Relative Price of Equipment, Investment Shocks and Oil-Food Prices

Oscar Jaulín-Méndez *

November 14, 2025

Abstract

This paper re-examines the empirical identification of investment-specific technology (IST) shocks by focusing on the influence of oil and food price fluctuations on the relative price of equipment (RPE), the key observable traditionally used to estimate such shocks. Standard approaches implicitly assume that movements in the RPE reflect technological forces alone, yet commodity prices disproportionately affect non-durable consumption goods—the denominator of the RPE—independently of investment-sector productivity. Using U.S. quarterly data and local projections, I document that conventional IST shock estimates are significantly correlated with several externally identified oil price shock series, and that these shocks generate counterintuitive impulse responses, including declines in real wages and increases in oil price indexes. After adjusting for oil and food price fluctuations, these anomalies disappear: real wages rise following IST shocks, oil prices do not react, and the forecast error variance attributed to IST shocks declines for GDP, consumption, and investment. To interpret these empirical patterns, I estimate a medium-scale DSGE model featuring an explicit commodity-producing sector that feeds into final-goods production. The model reproduces the qualitative changes observed in the empirical responses and illustrates how commodity price movements embed themselves into the RPE, thereby biasing IST shock identification. The results indicate that a substantial portion of the empirical influence attributed to IST shocks reflects omitted-variable distortions arising from oil and food price dynamics, suggesting a need to reassess their role in business cycle analysis.

Keywords— Relative Price of Equipment, Investment shocks, Oil prices.

*Junior Research Economist, Banco de la República de Colombia. ojaulime@banrep.gov.co

1 Introduction

The relative price of equipment (RPE)—defined as the price of equipment and durable consumption goods relative to non-durable consumption goods (Ben Zeev & Khan, 2015)—has been a central variable in macroeconomic research since the foundational work of Greenwood et al. (2000). The persistent decline in the U.S. RPE since the mid-1950s (Figure 1) is widely interpreted as evidence of sustained technological progress in the investment goods sector. This interpretation underlies the investment-specific technological change (IST) hypothesis, which posits that the downward trend reflects improvements in the productivity of investment goods producers. Within this framework, unexpected innovations to the underlying technology—so-called IST shocks—are commonly viewed as exogenous forces driving both long-run growth and business cycle fluctuations.

However, standard empirical methods for identifying IST shocks, which rely on the information contained in the RPE, may overlook important sources of relative price variation. The RPE is an imperfect proxy for investment technology and identifies IST shocks only under restrictive assumptions. In particular, I show that fluctuations in oil and food prices can disproportionately affect the price of non-durable consumption goods, thereby influencing the RPE independently of technological change. Because these components are often omitted from empirical specifications, the estimated impact of IST shocks may be overstated. In this paper, I document that conventional IST shock estimates exhibit a significant correlation with oil shocks identified in previous studies. The mechanism is straightforward: an increase in oil prices raises the cost of non-durable consumption goods—the denominator of the RPE—causing the relative price of equipment to fall. Once the estimation accounts for oil and food price fluctuations, the share of forecast error variance (FEV) attributed to IST shocks declines, particularly for GDP and investment over horizons from zero to ten quarters. This suggests that the empirical importance often ascribed to IST shocks may, in part, reflect the influence of commodity market dynamics.

Furthermore, IST shocks adjusted for oil and food price movements generate impulse response functions (IRFs) that align more closely with standard macroeconomic theory. When oil price fluctuations are ignored, IST shocks lead to a counterintuitive decline in real wages and an increase in oil price indexes. After accounting for oil and food price movements, however, real wages rise following the shock and oil prices remain unchanged, resolving these anomalies.

To further investigate these findings, I develop a DSGE model that explicitly incorporates commodity price shocks. The objective is to assess whether embedding oil and food price dynamics within a structural framework helps bridge the gap between two strands of the literature: Empirical studies that attribute a central role to IST shocks in driving business cycles (Chen & Wemy, 2015; Fisher, 2006; Galí & Gambetti, 2009), and empirical estimations of DSGE models, that discipline them with the RPE and instead find IST shocks to be far less influential (Ben Zeev & Khan, 2015; Justiniano et al., 2011; Schmitt-Grohé & Uribe, 2012).

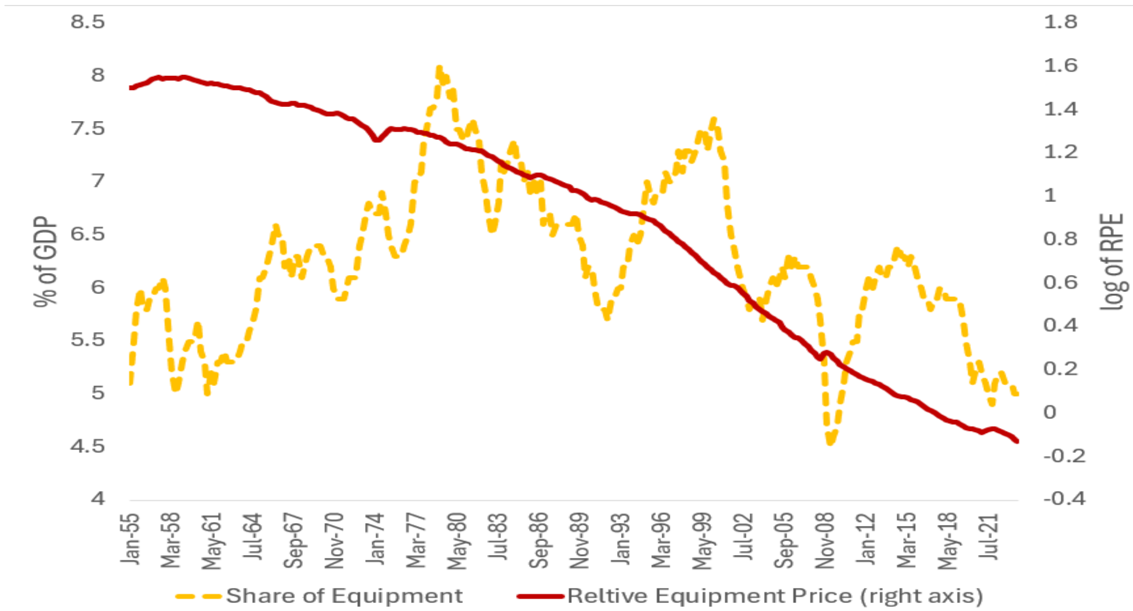


Figure 1: Relative price of equipment and share of equipment.

By introducing an explicit oil sector, the model allows me to evaluate whether accounting for commodity price movements can reconcile these contrasting views by improving the match between theoretical and empirical impulse responses.

I estimate a structural DSGE model that incorporates both IST shocks and oil- and food-price shocks. The model features a commodity-producing sector whose output is used by intermediate goods producers, who in turn supply consumption and investment goods producers as well as the final consumption goods sector. Through this production structure, commodity prices directly influence final-goods costs and shape the behavior of the relative price of investment. The model is estimated using Bayesian methods, with the RPE included as an observable to discipline its dynamics.

The estimated model delivers two main results. First, incorporating the oil sector generates theoretical IRFs to IST shocks that closely resemble those obtained in the empirical analysis, indicating that commodity prices are essential for reproducing the empirical transmission mechanism. Second, despite this improvement in fit, IST shocks remain quantitatively unimportant for explaining the variance of key macroeconomic variables. Thus, even after accounting for the role of commodity prices, the structural model continues to support the view that IST shocks play only a limited role in business cycle fluctuations.

Related Literature — The role of IST in business cycle fluctuations has been examined through both empirical analysis and structural modeling. Empirical studies consistently highlight the importance of IST shocks, typically relying on VAR frameworks to extract these shocks and using forecast error variance decompositions (FEVD) to assess their macroeconomic impact (Ramey, 2016). For instance, Fisher (2006), Galí and Gambetti (2009), and

Chen and Wemy (2015) employ long-run and medium-run restrictions to identify IST shocks and find strong evidence of their influence on macroeconomic dynamics. In addition, Ben Zeev and Khan (2015) investigates news shocks to IST—anticipated exogenous changes in investment-specific technology—and shows that these shocks explain a considerable share of the forecast error variance (FEV) in consumption, hours worked, investment, and GDP.

Structural models calibrated to the U.S. economy similarly suggest a prominent role for IST in shaping macroeconomic fluctuations. Studies such as Greenwood et al. (2000), Christensen and Dib (2008), Jaimovich and Rebelo (2009), and Justiniano et al. (2010) conclude that IST shocks account for a substantial portion of the variance in consumption, investment, and output at business cycle frequencies. More recent work—including Jaimovich and Rebelo (2009), Choi (2020), and Liao and Chen (2023)—emphasizes the contribution of news shocks to IST, finding that anticipated changes in investment-specific technology generate sizeable fluctuations in key macroeconomic indicators.

Despite the extensive evidence supporting the relevance of IST shocks, another strand of the literature questions their importance for explaining aggregate dynamics. This research employs medium-scale DSGE models with rich stochastic structures, following the methodology of Smets and Wouters (2007). These studies estimate the model using Bayesian techniques and include the RPE as a core observable used to discipline the model. Notable contributions include Justiniano et al. (2011), which analyzes non-news IST shocks, and Schmitt-Grohé and Uribe (2012) and Ben Zeev and Khan (2015), which focus on news shocks. The coexistence of these contrasting findings motivates a closer look at the empirical objects used to identify IST shocks. In this paper, I revisit the estimation of IST shocks by examining how oil and food price movements affect the RPE, and therefore the empirical identification strategies that rely on it.

The remainder of the paper is organized as follows. Section 2 presents the empirical methodology, examines the relationship between IST and oil price shocks, and studies the responses of real and price variables after adjusting for oil and food prices. Section 3 provides robustness checks. Section 4 introduces the medium-scale DSGE model, and Section 5 concludes.

2 RPE, IST shocks and oil-food prices

In this section, I follow the empirical literature by using the RPE to identify IST shocks. I then apply local projections, following Jordà (2005), to examine the impulse responses of several macroeconomic variables to these shocks. The results show that IST shocks are associated with increases in oil and food prices, which helps explain the observed decline in real wages. I also document that the estimated IST shocks exhibit a significant correlation with oil price shocks reported in the literature. Once oil and food prices are incorporated into the analysis, however, both the correlation with oil shocks and the rise in oil prices following an IST shock weaken substantially. In addition, the share of FEV attributed to IST shocks for GDP, consumption, and investment declines after controlling for these commodity price movements.

2.1 Identification of IST shocks

IST shocks are typically estimated by identifying structural innovations that explain the medium- or long-term variance of the RPE (Ramey, 2016). This approach involves estimating a VAR model and deriving orthogonal shocks from its reduced-form residuals to maximize the FEV of the RPE over h periods ahead (Barsky & Sims, 2011).

I estimate the IST shocks using quarterly data from the US from 1964:I to 209:IV. Following the estimation approach of Chen and Wemy (2015), consider the following VAR process, which is assumed to provide a sufficiently accurate approximation of the true data-generating process:

$$Y_t = \beta(L)Y_t + u_t \tag{1}$$

Where Y_t is an $(n \times 1)$ vector of macroeconomic variables at time t. I include the variables used in the baseline estimation of Chen and Wemy (2015): TFP (Fernald, 2014), log of RPE, log of real Output per capita (The sum of consumption, investment and government expenditure), log of real investment per capita (investment plus durable consumption), log of real non-durable consumption per capita, and log of total hours worked.[1] Notice that in the empirical literature, macroeconomic variables typically exclude both oil prices and food prices from the system of equations. All variables are in levels, following Sims et al. (1990). $\beta(L) = B_1(L) + B_2(L^2) + \cdots + B_P(L^P)$ is a lag polynomial, and u_t is an $(n \times 1)$ vector of reduced-form innovations. The latter is assumed to be a linear combination of structural shocks (ε_t):

$$u_t = A\varepsilon_t \tag{2}$$

Where the variance-covariance matrix of the reduced-form innovations is:

$$\Sigma_u = E[u_t u_t'] = E[A\varepsilon_t \varepsilon_t' A'] = AA' \tag{3}$$

However, it is well known that A cannot be uniquely identified. To see why, consider $A = \tilde{A}Q$ where Q is an orthonormal matrix. Note that $\tilde{A}$ satisfies (3) and therefore is also a matrix that can be used to obtain the structural shocks:

$$\Sigma_u = E[AA'] = E[\tilde{A}QQ'\tilde{A}'] = E[\tilde{A}\tilde{A}'] \tag{4}$$

Literature usually identifies the IST shocks by finding a column $\tilde{q}_1$ in Q that maximizes the FEV of the RPE at the horizon h:

[1] Data is obtained from the Bureau of Economic Analysis (real government expenditure, investment and consumption, chained dollars), Bureau of Labor Statistics, and the FRED.

$$\tilde{q}_1 = \arg\max q_1' S^h q_1 \tag{5}$$

subject to

$$q_1' q_1 = 1, \tag{6}$$

where S^h is the variance of the forecast error of the variable of interest h steps ahead, using the Cholesky decomposition on Σ to obtain $\tilde{A}$. Equation (6) guarantees that q_1 is a unit-length column vector that belongs to an orthonormal matrix. Then, the IST shock is obtained as the first value of the vector:

$$\epsilon_t = A^{-1} u_t = (Q\tilde{A})^{-1} u_t \tag{7}$$

Uhlig (2004) shows that the problem can also be written in a quadratic form where the q_1 is the eigenvector associated with the largest eigenvalue of the matrix S^h (Chen & Wemy, 2015). I estimate the VAR with standard OLS using quarterly data from 1964:I to 2019:IV.[2]

2.2 Local projections

Although impulse responses can be obtained in a standard VAR framework, I employ local projections as proposed by Jordà (2005) for two primary reasons. First, as emphasized by Olea et al. (2025), local projections tend to exhibit lower bias and provide more reliable uncertainty assessments, albeit at the cost of increased variance. This trade-off is often worthwhile, as biased IRFs arising from misspecified or overly restrictive VAR models can lead to misleading inferences.[3] Second, my empirical strategy involves estimating IST shocks using standard methods in the literature that do not incorporate food and oil prices in the estimation process. I then use these estimated shocks to analyze the responses of variables excluded from the original VAR—namely wages and oil–food price indexes. Examining how these variables react to the shocks allows me to assess whether the estimated IST shocks embed information beyond investment-specific technology.

Let y_t be the variable of interest, $\hat{\epsilon}_t^{IST}$ be the estimated measure of IST shock, X_t a vector of macroeconomic controls at time t. I include real activity variables: log consumption, investment (Gross capital formation plus durable consumption), output (consumption, investment and govenrment expenditure), and log of total hours worked; the shadow rate of FED fund's rate (Wu & Xia, 2016), log of cpi-deflected wages, log of food personal consumption expenditure price index and oil price index(WTI). I obtain the IRFs from the following

[2]Results are robust to the estimation technique in the VAR.

[3]See also Auerbach and Gorodnichenko (2013). For a survey of the local projections literature, see Jordà (2023).

OLS regression:

$$y_{t+h} - y_{t-1} = \alpha_h + \beta_h \hat{\epsilon}_t^{IST} + \sum_{j=1}^{4} \gamma_{h,j} X_{t-j} + u_t, \tag{8}$$

Where β_h is the value of the IRF at horizon h, and u_t are residuals. The confidence interval is computed by using HAC standard errors. (Jordà, 2023).

Figure 2 presents the IRFs for selected macroeconomic variables in response to the estimated IST shocks. The responses of real economic activity indicators align with conventional findings: investment, GDP, and hours worked rise immediately following a positive shock, while consumption exhibits a delayed increase, occurring some quarters later.

However, a puzzling response emerges in real wages, which decline following the shock.[4] Movements in oil and food prices offer a plausible explanation for this anomaly. Part of the decline in the RPE after the estimated IST innovations is driven by shocks that raise the prices of non-durable consumption goods—the denominator of the RPE—largely due to increases in energy and food prices. These shocks are fundamentally different from IST shocks, yet they contaminate the empirical identification by shifting the RPE in ways unrelated to investment-specific technology.

In the next section, I focus on the relationship between IST shocks and oil price shocks, given the extensive literature devoted to identifying exogenous oil price movements. In contrast, the identification of food price shocks has received comparatively little attention, and reliable estimates isolating food-specific shocks remain scarce. Nonetheless, rising food prices likely influence empirical IST shock estimates through similar channels as oil price fluctuations.[5]

2.3 IST shocks and oil price shocks

For IST shocks to be accurately identified, they must be uncorrelated with other exogenous disturbances, as structural shocks should exhibit no correlation with any other shocks (Ramey, 2016). This subsection examines the relationship between the identified IST shocks and several measures of oil price shocks. The results reveal a significant degree of correlation, suggesting that standard identification methods for IST shocks may inadvertently capture information unrelated to investment-specific technology.

I focus on five series of oil price shocks commonly used in the literature:

[4]Real wages are generally expected to rise in response to IST shocks, as predicted by DSGE models such as Justiniano et al. (2010) and Justiniano et al. (2011).

[5]Food prices also play an important role in understanding the U.S. and Euro-area economies. For recent studies, see De Winne and Peersman (2016), Peersman (2022), and Jo and Adjemian (2023).

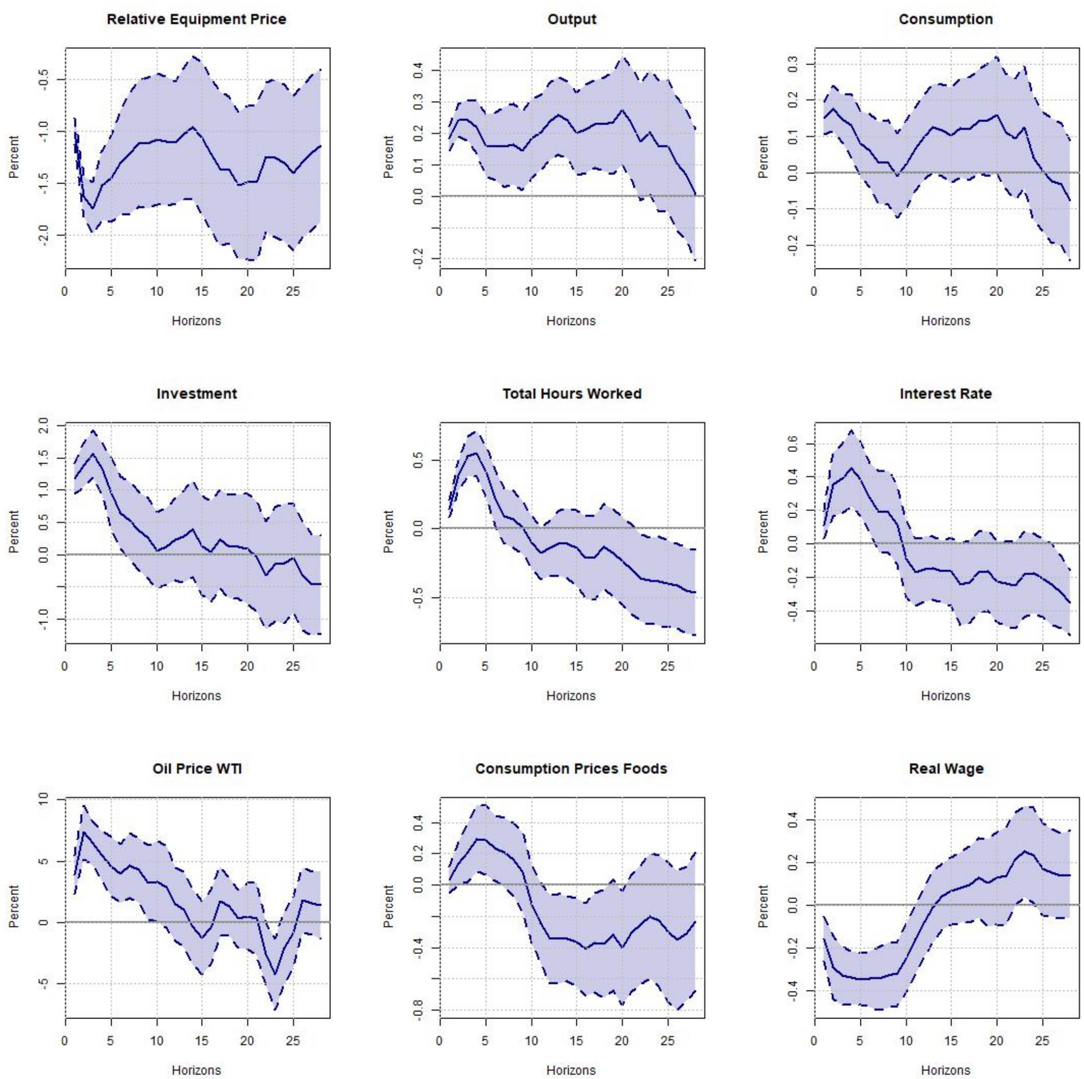


Figure 2: IRF to baseline IST shocks

Note: Impulse response of macroeconomic variables to baseline estimation of IST shocks. The shaded areas represent the 90.0% confidence interval.

1. O(1). Oil price surprises from Känzig (2021): This series is constructed using changes in oil futures prices in a narrow window around OPEC announcements. Oil futures provide a market-based measure of expectations, making them well suited to isolate the direct impact of these announcements. Although OPEC decisions respond to political and global economic conditions, the tight event window mitigates endogeneity concerns by assuming that broader macroeconomic information is already priced in and remains stable during the window. The resulting series therefore reflects revisions in oil price expectations attributable specifically to OPEC communication.

2. O(2). Oil price news from Känzig (2021): This second measure interprets OPEC announcements as news about future oil supply. To ensure that the surprise component does not reflect shifts in oil demand, global activity, or geopolitical developments, the construction relies on how the financial press reports these announcements—typically in terms of production quotas. Given the political nature of OPEC and the limited systematic relationship between its decisions and macroeconomic conditions, concerns about the information channel are less severe than in monetary policy identification. To further address this issue, the series is purified by removing the effects of revisions in OPEC's global demand forecasts, following an approach akin to Romer and Romer (2004) in the monetary policy literature.

3. O(3). "Pure" oil price expectation shocks from Baumeister and Hamilton (2019): This measure begins by identifying market-based oil price surprises as deviations between realized oil prices (e.g., WTI) and the level expected one month earlier. To isolate the pure expectation component, these surprises are regressed on a set of fundamental oil supply and demand shocks. The residuals—which strip out movements attributable to fundamentals—are interpreted as orthogonalized shifts in expectations. These shocks represent changes in oil prices driven exclusively by revisions in market beliefs rather than new information about underlying oil market conditions.

4. O(4) and O(5). Oil supply and oil demand shocks from Baumeister (2023): These shocks are identified using a Bayesian structural VAR framework that incorporates prior information on key parameters, including short-run elasticities of oil supply and demand. The approach accounts for measurement error—especially in global oil inventories—and uses historical data to refine the structural decomposition. The resulting impulse responses trace out the dynamic effects of supply and demand shocks on oil prices and economic activity. A historical decomposition further quantifies the contribution of each type of shock to major oil price movements, providing a detailed and robust characterization of the forces driving fluctuations in the oil market.

Taken together, these five measures capture distinct dimensions of oil market disturbances—ranging from high-frequency surprises around OPEC announcements to fundamental supply and demand shocks and shifts in market expectations. Their diversity is useful for assessing the robustness of the correlation between estimated IST shocks and oil price movements. If the identified IST shocks consistently comove with these different oil shock series, it suggests that standard IST identification strategies may be absorbing commodity-related information embedded in the RPE rather than isolating innovations to investment-specific technology.

Table 1 reports the correlations between the various oil price shocks and the IST shocks used in the analysis, along with benchmarks from the existing literature. IST(1) refers to the shock estimated in Section 2.1 based on Chen and Wemy (2015), while IST(2) corresponds to the series constructed by Drechsel (2023), who implements the identification strategy of Fisher (2006). Ben Zeev and Khan (2015), hereafter BZK, provide two additional IST shock series: IST(3), which captures unanticipated IST shocks that immediately affect the RPE and maximize its forecast error variance (FEV), and IST(4), a news component that maximizes the FEV of the RPE while remaining orthogonal to unanticipated IST shocks. This latter series is typically viewed as the most economically relevant IST component, as it explains the largest share of the variance in economic activity.

	O(1)	O(2)	O(3)	O(4)	O(5)
IST(1)	0.07	0.06	0.27***	0.13*	0.17**
IST(2)	0.15*	0.13*	0.18**	0.14*	0.16**
IST(3)	0.00	0.08	0.34***	0.18**	0.07
IST(4)	-0.11	-0.28***	-0.28***	-0.05	-0.28***

Table 1: IST and oil shocks: correlation.

Note: Correlation between IST shocks and Oil shocks. Correlations with O(4) are multiplied by -1 because the nature of the shock implies a decrease in the oil price. *p-value < 0.1, ** p-value < 0.05, *** p-value < 0.01.

The correlations reported in Table 1 reveal a clear relationship between oil price disturbances and several widely used IST shock measures. In particular, IST shocks identified using long-run restrictions (Drechsel, 2023) and medium-run restrictions (Chen & Wemy, 2015), as well as the BZK series, exhibit strong comovement with oil price shocks. The correlations between these IST shocks and oil price innovations O(3) and O(5) are positive, implying that exogenous increases in oil prices are systematically associated with higher estimated IST shocks. By contrast, the correlation with O(4)—which captures oil supply shocks that typically reduce oil prices—is negative, consistent with the notion that falling energy costs exert upward pressure on the RPE.

Turning to the BZK estimates, the correlation between unanticipated IST shocks and oil supply shocks remains sizeable, reinforcing the idea that movements in the RPE partly reflect commodity price fluctuations rather than pure investment-specific technology. The pattern for news shocks differs somewhat but remains statistically meaningful, suggesting that even forward-looking IST components may embed information related to oil market dynamics. Overall, these results indicate that standard empirical identification strategies for IST shocks capture more than technology-driven variation, absorbing instead part of the economic effects generated by shifts in global commodity prices.

Fisher (2006) also documents a significant correlation between IST shocks and oil price movements, but offers a different interpretation, suggesting that oil shocks could themselves be viewed as IST shocks. Specifically, Fisher (2006, p. 446) argues:

The oil shock result might not be surprising. Suppose that an exogenous increase in the price of oil induces substitution toward equipment that the United States is not good at producing, such as high-mileage cars. If this is the case, then the real price of equipment rises. From this perspective, a permanent oil shock is very much like an I-shock.

The findings in this paper suggest a different interpretation. My results indicate that the correlation between IST shocks and oil price movements arises primarily because oil shocks influence the price of non-durable consumption goods, which form the denominator of the RPE. These consumption-price responses mechanically shift the relative price of equipment, causing empirical IST measures to absorb the effects of commodity price fluctuations rather than reflecting changes in investment-specific technology. In this sense, the comovement documented by Fisher (2006) is better understood as a misidentification issue: oil shocks do not behave like IST shocks, but instead distort the empirical proxy used to identify them.

These findings underscore the importance of properly accounting for commodity price dynamics when identifying IST shocks through the relative price of equipment. Once movements in oil and food prices are recognized as key drivers of fluctuations in the denominator of the RPE, the empirical link between IST shocks and macroeconomic variables appears in a different light. Rather than reflecting technological innovations in the investment sector, part of the variation typically attributed to IST shocks is instead driven by shifts in commodity markets. The next section examines this issue more closely by incorporating oil and food prices directly into the empirical framework and evaluating how IST shock estimates and their associated impulse responses change once these commodity price effects are taken into account.

2.4 Re-estimating the IST shocks: oil-food prices.

The correlation patterns documented above motivate a refinement of the identification strategy. To mitigate the influence of commodity-induced movements in the RPE, I modify the VAR framework by augmenting it with two additional variables that capture the relevant price dynamics: the logarithm of the West Texas Intermediate (WTI) oil price and the food consumer price index (CPIF). These variables are incorporated into the VAR used to estimate IST shocks in Section 2.1. I apply this adjustment to both the identification strategy of Drechsel (2023) and the approach of BZK.[6]

[6]Appendix B provides details on how the BZK procedure is adapted to include information on oil and food prices.

After incorporating these additional series, the resulting IST shocks display a markedly weaker correlation with oil price innovations (Table 2).[7] This decline in comovement indicates that part of the previously observed relationship between oil price shocks and IST shocks stemmed from omitted commodity-price dynamics rather than genuine investment-specific technological disturbances.

	O(1)	O(2)	O(3)	O(4)	O(5)
IST(1)	-0.03	0.08	0.02	-0.03	-0.02
IST(2)	-0.01	0.07	-0.10	-0.04	-0.08
IST(3)	0.01	0.07	0.27***	0.17**	0.04
IST(4)	-0.14	-0.04	-0.08	-0.12	-0.12

Table 2: Corrected IST and oil shocks: correlation.

Note: Correlation between IST shocks including food and oil prices in the estimation and Oil shocks. Correlations with O(4) are multiplied by -1.0 because the nature of the shock implies a decrease in the oil price. *p-value < 0.1, ** p-value < 0.05.

Estimating IST shocks while controlling for oil and food prices makes it possible to reassess their macroeconomic effects using the LP strategy described in Section 2.2. Once these commodity-price variables are included in the VAR system, the resulting IST shocks no longer generate increases in food-oil consumer price indexes or declines in real wages (Figure 3). Importantly, the responses of other real variables remain qualitatively similar, indicating that the adjustment primarily affects the price-channel distortions embedded in the original IST estimates.

This refinement also reduces the apparent contribution of IST shocks to the dynamics of key macroeconomic variables. To illustrate this, I calculate the share of forecast error variance (FEV) explained by IST shocks in the VAR and compare results from the baseline specification with those from a VAR that includes oil and food prices. As reported in Table 3, the "Clean" specification attributes a smaller portion of short-term fluctuations (one to ten quarters) to IST shocks. At a five-quarter horizon, the adjustment lowers the FEV contribution of IST shocks by 15.5 percentage points for GDP, 17.5 p.p. for investment, and 10.1 p.p. for consumption. Appendix B shows that this pattern also holds for IST news shocks identified following the BZK methodology.

[7] The correlation between IST(3) and both O(3) and O(4) remains statistically significant. However, IST(3) accounts for only a small share of the variance in economic activity (Ben Zeev & Khan, 2015), so its macroeconomic relevance is limited.

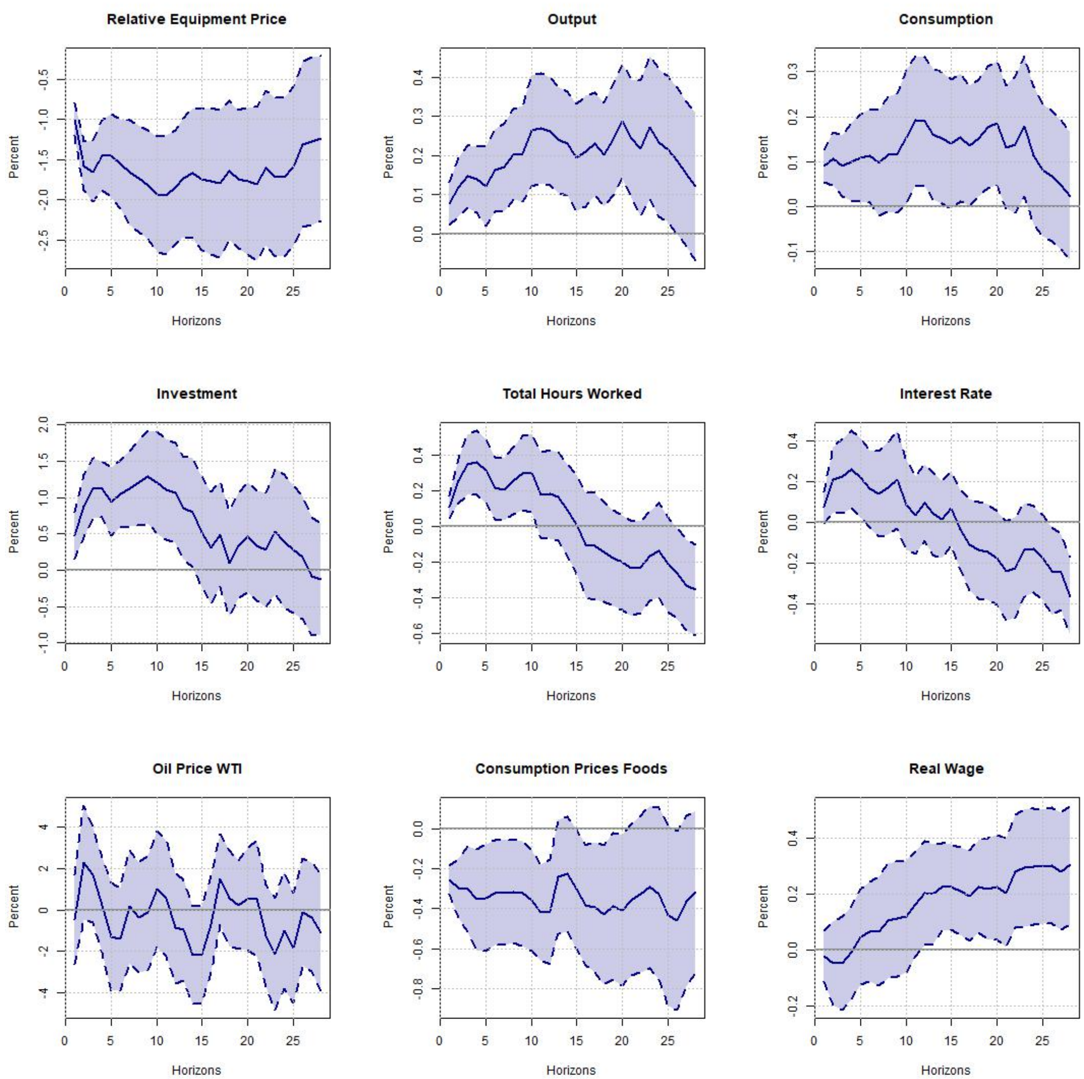


Figure 3: IRF to oil-food price corrected IST shocks.

Note: Impulse response of macroeconomic variables to oil and food price adjusted estimation of IST shocks. The shaded areas represent the 90.0% confidence interval.

	GDP	GDP	Inv.	Inv.	Cons.	Cons.	Hours	Hours
h	**Base**	**Clean**	**Base**	**Clean**	**Base**	**Clean**	**Base**	**Clean**
1	0.257	0.059	0.365	0.074	0.237	0.097	0.082	0.075
5	0.323	0.168	0.525	0.350	0.242	0.141	0.328	0.235
10	0.312	0.239	0.412	0.320	0.201	0.165	0.211	0.189
15	0.320	0.290	0.385	0.306	0.190	0.181	0.161	0.150
20	0.318	0.323	0.382	0.308	0.185	0.198	0.140	0.137

Table 3: FEV explained by IST shocks

3 Robustness analysis

The reduction in both the price-related anomalies and the FEV attributed to IST shocks suggests that commodity-price dynamics play a nontrivial role in shaping standard IST estimates. Before turning to the DSGE analysis, it is important to verify that these findings are not driven by particular modeling choices or by the specific measures of oil and food prices used in the analysis. This Section, therefore, presents a series of robustness checks that assess the stability of the results across alternative specifications and data constructions.

3.1 Modifying the RPE

An alternative way to control for exogenous movements in oil and food prices is to construct the RPE using a price index for non-durable consumption that excludes energy and food components. I refer to this adjusted measure as *RPENE*. Since energy and food prices are directly tied to commodity-price fluctuations, removing them from the denominator isolates the component of the RPE that is more closely related to investment-specific dynamics. This approach follows Beaudry et al. (2015), who propose RPENE when studying the cyclical properties of the RPE. Under this specification, it is no longer necessary to include oil and food prices directly in the VAR system, as in the previous section.

Figure A1 in Appendix A reports the IRFs to IST shocks when RPENE is used. The results closely mirror those in Figure 3: consumer price indexes do not rise in response to an IST shock, and the behavior of real variables remains consistent with the adjusted LP estimates. Table 4 presents the corresponding FEVDs, showing that the contribution of IST shocks to business-cycle fluctuations is smaller than in the baseline specification. The reduction is especially notable for GDP, consumption, and investment per capita, reinforcing the conclusion that commodity-price dynamics inflate the apparent importance of IST shocks when the standard RPE measure is used.

	GDP	GDP	Inv.	Inv.	Cons.	Cons.	Hours	Hours
h	**Base**	**Clean**	**Base**	**Clean**	**Base**	**Clean**	**Base**	**Clean**
1	0.257	0.108	0.365	0.172	0.237	0.150	0.082	0.075
5	0.323	0.239	0.525	0.448	0.242	0.209	0.328	0.318
10	0.312	0.285	0.412	0.407	0.201	0.213	0.211	0.249
15	0.320	0.323	0.385	0.400	0.190	0.220	0.161	0.202
20	0.318	0.341	0.382	0.400	0.185	0.223	0.140	0.176

Table 4: FEV explained by IST shocks (RPENE)

3.2 Joint price index of food and energy

The empirical results are also robust to alternative measures of food and oil price fluctuations. To verify this, I replace the CPIF and WTI series in the VAR with a single composite index of food and energy prices constructed from Personal Consumption Expenditures (*PCEFE*) data. This index aggregates energy and food components within non-durable consumption using expenditure-share weights from the national accounts, providing a unified measure of commodity-related price movements.

Figure A2 in Appendix A presents the IRFs to IST shocks identified using the PCEFE index. The responses are highly consistent with those reported in Figure 3, particularly for consumer price variables. Likewise, Table 5 confirms that the share of FEV attributed to IST shocks remains lower than in the baseline specification, with the reduction again most pronounced for GDP, consumption, and investment per capita at business-cycle frequencies.

	GDP	GDP	Inv.	Inv.	Cons.	Cons.	Hours	Hours
h	**Base**	**Clean**	**Base**	**Clean**	**Base**	**Clean**	**Base**	**Clean**
1	0.257	0.088	0.365	0.160	0.237	0.164	0.082	0.102
5	0.323	0.216	0.525	0.429	0.242	0.202	0.328	0.300
10	0.312	0.276	0.412	0.391	0.201	0.213	0.211	0.244
15	0.320	0.323	0.385	0.378	0.190	0.228	0.161	0.204
20	0.318	0.346	0.382	0.382	0.185	0.238	0.140	0.182

Table 5: FEV explained by IST shocks (PCEFE)

4 Evidence in a DSGE model

In this section, I examine whether incorporating oil and food price shocks into a DSGE framework helps clarify the role of IST shocks in business cycle fluctuations. The empirical analysis has shown that commodity-price movements contaminate standard measures of IST shocks through their influence on the RPE. A natural question is therefore whether a structural model that explicitly embeds these price channels can reconcile the contrasting views in the literature: empirical VAR studies typically attribute a substantial role to IST shocks, while estimated DSGE models disciplined by the RPE tend to downplay their importance.

DSGE models have long been used to assess the contribution of IST shocks to macroeconomic dynamics. Several influential studies find that once the RPE is included as an observable, IST shocks explain only a modest share of aggregate fluctuations. For instance, Justiniano et al. (2011) estimate a medium-scale DSGE model with both neutral and IST technology shocks and standard nominal rigidities, while Schmitt-Grohé and Uribe (2012) extend this framework by allowing for news shocks in both technology processes.[8] Building on these contributions, the goal here is to evaluate whether introducing an explicit channel for oil and food prices alters the conclusions drawn from such models.

To do so, I develop a model inspired by Justiniano et al. (2011) that incorporates a commodity-producing sector whose output price affects the conventional measure of the RPE. In this setting, shocks to commodity prices influence the RPE directly, creating a mechanism through which oil and food price movements can distort the empirical identification of IST shocks. This structure allows the model to assess whether explicitly modeling commodity-price dynamics changes the estimated importance of IST shocks in explaining business cycle variation.

4.1 The model

The model builds on the framework of Justiniano et al. (2011) but augments it with an additional commodity-goods sector. This sector differs from standard production blocks in that it does not employ capital or labor; instead, commodities are produced without cost, and their price is taken as exogenous. This assumption reflects the fact that commodities—such as oil and food—are traded on global markets, where price movements are largely driven by shifts in worldwide supply and demand rather than domestic production decisions. The commodity sector fulfills two roles in the model: commodities enter as an input in the production of investment and consumption goods, and they also form part of the final consumption bundle. Consequently, the price of commodities affects both the cost of producing investment goods and the price of consumption, implying that the RPE is influenced not only by IST shocks but also by fluctuations originating in commodity markets.

The structure of the model economy consists of two main blocks. The first block captures the interactions among households, firms, the government, and labor unions (or labor assemblers). Households finance government expenditures through taxes and bond holdings, supply labor to labor assemblers, and provide firms with utilization-adjusted capital. Labor assemblers bundle heterogeneous labor types and sell the composite input to firms. Firms

[8]Both studies adopt a rich stochastic structure and rely on Bayesian estimation following Smets and Wouters (2007).

combine this bundled labor input with utilization-adjusted capital and a share of the commodity good to produce investment and final consumption goods for households, as well as consumption goods for the government. Households receive interest payments from the government and use investment goods to accumulate the capital stock. Figure A3, Panel A, in Appendix A illustrates the flow of resources within this block.

The second block describes the production structure, which is organized into seven tiers: wholesalers, commodity-goods producers, intermediaries, non-commodity assemblers, investment-goods producers, capital producers, and final-consumption-goods producers. A key feature of this structure is the commodity-goods producer, which sets the commodity price exogenously and produces the quantity demanded by downstream sectors. Its output is sold to wholesalers and to the final consumption-goods producer as an input. This setup parallels the oil-sector representation in Guerrieri and Bodenstein (2012), but the model extends the commodity sector to capture both oil and food price movements, consistent with the empirical results in Section 3.

The economy features two sources of long-term growth: total factor productivity (TFP) and investment-specific technology (IST). Both technology processes are assumed to have permanent effects on their respective technological levels, with the growth rates of each following an AR(1) process.

Figure A3, Panel B, in Appendix A presents a diagram of this production block. The subsections that follow provide the detailed specification of each component in the production structure.

4.1.1 Wholesale sector

The wholesaler is a competitive producer that uses utilization-adjusted capital, labor from the labor assemblers, and the commodity good as inputs. These inputs are combined using a Cobb–Douglas production function with constant returns to scale to produce a single aggregate good. The wholesaler's optimization problem is given by

$$\max_{L_t, K_t, O_{p,t}} P_t^{\omega} Y_{\omega,t} - R_t^k K_t - W_t L_t - P_t^o O_{p,t} \tag{9}$$

subject to

$$Y_{\omega,t} = K_t^{\alpha} O_{p,t}^{\beta_o} (A_t L_t)^{1-\alpha-\beta_o}, \tag{10}$$

where $Y_{\omega,t}$ denotes wholesale output, L_t is labor, K_t is utilization-adjusted capital, $O_{p,t}$ is the quantity of the commodity input used in production, and P_t^{ω}, R_t^k, W_t, and P_t^o are, respectively, the prices of the wholesale good, capital services, labor, and the commodity. The final output of this sector is sold to a continuum of intermediaries in a competitive market; each intermediary uses it as an input to produce a differentiated good.

The growth rate of neutral technology, A_t, follows an AR(1) process in logs:

$$\log\left(\frac{A_t}{A_{t-1}}\right) = \log(g_{a,t}) = (1-\rho_a)\log(\bar{g}_a) + \rho_a \log(g_{a,t-1}) + \epsilon_{a,t}, \tag{11}$$

where $\epsilon_{a,t}$ is i.i.d. $\sim \mathcal{N}(0, \sigma_a)$.

4.1.2 Non-commodity assembler

The non-commodity assembler (NCA) combines the continuum of differentiated goods supplied by intermediary producers. Because each intermediary producer has monopoly power over its own variety, it is convenient to introduce the NCA first to determine total demand for each intermediary input. The NCA operates in a perfectly competitive market and solves

$$\max_{Y_t(i)} \; P_t Y_t - \int_0^1 P_t(i) Y_t(i)\, di \tag{12}$$

subject to

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_t}}\, di\right]^{1+\lambda_t}, \tag{13}$$

where Y_t is the total output of the non-commodity assembler and P_t is its associated price index, which serves as the model counterpart to the core consumer price index. The variables $Y_t(i)$ and $P_t(i)$ denote the quantity and price of each intermediary good i. The parameter λ_t governs the degree of substitutability across varieties.

Solving the NCA's problem yields the standard demand schedule for each intermediary good:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\frac{1+\lambda_t}{\lambda_t}} Y_t. \tag{14}$$

The elasticity shifter λ_t follows an AR(1) process:

$$\log(\lambda_t) = (1-\rho_\lambda)\log(\bar{\lambda}) + \rho_\lambda \log(\lambda_{t-1}) + \epsilon_{\lambda,t}, \tag{15}$$

where $\epsilon_{\lambda,t}$ is an i.i.d. shock.

4.1.3 Intermediaries

There is a mass-one continuum of intermediary producers indexed by $i \in [0, 1]$. Each intermediary produces a differentiated good $Y_t(i)$ using the wholesale output as its only input. These firms operate under monopolistic competition: they take the wholesale good as given and choose the price of their own variety, taking into account the downward-sloping demand curve in Equation (2.9). As in Rotemberg (1982), price adjustment is subject to quadratic costs, which introduces nominal rigidity and renders the pricing decision dynamic. The intermediary firm solves

$$\max_{P_t(i)} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \Big(P_t(i) - P^{\omega}_{t+s}\Big) Y_{t+s}(i) - \Upsilon_p\big(P_{t+s}(i)\big) \tag{16}$$

subject to

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\frac{1+\lambda_t}{\lambda_t}} Y_t. \tag{17}$$

Here, $\Lambda_{t,t+s}$ denotes the household's stochastic discount factor, derived below. The Rotemberg price-adjustment cost takes the form

$$\Upsilon_p(P_t(i)) = \frac{\phi_p}{2}\left[\frac{P_t(i)}{P_{t-1}(i)} - 1\right]^2 P_t Y_t, \tag{18}$$

which penalizes changes in the relative price of each variety.

4.1.4 Final consumption producing sector

The final consumption good is produced by a competitive firm that combines the non-commodity good and the commodity good through a constant elasticity of substitution (CES) technology. The firm solves

$$\max_{Y_{c,t}, O_{c,t}} P_t^f Y_t^f - P_t Y_{c,t} - P_t^o O_{c,t} \tag{19}$$

subject to

$$Y_t^f = \left[\omega_f Y_{c,t}^{\frac{1}{1+\lambda_f}} + (1-\omega_f) O_{c,t}^{\frac{1}{1+\lambda_f}}\right]^{1+\lambda_f}, \tag{20}$$

where Y_t^f is the final consumption good, with price P_t^f, the model counterpart of the consumer price index. The input $O_{c,t}$ is the amount of the commodity used directly in the consumption bundle (e.g., gasoline), purchased at price P_t^o, while $Y_{c,t}$ is the quantity of the non-commodity good used in consumption. The parameter λ_f governs the elasticity of substitution between commodity and non-commodity inputs.

4.1.5 Commodity good

The commodity good is produced without cost, and the commodity producer supplies whatever quantity is demanded by the wholesale sector and the final consumption–goods producer. The firm sells the commodity at price P_t^o, which follows an AR(1) process:

$$\log P_t^o = (1-\rho_o)\log \bar{P}^o + \rho_o \log P_{t-1}^o + \epsilon_{o,t}, \tag{21}$$

where $\epsilon_{o,t}$ is i.i.d. $\sim \mathcal{N}(0, \sigma_o)$. Since the commodity is supplied elastically at the prevailing price, equilibrium requires that total supply equals total demand. Thus, the quantity produced must match the sum of the commodity inputs used in the wholesale sector ($O_{p,t}$) and in the final consumption–goods sector ($O_{c,t}$).

4.1.6 Investment good producer

This sector uses the non-commodity good as input and transforms it into investment goods, which it sells competitively to the capital–goods producer. The efficiency of this transformation process is governed by an investment-specific technology (IST) term, γ_t, which determines how productively units of the non-commodity good are converted into investment goods. The firm solves:

$$\max_{Y_{i,t}} \; P_t^i I_t - P_t Y_{i,t} \tag{22}$$

subject to

$$I_t = \gamma_t Y_{i,t}, \tag{23}$$

where I_t is the flow of investment goods, P_t^i their price, and $Y_{i,t}$ the quantity of non-commodity goods used as input.

The growth rate of IST follows an AR(1) process:

$$\log\left(\frac{\gamma_t}{\gamma_{t-1}}\right) = \log(g_{\gamma,t}) = (1-\rho_i)\log \bar{g}_\gamma + \rho_i \log(g_{\gamma,t-1}) + \epsilon_{i,t}, \tag{24}$$

with $\epsilon_{i,t} \sim i.i.d.\ \mathcal{N}(0, \sigma_i)$. In this environment, the investment–good producer is naturally interpreted as the machinery and equipment sector in the data.

Profit maximization implies:

$$P_t^i \gamma_t = P_t, \tag{25}$$

so that

$$\gamma_t = \frac{P_t}{P_t^i}. \tag{26}$$

Equation (26) emphasizes that IST is tightly linked to the relative price of investment goods. Because P_t is the price of the non-commodity component of consumption, any distortion in P_t caused by commodity–price movements can spill into the measured IST process. This connection mirrors the empirical findings from Section 2, where oil and food price fluctuations were shown to contaminate conventional IST measures.

The usual empirical measure of the relative price of investment or equipment is:

$$rpi_t = \frac{P_t^i}{P_t^f}, \tag{27}$$

where P_t^f is the final consumption price index.

4.1.7 Capital good producer

The capital–goods producer purchases investment goods and transforms them into effective units of capital, subject to investment adjustment costs.[9] Because adjustment costs generate intertemporal trade-offs, the firm chooses an entire sequence of investment demands to maximize discounted profits:

$$\max_{\{I_{t+j}\}_{j=0}^{\infty}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[P_{t+j}^k I_{k,t+j} - P_{t+j}^i I_{t+j} \right], \tag{28}$$

subject to

$$I_{k,t} = \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t, \tag{29}$$

where $I_{k,t}$ is newly produced capital, P_t^k its price, $S(\cdot)$ denotes the adjustment–cost function, and μ_t is the marginal efficiency of investment (MEI). The MEI follows:

$$\log \mu_t = (1 - \rho_m) \log \bar{\mu} + \rho_m \log \mu_{t-1} + \epsilon_{m,t}, \tag{30}$$

with $\epsilon_{m,t} \sim i.i.d.\ \mathcal{N}(0, \sigma_m)$.

For simplicity, adjustment costs take the standard quadratic form:

$$S\left(\frac{I_t}{I_{t-1}} \right) = \frac{\psi_s}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2. \tag{31}$$

4.1.8 Household

Households choose consumption, labor supply, investment, capital utilization, and bond holdings. They rent out utilization–adjusted capital to wholesalers and incur a cost, expressed in units of final consumption goods, when utilizing capital more intensively. Their problem is:

$$\max_{\{C_t, L_{s,t}, I_t, B_t, u_t, \tilde{K}_t\}} \sum_{t=0}^{\infty} \beta^t b_t \left[\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \Gamma_t^{1-\sigma} \psi \frac{L_{s,t}^{1+\chi_L}}{1+\chi_L} \right] \tag{32}$$

subject to

$$P_t^f C_t + P_t^k I_{k,t} + B_t + T_t = R_{t-1} B_{t-1} + \tilde{W}_t L_{s,t} + R_t^k u_t \tilde{K}_{t-1} - P_t^f a(u_t) \tilde{K}_{t-1} + \Pi_t, \tag{33}$$

$$\tilde{K}_t = (1 - \delta) \tilde{K}_{t-1} + I_{k,t}, \tag{34}$$

$$\log b_t = (1 - \rho_b) \log \bar{b} + \rho_b \log b_{t-1} + \epsilon_{b,t}, \tag{35}$$

[9] This problem is analogous to the household's capital–accumulation decision, where investment goods are converted into productive capital subject to adjustment frictions.

where $L_{s,t}$ is labor supplied to unions, $\tilde{W}_t$ the real wage paid by unions, T_t taxes, u_t the utilization rate, $a(u_t)$ the utilization cost, and Π_t aggregate profits distributed to households. Utilization–adjusted capital is $K_t = u_t \tilde{K}_{t-1}$.

The utilization–cost function is:

$$a(u_t) = \frac{1}{\gamma_{ss}\mu_{ss}} \frac{R_{ss} - (1-\delta)}{\chi_u} \left[1 - e^{-\chi_u(u_t - 1)}\right]. \tag{36}$$

The term Γ_t captures long-run consumption growth and ensures a balanced–growth path.

4.1.9 GDP

Measured GDP in this economy is:

$$gdp_t = Y_t^f + \frac{P_t^k}{P_t^f} I_{k,t} - a(u_t)\hat{K}_{t-1}, \tag{37}$$

where Y_t^f reflects both private and government consumption and $I_{k,t}$ is final investment. The second term corresponds to national-accounts investment, while the final term subtracts the cost of utilization.

4.1.10 Labor market

There is a continuum of labor unions indexed by $l \in [0, 1]$. Each union hires labor from the household at the competitive wage $\tilde{W}_t$ and sells differentiated labor services to a labor packer at the monopolistically-set wage $W_t(l)$. Wage setting is subject to Rotemberg-style nominal rigidities (Rotemberg, 1982), captured by a quadratic adjustment cost $\Upsilon_w(W_t(l))$.

The labor packer aggregates the continuum of differentiated labor inputs into a composite labor service, which it sells to the wholesaler at price W_t. Its problem is analogous to that of the non-commodity assembler:

$$\max_{L_t(l)} W_t L_t - \int_0^1 W_t(l) L_t(l)\, dl \tag{38}$$

subject to

$$L_t = \left[\int_0^1 L_t(l)^{\frac{1}{1+\lambda_{l,t}}}\, dl\right]^{1+\lambda_{l,t}}, \tag{39}$$

where $\lambda_{l,t}$ governs the elasticity of substitution across labor varieties.

Solving this yields the labor demand faced by each union:

$$L_t(l) = \left[\frac{W_t(l)}{W_t}\right]^{-\frac{1+\lambda_{l,t}}{\lambda_{l,t}}} L_t. \tag{40}$$

The substitutability parameter follows an AR(1) process:

$$\log(\lambda_{l,t}) = (1-\rho_{\lambda_l})\log(\lambda_l) + \rho_{\lambda_l}\log(\lambda_{l,t-1}) + \epsilon_{\lambda_l,t}. \tag{41}$$

Unions, which possess monopoly power over their respective labor varieties, choose wages to maximize discounted profits:

$$\max_{W_t(l)} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\Big[(W_t(l)-\tilde{W}_t)L_t(l) - \Upsilon_w(W_t(l))\Big] \tag{42}$$

subject to the demand curve above.

Wage adjustment costs take the form:

$$\Upsilon_w(W_t(l)) = \frac{\phi_w}{2}\left[\frac{W_t(l)}{W_{t-1}(l)} - \Gamma\right]^2 P_t Y_t, \tag{43}$$

where Γ captures steady-state nominal wage growth.

The real wage relevant for households and firms is:

$$wp_t = \frac{W_t}{P_t^f}, \tag{44}$$

with W_t denoting the aggregate wage index associated with the bundle of differentiated labor services.

4.1.11 Government

The government is composed of a fiscal authority and a monetary authority. The fiscal authority finances spending through a combination of taxes and debt issuance.[10] Government purchases take the form of final consumption goods and follow a stochastic process. The fiscal authority's budget constraint and spending rule are:

$$P_t^f G_t + R_{t-1}B_{t-1} = B_t + \tau P_t^f Y_t, \tag{45}$$

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t^f, \tag{46}$$

$$\log(g_t) = (1-\rho_g)\log(\bar{g}) + \rho_g \log(g_{t-1}) + \epsilon_{g,t}, \tag{47}$$

with $\epsilon_{g,t} \sim i.i.d.\ \mathcal{N}(0,\sigma_g)$.

The monetary authority sets the nominal interest rate according to a standard Taylor-type rule:

$$\begin{aligned}\log(R_t) = {} & (1-\rho_r)\log(\bar{R}) + \rho_r \log(R_{t-1}) \\ & + (1-\rho_r)\theta_\pi\left[\log(\pi_t) - \log(\bar{\pi})\right] \\ & + (1-\rho_r)\theta_y\left[\log(Y_t) - \log(\bar{Y})\right] + \epsilon_{r,t},\end{aligned} \tag{48}$$

where $\epsilon_{r,t} \sim i.i.d.\ \mathcal{N}(0,\sigma_r)$ and $\pi_t = P_t/P_{t-1}$ is core inflation.

[10] Taxes are modeled as a constant proportion of household income earned from the non-commodity sector.

4.2 Solution

The model features two sources of long-run growth: neutral productivity and investment-specific technology. These trends imply that several variables grow over time even in the absence of shocks. To solve the model, I therefore rewrite all equilibrium conditions in terms of stationary, detrended variables consistent with the economy's Balanced Growth Path (BGP). This transformation ensures that the system can be handled using standard perturbation techniques.

Once the equilibrium conditions are expressed in stationary form, I solve the model using a first-order perturbation around the deterministic steady state, following the methodology described in Villemot (2011).[11] The resulting linearized system provides the basis for the subsequent Bayesian estimation and the impulse-response analysis reported below.

A complete description of the steady state, the transformed equilibrium conditions, and the detrending strategy is presented in Appendix C.

4.3 Estimation

I estimate the model using Bayesian methods, which combine prior beliefs about the parameters with the likelihood implied by the DSGE model.[12] The model is estimated on quarterly U.S. data spanning 1964–2019 and uses nine observable series.

This estimation strategy allows the data to discipline the role of commodity price fluctuations and investment-specific technology within a unified structural framework, providing a basis for comparing the model-implied responses to those obtained in the empirical analysis.

4.3.1 Priors and Fixed Parameters

Following the approach of Justiniano et al. (2011), I fix a small set of parameters that are standard in the DSGE literature. The depreciation rate is set to $\delta = 0.025$ and the intertemporal elasticity of substitution to $\sigma = 2.0$. The steady-state degree of substitutability in the labor market is fixed at $\lambda_l = 0.4$. The tax-to-output ratio is calibrated to $\tau = 25.4\%$, corresponding to the average general government tax-to-GDP ratio for the United States from 1965 to 2019.

[11]The model is implemented and estimated in Dynare 6.3; see Adjemian et al. (2024) for computational details.

[12]For an introduction to Bayesian estimation in DSGE models, see An and Schorfheide (2007); for a recent survey, see Fernández-Villaverde and Guerrón-Quintana (2021). The posterior distribution is sampled using a Random-Walk Metropolis-Hastings algorithm with 20,000 draws. The proposal density is Gaussian, with its covariance matrix calibrated from the inverse Hessian at the posterior mode. All computations are conducted in Matlab 2023a using Dynare 6.3. Identification is assessed at the posterior mean.

The weight of the commodity sector in the consumption bundle is set to $1-\omega_f = 0.177$, matching the average share of food and energy in final private expenditure in U.S. national accounts from 1947 to 2019. The elasticity of substitution in the final consumption aggregator is governed by $\lambda_f = 10.0$, which implies a near-unit elasticity in this tier of production.[13] The steady-state growth rate of investment-specific technology is fixed at 0.00701, which corresponds to the average quarterly growth rate of the inverse relative price of equipment in the data.

The priors for the remaining parameters follow Justiniano et al. (2011). I additionally impose a prior on the share of the commodity input in wholesale production, $\beta^o = 0.10$, ensuring that it remains smaller than the shares of capital and labor in that sector.

4.3.2 Data

The model is estimated using the following set of observable variables:

$$X_t = [\Delta gdp_t, \Delta C_t, \Delta I_{obs,t}, \Delta rpi_t, \Delta \pi_t, \Delta r_t, \Delta w_t, \Delta L_t, \Delta p_t^f].$$

Here, gdp_t, C_t, $I_{k,t}$, and rpi_t denote the logs of output, consumption, investment per capita, and the relative price of equipment, as defined in Section 2. The series r_t corresponds to the log of the real federal funds shadow rate constructed by Wu and Xia (2016). Real wages w_t are measured as the log of wages deflated by the price index of final private expenditure excluding food, energy, and durables. Inflation π_t is the quarterly change in the log of this same index. Labor input L_t is defined as the log of total hours worked. Finally, p_t^f denotes the log of the relative price of final private expenditure excluding durables, over the price index excluding food and energy. [14]

4.3.3 Estimation results

Table 6 presents the estimated parameters, along with their prior distributions and posterior results. The posterior means are consistent with findings in the related literature, further validating the model's calibration.

Qualitatively, the model reproduces several well-established stylized facts from the business cycle literature: (i) GDP is positively correlated with both consumption and investment; (ii) the standard deviation of consumption is lower than that of GDP, while the standard deviation of investment is higher; (iii) GDP and hours worked exhibit a positive correlation; and (iv) the relative price of equipment is negatively correlated with GDP. However, the model does not capture the relatively weak correlation between GDP and real wages,

[13] Larger values of λ_f bring the CES aggregator closer to a Cobb–Douglas specification.

[14] I demean the variables ΔL_t, $\Delta\pi_t$, Δr_t, and Δp_t^f to remove low-frequency movements whose origins lie outside the scope of the model. Demeaning these series improves the model's ability to match their short-run dynamics.

Par.	P. Dist.	Prior M.	Post. M.	90% Low	90% High	Prior SD.
$g_{a,ss}$	norm	1.010	1.0112	1.0111	1.0113	0.0150
α	beta	0.200	0.1759	0.1755	0.1763	0.0500
β^o	beta	0.100	0.0893	0.0889	0.0896	0.0500
λ	norm	0.150	0.1560	0.1557	0.1564	0.0500
ϕ_p	gamm	75.000	72.1885	71.9613	72.3866	20.0000
ϕ_w	gamm	75.000	72.6423	72.5538	72.7318	20.0000
ϕ_s	gamm	4.000	3.9795	3.9772	3.9815	1.0000
h	beta	0.500	0.4846	0.4828	0.4862	0.1000
χ_L	gamm	2.000	1.6995	1.6810	1.7146	0.7500
$100(\beta^{-1}-1)$	gamm	0.250	0.2565	0.2561	0.2570	0.1000
χ_u	gamm	5.000	5.2586	5.2467	5.2727	1.0000
θ_π	gamm	1.500	1.4537	1.4528	1.4545	0.3000
θ_y	gamm	0.500	0.4985	0.4983	0.4987	0.0500
ρ_a	beta	0.400	0.4495	0.4482	0.4506	0.2000
ρ_i	beta	0.200	0.1854	0.1849	0.1859	0.1000
ρ_m	beta	0.600	0.5895	0.5884	0.5907	0.2000
ρ_o	beta	0.600	0.6102	0.6099	0.6105	0.2000
ρ_g	beta	0.600	0.6119	0.6110	0.6130	0.2000
ρ_r	beta	0.600	0.5716	0.5691	0.5737	0.2000
ρ_w	beta	0.600	0.6023	0.6020	0.6027	0.2000
ρ_p	beta	0.600	0.6232	0.6227	0.6237	0.2000
ρ_b	beta	0.600	0.6424	0.6421	0.6427	0.2000
σ_a	invg	0.500	0.0648	0.0645	0.0653	1.0000
σ_i	invg	0.500	0.0648	0.0645	0.0653	1.0000
σ_m	invg	0.500	0.2142	0.2128	0.2156	1.0000
σ_o	invg	0.500	0.0651	0.0645	0.0659	1.0000
σ_g	invg	0.100	0.0890	0.0884	0.0897	1.0000
σ_r	invg	0.100	0.0858	0.0850	0.0867	1.0000
σ_w	invg	0.100	0.7818	0.7527	0.8072	1.0000
σ_p	invg	0.100	0.5765	0.5563	0.5976	1.0000
σ_b	invg	0.100	0.1213	0.1157	0.1280	1.0000

Table 6: Estimation results of parameters in the DSGE model.

suggesting that certain labor market frictions remain outside the model's structure. Table 7 compares the empirical moments with those generated by the simulated model.

Metric	**Data**	**Model**
S.D. Inv./S.D. GDP	5.043	3.141
S.D. Con/S.D. GDP	0.89	0.729
corr GDP-Inv	0.574	0.531
Corr GDP-Con	0.800	0.690
Corr Con-INV	0.408	0.527
Corr GDP-Labor	0.488	0.829
corr GDP-Wage	0.042	0.567
Corr GDP-RPI	-0.167	-0.062

Table 7: Comparison of moments between Data and Model

4.4 Model results

I organize the results into two parts. First, I present the impulse responses of the main macroeconomic variables to an IST shock and to a commodity price shock. Second, I examine the contribution of each structural shock to the variance of these variables.

The main findings are as follows:

1. The model-generated IRFs to IST shocks closely resemble those obtained in the empirical analysis once IST shocks are adjusted for oil and food price movements. The IRFs to commodity price shocks show that a fall in the commodity price raises the RPI, confirming that movements in the RPI are not solely driven by IST disturbances.
2. Given the specification and data used here, IST shocks explain only a modest share of investment fluctuations: they account for 7.4% of the variance of investment. As in Justiniano et al. (2011), investment is instead primarily shaped by marginal efficiency of investment (MEI) shocks.

Figure A4 in Appendix A reports the IRFs to a positive IST shock ($\epsilon_{i,t}$). Output, consumption, and investment rise on impact, and real wages increase as well. Labor input L_t initially expands before gradually declining after several quarters. These responses mirror the empirical patterns documented in Section 2.4, once IST shocks are purged of movements associated with food and energy prices.

Figure A5 in Appendix A shows the responses to a commodity price shock. An increase in the commodity price generates a decline in the RPE, highlighting how fluctuations in the commodity sector directly translate into movements in the relative price of investment. Inflation rises following the shock, while real wages fall, consistent with the empirical responses to oil price disturbances.

Table 8 reports the variance decomposition implied by the estimated DSGE model. Neutral technology, monetary policy, fiscal policy, MEI, and preference shocks explain most of the volatility in price-related variables and account for a substantial share of the variance in investment, consumption, and GDP. Commodity price shocks play a comparatively limited role: only 2.9% of the variance of the log-change in the RPI is attributed to these shocks. IST shocks contribute modestly to investment dynamics and even less to fluctuations in GDP and consumption, explaining 0.9% of GDP variance, 0.7% of consumption variance, and 7.4% of investment variance.[15]

	$\epsilon_{a,t}$	$\epsilon_{i,t}$	$\epsilon_{m,t}$	$\epsilon_{o,t}$	$\epsilon_{g,t}$	$\epsilon_{r,t}$	$\epsilon_{\lambda_l,t}$	$\epsilon_{\lambda,t}$	$\epsilon_{b,t}$
$\Delta(L_t)$	3.0	0.9	0.7	3.2	62.6	24.8	0.0	0.1	4.6
$\Delta(gdp_t)$	38.6	0.7	4.0	0.1	31.8	22.9	0.0	0.2	2.2
$\Delta(c_t)$	59.8	0.9	0.0	0.4	3.4	27.1	0.0	0.2	8.0
$\Delta(i_{obs,t})$	13.7	7.4	50.0	0.3	2.6	24.7	0.0	0.4	1.0
$\Delta(rpi_t)$	0.0	97.1	0.0	2.9	0.0	0.0	0.0	0.0	0.0
$\Delta(\pi_{n,t})$	15.2	11.9	2.7	0.8	2.5	7.1	4.7	50.3	4.7
$\Delta(R_t)$	0.5	0.0	0.0	0.1	2.8	95.3	0.0	0.6	0.6
$\Delta(w_t)$	87.0	0.7	0.3	0.0	0.0	1.2	2.6	6.7	0.6

Table 8: Variance Decomposition in the DSGE Model (%)

5 Conclusions

The relative price of equipment has long played a central role in macroeconomic research because of its connection to investment-specific technology A large empirical literature finds that IST shocks, identified through movements in the RPE, account for an important share of business cycle fluctuations. The analysis in this paper shows that standard empirical estimates of IST shocks remain partly contaminated by exogenous movements in oil and food prices. These fluctuations disproportionately affect the price of non-durable consumption goods, altering the denominator of the RPE and introducing bias into the inferred IST innovations. Once these price effects are controlled for, the resulting impulse responses display a more coherent pattern: IST shocks raise GDP, investment, and consumption on impact, while real wages increase with a short delay. The forecast error variance attributed to IST shocks also declines, particularly for GDP, and investment at business-cycle frequencies.

[15]The qualitative pattern is similar to the empirical FEVD results in Table 3: IST shocks matter more for investment than for GDP or consumption at business-cycle horizons, though the magnitudes differ.

To further examine the mechanism through which oil and food prices influence the RPE, I develop a medium-scale DSGE model that embeds an explicit commodity goods sector. In this framework, commodity prices are exogenous and their output is used directly in final consumption and as an input in production. As a result, movements in commodity prices affect both consumption and investment prices and therefore influence the RPE independently of IST. The estimated model replicates the qualitative features of the empirical impulse responses obtained after adjusting for oil and food price movements, reinforcing the idea that commodity prices are an important source of variation in the RPE.

Despite incorporating this channel, the DSGE model assigns only a limited role to IST shocks in accounting for macroeconomic fluctuations. Most of the variation in investment, output, and consumption is instead explained by other structural disturbances, such as marginal efficiency of investment shocks, neutral technology shocks, and policy shocks. This result helps clarify the gap between empirical studies that attribute substantial importance to IST shocks and DSGE analyses in which IST plays a more modest role.

The findings highlight that properly accounting for commodity price movements is essential when using the RPE to identify IST shocks. Ignoring these fluctuations risks overstating the role of IST and misinterpreting the underlying drivers of business cycle dynamics. Future work could explore alternative sources of measurement error in the RPE or investigate whether similar mechanisms arise in open-economy settings where commodity dependence is more pronounced.

References

Adjemian, S., Juillard, M., Karamé, F., Mutschler, W., Pfeifer, J., Ratto, M., Rion, N., & Villemot, S. (2024). *Dynare: Reference manual, version 6* (Dynare Working Papers No. 80). CEPREMAP. https://www.dynare.org/wp/

An, S., & Schorfheide, F. (2007). Bayesian analysis of dsge models. *Econometric reviews*, *26*(2-4), 113–172.

Auerbach, A. J., & Gorodnichenko, Y. (2013). Output spillovers from fiscal policy. *American Economic Review*, *103*(3), 141–146.

Barsky, R. B., & Sims, E. R. (2011). News shocks and business cycles. *Journal of monetary Economics*, *58*(3), 273–289.

Baumeister, C. (2023). Measuring market expectations. *Handbook of economic expectations*, 413–441.

Baumeister, C., & Hamilton, J. D. (2019). Structural interpretation of vector autoregressions with incomplete identification: Revisiting the role of oil supply and demand shocks. *American Economic Review*, *109*(5), 1873–1910.

Beaudry, P., Moura, A., & Portier, F. (2015). Reexamining the cyclical behavior of the relative price of investment. *Economics Letters*, *135*, 108–111.

Ben Zeev, N., & Khan, H. (2015). Investment-specific news shocks and us business cycles. *Journal of Money, Credit and Banking*, *47*(7), 1443–1464.

Chen, K., & Wemy, E. (2015). Investment-specific technological changes: The source of long-run tfp fluctuations. *European Economic Review*, *80*, 230–252.

Choi, Y. (2020). Investment shocks, consumption puzzle, and business cycles. *Economic Inquiry*, *58*(3), 1387–1400.

Christensen, I., & Dib, A. (2008). The financial accelerator in an estimated new keynesian model. *Review of economic dynamics*, *11*(1), 155–178.

De Winne, J., & Peersman, G. (2016). Macroeconomic effects of disruptions in global food commodity markets: Evidence for the united states. *Brookings Papers on Economic Activity*, *2016*(2), 183–286.

Drechsel, T. (2023). Earnings-based borrowing constraints and macroeconomic fluctuations. *American Economic Journal: Macroeconomics*, *15*(2), 1–34.

Fernald, J. (2014). A Quarterly Utilization-Adjusted Series on Total Factor Productivity. *Working Paper Series 2012-19, Federal Reserve Bank of San Francisco.*

Fernández-Villaverde, J., & Guerrón-Quintana, P. A. (2021). Estimating dsge models: Recent advances and future challenges. *Annual Review of Economics*, *13*(1), 229–252.

Fisher, J. D. (2006). The dynamic effects of neutral and investment-specific technology shocks. *Journal of political Economy*, *114*(3), 413–451.

Galí, J., & Gambetti, L. (2009). On the sources of the great moderation. *American Economic Journal: Macroeconomics*, *1*(1), 26–57.

Greenwood, J., Hercowitz, Z., & Krusell, P. (2000). The role of investment-specific technological change in the business cycle. *European Economic Review*, *44*(1), 91–115.

Guerrieri, L., & Bodenstein, M. (2012). Oil efficiency, demand, and prices: A tale of ups and downs. *2012 Meeting Papers*, (25).

Jaimovich, N., & Rebelo, S. (2009). Can news about the future drive the business cycle? *American Economic Review*, *99*(4), 1097–1118.

Jo, J., & Adjemian, M. K. (2023). Measuring the effects of agricultural supply news shocks.

Jordà, Ò. (2005). Estimation and inference of impulse responses by local projections. *American economic review*, *95*(1), 161–182.

Jordà, Ò. (2023). Local projections for applied economics. *Annual Review of Economics*, *15*(1), 607–631.

Justiniano, A., Primiceri, G. E., & Tambalotti, A. (2010). Investment shocks and business cycles. *Journal of Monetary Economics*, *57*(2), 132–145.

Justiniano, A., Primiceri, G. E., & Tambalotti, A. (2011). Investment shocks and the relative price of investment. *Review of Economic Dynamics*, *14*(1), 102–121.

Känzig, D. R. (2021). The macroeconomic effects of oil supply news: Evidence from opec announcements. *American Economic Review*, *111*(4), 1092–1125.

Liao, S.-Y., & Chen, B.-L. (2023). News shocks to investment-specific technology in business cycles. *European Economic Review*, *152*, 104363. https://doi.org/https://doi.org/10.1016/j.euroecorev.2022.104363

Olea, J. L. M., Plagborg-Møller, M., Qian, E., & Wolf, C. K. (2025). *Local projections or vars? a primer for macroeconomists* (tech. rep.). National Bureau of Economic Research.

Peersman, G. (2022). International food commodity prices and missing (dis) inflation in the euro area. *Review of Economics and Statistics*, *104*(1), 85–100.

Ramey, V. A. (2016). Macroeconomic shocks and their propagation. *Handbook of macroeconomics*, *2*, 71–162.

Romer, C. D., & Romer, D. H. (2004). A new measure of monetary shocks: Derivation and implications. *American economic review*, *94*(4), 1055–1084.

Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. *The Review of Economic Studies*, *49*(4), 517–531.

Schmitt-Grohé, S., & Uribe, M. (2012). What's news in business cycles. *Econometrica*, *80*(6), 2733–2764.

Sims, C. A., Stock, J. H., & Watson, M. W. (1990). Inference in linear time series models with some unit roots. *Econometrica: Journal of the Econometric Society*, 113–144.

Smets, F., & Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American economic review*, *97*(3), 586–606.

Uhlig, H. (2004). What moves gnp? *Econometric Society 2004 North American Winter Meetings*, (636).

Villemot, S. (2011). *Solving rational expectations models at first order: what Dynare does* (Dynare Working Papers No. 2). CEPREMAP. https://ideas.repec.org/p/cpm/dynare/002.html

Wu, J. C., & Xia, F. D. (2016). Measuring the macroeconomic impact of monetary policy at the zero lower bound. *Journal of Money, Credit and Banking*, *48*(2-3), 253–291.

Appendices

A Figures

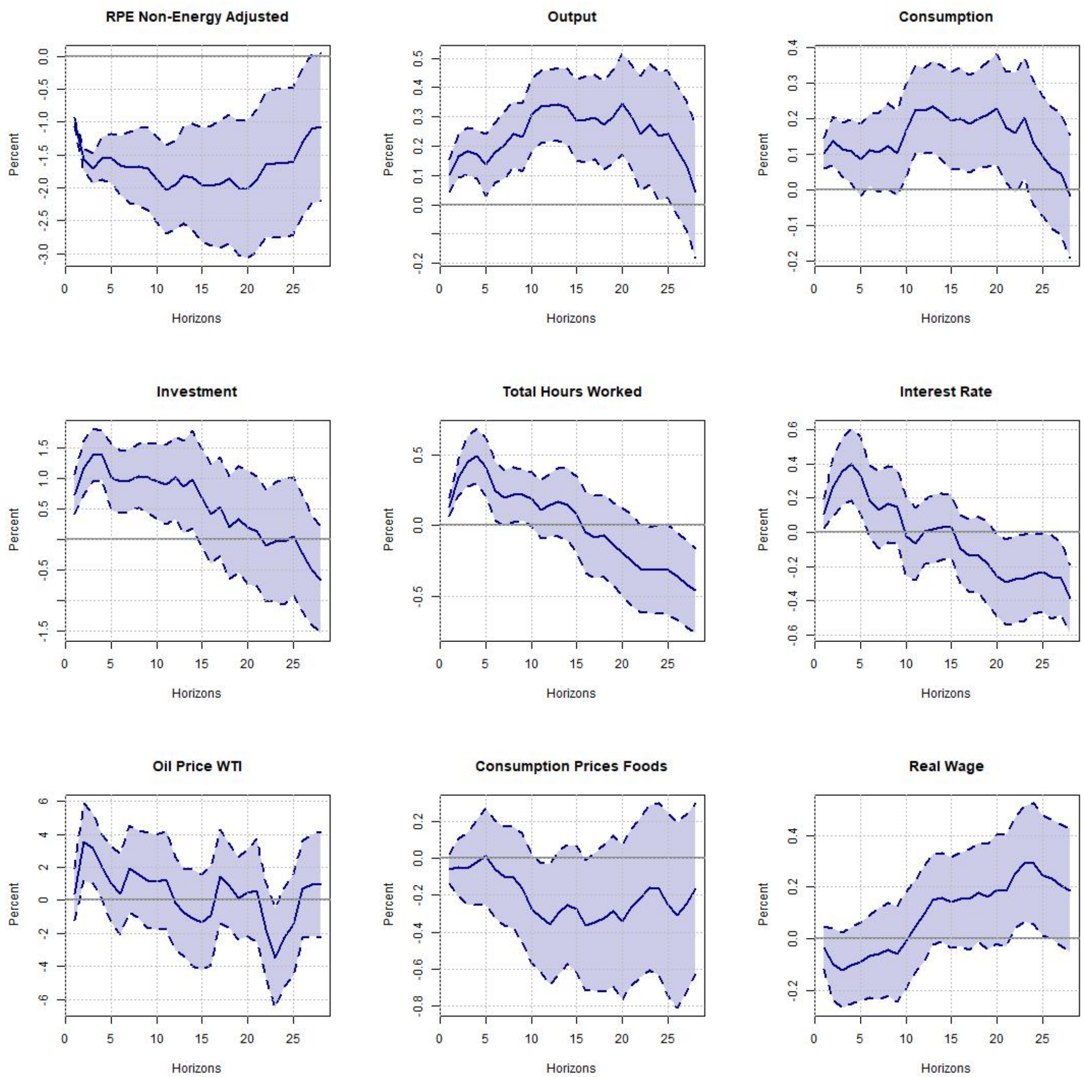


Figure A1: IRF to IST shocks using RPENE.

Note: Impulse response of macroeconomic variables to IST shocks estimated using RPENE. The shaded areas represent the 90.0% confidence interval.

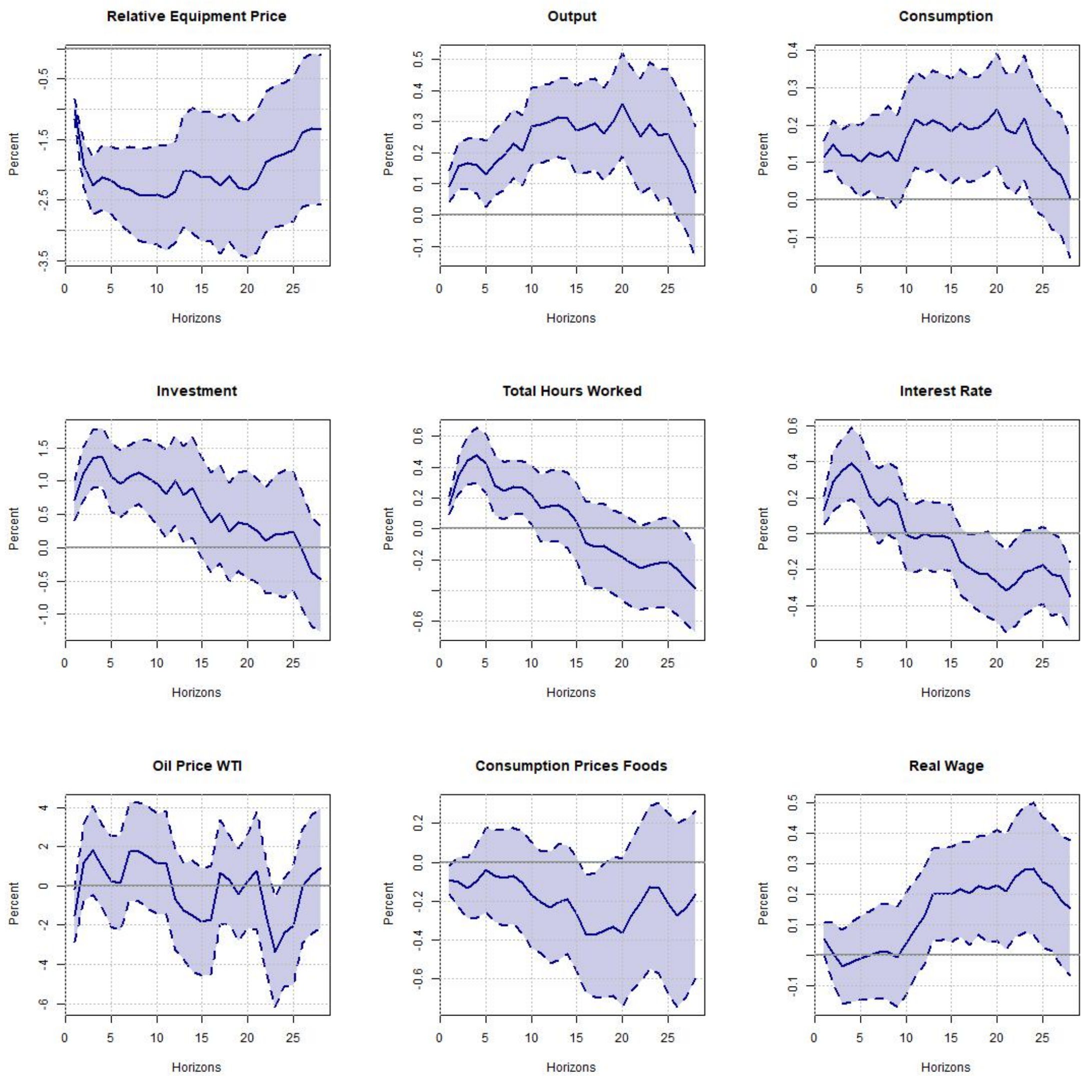
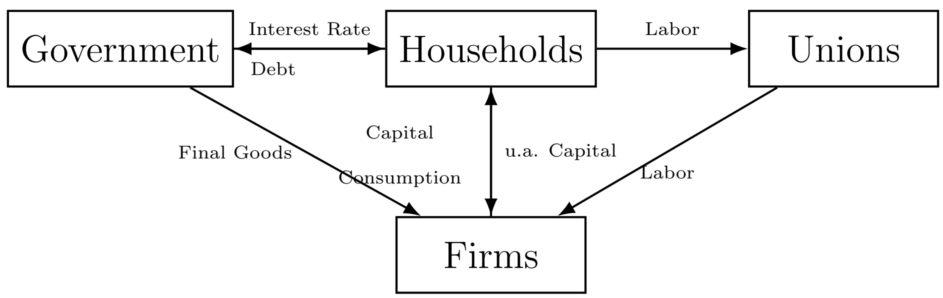
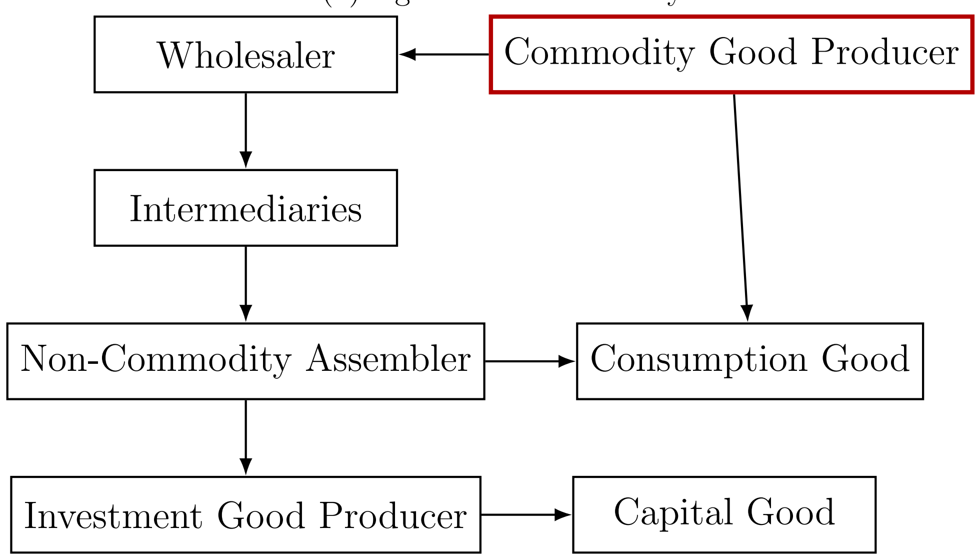


Figure A2: IRF to IST shocks using PCEFE.

Note: Impulse response of macroeconomic variables to IST shocks estimated using PCEFE. The shaded areas represent the 90.0% confidence interval.



(a) Agents in the economy.



(b) Producing sectors.

Figure A3: Diagram of the flows in the model economy.

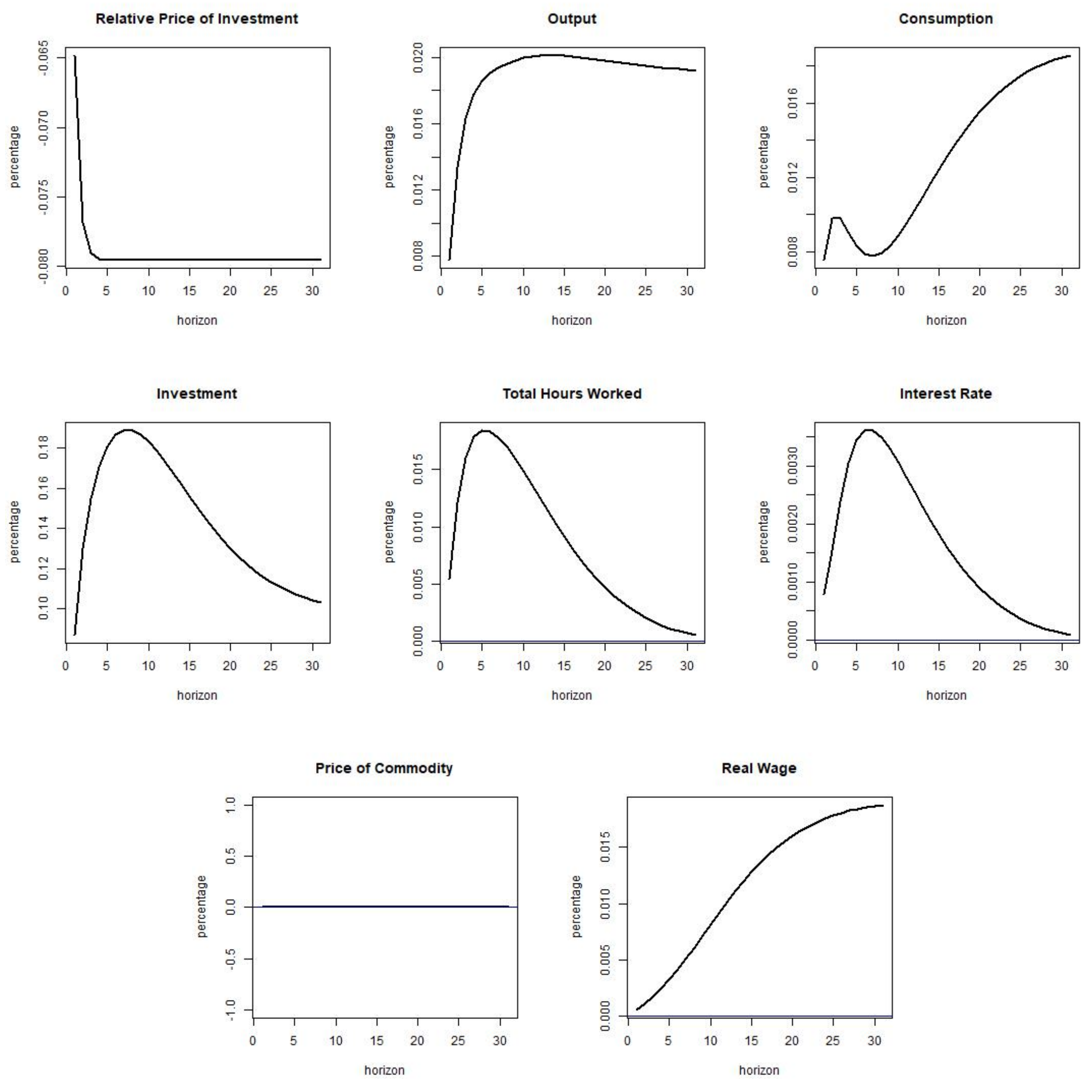


Figure A4: IRF after an IST shock in the DSGE model.

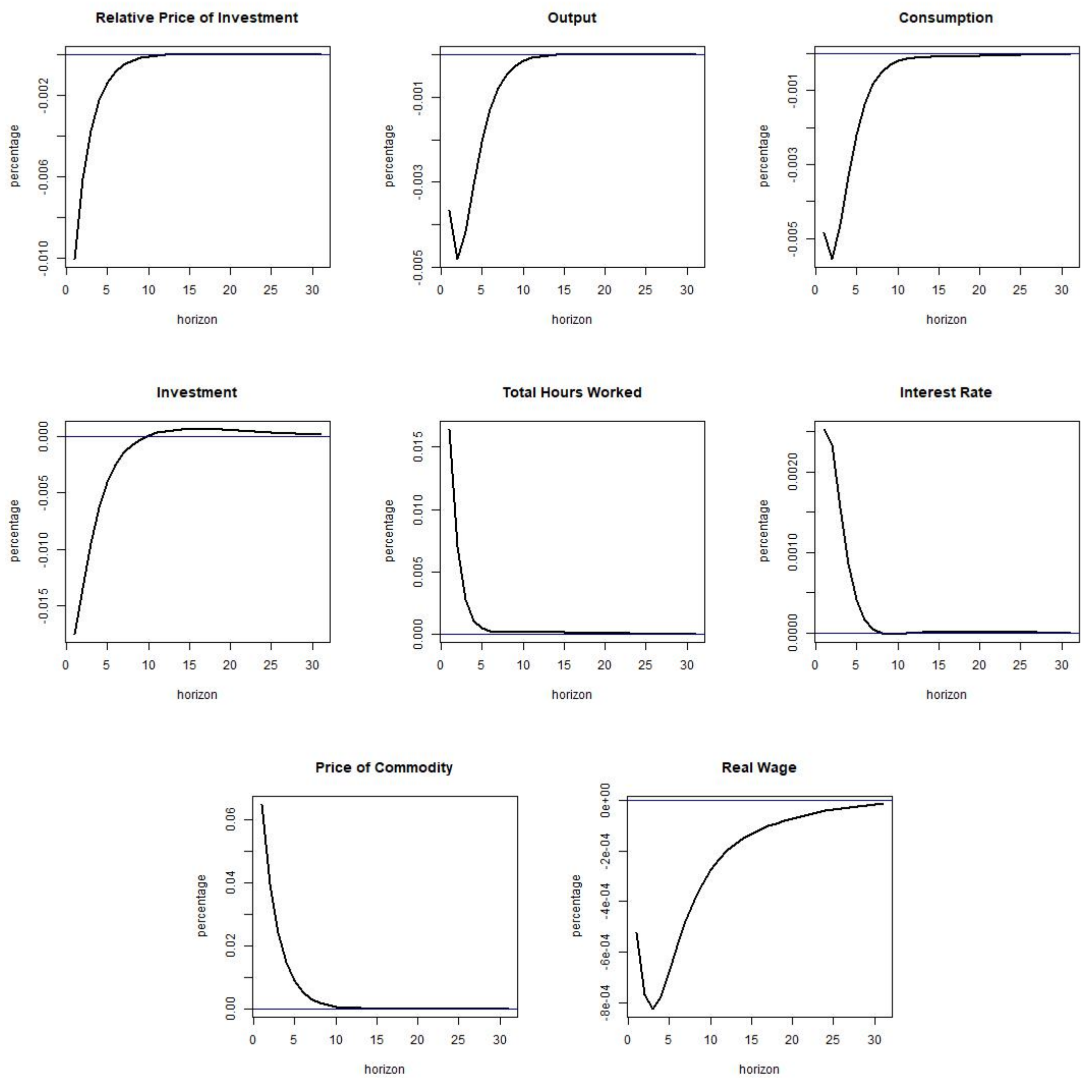


Figure A5: IRF after a commodity price shock in the DSGE model.

B Importance of BZK's IST news shocks

I compute the percentage of the forecast error variance explained by BZK's news shocks, both with and without the inclusion of oil and food price indexes. It is important to note that, according to the original document, the estimation already incorporates inflation (measured as the annual change in the CPI) into the system of equations, yet there is still a correlation with oil price shocks. To better account for the effects of food prices and WTI (West Texas Intermediate crude oil prices), I remove inflation from the estimation and instead introduce three components that are part of the CPI: Core CPI, Food CPI, and WTI.

Since Core CPI data is only available after 1957, the forecast error variance decomposition (FEVD) is calculated using data from that period onward for both the adjusted system (the clean one) and the baseline model. This adjustment allows for a more accurate assessment of how food and oil prices contribute to the variance explained by BZK's news shocks, ensuring that the results account for key components of the CPI while isolating their effects.

Table 4 shows the estimation with (Clean) and without (Base) the variables in the system. Notice that there is decrease in the forecast error variance of GDP, consumption and hours worked explained by the IST news shocks once I introduce both variables in the system.

	GDP	GDP	Inv.	Inv.	Cons.	Cons.	Hours	Hours
h	**Base**	**Clean**	**Base**	**Clean**	**Base**	**Clean**	**Base**	**Clean**
1	0.115	0.056	0.125	0.233	0.403	0.086	0.94	0.119
5	0.543	0.225	0.506	0.492	0.602	0.258	0.605	0.296
10	0.573	0.244	0.488	0.440	0.635	0.290	0.678	0.348
15	0.562	0.226	0.444	0.389	0.629	0.270	0.621	0.281
20	0.593	0.221	0.475	0.391	0.644	0.256	0.609	0.258

Table 9: FEV explained by IST news shock as in BZK

C Equilibrium Equations

To solve the model, I obtain the equations that characterize the equilibrium and redefine the nominal prices using the price of the non-commodity assembler as a numeraire. Price variables in lower case are the ratio of each price over the numeraire. All the variables are stationary around the BGP. The system of equations that characterize the equilibrium is as follows:

- Equations from the Wholesaler

$$
\begin{aligned}
p_t^w[1-\alpha-\beta_o]A_t^{1-\alpha-\beta_o}K_t^{\alpha}O_{p,t}^{\beta_o}L_t^{-\alpha-\beta_o} &= w_t && (49)\\
p_t^w[\alpha]A_t^{1-\alpha-\beta_o}K_t^{\alpha-1}O_{p,t}^{\beta_o}L_t^{1-\alpha-\beta_o} &= r_t^k && (50)\\
p_t^w[\beta_o]A_t^{1-\alpha-\beta_o}K_t^{\alpha}O_{p,t}^{\beta_o-1}L_t^{1-\alpha-\beta_o} &= p_t^o && (51)
\end{aligned}
$$

- New-Keynesian Phillips curve:

$$
\phi_p[\pi_t-1]\pi_t = \frac{1}{\lambda_t}[(1+\lambda_t)p_t^w-1] + E_t\left[\Lambda_{t,t+1}\phi_p[\pi_{t+1}-1]\pi_{t+1}g_{a,t+1}^{\psi_A}g_{\gamma,t+1}^{\psi_i}\frac{y_{t+1}}{y_t}\right] \quad (52)
$$

- Equations from the final good producer

$$
\begin{aligned}
1 &= p_t^f Y_{f,t}^{\frac{\lambda_f}{1+\lambda_f}}\omega_f Y_{c,t}^{\frac{-\lambda_f}{1+\lambda_f}} && (53)\\
p_t^o &= p_t^f Y_{f,t}^{\frac{\lambda_f}{1+\lambda_f}}[1-\omega_f]O_{c,t}^{\frac{-\lambda_f}{1+\lambda_f}} && (54)\\
Y_t^f &= \left[\omega_f Y_{c,t}^{\frac{1}{1+\lambda_f}} + (1-\omega_f)O_{c,t}^{\frac{1}{1+\lambda_f}}\right]^{1+\lambda_f} && (55)
\end{aligned}
$$

- Equations from the investment producer

$$
\begin{aligned}
p_t^i &= 1 && (56)\\
I_t &= Y_{i,t} && (57)
\end{aligned}
$$

- Equations from the capital producer

$$
\begin{aligned}
&p_t^k\mu_t\left[1-s_t-s_{p,t}\,g_{a,t}^{\psi_A}g_{\gamma,t}^{\psi_i+1}\frac{I_t}{I_{t-1}}\right]\\
&\quad + E_t\left[\Lambda_{t,t+1}g_{\gamma,t+1}^{-1}\,p_{t+1}^k\mu_{t+1}s_{p,t+1}g_{a,t+1}^{\psi_A}g_{\gamma,t+1}^{\psi_i+1}\left(\frac{I_t}{I_{t-1}}\right)^2\right] = p_t^i. \quad (58)
\end{aligned}
$$

$$
I_{k,t} = \mu_t[1-s_t]I_t \quad (59)
$$

- Equations from the household

$$\lambda_t^c = \psi \frac{L_{s,t}^{\chi_l}}{\tilde{w}_t} \tag{60}$$

$$p_t^f \lambda_t^c = [C_t - h\frac{C_{t-1}}{g_{a,t}^{\psi_A} g_{\gamma,t}^{\psi_i}}]^{\sigma} - E_t\Big[\beta_c h \frac{b_{t+1}}{b_t}[g_{a,t+1}^{\psi_A} g_{\gamma,t+1}^{\psi_i} C_{t+1} - hC_t]^{\sigma}\Big] \tag{61}$$

$$\Lambda_{t,t+1} = E_t\Big[\beta_c g_{a,t}^{-\sigma\psi_A} g_{\gamma,t}^{-\sigma\psi_i} \frac{\lambda_{t+1}^c}{\lambda_t^c} \pi_t^{-1}\Big] \tag{62}$$

$$1 = E_t\Big[R_{t+1}\Lambda_{t,t+1}\Big] \tag{63}$$

$$r_t^k = p_t^f a_{up,t} \tag{64}$$

$$\lambda_t^{c2} = \lambda_t^c p_t^f \tag{65}$$

$$\lambda_t^{c2} = E_t\Big[\beta_c g_{a,t+1}^{-\sigma\psi_A} g_{\gamma,t+1}^{-\sigma\psi_i} \lambda_{t+1}^c \frac{[r_{t+1}^k u_{t+1} - p_{t+1}^f a_{u,t+1}]}{g_{\gamma,t+1}} + \beta_c g_{a,t}^{-\sigma\psi_A} g_{\gamma,t}^{-\sigma(\psi_i+1)} \lambda_{t+1}^{c2}[1-\delta]\Big] \tag{66}$$

$$K_t = u_t \frac{\tilde{K}_{t-1}}{g_{a,t}^{\psi_A} g_{\gamma,t}^{\psi_i+1}} \tag{67}$$

$$\tilde{K}_t = [1-\delta]\frac{\tilde{K}_{t-1}}{g_{a,t}^{\psi_A} g_{\gamma,t}^{\psi_i+1}} + I_{k,t} \tag{68}$$

- Fiscal Authority Equations

$$P_t^f G_t + R_{t-1}\frac{B_{t-1}}{g_{a,t}^{\psi_A} g_{\gamma,t}^{\psi_i}} = B_t + \tau P_t^f Y_t \tag{69}$$

$$G_t = \Big[1 - \frac{1}{g_t}\Big] Y_t^f \tag{70}$$

- Wages Phillips curve

$$\phi_w \left[\left(\frac{w_t}{w_{t-1}}\right) g_{a,t}^{\psi_a} g_{\gamma,t}^{\psi_i} \pi_t - g_{a,ss}^{\psi_a} g_{\gamma,ss}^{\psi_i}\right] \left(\frac{y_t}{L_t}\right) \left(\frac{\pi_t}{w_{t-1}}\right)$$
$$- \frac{1}{\lambda_{l,t}}\left[(1+\lambda_{l,t})\left(\frac{\tilde{w}_t}{w_t}\right) - 1\right]$$
$$= E_t\Bigg[\Lambda_{t,t+1}\,\phi_w \left(\left(\frac{w_{t+1}}{w_t}\right) g_{a,t+1}^{\psi_a} g_{\gamma,t+1}^{\psi_i} \pi_{t+1} - g_{a,ss}^{\psi_a} g_{\gamma,ss}^{\psi_i}\right) \left(\frac{y_{t+1}}{L_{t+1}}\right) \left(\frac{\pi_{t+1}}{w_t}\right) \left(\frac{w_{t+1}}{w_t} g_{a,t+1}^{\psi_a} g_{\gamma,t+1}^{\psi_i}\right)\Bigg]. \tag{71}$$

- Other Market Clearing Conditions

$$Y_t = Y_{c,t} + Y_{i,t} \tag{72}$$
$$e_{o,t} = O_{p,t} + O_{c,t} \tag{73}$$
$$Y_{f,t} = C_t + G_t + a_{u,t}\hat{K}_t \tag{74}$$

- Function Definitions

$$s_t = \frac{\psi_s}{2}\left[g_{a,t}^{\psi_a} g_{\gamma,t}^{\psi_i+1}\frac{I_t}{I_{t-1}} - g_{a,ss}^{\psi_a} g_{\gamma,ss}^{\psi_i+1}\right]^2 \tag{75}$$
$$s_{p,t} = \psi_s\left[g_{a,t}^{\psi_a} g_{\gamma,t}^{\psi_i+1}\frac{I_t}{I_{t-1}} - g_{a,ss}^{\psi_a} g_{\gamma,ss}^{\psi_i+1}\right] \tag{76}$$
$$a_{u,t} = \frac{1}{\gamma_{ss}\mu_{ss}}\frac{R_{ss} - (1-\delta)}{\chi_u}\left[1 - e^{-\chi_u(u_t-1)}\right] \tag{77}$$
$$a_{up,t} = \frac{1}{\gamma_{ss}\mu_{ss}}[R_{ss} - (1-\delta)]e^{-\chi_u(u_t-1)} \tag{78}$$

- Law of motion of exogenous processes

$$log(g_{a,t}) = (1-\rho_a)log\bar{g_{a,ss}} + \rho_a log(g_{a,t-1}) + \epsilon_{a,t} \tag{79}$$
$$logP_t^o = (1-\rho_o)log\bar{P}^o + \rho_o logP_{t-1}^o + \epsilon_{o,t} \tag{80}$$
$$log(g_{\gamma,t}) = (1-\rho_i)log\bar{g_\gamma} + \rho_i log(g_{\gamma,t-1}) + \epsilon_{i,t} \tag{81}$$
$$log\mu_t = (1-\rho_m)log\bar{\mu} + \rho_m log\mu_{t-1} + \epsilon_{m,t} \tag{82}$$
$$logb_t = (1-\rho_b)log\bar{b} + \rho_b logb_{t-1} + \epsilon_{b,t} \tag{83}$$
$$log(g_t) = (1-\rho_g)log(\bar{g}) + \rho_g log(g_{t-1}) + \epsilon_{g,t} \tag{84}$$
$$log(\lambda_t) = (1-\rho_\lambda)log(\lambda) + \rho_\lambda log(\lambda_{t-1}) + \epsilon_{\lambda,t} \tag{85}$$
$$log(\lambda_{l,t}) = (1-\rho_{\lambda_l})log(\lambda_l) + \rho_{\lambda_l} log(\lambda_{l,t-1}) + \epsilon_{\lambda_l,t} \tag{86}$$

$$\begin{aligned}\log(R_t) = (1-\rho_r)\log(\bar{R}) + \rho_r\log(R_{t-1}) \\ + (1-\rho_r)\theta_\pi[\log(\pi_t) - \log(\bar{\pi})] \\ + (1-\rho_r)\theta_y[\log(Y_t) - \log(\bar{Y})] + \epsilon_{r,t}\end{aligned} \tag{87}$$

Steady State

To compute the Steady State (SS) I need to assume the SS values of some of the endogenous variables in the model. In particular, the SS of the technology variables ($\bar{g}_A, \bar{\mu}$, g_γ), the core inflation ($\bar{\pi}$), the government expenditure exogenous variable ($\bar{g}$) and the labor demanded by the wholesaler ($\bar{L}$) are set to be 1. Using these restrictions I find the SS of all the other variables by finding their time-invariant values that satisfy the equilibrium equations, and the SS defined above.