EXERCISE 4

1. Solve the inequation, $3x - 11 < 3$ where $x \in \{1, 2, 3, \ldots\ldots, 10\}$. Also, represent its solution on a number line.
Solution:

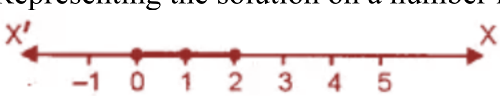
Given inequation, $3x - 11 < 3$
$3x < 3 + 11$
$3x < 14$
$\Rightarrow x < 14/3$
But, $x \in \{1, 2, 3, \ldots\ldots, 10\}$
Hence, the solution set is $\{1, 2, 3, 4\}$.
Representing the solution on a number line:

2. Solve $2(x - 3) < 1$, $x \in \{1, 2, 3, \ldots. 10\}$
Solution:

Given inequation, $2(x - 3) < 1$
$2x - 6 < 1$
$2x < 7$
$\Rightarrow x < 7/2$
But, $x \in \{1, 2, 3, \ldots. 10\}$
Hence, the solution set is $\{1, 2, 3\}$

3. Solve $5 - 4x > 2 - 3x$, $x \in W$. Also represent its solution on the number line.
Solution:

Given inequation, $5 - 4x > 2 - 3x$
$- 4x + 3x > 2 - 5$
$-x > -3$
On multiplying both sides by -1, the inequality reverses
$\Rightarrow x < 3$
Since, $x \in W$
The solution set is $\{0, 1, 2\}$
Representing the solution on a number line:



4. List the solution set of $30 - 4(2x - 1) < 30$, given that x is a positive integer.
Solution:

Given inequation, $30 - 4(2x - 1) < 30$
$30 - 8x + 4 < 30$

$34 - 8x < 30$
$-8x < 30 - 34$
$-8x < -4$ [On multiplying both sides by -1, the inequality reverses]
$8x > 4$
$x > 4/8$
$\Rightarrow x > 1/2$
As x is a positive integer
The solution set is {1, 2, 3, … }

5. Solve: 2 (x – 2) < 3x – 2, x ∈ {– 3, – 2, – 1, 0, 1, 2, 3}.
Solution:

Given inequation, $2(x - 2) < 3x - 2$
$2x - 4 < 3x - 2$
$2x - 3x < -2 + 4$
$-x < 2$
$\Rightarrow x > -2$
But, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$
Hence, the solution set is {– 1, 0, 1, 2, 3}.

6. If x is a negative integer, find the solution set of 2/3 + 1/3 (x + 1) > 0.
Solution:

Given inequation, $2/3 + 1/3 (x + 1) > 0$.
$2/3 + x/3 + 1/3 > 0$
$x/3 + 1 > 0$
$x/3 > -1$
$\Rightarrow x > -3$
As x is a negative integer
The solution set is {-1, -2}.

7. Solve x – 3 (2 + x) > 2 (3x – 1), x ∈ { – 3, – 2, – 1, 0, 1, 2, 3}. Also represent its solution on the number line.
Solution:

Given inequation, $x - 3(2 + x) > 2(3x - 1)$
$x - 6 - 3x > 6x - 2$
$-2x - 6 > 6x - 2$
$-6x - 2x > -2 + 6$
$-8x > 4$
$x < -4/8$
$\Rightarrow x < -1/2$
But, $x \in \{-3, -2, -1, 0, 1, 2, 3\}$
Hence, the solution set is {-3, -2, -1}

8. Given $x \in \{1, 2, 3, 4, 5, 6, 7, 9\}$ solve $x - 3 < 2x - 1$.
Solution:

Given inequation, $x - 3 < 2x - 1$
$x - 2x < -1 + 3$
$-x < 2$
$\Rightarrow x > -2$
But, $x \in \{1, 2, 3, 4, 5, 6, 7, 9\}$
Hence, the solution set is $\{1, 2, 3, 4, 5, 6, 7, 9\}$.

9. List the solution set of the inequation $\frac{1}{2} + 8x > 5x - 3/2$, $x \in Z$
Solution:

Given inequation, $\frac{1}{2} + 8x > 5x - 3/2$
$8x - 5x > -3/2 - \frac{1}{2}$
$3x > -4/2$
$\Rightarrow x > -2/3$
As $x \in Z$
The solution set is $\{0, 1, 2, 3, 4, 5, \ldots\}$

10. List the solution set of $(11 - 2x)/5 \geq (9 - 3x)/8 + 3/4$, $x \in N$
Solution:

Given inequation, $(11 - 2x)/5 \geq (9 - 3x)/8 + \frac{3}{4}$
$(11 - 2x)/5 \geq (9 - 3x + 6)/8$
$8\,(11 - 2x) \geq 5\,(15 - 3x)$
$88 - 16x \geq 75 - 15x$
$15x - 16x \geq 75 - 88$
$-x \geq -13$
$\Rightarrow x \leq 13$
As $x \in N$
Hence, the solution set is $\{1, 2, 3, 4, \ldots, 13\}$.

11. Find the values of x, which satisfy the inequation : $-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6}$, $x \in N$. Graph the solution set on the number line.
Solution:

Given inequation, $-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6}$
$-2 \leq (3 - 4x)/6 \leq 11/6$
$-12 \leq 3 - 4x \leq 11$
$-12 - 3 \leq -4x \leq 11 - 3$
$-15 \leq -4x \leq 8$
$-15/4 \leq -x \leq 8/4$

⇒ 15/4 ≥ x ≥ -2
As x ∈ N,
The solution set is {1, 2, 3}.
Representing the solution on a number line:

12. If x ∈ W, find the solution set of 3/5 x – (2x - 1)/3 > 1. Also graph the solution set on the number line, if possible.
Solution:

Given inequation, 3/5 x – (2x - 1)/3 > 1
9/15 x – 5(2x - 1)/15 > 1 [Taking L.C.M]
9x – 5(2x - 1) > 15 [Multiplying by 15 on both sides]
9x – 10x + 5 > 15
-x > 15 – 5
-x > 10
⇒ x < -10
But, x ∈ W
Hence, the solution set is a null set.
Thus, it can't be represented on number line.

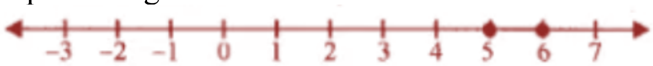
13. Solve:
(i) x/2 + 5 ≤ x/3 + 6, where x is a positive odd integer.
(ii) (2x + 3)/3 ≥ (3x - 1)/4, where x is positive even integer.
Solution:

(i) Given inequation, x/2 + 5 ≤ x/3 + 6
(x + 10)/2 ≤ (x + 18)/3 [Taking L.C.M on both sides]
3 (x + 10) ≤ 2 (x + 18) [On cross-multiplying]
3x + 30 ≤ 2x + 36
3x – 2x ≤ 36 – 30
⇒ x ≤ 6
As x is a positive odd integer.
Hence, the solution set is {1, 3, 5}.

(ii) Given inequation, (2x + 3)/3 ≥ (3x - 1)/4
4 (2x + 3) ≥ 3 (3x - 1) [On cross-multiplying]
8x + 12 ≥ 9x – 3
-9x + 8x ≥ -12 – 3
-x ≥ -15
⇒ x ≤ 15
As x is positive even integer.
Hence, the solution set is {2, 4, 6, 8, 10, 12, 14}.

14. Given that $x \in I$, solve the inequation and graph the solution on the number line:
$3 \geq (x - 4)/2 + x/3 \geq 2$
Solution:

Given inequation, $3 \geq (x - 4)/2 + x/3 \geq 2$
Now, let's take
$3 \geq (x - 4)/2 + x/3$, we have
$3 \geq (3x - 12 + 2x)/6$ [Taking L.C.M]
$18 \geq 5x - 12$
$30 \geq 5x$
$\Rightarrow x \leq 6$ …. (i)
Next,
$(x - 4)/2 + x/3 \geq 2$
$(3x - 12 + 2x)/6 \geq 2$
$5x - 12 \geq 12$
$5x \geq 24$
$x \geq 24/5 \Rightarrow x \geq 4.8$ … (ii)
Hence, from (i) and (ii) we have
Solution of $x = \{5, 6\}$
Representing the solution on a number line:



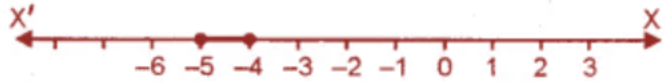
15. Solve: $1 \geq 15 - 7x > 2x - 27$, $x \in N$
Solution:

Given inequation, $1 \geq 15 - 7x > 2x - 27$,
So, we have
$1 \geq 15 - 7x$ and $15 - 7x > 2x - 27$
$7x \geq 15 - 1$ and $-2x - 7x > -27 - 15$
$7x \geq 14$ and $-9x > -42$
$x \geq 2$ and $-x > -42/9$
$x \geq 2$ and $x < 14/3$
$\Rightarrow 2 \leq x < 14/3$
But as $x \in N$
The solution set is $\{2, 3, 4\}$.

16. If $x \in Z$, solve $2 + 4x < 2x - 5 \leq 3x$. Also represent its solution on the number line.
Solution

Given inequation, $2 + 4x < 2x - 5 \leq 3x$
So, we have
$2 + 4x < 2x - 5$ and $2x - 5 \leq 3x$
$4x - 2x < -5 - 2$ and $2x - 3x \leq 5$
$2x < -7$ and $-x \leq 5$
$x < -7/2$ and $x \geq -5$

⇒ $-5 \leq x < -7/2$
As $x \in Z$
The solution set is {-5, -4}.
Representing the solution on a number line:

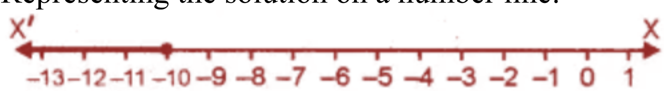


17. Solve: $(4x - 10)/3 \leq (5x - 7)/2$, $x \in R$ and represent the solution set on the number line.
Solution:

Given inequation, $(4x - 10)/3 \leq (5x - 7)/2$
$2(4x - 10) \leq 3(5x - 7)$ [On cross-multiplying]
$8x - 20 \leq 15x - 21$
$8x - 15x \leq -21 + 20$
$-7x \leq -1$
$-x \leq -1/7$
$x \geq 1/7$
As $x \in R$
Hence, the solution set is $\{x: x \in R, x \geq 1/7\}$
Representing the solution on a number line:

18. Solve $3x/5 - (2x - 1)/3 > 1$, $x \in R$ and represent the solution set on the number line.
Solution:

Given inequation, $3x/5 - (2x - 1)/3 > 1$
$(9x - 10x + 5)/15 > 1$ [Taking L.C.M]
$-x + 5 > 15$
$-x > 15 - 5$
$-x > 10$
$x < -10$
As $x \in R$
Hence, the solution set is $\{x: x \in R, x < -10\}$
Representing the solution on a number line:



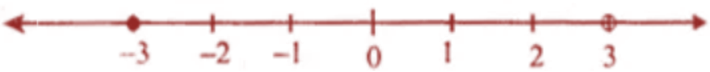
19. Given that $x \in R$, solve the following inequation and graph the solution on the number line: $-1 \leq 3 + 4x < 23$.
Solution:

Given inequation, $-1 \leq 3 + 4x < 23$
$-1 - 3 \leq 4x < 23 - 3$

-4 ≤ 4x < 20
-4/4 ≤ x < 20/4
-1 ≤ x < 5
Hence, the solution set is {-1 ≤ x < 5; x ∈ R}
Representing the solution on a number line:

20. Solve the following inequation and graph the solution on the number line.
$-2\frac{2}{3} \leq x + \frac{1}{3} < 3 + \frac{1}{3}$**, x ∈ R**
Solution:

Given inequation,
$-2\frac{2}{3} \leq x + \frac{1}{3} < 3 + \frac{1}{3}$
-8/3 ≤ (3x + 1)/3 < 10/3
-8 ≤ 3x + 1 < 10 [Multiplying by 3]
-8 – 1 ≤ 3x < 10 – 1
-9 ≤ 3x < 9
-3 ≤ x < 3 [Dividing by 5]
Thus, the solution set is {x: x ∈ R, -3 ≤ x < 3}
Representing the solution on a number line:



21. Solve the following inequation and represent the solution set on the number line:
$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in R$
Solution:

Given in equation,
$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in R$
-3 < -(3 + 4x)/6 ≤ 5/6 [Taking L.C.M]
-18 < -3 – 4x ≤ 5 [Multiplying by 6]
-18 + 3 < -4x ≤ 5 + 3
-15 < -4x ≤ 8
-15/4 < -x ≤ 8/4
-2 ≤ x < 15/4
Hence, the solution set is {x : x ∈ R, -2 ≤ x < 15/4}
Representing the solution on a number line:

22. Solving the following inequation, write the solution set and represent it on the number line

$-3(x - 7) \geq 15 - 7x > \frac{x+1}{3}$, x ∈ R
Solution:

Given inequation, $-3(x - 7) \geq 15 - 7x > \frac{x+1}{3}$
$-3x + 21 \geq 15 - 7x > (x + 1)/3$
So,
$-3x + 21 \geq 15 - 7x$
$7x - 3x \geq 15 - 21$
$4x \geq -6$
$x \geq -6/4$
$x \geq -3/2$
And,
$15 - 7x > (x + 1)/3$
$3(15 - 7x) > x + 1$
$45 - 21x > x + 1$
$-21x - x > 1 - 45$
$-22x > -44$
$-x > -44/22$
$x < 2$
Hence, the solution set is $\{x : x \in R, -3/2 \leq x < 2\}$
Representing the solution on a number line:

23. Solve the following inequation, write down the solution set and represent it on the real number line:
$-2 + 10x \leq 13x + 10 \leq 24 + 10x$, x ∈ Z
Solution:

Given inequation, $-2 + 10x \leq 13x + 10 \leq 24 + 10x$
So, we have
$-2 + 10x \leq 13x + 10$ and $13x + 10 \leq 24 + 10x$
$10x - 13x \leq 10 + 2$ and $13x - 10x \leq 24 - 10$
$-3x \leq 12$ and $3x \leq 14$
$x \geq -12/3$ and $x \leq 14/3$
$x \geq -4$ and $x \leq 14/3$
So, $-4 \leq x \leq 14/3$
As x ∈ Z
Thus, the solution set is {-4, -3, -2, -1, 0, 1, 2, 3, 4}
Representing the solution on a number line:

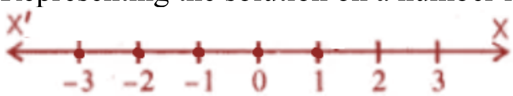
24. Solve the inequation $2x - 5 \leq 5x + 4 < 11$, where x ∈ I. Also represent the solution set on the number line.

Solution:

Given inequation, $2x - 5 \leq 5x + 4 < 11$
So, we have

$2x - 5 \leq 5x + 4$	and	$5x + 4 < 11$
$2x - 5x \leq 4 + 5$	and	$5x < 11 - 4$
$-3x \leq 9$	and	$5x < 7$
$-x \leq 9/3$	and	$x < 7/5$
$x \geq -3$	and	$x < 7/5$

$-3 \leq x < 7/5$
As $x \in I$
Thus, the solutions set is {-3, -2, -1, 0, 1}
Representing the solution on a number line:



25. If $x \in I$, A is the solution set of $2(x - 1) < 3x - 1$ and B is the solution set of $4x - 3 \leq 8 + x$, find $A \cap B$.
Solution:

Given inequations,
$2(x - 1) < 3x - 1$ and $4x - 3 \leq 8 + x$ for $x \in I$
Solving for both, we have

$2x - 3x < 2 - 1$	and	$4x - x \leq 8 + 3$
$-x < 1$	and	$3x \leq 11$
$x > -1$	and	$x \leq 11/3$

Hence,
Solution set A = {0, 1, 2, 3, …}
Solution set B = {3, 2, 1, 0, -1, … }
Thus, $A \cap B$ = {0, 1, 2, 3}

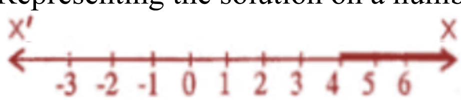
26. If P is the solution set of $-3x + 4 < 2x - 3$, $x \in N$ and Q is the solution set of $4x - 5 < 12$, $x \in W$, find
(i) $P \cap Q$
(ii) $Q - P$.
Solution:

Given inequations,
$-3x + 4 < 2x - 3$ where $x \in N$ and $4x - 5 < 12$ where $x \in W$
So, solving
$-3x + 4 < 2x - 3$ where $x \in N$
$-3x - 2x < -3 - 4$
$-5x < -7$
$x > 7/5$
Hence, the solution set P is {2, 3, 4, 5, …}

And, solving
$4x - 5 < 12$ where $x \in W$
$4x < 12 + 5$
$4x < 17$
$x < 17/4$
Hence, the solution set Q is {0, 1, 2, 3, 4}
Therefore,
(i) $P \cap Q = \{2, 3, 4\}$
(ii) $Q - P = \{0, 1\}$

27. A = {x : 11x – 5 > 7x + 3, x ∈ R} and
B = {x : 18x – 9 ≥ 15 + 12x, x ∈ R}
Find the range of set A ∩ B and represent it on a number line
Solution:

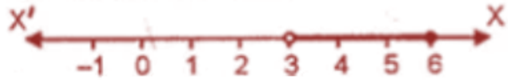
Given, $A = \{x : 11x - 5 > 7x + 3, x \in R\}$ and $B = \{x : 18x - 9 \geq 15 + 12x, x \in R\}$
Solving for A,
$11x - 5 > 7x + 3$
$11x - 7x > 3 + 5$
$4x > 8$
$x > 2$
Hence, $A = \{x : x > 2, x \in R\}$
Next, solving for B
$18x - 9 \geq 15 + 12x$
$18x - 12x \geq 15 + 9$
$6x \geq 24$
$x \geq 4$
Hence, $B = \{x : x \geq 4, x \in R\}$
Thus, $A \cap B = x \geq 4$
Representing the solution on a number line:



28. Given: P {x : 5 < 2x – 1 ≤ 11, x ∈ R}
Q {x : – 1 ≤ 3 + 4x < 23, x ∈ I} where
R = (real numbers), I = (integers)
Represent P and Q on number line. Write down the elements of P ∩ Q.
Solution:

Given, $P \{x : 5 < 2x - 1 \leq 11, x \in R\}$ and $Q \{x : -1 \leq 3 + 4x < 23, x \in I\}$
Solving for P,
$5 < 2x - 1 \leq 11$
$5 + 1 < 2x \leq 11 + 1$
$6 < 2x \leq 12$
$3 < x \leq 6$

Hence, P = P {x : 3 < x ≤ 6, x ∈ R}
Representing the solution on a number line:



Next, solving for Q
– 1 ≤ 3 + 4x < 23
-1 – 3 ≤ 4x < 23 - 3
-4 ≤ 4x < 20
-1 ≤ x < 5
Hence, solution Q = {-1, 0, 1, 2, 3, 4}
Representing the solution on a number line:

Therefore, P ∩ Q = {4}

29. If x ∈ I, find the smallest value of x which satisfies the inequation $2x+\frac{5}{2}>\frac{5x}{3}+2$
Solution:

Given inequation, $2x+\frac{5}{2}>\frac{5x}{3}+2$
(4x + 5)/2 > (5x + 6)/3 [Taking L.C.M]
3 (4x + 5) > 2 (5x + 6) [On cross-multiplication]
12x + 15 > 10x + 12
12x – 10x > 12 – 15
2x > -3
x > -3/2
Hence, for x ∈ I the smallest value of x is -1.

30. Given 20 – 5 x < 5 (x + 8), find the smallest value of x, when
(i) x ∈ I
(ii) x ∈ W
(iii) x ∈ N.
Solution:

Given inequation, 20 – 5 x < 5 (x + 8)
20 – 5x < 5x + 40
-5x – 5x < 40 – 20
-10x < 20
-x < 20/10
x > -2
Thus,
(i) For x ∈ I, the smallest value = -1
(ii) For x ∈ W, the smallest value = 0
(iii) For x ∈ N, the smallest value = 1

31. Solve the following inequation and represent the solution set on the number line:

$$4x - 19 < \frac{3x}{5} - 2 \leq -\frac{2}{5} + x, x \in R$$

Solution:

Given inequation,

$$4x - 19 < \frac{3x}{5} - 2 \leq -\frac{2}{5} + x, x \in R$$

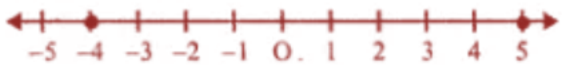
So, we have

$4x - 19 < 3x/5 - 2$	and	$3x/5 - 2 \leq -2/5 + x$	
$4x - 3x/5 < 19 - 2$	and	$3x/5 - x \leq 2 - 2/5$	
$(20x - 3x)/5 < 17$	and	$(3x - 5x)/5 \leq (10 - 2)/5$	
$17x < 35$	and	$-2x \leq 8$	[Multiplying by 5]
$x < 5$	and	$-x \leq 4$	
$x < 5$	and	$x \geq 4$	

$-4 \leq x < 5, x \in R$

Hence, the solution set is $\{x : -4 \leq x < 5, x \in R\}$

Representing the solution on a number line:



32. Solve the given inequation and graph the solution on the number line:

$2y - 3 < y + 1 \leq 4y + 7$; $y \in R$.

Solution:

Given inequation, $2y - 3 < y + 1 \leq 4y + 7$

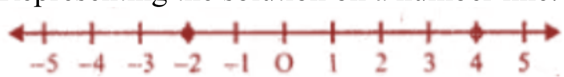
So, we have

$2y - 3 < y + 1$	and	$y + 1 \leq 4y + 7$
$2y - y < 1 + 3$	and	$y - 4y \leq 7 - 1$
$y < 4$	and	$-3y \leq 6$
$y < 4$	and	$-y \leq 2 \Rightarrow y \geq -2$

Thus, $-2 \leq y < 4$

The solution set is $\{y : -2 \leq y < 4, y \in R\}$

Representing the solution on a number line:



33. Solve the inequation and represent the solution set on the number line.

$$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x, Where \quad x \in I$$

Solution:

Given inequation,

$$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x, Where \quad x \in I$$

So, we have

$-3 + x \leq 8x/3 + 2$	and	$8x/3 + 2 \leq 14/3 + 2x$	
$x - 8x/3 \leq 2 + 3$	and	$8x/3 - 2x \leq 14/3 - 2$	
$(3x - 8x)/3 \leq 5$	and	$(8x - 6x)/3 \leq (14 - 6)/3$	[Taking L.C.M]
$-5x/3 \leq 5$	and	$2x \leq 8$	
$-5x \leq 15$	and	$x \leq 8/2$	
$-x \leq 3$	and	$x \leq 4$	
$x \geq -3$	and	$x \leq 4$	

$\Rightarrow -3 \leq x \leq 4$

Thus, the solution set is {-3, -2, -1, 0, 1, 2, 3, 4}

Representing the solution on a number line:

34. Find the greatest integer which is such that if 7 is added to its double, the resulting number becomes greater than three times the integer.

Solution:

Let's consider the greatest integer to be x

Then according to the given condition, we have

$2x + 7 > 3x$

$2x - 3x > -7$

$-x > -7$

$x < 7$, $x \in R$

Hence, the greatest integer value is 6.

35. One-third of a bamboo pole is buried in mud, one-sixth of it is in water and the part above the water is greater than or equal to 3 metres. Find the length of the shortest pole.

Solution:

Let's assume the length of the shortest pole = x metre

Now,

Length of the pole which is buried in mud = x/3

Length of the pole which is in the water = x/6

Then according to the given condition, we have

$x - [x/3 + x/6] \geq 3$

$x - [(2x + x)/6] \geq 3$

$x - 3x/6 \geq 3$

$x - x/2 \geq 3$

$x/2 \geq 3$

$x \geq 6$ [Multiplying by 6]

Therefore, the length of the shortest pole is 6 metres.

CHAPTER TEST

1. Solve the inequation: $5x - 2 \leq 3(3 - x)$ where $x \in \{-2, -1, 0, 1, 2, 3, 4\}$. Also represent its solution on the number line.
Solution:

Given inequation, $5x - 2 \leq 3(3 - x)$
$5x - 2 \leq 9 - 3x$
$5x + 3x \leq 9 + 2$
$8x \leq 11$
$x \leq 11/8$
As $x \in \{-2, -1, 0, 1, 2, 3, 4\}$
The solution set is $\{-2, -1, 0, 1\}$
Representing the solution on a number line:

2. Solve the inequation: $6x - 5 < 3x + 4$, $x \in I$
Solution:

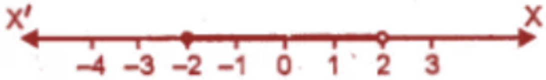
Given inequation, $6x - 5 < 3x + 4$
$6x - 3x < 4 + 5$
$3x < 9$
$x < 9/3$
$x < 3$
As $x \in I$
The solution set is $\{2, 1, 0, -1, -2, \ldots\}$

3. Find the solution set of the inequation $x + 5 \leq 2x + 3$; $x \in R$
Graph the solution set on the number line.
Solution:

Given inequation, $x + 5 \leq 2x + 3$
$x - 2x \leq 3 - 5$
$-x \leq -2$
$x \geq 2$
As $x \in R$
Thus, the solution set is $\{2, 3, 4, 5, \ldots\}$
Representing the solution on a number line:

4. If $x \in R$ (real numbers) and $-1 < 3 - 2x \leq 7$, find solution set and present it on a number line.
Solution:

Given inequation, -1 < 3 – 2x ≤ 7
-1 – 3 < -2x ≤ 7 - 3
-4 < -2x ≤ 4
-4/2 < -x ≤ 4/2
-2 < -x ≤ 2
Thus, -2 ≤ x < 2
The solution set is {x : x ∈ R, -2 ≤ x < 2}
Representing the solution on a number line:



5. Solve the inequation:

$$\frac{5x+1}{7} - 4\left(\frac{x}{7} + \frac{2}{5}\right) \leq 1\frac{3}{5} + \frac{3x-1}{7}, x \in R$$

Solution:

Given inequation,

$$\frac{5x+1}{7} - 4\left(\frac{x}{7} + \frac{2}{5}\right) \leq 1\frac{3}{5} + \frac{3x-1}{7}, x \in R$$

(5x + 1)/7 – 4 (5x + 14)/35 ≤ 8/5 + (3x - 1)/7
[5(5x + 1) - 4 (5x + 14)]/ 35 ≤ [56 + 5 (3x – 1)]/ 35 [Taking L.C.M]
(25x + 5 – 20x – 56) ≤ 56 + 15x – 5
5x – 51 ≤ 51 + 15x
5x – 15x ≤ 51 + 51
-10x ≤ 102
-x ≤ 102/10
x ≥ - 51/5
Hence, the solution set is {x : x ∈ R , x ≥ - 51/5}

6. Find the range of values of a, which satisfy 7 ≤ – 4x + 2 < 12, x ∈ R. Graph these values of a on the real number line.
Solution:

7 < – 4x + 2 < 12
7 < – 4x + 2 and – 4x + 2 < 12

7. If x ∈ R, solve 2x – 3 ≥ x + (1 - x)/3 > 2x/5
Solution:

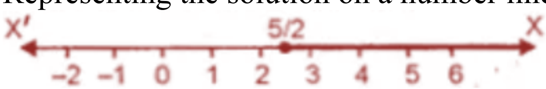
Given inequation, 2x – 3 ≥ x + (1 - x)/3 > 2x/5
So, we have

2x – 3 ≥ x + (1 - x)/3	and	x + (1 - x)/3 > 2x/5	
2x – 3 ≥ (3x + 1 - x)/3	and	(3x + 1 - x)/3 > 2x/5	[On taking L.C.M]
3(2x - 3) ≥ 2x + 1	and	(2x + 1) × 5 > 2x × 3	[Upon cross multiplication]
6x – 9 ≥ 2x + 1	and	10x + 5 > 6x	

$6x - 2x \geq 1 + 9$ and $10x - 6x > -5$
$4x \geq 10$ and $4x > -5$
$x \geq 10/4$ and $x > -5/4$
$x \geq 5/2$
As $x \in R$
Thus, the solution set is $\{x: x \in R, x \geq 5/2\}$
Representing the solution on a number line:

7. If x ∈ R, solve 2x – 3 ≥ x + (1 - x)/3 > 2x/5. Also represent the solution on the number line.
Solution:

Given inequation, $2x - 3 \geq x + (1 - x)/3 > 2x/5$
So, we have
$2x - 3 \geq x + (1 - x)/3$ and $x + (1 - x)/3 > 2x/5$
$2x - 3 \geq (3x + 1 - x)/3$ and $(3x + 1 - x)/3 > 2x/5$ [On taking L.C.M]
$3(2x - 3) \geq 2x + 1$ and $5 \times (2x + 1) > 3 \times 2x$
$6x - 9 \geq 2x + 1$ and $10x + 5 > 6x$
$6x - 2x \geq 1 + 9$ and $10x - 6x > -5$
$4x \geq 10$ and $4x > -5$
$x \geq 10/4$ and $x > -5/4$
$x \geq 5/2$
As $x \in R$
The solution set = $\{x: x \in R, x \geq 5/2\}$
Representing the solution on a number line:



8. Find positive integers which are such that if 6 is subtracted from five times the integer then the resulting number cannot be greater than four times the integer.
Solution:

Let's consider the positive integer be x
Then according to the problem, we have
$5a - 6 < 4x$
$5a - 4x < 6$
$\Rightarrow x < 6$
Hence, the solution set = $\{x : x < 6\}$
= $\{1, 2, 3, 4, 5, 6\}$

9. Find three smallest consecutive natural numbers such that the difference between one-third of the largest and one-fifth of the smallest is at least 3.
Solution:

Let’s consider the first least natural number as x
Then, second number = x + 1
And third number = x + 2
So, according the conditions given in the problem, we have
$1/3 \times (x + 2) - x/5 \geq 3$
$5x + 10 - 3x \geq 3 \times 15$
[Multiplying by 15 the L.C.M of 3 and 5]
$2x \geq 45 - 10$
$2x \geq 35$
$x \geq 35/2$
$x \geq 17.5$
As x is a natural least number
Thus, first least natural number = 18
Second number = 18 + 1 = 19
And, third number = 18 + 2 = 20
Hence, the least natural numbers are 18, 19 and 20