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I. Aggregating Bilateral Export Functions

1. It can be argued that bilateral import and export equations are more basic than totally aggregated equations, even though the former is presumably an aggregate of the import demands of all residents of a given country. In any case, for purposes of the argument, we will assume the existence of well-fitted bilateral export equations for the United States, expressed as the imports, M, of country *i* from the US. Most statistical tests show that a double log functional form is superior to others (despite its theoretical problems):

$$\ln M_i = a_i + \varepsilon^Y_i \ln Y_i + \varepsilon^P_i \ln\left(\frac{P_i}{P_{US}}\right) + \varepsilon^P_{ik} \ln\left(\frac{P_k}{P_{US}}\right), \quad (1)$$

where: M_i = real imports of country i from the US (in $)

Y_i = some real income measure for country i.

P_{US} = some sort of U.S. export price.

P_i = some sort of price deflator for country i. (expressed in dollars)

P_k = an export price from a third country k, producing substitutes for US exports. (in $)

Such equations sometimes work quite well, i.e. in the original version of the MCM. They also naturally allow the ability to enter the prices of exports from competing countries. For world models, such as the MCM, bilateral equations, if they can be used, allow the avoidance of allocative devices such as trade-share matrices.

2. Supposing we have a complete set of such log-linear equations estimated for the United States, can we justifiably *aggregate* these – getting a consistent aggregate export equation for the United States?

a. Of course, if we have all of the estimated coefficients of the bilateral equations, we can aggregate over the individual country variables forming aggregate variables as the sums of logs of the bilateral variables. But these aggregates of logs are the logs of *geometric means,* and likely not very interesting: e.g. $\sum_{i=1}^{N} \ln M_i = \ln\left\{\prod_{i=1}^{N} M_i\right\}$

b. Suppose we want at least our *dependent* variable to be expressed as the arithmetic sum of bilateral exports or the log thereof? There is a nice theorem that links the arithmetic and geometric means of a variable, given that the variable is normally distributed (See, e.g. L. Klein, *An Introduction to Econometrics (1962), p. 154-156*).

$$\frac{1}{N}\sum_{i=1}^{N} M_i = \left[\prod M_i\right]^{\frac{1}{N}} \cdot e^{\sigma^2/2}$$

$$\text{Hence: } \ln\left(\sum_i M_i\right) = \frac{1}{N}\sum_i \ln M_i + \frac{1}{2}\sigma^2_{\ln M_i} - \ln\frac{1}{N} \quad (2)$$

3. Assuming the underlying normality of the distribution of bilateral exports of the United States, the above transformation would allow us to get from the *estimated* bilateral export equations to aggregate U.S. exports. We would, e.g., use the bilateral equations to forecast each $\ln M_i^e$, add these up, subtract ½ of the variance of the (normally distributed) ln exports and derive the desired log of aggregate exports. $\ln \sum_i M_i$, adding along the way the corrections for the terms in "N" in (2).

B. Wanting More: Justifying an Aggregate Export Function Given the Bilateral Functions

1. Assuming normality, we can apply the theorem to any of the other variables on the right hand side of the equation. However, each set of logs on the r.h.s. is complicated by the fact that each individual log is weighted by an elasticity that must be estimated. If we *have* these elasticities, we would do best by forming an index variable, weighting each individual log by its respective elasticity.

2. Assuming we must estimate these elasticities, consider the following:

Express: $$\sum_{i=1}^{N} \varepsilon_i^y \ln Y_i \equiv \sum_{i=1}^{N} (\bar{e}^y + \Delta\varepsilon_i^y) \ln Y_i \qquad (3)$$

where $\Delta e_i^y = \varepsilon_i^y - \bar{e}^y$, and $\bar{e}^y$ is the mean of the elasticities, e_i^y.

3. IF we can assume that the ε_i^y, bilateral income elasticities are also normally distributed (with mean $\bar{e}^y$), then the r.h.s. of (3) becomes tractable:

$$\sum_i (\bar{e}^y + \Delta\varepsilon_i^y) \ln Y_i = \bar{e}^y \sum_i \ln Y_i + \sum_i \left[\Delta\varepsilon_i^y \cdot \ln Y_i\right] \qquad (4)$$

4. The Klein theorem (2) can be applied to

$$\bar{e}^y \sum_i \ln Y_i = N\bar{e}^y \left[\ln \sum_i Y_i - \tfrac{1}{2}\sigma^2_{\ln Y} + \ln \tfrac{1}{N}\right]$$

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5. The second term on the r.h.s of (4) is the sum of the product of normally distributed variables. Without further information, this term must go into the error term of the aggregate export equation. However, one can calculate its mean and variance, even if the variables are correlated ($\rho \neq 0$)

Using the formula for the covariance between two random variables, we have that:

$$E\left(\Delta\varepsilon_i^y \cdot \ln Y_i\right) = 0 \cdot E(\ln Y_i) + \rho\sqrt{\sigma^2_{\Delta\varepsilon^y} \cdot \sigma^2_{\ln Y_i}}$$
$$= 0, \text{ if } \rho = 0. \qquad (5)$$

If we can assume that the estimated income elasticities and income size are independent, then the last term in (4) goes into the error term & has mean zero. If $\rho \neq 0$, then we can add another term to the aggregate regression to correct for this.

6. *Summarizing:* If the correlation between $\Delta\varepsilon_i^y$ and $\ln Y_i$ Can be assumed to be 0, then:

$$\sum_{i=1}^{N} \varepsilon_i^y \ln Y_i \approx \bar{\varepsilon}^y \sum_{i=1}^{N} \ln Y_i \approx N\bar{\varepsilon}^y \left[\ln \sum_i Y_i - \tfrac{1}{2}\sigma^2_{\ln Y} + \ln \tfrac{1}{N}\right] \quad (6)$$

7. We can also make similar substitutions for the aggregates of the price terms on the right hand side of the bilateral equations (1). Thus, assuming normality for each of the variables in (1), both dependent and independent, we can derive an equation in the log of the arithmetic means of these variables – requiring, however, the addition to the equation of terms in the variances of the original variables and a correction factor for the number (N) of bilateral equations.

II. Exports, Imports, and Direct Investment

1. Direct Investment affecting supply. Assembly abroad with intermediate exports. Full production abroad.

2. Direct Investment affecting demand?

A. Assume previous Log-linear Demand curve for U.S. good. As before assume some degree of monopoly power and pricing. Thus, get previous pricing rule MR = MC.

$$P_{\$} = \left(1 - \frac{1}{\eta_d}\right)^{-1} MC = \bar{K} Q^{-\frac{1}{\eta_d}} \qquad (1)$$

B. Direct Investment Production and the Firm's Cost Curve for Exports.

1. Assume two possibilities for production abroad: assembly with intermediates (engines) exported from US; and full production abroad: assembly and engine production abroad.

2. Both operations involve fixed costs, unrelated to level of production, and constant marginal costs of production (so-called increasing returns production). For simplicity, think of fixed costs as land or plant rental; rent, so don't have to solve intertemporal problem.

a. costs saved by producing abroad: transportation costs (high for assembled autos), possible tariffs, possibly lower labor costs in developing countries.

C. Optimal Exports When Exporting from US (No Direct Investment).

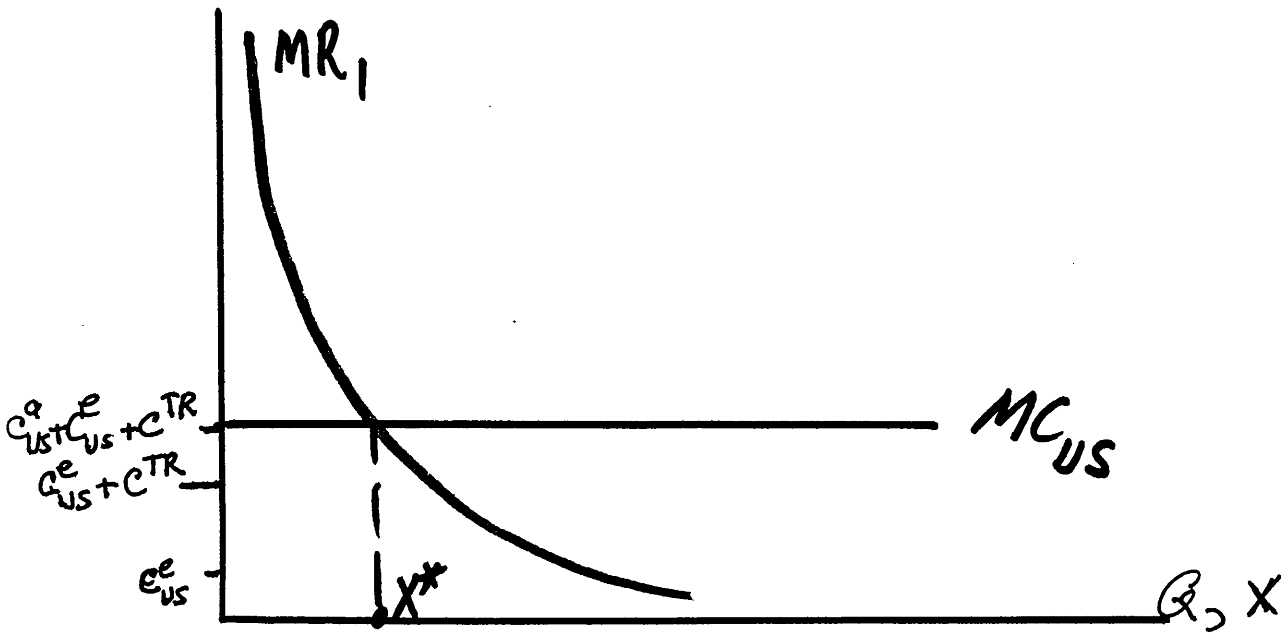
1. Constant marginal cost of production, which includes US cost of assembly, engine production, and transportation abroad (could also include tariffs, etc.):

$$MC_{US} = c^{a}_{US} + c^{e}_{US} + c^{tR} \qquad (2)$$

2. Assuming no fixed costs for US production,

$$TC(X) = \{c^{a}_{US} + c^{e}_{US} + c^{tR}\} X \qquad (3)$$

3. With all production in the US, the optimal level of real exports (X*) is determined by equating (1) and (2) and solving for Q; in this case, with all production shipped from the US, X* equals Q. The *value* of exports is PQ.



D. Production Alternatives: Assembly Abroad and/or Full Production Abroad.

Assembly: save on transportation (tariff) costs; possibly have lower assembly costs. Incur annual fixed costs: F^a.

2. Assume, transport costs ½ of previous case, with overall marginal cost less than full export from the United States. Engines still exported at US marginal cost c , incurring half the previous transportation costs. Total Costs become:

$$TC(Q) = F^a + \left\{ C^a_f + \left(C^e_{US} + \tfrac{1}{2} C^{TR}_{US}\right) \right\} Q \quad (4)$$

Full Production Abroad: save all transport costs with possibly lower assembly and engine production costs. Incur addition annual fixed costs, F^e, in addition to F^a. Total Cost becomes:

$$TC(Q) = F^a + F^e + \left\{ C^a_f + C^e_f \right\} Q \quad (5)$$

EXPORTS AND FOREIGN PRODUCTION UNDER ALTERNATIVE COST SCENARIOS