocks and the rise in consumer prices after an IST shock diminish. Moreover, the share of FEV attributed to IST shocks for GDP, consumption, and investment declines after this adjustment.

## 2.1 Identification of IST shocks

IST shocks are typically estimated by identifying structural innovations that explain the medium- or long-term variance of the RPE (Ramey, 2016). This approach involves estimating a VAR model and deriving orthogonal shocks from its reduced-form residuals to maximize the FEV of the RPE over $h$ periods ahead (Barsky & Sims, 2011).

I estimate the IST shocks using quarterly data from the US from 1964:I to 2022:IV. Following the estimation approach of Chen and Wemy (2015), consider the following VAR process, which is assumed to provide a sufficiently accurate approximation of the true data-generating process:

$$Y_t = \beta(L)Y_t + u_t \tag{1}$$

Where $Y_t$ is an $(n \times 1)$ vector of macroeconomic variables at time $t$. I include the variables used in the baseline estimation of Chen and Wemy (2015): TFP (Fernald, 2014), log of RPE, log of real GDP per capita, log of real investment per capita, log of real consumption per capita, and log of total hours worked.[1] Notice that in the empirical literature, macroeconomic variables typically exclude both oil prices and food prices from the system of equations. All variables are in levels, following Sims et al. (1990). $\beta(L) = B_1(L) + B_2(L^2) + \cdots + B_P(L^P)$ is a lag polynomial, and $u_t$ is an $(n \times 1)$ vector of reduced-form innovations. The latter is assumed to be a linear combination of structural shocks $(\varepsilon_t)$:

$$u_t = A\varepsilon_t \tag{2}$$

Where the variance-covariance matrix of the reduced-form innovations is:

$$\Sigma_u = E[u_t u_t'] = E[A\varepsilon_t \varepsilon_t' A'] = AA' \tag{3}$$

However, it is well known that $A$ cannot be uniquely identified. To see why, consider $A = \tilde{A}Q$ where $Q$ is an orthonormal matrix. Note that $\tilde{A}$ satisfies (3) and therefore is also a matrix that can be used to obtain the structural shocks:

$$\Sigma_u = E[AA'] = E[\tilde{A}QQ'\tilde{A}'] = E[\tilde{A}\tilde{A}'] \tag{4}$$

Literature usually identifies the IST shocks by finding a column $\tilde{q}_1$ in $Q$ that maximizes the FEV of the RPE at the horizon $h$:

$$\tilde{q}_1 = \arg\max q_1' S^h q_1 \tag{5}$$

subject to

$$q_1' q_1 = 1, \tag{6}$$

where $S^h$ is the variance of the forecast error of the variable of interest $h$ steps ahead, using the Cholesky decomposition on $\Sigma$ to obtain $\tilde{A}$. Equation (6) guarantees that $q_1$ is a unit-length column vector that belongs to an orthonormal matrix. Then, the IST shock is obtained as the first value of the vector:

$$\epsilon_t = A^{-1}u_t = (Q\tilde{A})^{-1}u_t \tag{7}$$

[1] Data is obtained from the Bureau of Economic Analysis (real GDP, investment and consumption, chained dollars), Bureau of Labor Statistics, and the FRED.

Uhlig (2004) shows that the problem can also be written in a quadratic form where the $q_1$ is the eigenvector associated with the largest eigenvalue of the matrix $S^h$ (Chen & Wemy, 2015). I estimate the VAR with standard OLS using quarterly data from 1964:I to 2019:IV.[2]

## 2.2 Local projections

Although computing the IRF within the VAR framework is relatively simple, I employ local projections as proposed by Jordà (2005) for two primary reasons. First, as highlighted by Olea et al. (2025), local projections tend to exhibit lower bias and offer more reliable uncertainty assessments, albeit at the cost of increased variance. However, this trade-off is often worthwhile to prevent misleading inferences that can arise from biased estimates produced by under-specified VAR models.[3] Second, my approach entails estimating IST shocks using standard methods in the literature that do not consider food and oil prices during the estimation process. I then utilize these estimated shocks to analyze the responses of several consumer price variables, which were excluded from the original estimation. By examining how these variables respond to the shocks, I can assess whether the shocks contain additional information.

Let $y_t$ be the variable of interest, $\hat{\epsilon}_t^{IST}$ be the estimated measure of IST shock, $X_t$ a vector of macroeconomic controls at time t. I include real activity variables (log of GDP, consumption, investment, and hours worked), the shadow rate of FED fund's rate (Wu & Xia, 2016), log of cpi deflected wages, logs of consumer price index, core consumer price index, food consumer price index and oil price (WTI). I obtain the IRFs from the following OLS regression:

$$y_{t+h} - y_{t-1} = \alpha_h + \beta_h \hat{\epsilon}_t^{IST} + \sum_{j=1}^{4} \gamma_{h,j} X_{t-j} + u_t, \tag{8}$$

Where $\beta_h$ is the value of the IRF at horizon $h$, and $u_t$ are residuals. The confidence interval is computed by using HAC standard errors (Jordà, 2023).

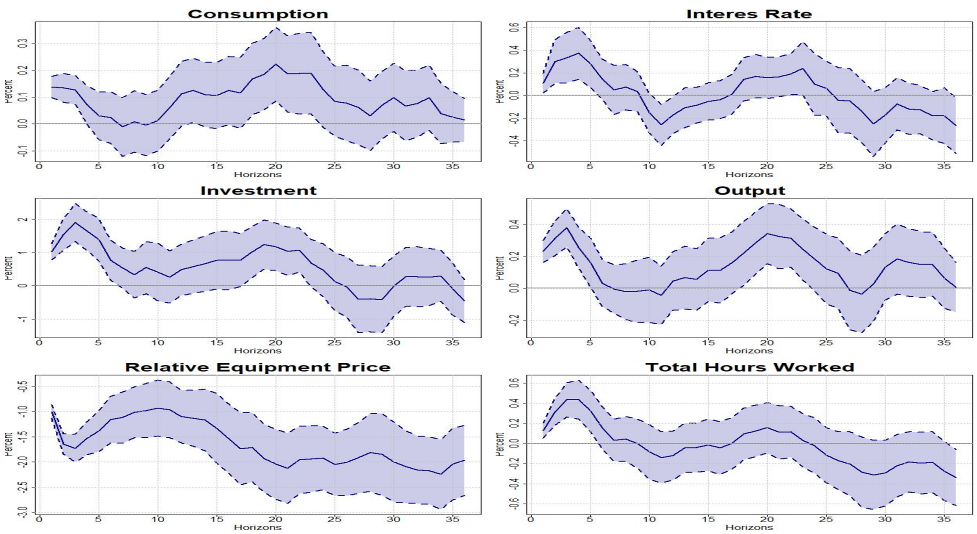
Figure 2 presents the IRFs for selected macroeconomic variables in response to the estimated IST shocks. The responses of real economic activity indicators align with conventional findings: investment, GDP, and hours worked rise immediately following a positive shock, while consumption exhibits a delayed increase, occurring some quarters later.

However, a puzzling reaction emerges in real wages, which decline after the shock.[4] This anomaly is closely tied to the behavior of nominal variables, including the consumer price

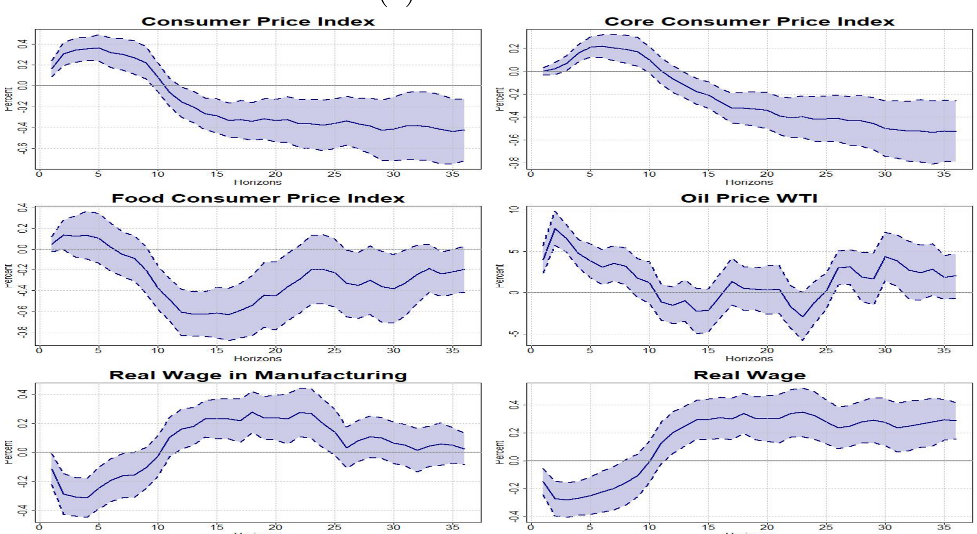
[2]Results are robust to the estimation technique in the VAR.

[3]See also, Auerbach and Gorodnichenko (2013). For a survey in Local Projections literature, see Jordà (2023).

[4]Real wages are generally expected to rise in response to IST shocks, as noted in DSGE models by Justiniano et al. (2010) and Justiniano et al. (2011).




(a) Real Variables.




(b) Price Variables

Figure 2: IRF to baseline IST shocks

Note: Impulse response of macroeconomic variables to baseline estimation of IST shocks. The shaded areas represent the 90.0% confidence interval.

index (CPI), core CPI, and food CPI, which display non-monotonic patterns. In particular, CPI measures initially rise after the shock before subsequently declining.

Changes in oil and food prices offer a potential explanation for these puzzling responses. A portion of the decline in the RPE following IST shocks is driven by shocks that raise the prices of non-durable consumption goods (the denominator in the RPE ratio) primarily due to increases in energy and food prices. These shocks, however, are distinct from IST shocks and represent a separate economic phenomenon. In the following section, I focus on the relationship between IST shocks and oil price shocks, given the extensive literature on identifying exogenous oil price movements. In contrast, the identification of food price shocks has received comparatively little attention, and reliable estimates isolating food-specific shocks remain scarce. Nevertheless, rising food prices likely affect IST shock estimates through similar mechanisms as oil price fluctuations.[5]

## 2.3 IST shocks and oil price shocks

For IST shocks to be accurately identified, they must be uncorrelated with other exogenous disturbances, as structural shocks should exhibit no correlation with any other shocks (Ramey, 2016). This subsection examines the relationship between the identified IST shocks and oil price shocks. The findings reveal a significant correlation, suggesting that current identification methods for IST shocks may inadvertently capture information beyond investment technology changes.

I focus on five series of oil price shocks found in the literature:

1. O(1). Oil price surprises from Känzig (2021): This paper uses the change in oil futures prices around OPEC announcements. Oil futures serve as a market-based proxy for oil price expectations, making them suitable for measuring the impact of these announcements. Although OPEC's decisions may be influenced by political and global economic conditions, using a tight window around the announcements helps isolate their impact and mitigate endogeneity concerns. This approach assumes that global economic conditions are already priced in by the market and remain stable within the window, ensuring that the series captures changes in oil price expectations due to OPEC's decisions.

2. O(2). Oil Price news from Känzig (2021): To interpret OPEC announcements as news about future oil supply, the announcements mustn't introduce new information about other factors like oil demand, global economic activity, or geopolitical developments. To address this, one alternative is to see how OPEC announcements are covered in the financial press, typically focusing on production quotas. Given the political nature

[5]Food prices also play a crucial role in understanding the US and Euro-area economies. For recent studies, see De Winne and Peersman (2016), Peersman (2022), and Jo and Adjemian (2023).

of OPEC and its less systematic response to economic developments, the information channel problem may be less significant compared to monetary policy shocks. To further mitigate this concern, this measure constructs an informationally robust surprise series by removing the effects of revisions in OPEC's global demand forecasts, similar to the refinement used by Romer and Romer (2004) in the monetary policy context, ensuring the robustness of the results.

3. O(3). "Pure" oil price expectation shocks from Baumeister and Hamilton (2019): The shocks are obtained by first identifying market-based oil price surprises, which are the deviations between the realized price of oil (such as WTI) and what market participants had expected the price to be a month before. To isolate the "pure" expectation component, the authors regress these market-based surprises on a set of fundamental oil supply and demand shocks. By filtering out the influence of these fundamental shocks, the residuals from this regression are interpreted as the orthogonalized, or "pure" oil price expectation shocks. These shocks represent changes in oil prices driven solely by shifts in market expectations, independent of new information about underlying oil market fundamentals.

4. Oil supply O(4), and oil demand O(5) shocks from Baumeister (2023): The authors propose a Bayesian approach to estimate oil supply and demand shocks using structural vector autoregressions (SVARs). Their method incorporates prior information about the parameters in the model, including the short-run price elasticities of oil supply and demand. They account for measurement errors, particularly in global oil inventories, and utilize historical data to refine their estimates. By generating impulse-response functions, they analyze the dynamic effects of these shocks on oil prices and economic activity. Their approach also includes a historical decomposition to assess the contributions of supply and demand shocks to significant oil price movements, ensuring a robust and nuanced understanding of the underlying factors driving these fluctuations.

Table 1 presents the correlations between oil price shocks and IST shocks, including comparisons with IST shocks estimated in previous studies. IST(1) refers to the shock estimated in Section 2.1, while IST(2) corresponds to the estimation by Drechsel (2023), who follows the identification approach of Fisher (2006). Ben Zeev and Khan (2015), hereafter BZK, estimate two series of IST shocks: IST(3), which captures unanticipated IST shocks that immediately affect the RPE and maximize its FEV, and IST(4), which represents news shocks that maximize the FEV of the RPE while remaining orthogonal to unanticipated IST shocks. The latter is considered the most significant component of IST, as it accounts for the largest share of economic activity variance.

The results reveal a significant correlation between IST shocks identified using long-run restrictions (Drechsel, 2023) and medium-run restrictions (as in Chen and Wemy, 2015; and BZK) with oil price shocks. Specifically, the correlation between the first two IST shock estimates and oil price innovations (O(3) and O(5)) is positive, indicating that an exogenous increase in oil prices is strongly associated with higher IST shocks. Conversely, the correlation

| | O(1) | O(2) | O(3) | O(4) | O(5) |
|---|---|---|---|---|---|
| IST(1) | 0.04 | 0.10 | 0.25*** | 0.13* | 0.16** |
| IST(2) | 0.15* | 0.13* | 0.18** | 0.14* | 0.16** |
| IST(3) | 0.00 | 0.08 | 0.34*** | 0.18** | 0.07 |
| IST(4) | -0.11 | -0.28*** | -0.28*** | -0.05 | -0.28*** |

Table 1: IST and oil shocks: correlation.

Note: Correlation between IST shocks and Oil shocks. Correlations with O(4) are multiplied by -1 because the nature of the shock implies a decrease in the oil price. *p-value $< 0.1$, ** p-value $< 0.05$, *** p-value $< 0.01$.

with O(4), which captures oil supply shocks (i.e., reductions in oil prices), is negative. When analyzing the estimates from BZK, the correlation between unanticipated IST shocks and oil supply shocks remains strong, whereas the correlation with news shocks follows a distinct but still statistically significant pattern.

For IST(4), there is no clear theoretical expectation regarding its correlation with contemporaneous oil price shocks, as IST news shocks are forward-looking and relate to anticipated future changes in the relative price of investment. However, as shown in Table 1, the results indicate a statistically significant negative correlation between the estimated news shocks and current oil price shocks.

Fisher (2006) also observes a significant correlation between IST shocks and oil price movements but offers a different interpretation, suggesting that oil shocks could be regarded as IST shocks. Specifically, Fisher (2006, p.446) argues:

*The oil shock result might not be surprising. Suppose that an exogenous increase in the price of oil induces substitution toward equipment that the United States is not good at producing, such as high-mileage cars. If this is the case, then the real price of equipment rises. From this perspective, a permanent oil shock is very much like an I-shock.*

The findings in this paper suggest an alternative interpretation. My analysis shows that oil shocks affect fluctuations in the RPE mainly by directly influencing the price of consumption goods, which form the denominator of the RPE ratio. These price movements distort the estimated measures of IST shocks, leading to potential misidentification.

## 2.4 Re-estimating the IST shocks: oil and food prices.

The observed relationship between oil prices and IST shocks underscores the need to refine the identification strategy. In this paper, I adjust the VAR framework by incorporating two additional variables to account for the influence of oil and food prices. Specifically, I include the logarithms of the West Texas Intermediate (WTI) oil price and the food consumer price index (CPIF) in the VAR model used to estimate IST shocks, as described in Section 2.1.

This modification is applied to the identification strategies of Drechsel (2023) and BZK.[6] After incorporating these variables, the identified IST shocks exhibit a significantly lower correlation with oil price shocks (Table 2).[7]

| | O(1) | O(2) | O(3) | O(4) | O(5) |
|---|---|---|---|---|---|
| IST(1) | -0.05 | 0.09 | 0.01 | 0.05 | -0.03 |
| IST(2) | -0.01 | 0.07 | -0.10 | 0.04 | -0.08 |
| IST(3) | 0.01 | 0.07 | 0.27*** | 0.17** | 0.04 |
| IST(4) | -0.14 | -0.04 | -0.08 | 0.12 | -0.13 |

Table 2: Corrected IST and oil shocks: correlation.

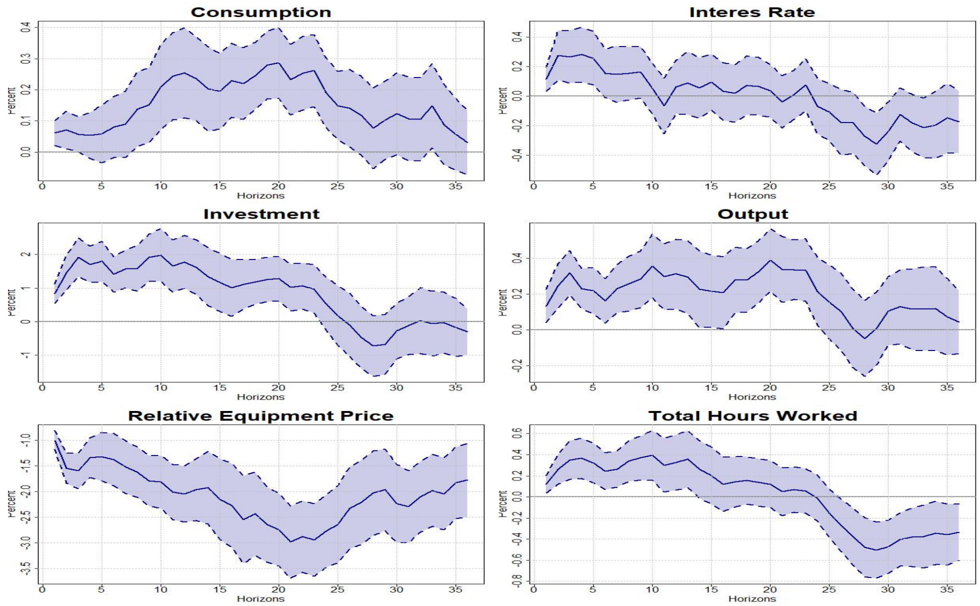
Note: Correlation between IST shocks including food and oil prices in the estimation and Oil shocks. Correlations with O(4) are multiplied by -1.0 because the nature of the shock implies a decrease in the oil price. *p-value $< 0.1$, ** p-value $< 0.05$.

Estimating IST shocks while controlling for oil and food prices allows for the analysis of impulse responses using the LP strategy outlined in Section 2.2. Notably, incorporating these price variables into the VAR system yields IST shocks that no longer generate increases in consumer price variables or declines in real wages (Figure 3). Meanwhile, the responses of other real economic variables remain qualitatively unchanged.

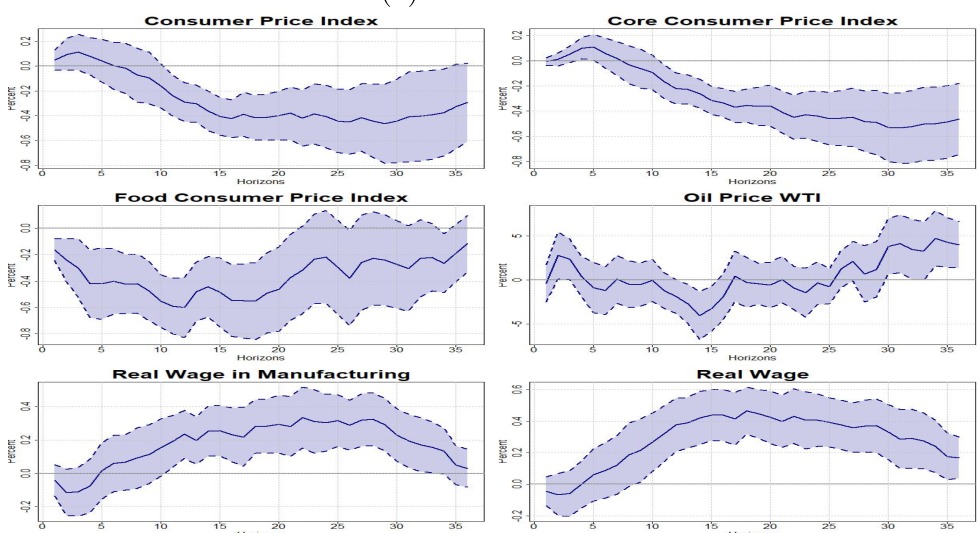
The adjustment described above reduces the relative contribution of identified IST shocks to the dynamics of key macroeconomic variables. To illustrate this, I examine the fraction of FEV attributed to IST shocks within a VAR framework, comparing results with and without the inclusion of oil and food prices. As shown in Table 3, incorporating these variables (”Clean”) lowers the estimated share of FEV explained by IST shocks, particularly over small frequencies (one to ten quarters). For instance, at the five-quarter horizon, the adjustment reduces the FEV of GDP attributed to IST shocks by 16.2 percentage points (p.p.), investment by 10.6 p.p., and consumption by 14.9 p.p. Furthermore, Appendix B confirms this pattern for IST news shocks, as identified by BZK.

[6] In Appendix B, I detail how I adapt the BZK methodology to incorporate information on oil and food prices.

[7] The correlation between IST(3) and both O(3) and O(4) remains statistically significant. However, the macroeconomic impact of IST(3) is minimal, as it explains only a small fraction of the variance in economic activity (Ben Zeev & Khan, 2015).




(a) Real Variables.




(b) Price Variables

Figure 3: IRF to oil-food price corrected IST shocks.

Note: Impulse response of macroeconomic variables to oil and food price adjusted estimation of IST shocks. The shaded areas represent the 90.0% confidence interval.

| | GDP | GDP | Inv. | Inv. | Cons. | Cons. | Hours | Hours |
|---|---|---|---|---|---|---|---|---|
| **h** | **Base** | **Clean** | **Base** | **Clean** | **Base** | **Clean** | **Base** | **Clean** |
| 1 | 0.192 | 0.053 | 0.181 | 0.073 | 0.237 | 0.063 | 0.061 | 0.058 |
| 5 | 0.373 | 0.212 | 0.452 | 0.346 | 0.255 | 0.116 | 0.268 | 0.219 |
| 10 | 0.318 | 0.214 | 0.368 | 0.280 | 0.224 | 0.140 | 0.176 | 0.167 |
| 15 | 0.298 | 0.230 | 0.342 | 0.258 | 0.209 | 0.163 | 0.138 | 0.131 |
| 20 | 0.277 | 0.246 | 0.332 | 0.261 | 0.191 | 0.182 | 0.110 | 0.117 |

Table 3: FEV explained by IST shocks

# 3 Robustness analysis

## 3.1 Modifying the RPE

An alternative approach to controlling for exogenous fluctuations in oil and food prices is to compute the RPE as the ratio of the price of equipment and durable consumption to the price of non-durable consumption, excluding energy and food (*RPENE*). These excluded components are directly linked to oil and food price dynamics. This adjustment was first proposed by Beaudry et al. (2015) in their study of the cyclical behavior of the RPE. By adopting this measure, it is no longer necessary to explicitly include oil and food prices in the VAR system, as done in the previous section.

Figure A1 in Appendix A presents the IRFs to IST shocks estimated using RPENE. The results closely resemble those in Figure 3, with consumer price variables showing no significant increases after the shock. Similarly, Table 4 reports the FEVD under this specification, revealing that the share of forecast error variance attributed to IST shocks at business-cycle frequencies is lower than in the baseline scenario. This reduction is particularly pronounced for GDP, consumption, and investment per capita.

| | GDP | GDP | Inv. | Inv. | Cons. | Cons. | Hours | Hours |
|---|---|---|---|---|---|---|---|---|
| **h** | **Base** | **Clean** | **Base** | **Clean** | **Base** | **Clean** | **Base** | **Clean** |
| 1 | 0.192 | 0.063 | 0.181 | 0.088 | 0.237 | 0.113 | 0.061 | 0.032 |
| 5 | 0.373 | 0.265 | 0.452 | 0.387 | 0.255 | 0.183 | 0.268 | 0.243 |
| 10 | 0.318 | 0.267 | 0.368 | 0.356 | 0.224 | 0.204 | 0.176 | 0.182 |
| 15 | 0.298 | 0.280 | 0.342 | 0.354 | 0.209 | 0.212 | 0.138 | 0.148 |
| 20 | 0.277 | 0.277 | 0.332 | 0.352 | 0.191 | 0.207 | 0.110 | 0.126 |

Table 4: FEV explained by IST shocks (RPENE)

## 3.2 Joint price index of food and energy

The empirical findings remain robust to the choice of variables used to capture fluctuations in food and oil prices. To verify this, I introduce an alternative measure into the VAR framework. Instead of using the CPIF and WTI price indices, I incorporate a combined index of food and oil prices derived from Personal Consumption Expenditures (*PCEFE*) data. This alternative variable is constructed as a weighted average of energy and food prices within non-durable consumption, with weights based on national accounts data.

Figure A2 in Appendix A displays the IRFs to IST shocks identified using the PCEFE index within the VAR system. The response patterns closely resemble those in Figure 3. Furthermore, Table 5 shows that the share of FEV explained by IST shocks remains lower than in the baseline scenario, particularly for GDP, consumption, and investment per capita at business-cycle frequencies.

| | GDP | GDP | Inv. | Inv. | Cons. | Cons. | Hours | Hours |
|---|---|---|---|---|---|---|---|---|
| **h** | **Base** | **Clean** | **Base** | **Clean** | **Base** | **Clean** | **Base** | **Clean** |
| 1 | 0.192 | 0.064 | 0.181 | 0.084 | 0.237 | 0.121 | 0.061 | 0.060 |
| 5 | 0.373 | 0.261 | 0.452 | 0.367 | 0.255 | 0.173 | 0.268 | 0.247 |
| 10 | 0.318 | 0.247 | 0.368 | 0.312 | 0.224 | 0.186 | 0.176 | 0.185 |
| 15 | 0.298 | 0.256 | 0.342 | 0.298 | 0.209 | 0.200 | 0.138 | 0.152 |
| 20 | 0.277 | 0.266 | 0.332 | 0.303 | 0.191 | 0.209 | 0.110 | 0.136 |

Table 5: FEV explained by IST shocks (PCEFE)

# 4 Evidence in a DSGE model

In this section, I provide evidence that incorporating oil and food price shocks into the DSGE framework influences the estimation of IST shocks and its iimportance in business cycles. DSGE models have been widely used to assess the contribution of IST shocks to business-cycle dynamics, with some influential studies suggesting that, once the RPE is used to discipline the model, IST shocks account for only a limited portion of macroeconomic variance. For example, Justiniano et al. (2011) estimate a medium-scale DSGE model with both neutral and IST technology shocks, incorporating standard nominal frictions, while Schmitt-Grohé and Uribe (2012) extend the analysis by introducing news shocks into both types of technologies.[8] The results in this section find that IST shocks play a role in explaining business cycle when introducing RPE as an observable, but their importance is small. It also shows that commodity prices affect the estimation of IST shocks.

[8]Both studies adopt a rich stochastic structure and estimate their models using Bayesian techniques, following the methodology of Smets and Wouters (2007).

I develop a model inspired by Justiniano et al. (2011), integrating a production sector (referred to as the commodity sector) whose price influences the conventional measure of the RPE. As a result, shocks to the price of this sector also affect the RPE, introducing a potential channel through which oil and food price fluctuations distort IST shock estimates.

## 4.1 The model

The model builds on the framework of Justiniano et al. (2011) but introduces an additional commodity goods sector. Unlike standard production sectors, this sector does not rely on capital or labor for production; instead, commodities are assumed to be produced without cost, with their price determined exogenously. This simplification reflects the reality that commodities are traded on global markets, where price fluctuations primarily stem from shifts in global demand and supply. The commodity sector plays a dual role in the economy, functioning as both a production input and a consumption good. As a result, the price of this commodity affects both the price of investment goods—through its influence on production costs—and the price of the consumption bundle, directly impacting consumer expenditures. Consequently, the RPE, defined as the price of investment goods relative to the consumption bundle, is not exclusively driven by IST shocks but also reflects fluctuations in the commodity sector.

The economy consists of two primary components. The first block captures the interactions among the government, firms, households, and labor unions (or labor assemblers). In this block, households finance government expenditures through taxes and short-term bond holdings while supplying firms with utilization-adjusted capital and providing labor to labor assemblers. Firms acquire a bundled labor input from labor assemblers and combine it with utilization-adjusted capital and a portion of the commodity good to produce both investment and final consumption goods, which they sell to households. Additionally, firms supply consumption goods to the government. Households receive interest payments from the government and use final investment goods to generate and supply utilization-adjusted capital. Figure A3, Panel A, in Appendix A illustrates the flow of resources within this first block of the model economy.

The second block represents the production structure of the economy, which is organized into seven tiers: wholesalers, commodity goods producers, intermediaries, non-commodity assemblers, investment goods producers, capital producers, and final consumption goods producers. A key feature of the model is the inclusion of a commodity goods producer. This sector operates under the assumption that the commodity price is exogenously determined and produces the required quantity to meet demand. The commodity goods producer sells its output to both wholesalers and the final consumption goods producer as an input. This setup mirrors the oil sector specification in Guerrieri and Bodenstein (2012), but based on the findings in Section 3, the model extends the commodity sector to include both oil and food.

Figure A3, Panel B, in Appendix A presents a diagram illustrating this block within the economy. The following subsections provide a detailed specification of each component of the production structure.

### 4.1.1 Wholesale sector

The wholesaler is a competitive producer that uses utilization-adjusted capital, labor from the labor assemblers, and the commodity good as inputs. These inputs are combined using a Cobb-Douglas production function that exhibits constant returns to scale, resulting in the production of a single aggregate good. The optimization problem faced by the wholesaler is as follows:

$$\max_{L_t, K_t, O_t} P_t^{\omega} Y_{\omega,t} - R_t^k K_t - W_t L_t - P_t^o O_{p,t} \tag{9}$$

subject to

$$Y_{\omega,t} = K_t^{\alpha} O_{p,t}^{\beta_o} (A_t L_t)^{1-\alpha-\beta_o}, \tag{10}$$

where $Y_{\omega,t}$ is the quantity of wholesale sector, $L_t$ is the labor, $K_t$ is the utilization-adjusted capital, $O_{p.t}$ is the quantity of commodity that is demanded for production of the wholesale product, and $P_t^{\omega}$, $R_t^k$, $W_t$, and $P_t^o$ are the prices of each good. The final product of this sector is sold to each intermediary in a competitive market that uses it as input to produce an intermediary-specific good. The neutral technology variable ($A_t$) follows an $AR(1)$ in logs:

$$logA_t = (1-\rho_a) log\bar{A} + \rho_a logA_{t-1} + \epsilon_{a,t}, \tag{11}$$

with $\epsilon_{a,t}$ is $i.i.d \sim \mathcal{N}(0, \sigma_a)$.

### 4.1.2 Non-commodity assembler

The non-commodity assembler (NCA) uses the output of all intermediary producers as inputs. Since each intermediary producer holds monopoly power over its respective good, I introduce the non-commodity sector first to determine the total demand for each intermediary good. The NCA operates in a perfectly competitive market and faces the following optimization problem:

$$\max_{Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) d_i \tag{12}$$

subject to

$$Y_t = \left[ \int_0^1 Y_t(i)^{\frac{1}{1+\lambda}} d_i \right]^{1+\lambda} \tag{13}$$

where $Y_t$ is the total quantity of the non-commodity assembler. Its price, $P_t$, is the model equivalence of the core consumer price index. $Y_t(i)$ and $P_t(i)$ are the output and price of each intermediary producer. The parameter $\lambda$ governs the substitutability degree among the intermediary goods.[9]

Solving the non-commodity assembler problem allows me to know the demand for each intermediary good $i$:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\frac{1+\lambda}{\lambda}} Y_t \tag{14}$$

### 4.1.3 Intermediaries

There is a mass-one continuum of intermediaries indexed by $i \in [0,1]$. Each intermediary good $Y_t(i)$ is produced by a monopolist and uses the output from the wholesaler as input. They take the quantity of the wholesaler and sell it in a market with monopolistic power. They choose their optimal price given the downward sloping demand curve Equation 2.9 for their good. In doing so, they take into account that, as in Calvo (1983), they can only reset their price with probability $1-\phi_p$. This sticky price structure introduces a dynamic problem for the intermediary, as any present change in price has implications for the future profit of the firm:

$$\max_{P_t(i)} E_t \sum_{s=0}^{\infty} \phi_p^s \Lambda_{t,t+s} \left[P_t(i) - P_t^{\omega}\right] Y_t(i) \tag{15}$$

subject to

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\frac{1+\lambda}{\lambda}} Y_t \tag{16}$$

where $\Lambda_{t,t+s}$ is the stochastic discount factor of the household derived below.

### 4.1.4 Final consumption producing sector

The final consumption good in this economy is produced by a competitive firm that combines a non-commodity good and a commodity good as inputs, using a constant elasticity of substitution (CES) production function. The optimization problem faced by this producer is as follows:

[9] To simplify the solution of the model, I consider a version in which neither price nor wage markup shocks exists.

$$\max_{Y_t^c, O_{c,t}} P_t^f Y_t^f - P_t Y_{c,t} - P_t^o O_{c,t} \tag{17}$$

subject to

$$Y_{f,t} = \left[\omega_f Y_{c,t}^{\frac{1}{1+\lambda_f}} + (1-\omega_f) O_{c,t}^{\frac{1}{1+\lambda_f}}\right]^{1+\lambda_f} \tag{18}$$

where $Y_t^f$ is the quantity of the final consumption good, $P_t^f$ its price, which is the model counterpart of the consumer price index, $o_{c,t}$ the quantity of commodity good that is used in the creation of the final consumption good (e.g. the gasoline that uses directly the oil), $P_t^o$ its price, and $Y_{c,t}$ is the quantity of the non-commodity good that is used to produce consumption goods.

### 4.1.5 Commodity good

The commodity is produced without any cost and the firm sells it to the final consumption-producing sector and to the wholesale sector at a price $P_t^o$. The price follows an $AR(1)$ process:

$$logP_t^o = (1-\rho_o)log\bar{P^o} + \rho_o logP_{t-1}^o + \epsilon_{o,t}, \tag{19}$$

with $\epsilon_{o,t}$ is $i.i.d \sim \mathcal{N}(0, \sigma_o)$. In equilibrium, the supply of this good must be equal to the total amount of commodity good demanded by the wholesale sector ($O_{p,t}$) and the final good producer ($O_{c,t}$) at the given price.

### 4.1.6 Investment good producer

This sector uses the non-commodity good as an input and transforms it into an investment good, which is then sold in a competitive market to the capital good producer. The efficiency of this transformation process is governed by an investment-specific technology process ($\gamma_t$), which influences the productivity of converting the non-commodity good into the investment good. The firm's optimization problem is as follows:

$$\max_{Y_{i,t}} P_t^i I_t - P_t Y_{i,t} \tag{20}$$

subject to

$$I_t = \gamma_t Y_{i,t} \tag{21}$$

where $I_t$ is the quantity of the investment good, $P_t^i$ its price, $Y_{i,t}$ is the quantity of the non-commodity good that is used in the production of investment goods, and $\gamma_t$ is the IST. The latter follows an $AR(1)$ process:

$$log\gamma_t = (1-\rho_i)log\bar{\gamma} + \rho_i log\gamma_{t-1} + \epsilon_{i,t} \tag{22}$$

with $\epsilon_{i,t}$ is $i.i.d \sim \mathcal{N}(0, \sigma_i)$. In this economy, this sector is best thought of as the equipment and machinery in real data. Notice that, solving the problem of the investment good producer yields to:

$$P_t^i \gamma_t = P_t \tag{23}$$

hence,

$$\gamma_t = \frac{P_t}{P_t^i} \tag{24}$$

Equation 24 highlights that the IST process is inversely related to the relative price of investment to the non-commodity good. This adjustment reflects the empirical findings presented in Section 2, where fluctuations in the prices of oil and food may distort the estimation of IST shocks by influencing the consumption price.

The usual measure of the relative price of equipment (i.e. price of equipment relative to the price of consumption expenditure) is:

$$rpi_t = \frac{P_t^i}{P_t^f} \tag{25}$$

#### 4.1.7 Capital good producer

The capital good producer uses the investment good as its input and determines the level of capital production, but is subject to investment adjustment costs.[10] As a result, the firm faces a dynamic optimization problem: changing the level of investment today affects future adjustment costs. Therefore, the firm must decide on a sequence of investment good demands that maximizes the present value of its future profits. The firm's optimization problem is as follows:

$$\max_{\{I_{t+j}\}_0^\infty} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[ P_{t+j}^k I_{k,t+j} - P_{t+j}^i I_{t+j} \right] \tag{26}$$

[10]This firm's problem is analogous to that of a household, which uses the investment good to produce capital for the next period, subject to adjustment costs.

subject to

$$I_{k,t} = \mu_t \Big[ 1 - S\big(\frac{I_t}{I_{t-1}}\big) \Big] I_t, \tag{27}$$

where $I_{k,t}$ is the quantity of capital good produced, $P_t^k$ is its price, $S\big(\frac{I_t}{I_{t-1}}\big)$ is a function that governs the investment adjustment cost of the firm, and $\mu_t$ is the marginal efficiency of investment (MEI), which indicates the efficiency by which the investment goods are converted into capital goods.[11] The latter, follows an $AR(1)$ process:

$$log\mu_t = (1-\rho_m)log\bar{\mu} + \rho_m log\mu_{t-1} + \epsilon_{m,t}, \tag{28}$$

where $\epsilon_{m,t}$ is $i.i.d \sim \mathcal{N}(0, \sigma_m)$. For simplicity, I assume that the cost of adjustment is:

$$S\Big(\frac{I_t}{I_{t-1}}\Big) = \frac{\psi_s}{2}\Bigg[\Big[\frac{I_t}{I_{t-1}}\Big] - 1\Bigg]^2 \tag{29}$$

### 4.1.8 Household

The household decides on the quantity of final consumption goods to demand from the final consumption good producer, the amount of labor to supply to labor unions, and the capital goods to demand from the capital goods producer, which it uses to build its capital stock. The household also chooses the level of capital utilization, which is rented to the wholesaler, and the amount of debt to borrow from the monetary authority. Since the household controls the level of capital utilization, higher utilization levels result in higher costs, which are incurred in units of final consumption goods. The household's optimization problem is as follows:

$$\max_{C_t, L_{s,t}, I_t, B_t, u_t, \hat{K}_t} \sum_{t=0}^{\infty} \beta^t b_t \Bigg[ \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \psi \frac{L_{s,t}^{1+\chi_L}}{1+\chi_L} \Bigg] \tag{30}$$

subject to

$$P_t^f C_t + P_t^k I_{k,t} + B_t + T_t = R_{t-1}B_{t-1} + \tilde{W}_t L_{s,t} + R_t^k u_t \tilde{K}_{t-1} - P_t^f a(u_t)\tilde{K}_{t-1} + \Pi_t \tag{31}$$

$$\tilde{K}_t = (1-\delta)\tilde{K}_{t-1} + I_{k,t} \tag{32}$$

$$logb_t = (1-\rho_b)log\bar{b} + \rho_b logb_{t-1} + \epsilon_{b,t}, \tag{33}$$

where $C_t$ is the consumption of the final good, $L_{s,t}$ is the labor supplied to the labor unions which pays $\tilde{W}_t$, $I_{k,t}$ is the investment good, $T_t$ are taxes paid to the fiscal authority, $u_t$ is

[11] Ramey (2016) and Justiniano et al. (2011) provide a comprehensive discussion regarding the importance of the difference between MEI and IST.

the level of utilization of capital, and $a(u_t)$ is a function that indicates the cost in final consumption goods units that must be paid for each level of utilization. $\Pi_t$ is the total quantity of profits. $B_t$ is the amount of one-period risk-free bond held by the household, while $R_t$ is the interest rate that it pays. Finally, $\tilde{K}_t$ is the stock of physical capital held by the household. Notice that according to this specification, $K_t = u_t \tilde{K}_{t-1}$.

The cost of the utilization of capital follows:

$$a(u_t) = \frac{R^K_{ss}}{\chi u}\left[1 - e^{-\chi_u(u_t-1)}\right] \tag{34}$$

Finally, the measure of GDP in this economy is as follows:

$$gdp_t = Y^f_t + \frac{P^k_t}{P^f_t} \times I_{k,t} - a(u_t)\hat{K}_{t-1} \tag{35}$$

$Y^f_t$ contains both the consumption from households and fiscal authority and $I_{k,t}$ is the final amount of investment.

### 4.1.9 Labor market

There is a continuum of labor unions indexed by $l \in [0, 1]$ that hire labor from the household at $\tilde{W}_t$ in perfect competition and sell labor to a labor packer at price $W_t(l)$ in monopolistic competition. These wages are updated with sticky price frictions a la Calvo (1983) with a probability of updating wages being $(1 - \phi_w)$. The labor packer sells the bundle of labor to the wholesaler at a price $W_t$. The problem of the labor packer is analogous to the non-commodity assembler:

$$\max_{L_t(l)} W_t L_t - \int_0^1 W_t(l) L_t(l) d_l \tag{36}$$

subject to

$$L_t = \left[\int_0^1 L_t(l)^{\frac{1}{1+\lambda_l}} d_l\right]^{1+\lambda_l} \tag{37}$$

$$\tag{38}$$

As in the case of non-commodity assemblers, the demand for each union's product is:

$$L_t(l) = \left[\frac{W_t(l)}{W_t}\right]^{-\frac{1+\lambda_l}{\lambda_l}} L_t. \tag{39}$$

Using this demand, the unions (with monopolistic competition) face the following problem:

$$\max_{W_t(l)} E_t \sum_{s=0}^{\infty} \phi_w^s \Lambda_{t,t+s} \Big[ W_t(l) - \tilde{W}_t \Big] L_t(l) \tag{40}$$

subject to

$$L_t(l) = \left[ \frac{W_t(l)}{W_t} \right]^{-\frac{1+\lambda_l}{\lambda_l}} L_t \tag{41}$$

Finally, the real wage (wage in final good terms) is computed as:

$$wp_t = \frac{W_t}{P_t^f} \tag{42}$$

Where $W_t$ is the aggregated wage index received by the labor unions.

#### 4.1.10 Government

The government consists of two distinct entities: fiscal and monetary authorities. The fiscal authority finances its activities through a combination of debt issuance and taxes paid by the household.[12] The authority allocates its revenue to purchase a portion of the final consumption good as government expenditure and to repay the debt from the previous period. Government expenditure is assumed to follow a stochastic process. The following equations characterize the behavior of the fiscal authority:

$$P_t^f G_t + R_{t-1} B_{t-1} = B_t + \tau P_t^f Y_t \tag{43}$$

$$G_t = \left[ 1 - \frac{1}{g_t} \right] Y_t^f \tag{44}$$

$$log(g_t) = (1 - \rho_g) log(\bar{g}) + \rho_g log(g_{t-1}) + \epsilon_{g,t} \tag{45}$$

where $\epsilon_{g,t}$ is $i.i.d \sim \mathcal{N}(0, \sigma_g)$.

Meanwhile, the monetary authority fixes the interest rate by following a Taylor-type monetary policy rule:

$$\begin{aligned} \log(R_t) = (1 - \rho_r) \log(\bar{R}) + \rho_r \log(R_{t-1}) \\ + (1 - \rho_r)\theta_\pi [\log(\pi_t) - \log(\bar{\pi})] \\ + (1 - \rho_r)\theta_y [\log(Y_t) - \log(\bar{Y})] + \epsilon_{r,t} \end{aligned} \tag{46}$$

where $\epsilon_{r,t}$ is $i.i.d \sim \mathcal{N}(0, \sigma_r)$, and $\pi_t = \frac{P_t}{P_{t-1}}$ is the headline (core) inflation.

[12] I assume that taxes are a constant proportion of the household's income derived from the non-commodity good.

## 4.2 Solution

I use the perturbation method of order one around the steady state as in Villemot (2011).[13] I relegate the specification of the steady state and the equations that characterize the equilibrium to Appendix C.

## 4.3 Estimation

I employ Bayesian methods to estimate the model parameters' posterior mean values and distributions. This approach combines the likelihood function with prior parameter distributions to perform the estimation.[14] The estimation uses quarterly data from 1964 to 2019 for nine key economic variables. Since the model does not account for long-term trends, I follow the detrending approach recommended by Liao and Chen (2023) and Born and Pfeifer (2021). Instead of applying the one-sided Hodrick-Prescott filter, I use polynomial detrending.[15]

### 4.3.1 Priors and fixed parameters

Following the approach of Justiniano et al. (2011), I fix a small set of parameters commonly used in the literature. Specifically, I set the depreciation rate $\delta$ to 0.025, the intertemporal elasticity of substitution parameter $\sigma$ to 2.0, and the investment adjustment cost parameter $\psi_s$ to 1.0. Additionally, I fix the weight of the commodity sector in the consumption bundle $(1 - \omega_f)$ to 0.177. This value aligns with the average share of food and energy in final private expenditure in the U.S. national accounts from 1947 to 2019, ensuring consistency with historical data.

Most of the priors for the estimated parameters follow the values proposed by Justiniano et al. (2011). For parameters specific to the commodity sector, I set the prior for the weight of commodities in the wholesale sector ($\beta_o$) to follow a beta distribution with a mean of 0.10. The prior for the parameter governing the elasticity of substitution in the final production sector ($\lambda_f$) follows a normal distribution with a mean of 10.0.[16]

[13]I use the software platform Dynare 5.3 to solve the model and to estimate it. See Adjemian et al. (2024).

[14]For a detailed explanation of Bayesian estimation methods, see An and Schorfheide (2007); for a review, refer to Fernández-Villaverde and Guerrón-Quintana (2021). I use 20,000 draws for the MCMC algorithm, with the Random-Walk-Metropolis-Hastings sampler. The proposal density is the normal distribution with a covariance matrix from the inverse hessian at the mode. I compute the identification using the posterior mean of the parameter space. I use Matlab 2022a and Dynare version 5.3.

[15]The Hodrick-Prescott filter has been criticized for its limitations in estimating cyclical components (Hamilton, 2018). Polynomial detrending has been employed as an alternative in studies such as Uribe and Schmitt-Grohé (2017) and Canova (2020).

[16]A higher value of $\lambda_f$ implies that the elasticity of substitution in this sector approaches 1.0.

### 4.3.2 Data

I estimate the model using the following data:

$$X_t = [gdp_t, C_t, I_{k,t}, rpi_t, \pi_t, r_t, w_t]$$

where $gdp_t$, $C_t$, and $I_{k,t}$ are the logs of GDP, consumption and investment per càpita used in Section 2. $rpi_t$ is the log of the relative price of equipment. $r_t$ is the FED interest shadow rate from Wu and Xia (2016). $w_t$ is the log of real wage divided by the final private expenditure price without food, energy and durables, and $\pi_t$ is the quarterly change of the log of final private expenditure without food and energy prices.

### 4.3.3 Estimation results

Table 6 presents the estimated parameters, along with their prior distributions and posterior results. The posterior means are consistent with findings in the related literature, further validating the model's calibration.

Qualitatively, the model replicates several well-established stylized facts from the business cycle literature: (i) GDP is positively correlated with both consumption and investment; (ii) the standard deviation of consumption is lower than that of GDP, while the standard deviation of investment is higher; and (iii) GDP and hours worked exhibit a positive correlation. However, the quantitative values of these correlations and standard deviations deviate from those observed in the data. Additionally, the model fails to reproduce the high correlation between consumption and investment. Table 7 compares the empirical moments with those generated by the simulated model.

## 4.4 Model results

I structure the results into three main analyses. First, I present the IRFs of key macroeconomic variables in response to both an IST shock and a commodity good price shock. Second, I evaluate the relative contribution of different shocks to the variance of these variables within the model. Finally, I simulate the model to assess its ability to replicate the responses observed in the empirical data.

The findings can be summarized as follows:

1. The model's IRFs to IST shocks are broadly consistent with those observed in the empirical analysis of IST shocks that control for oil and food price movements. Additionally, the IRFs to commodity good supply shocks indicate that a decrease in the price of the commodity good leads to an increase in the RPI, confirming that IST shocks are not the only factor driving movements in the RPI.
2. Based on the model specification, estimation, and data used in this analysis, IST shocks explain only a small fraction of the variance in consumption, GDP, and investment,

| Par. | P. Dist. | Prior M. | Post. M. | 90% Low | 90% High | Prior SD. |
|---|---|---|---|---|---|---|
| $\alpha$ | beta | 0.300 | 0.2991 | 0.2969 | 0.3009 | 0.0500 |
| $\beta^o$ | beta | 0.100 | 0.0801 | 0.0794 | 0.0809 | 0.0250 |
| $\lambda$ | normal | 0.100 | 0.1074 | 0.1064 | 0.1085 | 0.0250 |
| $\phi_p$ | beta | 0.660 | 0.6316 | 0.6302 | 0.6334 | 0.0500 |
| $\lambda_f$ | norm | 10.000 | 9.9614 | 9.9322 | 9.9807 | 0.5000 |
| $h$ | beta | 0.500 | 0.9525 | 0.9520 | 0.9528 | 0.1000 |
| $\chi_L$ | gamma | 2.000 | 0.7747 | 0.7570 | 0.7891 | 0.7500 |
| $100(\beta^{-1} - 1)$ | gamma | 0.250 | 0.4114 | 0.4055 | 0.4169 | 0.1000 |
| $\lambda_l$ | normal | 0.250 | 0.2484 | 0.2468 | 0.2495 | 0.0500 |
| $\phi_w$ | beta | 0.760 | 0.9128 | 0.9104 | 0.9155 | 0.0500 |
| $\chi_u$ | gamma | 5.000 | 3.0612 | 3.0342 | 3.0949 | 1.0000 |
| $\theta_\pi$ | gamma | 1.500 | 1.8092 | 1.7975 | 1.8170 | 0.3000 |
| $\theta_y$ | gamma | 0.500 | 0.3670 | 0.3650 | 0.3695 | 0.0500 |
| $\tau$ | beta | 0.200 | 0.4941 | 0.4894 | 0.4981 | 0.1000 |
| $\rho_a$ | beta | 0.400 | 0.4768 | 0.4705 | 0.4827 | 0.2000 |
| $\rho_i$ | beta | 0.400 | 0.4077 | 0.4034 | 0.4142 | 0.2000 |
| $\rho_m$ | beta | 0.400 | 0.8685 | 0.8613 | 0.8783 | 0.2000 |
| $\rho_o$ | beta | 0.400 | 0.9090 | 0.9054 | 0.9142 | 0.2000 |
| $\rho_g$ | beta | 0.400 | 0.8552 | 0.8502 | 0.8595 | 0.2000 |
| $\rho_r$ | beta | 0.400 | 0.9489 | 0.9445 | 0.9547 | 0.2000 |
| $\rho_b$ | beta | 0.400 | 0.4816 | 0.4736 | 0.4916 | 0.2000 |
| $\sigma_a$ | invgam | 0.900 | 0.5298 | 0.4983 | 0.5633 | 1.0000 |
| $\sigma_i$ | invgam | 0.500 | 0.5077 | 0.5067 | 0.5085 | 1.0000 |
| $\sigma_m$ | invgam | 0.500 | 0.2728 | 0.2519 | 0.3058 | 1.0000 |
| $\sigma_o$ | invgam | 0.500 | 0.4628 | 0.4380 | 0.4907 | 1.0000 |
| $\sigma_g$ | invgam | 0.100 | 0.0123 | 0.0118 | 0.0129 | 1.0000 |
| $\sigma_r$ | invgam | 0.100 | 0.3619 | 0.3518 | 0.3696 | 1.0000 |
| $\sigma_b$ | invgam | 0.100 | 0.9682 | 0.9304 | 1.0094 | 1.0000 |

Table 6: Estimation Results of parameters in the DSGE model.

| Metric | Data | Model |
|---|---|---|
| S.D. Inv./S.D. GDP | 3.280 | 4.725 |
| S.D. Con/S.D. GDP | 0.734 | 0.447 |
| corr GDP-Inv | 0.817 | 0.863 |
| Corr GDP-Con | 0.925 | 0.446 |
| Corr Con-INV | 0.661 | 0.057 |
| Corr GDP-Labor | 0.773 | 0.358 |
| Corr GDP-RPI | 0.011 | -0.158 |

Table 7: Comparison of moments between Data and Model

although the importance in the latter is higher. Specifically, IST shocks contribute to 10.1% of GDP variance, 0.6% of consumption variance, and 17.7% of investment variance.

3. When computing IRFs using simulated data and applying the empirical methodology outlined in Sections 2.2 and 3.2, the results align with those found in actual data: relying on the RPI without adjustments leads to short-run increases in price variables and a decline in real wages. This highlights the challenges associated with estimating IST shocks using the RPI.

Figure A4 in Appendix A presents the model's IRFs in response to a positive IST shock ($\epsilon_{i,t}$). Following the shock, real wages increase, while both non-commodity inflation and the final consumption good inflation rate decline. Simultaneously, GDP and investment initially rise before the effect dissipates after several periods. Consumption does not increase just after the shock but increases after some lags. Additionally, labor demand from wholesalers ($ld$) initially rises before declining after a few quarters. These responses closely align with those analyzed in Section 2.4, once IST shocks are adjusted for food and oil price fluctuations.

In contrast, Figure A5 in Appendix A illustrates the macroeconomic responses to a commodity price shock. A key insight is that an increase in commodity prices leads to a decline in the RPE, reinforcing the idea that RPE movements reflect fluctuations in the commodity sector. Notably, inflation measures rise, while real wages decline in response to the shock.

Table 8 presents the variance decomposition from the DSGE model. The columns represent the shocks incorporated into the model, while the rows display the macroeconomic variables under analysis. The results indicate that the neutral technology shock, commodity price shock, monetary policy shock, MEI shock, and preferences shock account for a significant portion of the variance in price-related variables, as well as a share of the variance in investment, consumption, and GDP. In fact, 13.1% of the variance of the RPi is related to commodity price shocks. The IST shock plays a small role in explaining investment variance but contributes much less to GDP and consumption, accounting for 10.1% of GDP variance, 0.6% of consumption variance, and 17.7% of investment variance.[17] Finally, the commodity price shock emerges as a key driver of inflation dynamics, explaining 48.5% of CPI variance.

Finally, I simulate time series for the same macroeconomic variables used in Section 2, generating 10,000 draws of GDP, consumption, investment, labor demand from wholesalers, interest rates, wages, CPI, core CPI inflation, commodity prices, RPI, and the RPI relative to core prices.[18] Using the same identification strategy as in Section 2, where the RPI measure still captures information from commodity price shocks, the real variables exhibit behavior qualitatively similar to the empirical findings: GDP and investment rise immediately after the shock, while consumption initially declines. Meanwhile, the price variables respond with an immediate increase in CPI and commodity prices (Figure A6 in Appendix A).

In contrast, when the RPI is measured relative to core prices (as in the empirical analysis in Section 3.2) the impulse response of consumption shows a delayed increase a few quarters after the IST shock, while the responses of other real variables remain qualitatively similar. The behavior of price-related variables, however, changes more noticeably: both CPI and core CPI inflation decline following the shock, and the real wage rises, consistent with the empirical evidence. This contrast underscores the importance of properly accounting for commodity price influences when interpreting the RPI and its role in identifying IST shocks (see Figure A7 in Appendix A).

| | $\epsilon_{a,t}$ | $\epsilon_{i,t}$ | $\epsilon_{m,t}$ | $\epsilon_{o,t}$ | $\epsilon_{g,t}$ | $\epsilon_{r,t}$ | $\epsilon_{b,t}$ |
|---|---|---|---|---|---|---|---|
| rpi | 0.0 | 86.9 | 0.0 | 13.1 | 0.0 | 0.0 | 0.0 |
| gdp | 4.9 | 10.4 | 2.2 | 8.0 | 0.3 | 67.5 | 6.7 |
| cons. | 0.0 | 0.6 | 5.5 | 8.0 | 0.0 | 29.0 | 56.8 |
| inv. | 3.7 | 17.7 | 20.5 | 2.9 | 0.0 | 54.5 | 0.7 |
| labor | 38.7 | 3.4 | 3.0 | 23.3 | 0.1 | 28.6 | 2.8 |
| wages | 15.2 | 2.1 | 10.7 | 52.9 | 0.0 | 17.9 | 1.2 |
| Inflation | 34.8 | 1.1 | 6.0 | 49.6 | 0.0 | 8.0 | 0.3 |
| c. Inflation | 59.8 | 2.0 | 10.3 | 13.5 | 0.0 | 13.8 | 0.6 |
| Com. Price. | 0.0 | 0.0 | 0.0 | 100.0 | 0.0 | 0.0 | 0.0 |

Table 8: Variance Decomposition in the DSGE model (%)

[17] These results exhibit a similar pattern to the empirical FEVD results in Table 3: IST explains a larger share of investment's FEV and a smaller share of GDP and consumption variance at longer horizons, though the magnitudes differ.

[18] Note that the RPI relative to core prices is the inverse of the IST variable, defined as $\frac{1}{\gamma_t} = \frac{P_t^i}{P_t}$.

# 5 Conclusions

The RPE has been a central topic in macroeconomic research, primarily due to its relationship with IST. Empirical studies suggest that IST shocks, identified through the RPE, play a key role in explaining the behavior of macroeconomic variables during business cycle fluctuations. This paper demonstrates that the estimated IST shocks remain influenced by exogenous movements in oil and food prices. These price fluctuations disproportionately affect the cost of non-durable consumption goods relative to equipment prices, introducing potential biases in the estimation of IST shocks.

After controlling for these price effects, IRFs indicate that IST shocks lead to a decline in price levels, while GDP and investment rise immediately, and consumption responds with a lag. Moreover, the share of FEV attributed to IST shocks is significantly reduced for GDP, consumption, and investment compared to estimates that do not account for oil and food price fluctuations.

A medium-scale DSGE model with a rich stochastic structure helps explain these empirical findings. The model incorporates a commodity goods sector, where prices are determined exogenously, and its output serves as an input for final consumption goods production. As a result, the RPE reflects information about commodity sector price fluctuations. Simulations from the DSGE model qualitatively reproduce the IRFs observed in the empirical data, reinforcing the importance of properly accounting for commodity price movements in IST shock identification.

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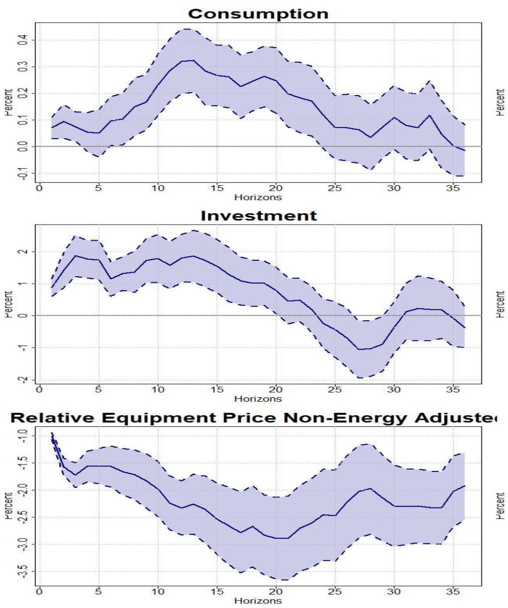
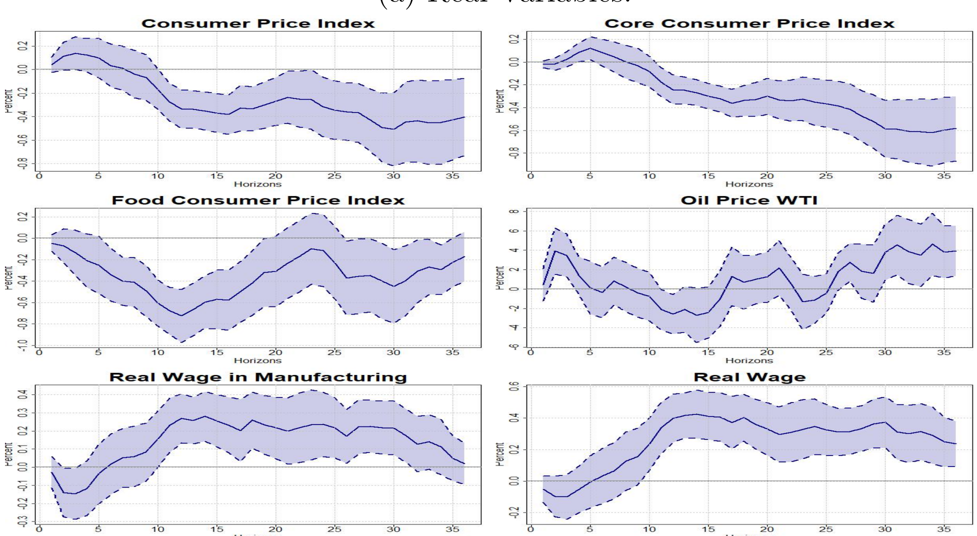
# Appendices

## A Figures



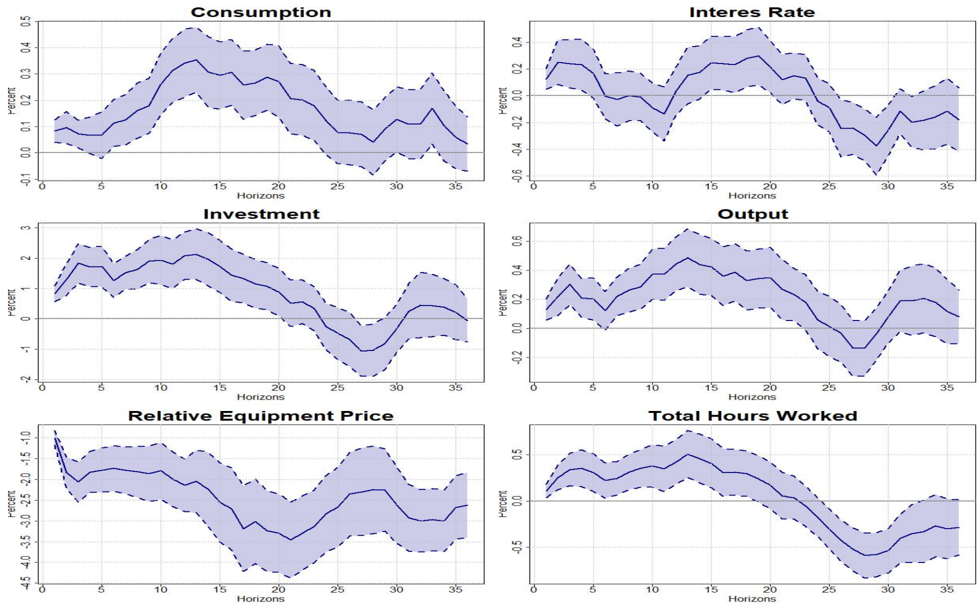

(a) Real Variables.



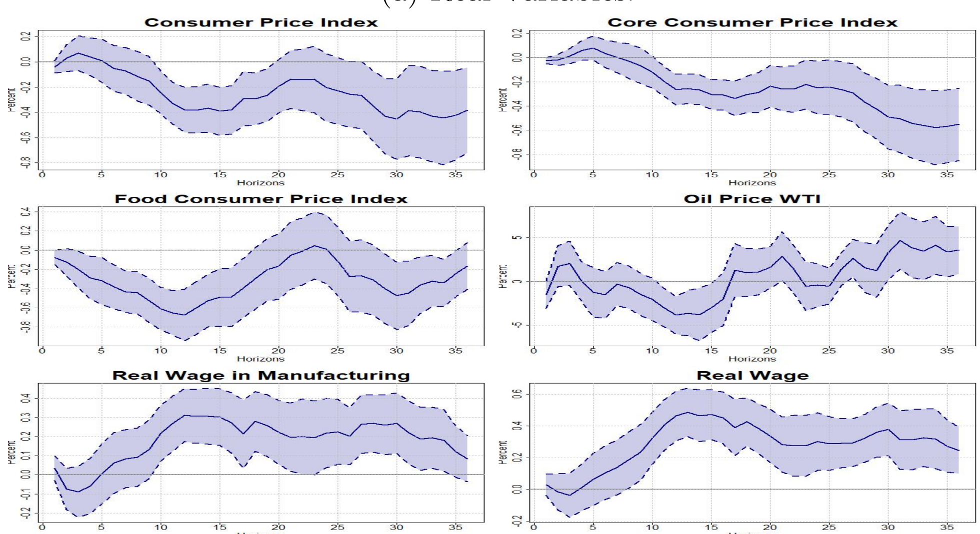

(b) Price Variables

Figure A1: IRF to IST shocks using RPENE.

Note: Impulse response of macroeconomic variables to IST shocks estimated using RPENE.The shaded areas represent the 90.0% confidence interval.



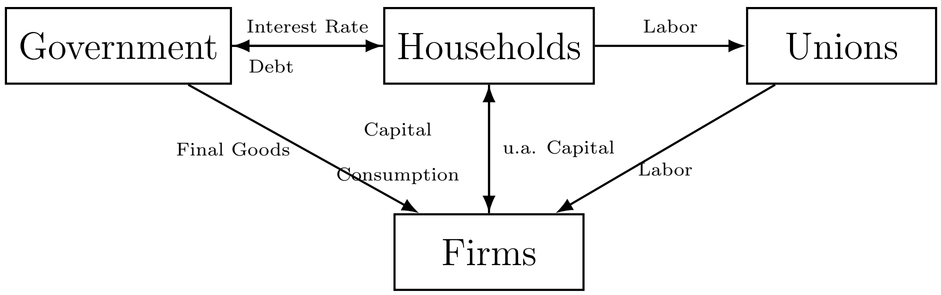

(a) Real Variables.



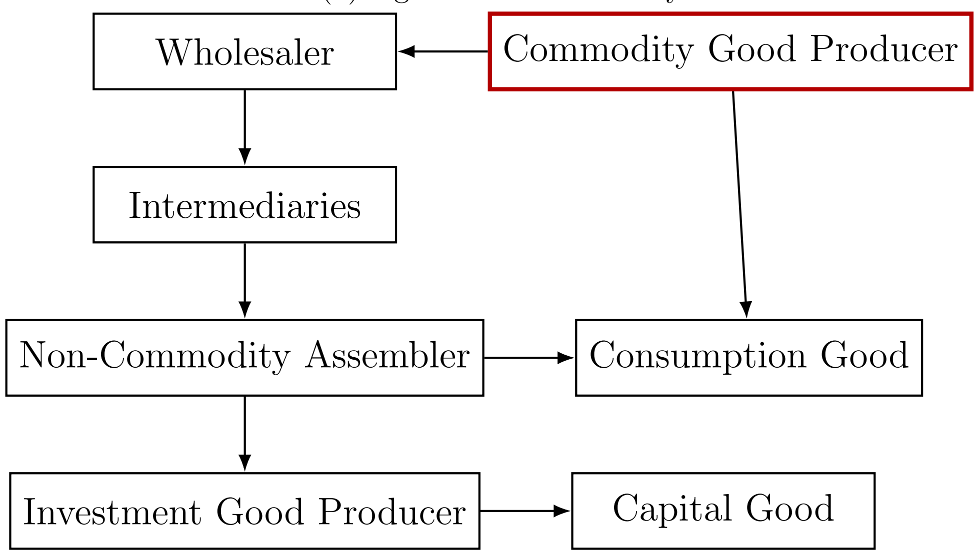

(b) Price Variables

Figure A2: IRF to IST shocks using PCEFE.

Note: Impulse response of macroeconomic variables to IST shocks estimated using PCEFE. The shaded areas represent the 90.0% confidence interval.




(a) Agents in the economy.




(b) Producing sectors.

Figure A3: Diagram of the flows in the model economy.

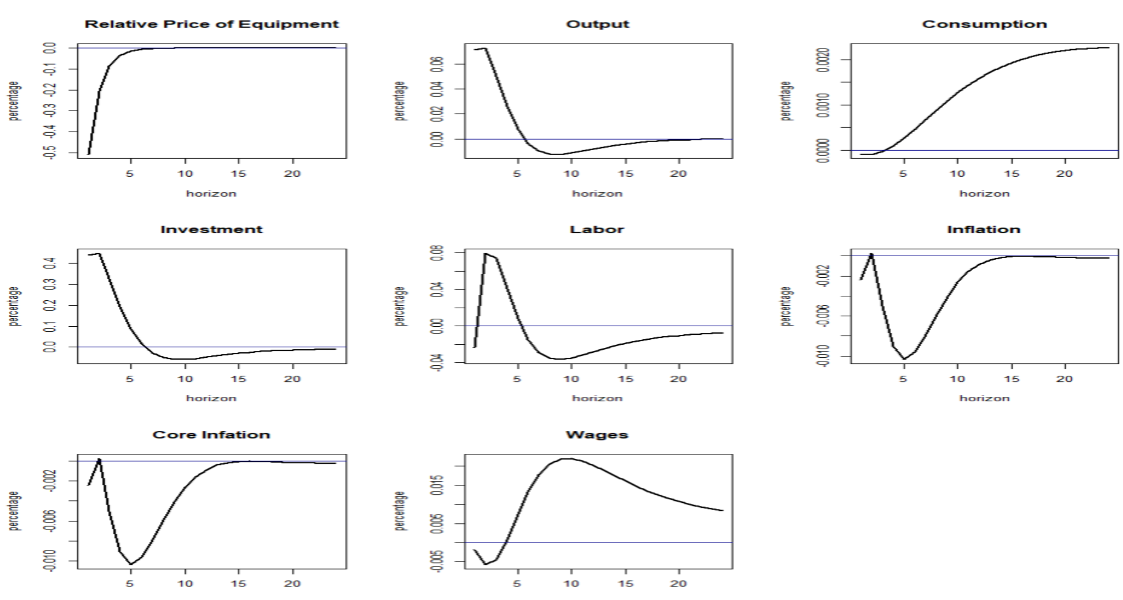



Figure A4: IRF after an IST shock in the DSGE model.

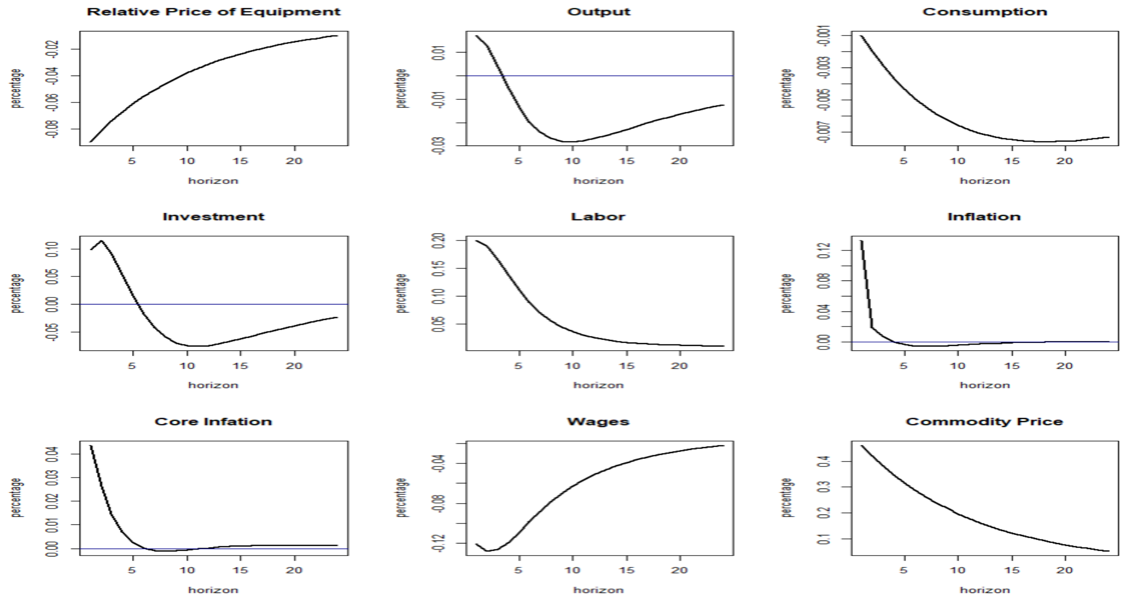
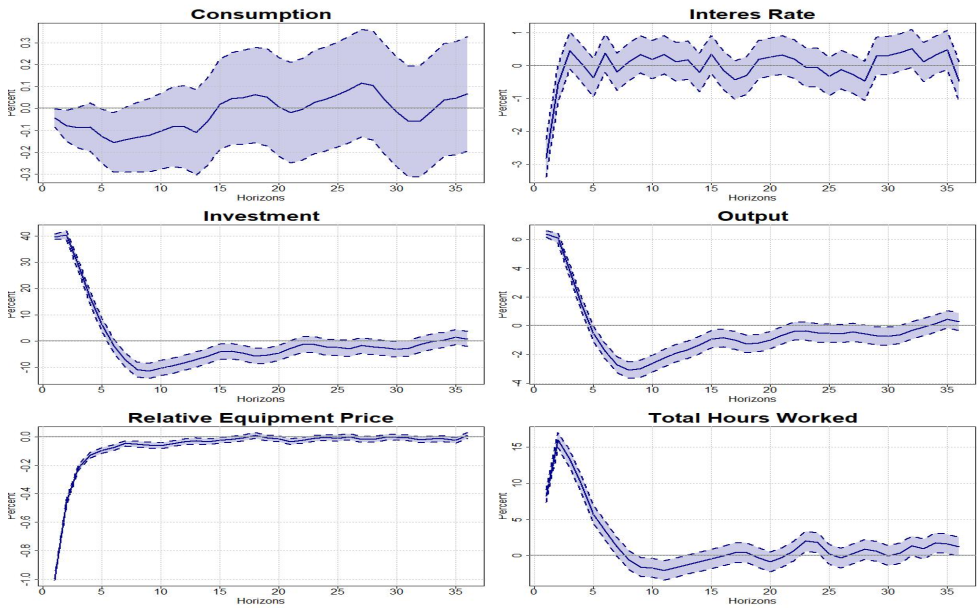
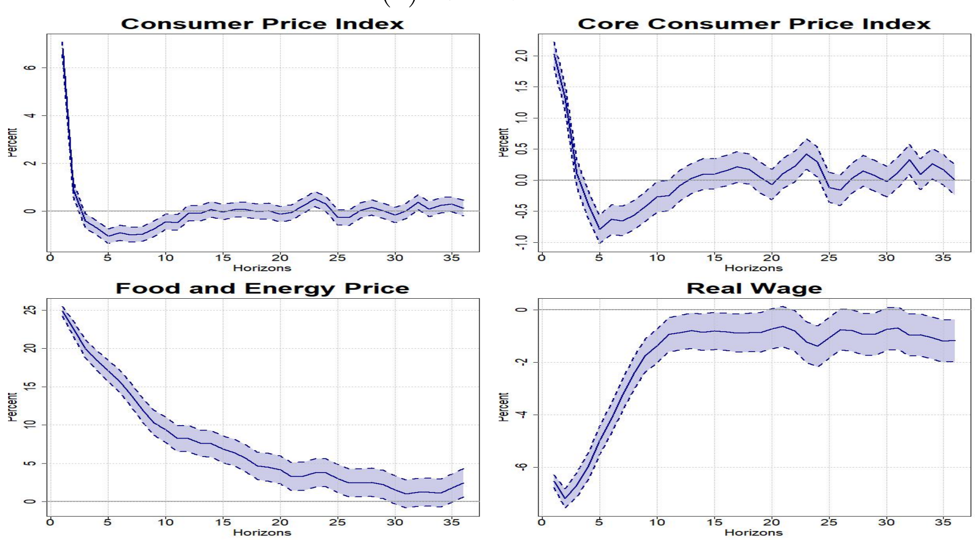



Figure A5: IRF after a commodity price shock in the DSGE model.



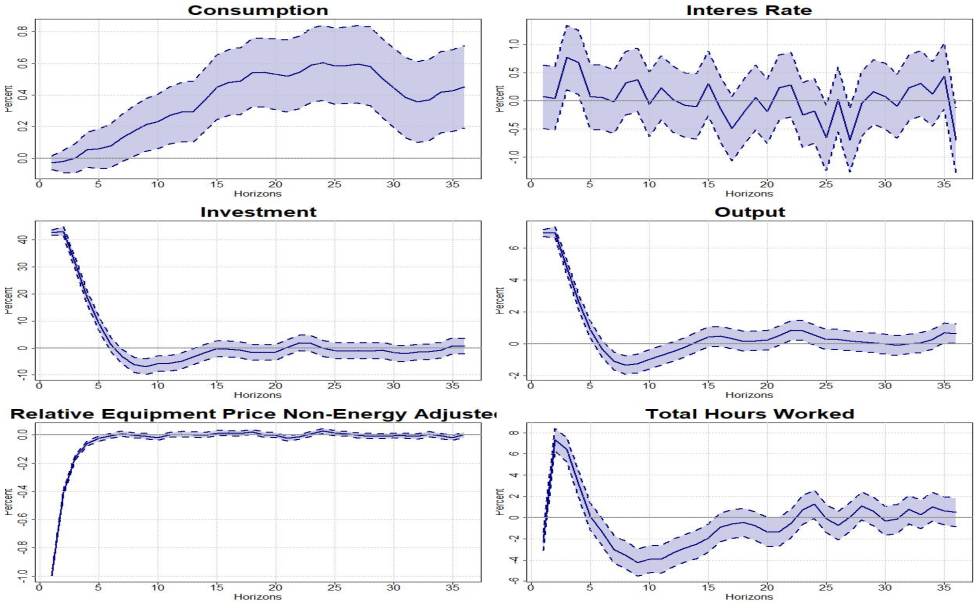

(a) Real Variables.



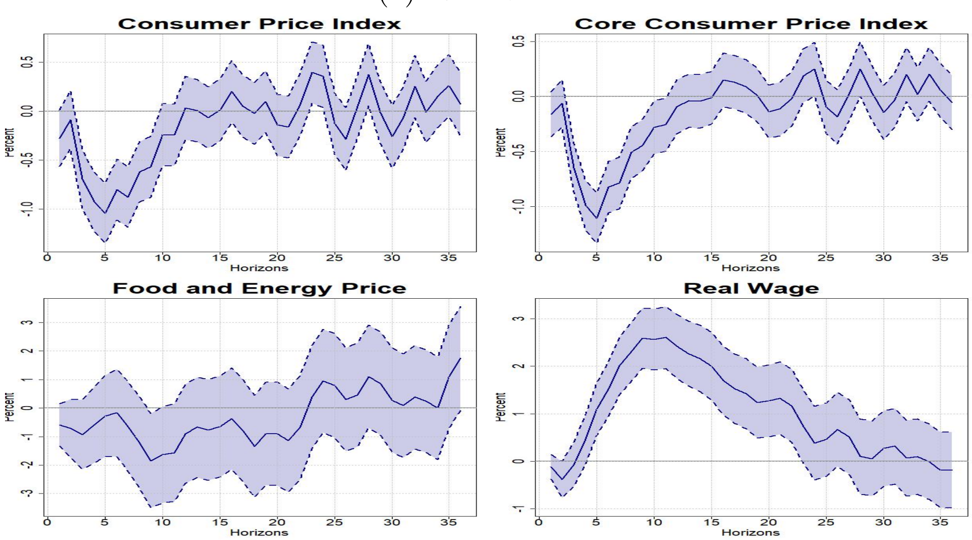

(b) Price Variables

Figure A6: IRF to IST shocks with simulated variables.

Note: Impulse response of macroeconomic variables to IST shocks estimated with the RPI using the simulated data from the DSGE model. The shaded areas represent the 90.0% confidence interval.




(a) Real Variables.




(b) Price Variables

Figure A7: IRF to IST shocks using RPENE with simulated variables.

Note: Impulse response of macroeconomic variables to IST shocks estimated with the RPI with respect to core price (analog to RPENE in the data), using the simulated data from the DSGE model. The shaded areas represent the 90.0% confidence interval.

# B Importance of BZK's IST news shocks

I compute the percentage of the forecast error variance explained by BZK's news shocks, both with and without the inclusion of oil and food price indexes. It is important to note that, according to the original document, the estimation already incorporates inflation (measured as the annual change in the CPI) into the system of equations, yet there is still a correlation with oil price shocks. To better account for the effects of food prices and WTI (West Texas Intermediate crude oil prices), I remove inflation from the estimation and instead introduce three components that are part of the CPI: Core CPI, Food CPI, and WTI.

Since Core CPI data is only available after 1957, the forecast error variance decomposition (FEVD) is calculated using data from that period onward for both the adjusted system (the clean one) and the baseline model. This adjustment allows for a more accurate assessment of how food and oil prices contribute to the variance explained by BZK's news shocks, ensuring that the results account for key components of the CPI while isolating their effects.

Table 4 shows the estimation with (Clean) and without (Base) the variables in the system. Notice that there is decrease in the forecast error variance of GDP, consumption and hours worked explained by the IST news shocks once I introduce both variables in the system.

| | GDP | GDP | Inv. | Inv. | Cons. | Cons. | Hours | Hours |
|---|---|---|---|---|---|---|---|---|
| **h** | **Base** | **Clean** | **Base** | **Clean** | **Base** | **Clean** | **Base** | **Clean** |
| 1 | 0.115 | 0.056 | 0.125 | 0.233 | 0.403 | 0.086 | 0.94 | 0.119 |
| 5 | 0.543 | 0.225 | 0.506 | 0.492 | 0.602 | 0.258 | 0.605 | 0.296 |
| 10 | 0.573 | 0.244 | 0.488 | 0.440 | 0.635 | 0.290 | 0.678 | 0.348 |
| 15 | 0.562 | 0.226 | 0.444 | 0.389 | 0.629 | 0.270 | 0.621 | 0.281 |
| 20 | 0.593 | 0.221 | 0.475 | 0.391 | 0.644 | 0.256 | 0.609 | 0.258 |

Table 9: FEV explained by IST news shock as in BZK

# C Equilibrium Equations

To solve the model, I obtain the equations that characterize the equilibrium and redefine the nominal prices using the price of the non-commodity assembler as a numeraire. Nominal variables in lower case are the ratio of each price over the numeraire. The system of equations that characterize the equilibrium is as follows:

- Equations from the Wholesaler

$$p_t^w[1-\alpha-\beta_o]A_t^{1-\alpha-\beta_o}K_t^{\alpha}O_{p,t}^{\beta_o}L_t^{-\alpha-\beta_o} = w_t \tag{47}$$
$$p_t^w[\alpha]A_t^{1-\alpha-\beta_o}K_t^{\alpha-1}O_{p,t}^{\beta_o}L_t^{1-\alpha-\beta_o} = r_t^k \tag{48}$$
$$p_t^w[\beta_o]A_t^{1-\alpha-\beta_o}K_t^{\alpha}O_{p,t}^{\beta_o-1}L_t^{1-\alpha-\beta_o} = p_t^o \tag{49}$$

- Three Equations Phillips curve:

$$\tilde{p}_t = (1+\lambda)\frac{x_{2,t}}{x_{1,t}} \tag{50}$$
$$x_{1,t} = Y_t + E_t\Big[\phi_p\Lambda_{t,t+1}\pi^{\frac{1+\lambda}{\lambda}}x_{1,t+1}\Big] \tag{51}$$
$$x_{2,t} = p_t^w Y_t + E_t\Big[\phi_p\Lambda_{t,t+1}\pi^{\frac{1+\lambda}{\lambda}+1}x_{2,t+1}\Big] \tag{52}$$

- Equations from the final good producer

$$1 = p_t^f Y_{f,t}^{\frac{\lambda_f}{1+\lambda_f}}\omega_f Y_{c,t}^{\frac{-\lambda_f}{1+\lambda_f}} \tag{53}$$
$$p_t^o = p_t^f Y_{f,t}^{\frac{\lambda_f}{1+\lambda_f}}[1-\omega_f]O_{c,t}^{\frac{-\lambda_f}{1+\lambda_f}} \tag{54}$$
$$Y_t^f = \Big[\omega_f Y_{c,t}^{\frac{1}{1+\lambda_f}} + (1-\omega_f)O_{c,t}^{\frac{1}{1+\lambda_f}}\Big]^{1+\lambda_f} \tag{55}$$

- Equations from the investment producer

$$p_t^i\gamma_t = 1 \tag{56}$$
$$I_t = \gamma_t Y_{i,t} \tag{57}$$

- Equations from the capital producer

$$p_t^k \mu_t [1 - s_t - s_{p,t} \frac{I_t}{I_{t-1}}] + E_t \Big[ \Lambda_{t,t+1} [p_{t+1}^k \mu_{t+1} s_{p,t+1} [\frac{I_t}{I_{t-1}}]^2 \pi_{t+1}] \Big] = p_t^i \tag{58}$$

$$I_{k,t} = \mu_t [1 - s_t] I_t \tag{59}$$

- Equations from the household

$$\lambda_t^c = \psi \frac{L_{s,t}^{\chi_l}}{\tilde{w}_t} \tag{60}$$

$$p_t^f \lambda_t^c = b_t [C_t - hC_{t-1}]^\sigma - E_t \Big[ \beta_c h b_{t+1} [C_{t+1} - hC_t]^\sigma \Big] \tag{61}$$

$$\Lambda_{t,t+1} = E_t \Big[ \beta_c \frac{\lambda_{t+1}^c}{\lambda_t^c} \Big] \tag{62}$$

$$1 = E_t \Big[ R_{t+1} \Lambda_{t,t+1} \Pi_{t+1}^{-1} \Big] \tag{63}$$

$$r_t^k = p_t^f a_{up,t} \tag{64}$$

$$\lambda_t^{c2} = \lambda_t^c p_t^f \tag{65}$$

$$\lambda_t^{c2} = E_t \Big[ \beta_c \lambda_{t+1}^c [r_{t+1}^k u_{t+1} - p_{t+1}^f a_{u,t+1}] + \beta_c \lambda_{t+1}^{c2} [1 - \delta] \Big] \tag{66}$$

$$K_t = u_t \tilde{K}_{t-1} \tag{67}$$

$$\tilde{K}_t = [1 - \delta] \tilde{K}_{t-1} + I_{k,t} \tag{68}$$

- Fiscal Authority Equations

$$P_t^f G_t + R_{t-1} B_{t-1} = B_t + \tau P_t^f Y_t \tag{69}$$

$$G_t = \Big[ 1 - \frac{1}{g_t} \Big] Y_t^f \tag{70}$$

- Wages Three Equations

$$\tilde{w}_t = [1 + \lambda_l] \frac{f_{1,t}}{f_{2,t}} \tag{71}$$

$$f_{1,t} = \tilde{w}_t [w_t^{\frac{1+\lambda_l}{\lambda_l}}] L_t + E_t \Big[ \phi_w \Lambda_{t,t+1} \pi_{t+1}^{\frac{1+\lambda_l}{\lambda_l} + 1} f_{1,t+1} \Big] \tag{72}$$

$$f_{2,t} = [w_t^{\frac{1+\lambda_l}{\lambda_l}}] L_t + E_t \Big[ \phi_w \Lambda_{t,t+1} \pi_{t+1}^{\frac{1+\lambda_l}{\lambda_l}} f_{2,t+1} \Big] \tag{73}$$

- Aggregate price and dispersion

$$1 = [1-\phi_p]\tilde{p}_t^{-\frac{1}{\lambda}} + \phi_p \pi_t^{\frac{1}{\lambda}} \tag{74}$$

$$v_{p,t} = [1-\phi_p]\tilde{p}_t^{-\frac{1+\lambda}{\lambda}} + \phi_p \pi_t^{\frac{1+\lambda}{\lambda}} v_{p,t-1} \tag{75}$$

$$Y_{w,t} = v_{p,t} Y_t \tag{76}$$

- Aggregate wage and dispersion

$$w_t^{-\frac{1}{\lambda_l}} = [1-\phi_w]\tilde{w}_t^{-\frac{1}{\lambda_l}} + \phi_w w_{t-1}^{-\frac{1}{\lambda_l}} \pi_t^{\frac{1}{\lambda_l}} \tag{77}$$

$$v_{w,t} = [1-\phi_w][\frac{\tilde{w}_t}{w_t}]^{-\frac{1+\lambda_l}{\lambda_l}} + \phi_w [\frac{w_t}{w_{t-1}}]^{\frac{1+\lambda_l}{\lambda_l}} \pi_t^{\frac{1+\lambda_l}{\lambda_l}} v_{w,t-1} \tag{78}$$

$$L_{s,t} = v_{w,t} L_t \tag{79}$$

- Other Market Clearing Conditions

$$Y_t = Y_{c,t} + Y_{i,t} \tag{80}$$

$$e_{o,t} = O_{p,t} + O_{c,t} \tag{81}$$

$$Y_{f,t} = C_t + G_t + a_{u,t}\hat{K}_t \tag{82}$$

- Function Definitions

$$s_t = \frac{\psi_s}{2}\Big[\frac{I_t}{I_{t-1}} - 1\Big]^2 \tag{83}$$

$$s_{p,t} = \psi_s\Big[\frac{I_t}{I_{t-1}} - 1\Big] \tag{84}$$

$$a_{u,t} = \frac{1}{\gamma_{ss}\mu_{ss}} \frac{R_{ss} - (1-\delta)}{\chi_u}\Big[1 - e^{-\chi_u(u_t - 1)}\Big] \tag{85}$$

$$a_{up,t} = \frac{1}{\gamma_{ss}\mu_{ss}}[R_{ss} - (1-\delta)]e^{-\chi_u(u_t-1)} \tag{86}$$

- Law of motion of exogenous processes

$$logA_t = (1-\rho_a)log\bar{A} + \rho_a logA_{t-1} + \epsilon_{a,t} \tag{87}$$

$$logP_t^o = (1-\rho_o)log\bar{P}^o + \rho_o logP_{t-1}^o + \epsilon_{o,t} \tag{88}$$

$$log\gamma_t = (1-\rho_i)log\bar{\gamma} + \rho_i log\gamma_{t-1} + \epsilon_{i,t} \tag{89}$$

$$log\mu_t = (1-\rho_m)log\bar{\mu} + \rho_m log\mu_{t-1} + \epsilon_{m,t} \tag{90}$$

$$logb_t = (1-\rho_b)log\bar{b} + \rho_b logb_{t-1} + \epsilon_{b,t} \tag{91}$$

$$log(g_t) = (1-\rho_g)log(\bar{g}) + \rho_g log(g_{t-1}) + \epsilon_{g,t} \tag{92}$$

$$
\begin{aligned}
\log(R_t) = &(1-\rho_r)\log(\bar{R}) + \rho_r \log(R_{t-1}) \\
&+ (1-\rho_r)\theta_\pi[\log(\pi_t) - \log(\bar{\pi})] \\
&+ (1-\rho_r)\theta_y[\log(Y_t) - \log(\bar{Y})] + \epsilon_{r,t}
\end{aligned} \tag{93}
$$

# Steady State

To compute the Steady State (SS) I need to assume the SS values of some of the endogenous variables in the model. In particular, the SS of the technology variables ($\bar{A}, \bar{\mu}$, $\gamma$), the core inflation ($\bar{\pi}$), the government expenditure exogenous variable ($\bar{g}$) and the labor demanded by the wholesaler ($\bar{L}$) are set to be 1. Using these restrictions I find the SS of all the other variables by finding their time-invariant values that satisfy the equilibrium equations, and the SS defined above.