SKJ Education

LC HL MATHS

FOUNDATION PROGRAM:

WEEK 5

COORDINATE GEOMETRY

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LC HL MATHEMATICS – FOUNDATION PROGRAM

Week 5: Coordinate Geometry

Learning Objectives

- **5.1:** To find equations of lines (slope-intercept, point-slope) and circles (centre-radius form).
- **5.2:** To solve problems involving parallel and perpendicular lines, and the perpendicular distance from a point to a line.
- **5.3:** To find the equation of a tangent to a circle at a given point.
- **5.4:** To determine the relationship between two circles (touching externally/internally, intersecting).

Key Terms - Week 5

- **Slope-Intercept Form:** The equation of a line in the form $y = mx + c$, where m is the slope and c is the y-intercept.
- **Point-Slope Form:** The equation of a line in the form $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope.
- **Centre-Radius Form:** The equation of a circle in the form $(x-h)^2+(y-k)^2 = r^2$, where (h, k) is the centre and r is the radius.
- **Perpendicular Distance:** The shortest distance from a point to a line, calculated using the formula $d = \frac{|Ax+By+C|}{\sqrt{A^2+B^2}}$.
- **Tangent to a Circle:** A line that touches a circle at exactly one point, perpendicular to the radius at the point of contact.
- **Relationship Between Circles:** The relative positions of two circles, including touching externally/internally, intersecting, or being disjoint.

Weekly *Challenge*: A satellite is orbiting the Earth in a circular path. Using coordinate geometry, derive the equation of the circle representing the satellite's orbit. ***Discuss the assumptions made*** and explain how this model can be used to predict the satellite's position at any given time.

WEEK 2 STUDY PLAN

Day	Activities & Time Commitment	✓	Rating (1-10)
Monday	- Review Learning Objectives (5 min) - Rank your current ability (5 min) - Review Key Terms (10 min) - Watch video (10-20 min) *Focus: PREPARATION*		
Tuesday	- Complete Exercises A1 & A2 (60 min) *Focus: EXPLORING*		
Wednesday	- Complete Exercise A3 (30 min) - Correct Exercises A1-3 and reattempt difficult questions (45 min) *Focus: PROCESSING*		
Thursday	- 1-hour online lesson (60 min) *Focus: QUESTIONING*		
Friday	- Complete Exercise B (40 min) *Focus: ERROR ANALYSIS*		
Saturday	- Complete Exam Question Assessment (C) (60 min) *Focus: EXECUTION*		
Sunday	- Correct assessment (30 min) - Complete self-reflection (15 min) - Plan next week (15 min) *Focus: REFLECTION & RECHARGING*		

Study Tips for Success

- **Active Recall:** After studying, close your notes and write down **everything you remember**. Force your brain to grow.
- **Spaced Repetition:** Review concepts **multiple times** over several days.
- **Mathematics in Action:** Look for **real-world examples** of the concepts you're learning.
- **Ask Questions:** Don't hesitate to ask for help when concepts are unclear. Reach out via *Google Classroom* or email; *steven@skjeducation.com*.
- **Celebrate Progress:** **Acknowledge your improvements**, no matter how small.

A2. Worked Example & Questions

Learning Focus: Understand and apply the concept of the equation of a circle given the endpoints of a diameter.

Scenario

Find the equation of the circle with endpoints of a diameter at $A(2, 3)$ and $B(6, 7)$.

Solution Framework:

1. **Decode & Define:** "The equation of a circle with center (h, k) and radius r is given by $(x - h)^2 + (y - k)^2 = r^2$. The center is the midpoint of the diameter, and the radius is half the length of the diameter."
2. **Plan:**
   - Find the center of the circle by calculating the midpoint of AB.
   - Calculate the radius by finding half the distance between A and B.
   - Use the center and radius to write the equation of the circle.
3. **Execute:**
   - The midpoint (center) is $(\frac{2+6}{2}, \frac{3+7}{2}) = (4, 5)$.
   - The distance between A and B is $\sqrt{(6 - 2)^2 + (7 - 3)^2} = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$, so the radius is $2\sqrt{2}$.
   - The equation of the circle is $(x - 4)^2 + (y - 5)^2 = (2\sqrt{2})^2$, simplifying to $(x - 3)^2 + (y - 5)^2 = 8$.
4. **Evaluate:** "The equation of the circle is $(x-4)^2+(y-5)^2 = 8$. This demonstrates the application of the midpoint and distance formulas to find the equation of a circle."

Test Yourself

1. **(Knowledge)** What is the general form of the equation of a circle? How does it relate to the center and radius?
2. **(Application)** Find the equation of the circle with a center at $(3, -2)$ and passing through $(1, 4)$.
3. **(Analysis)** Consider a circle with the equation $(x+1)^2+(y-2)^2 = 16$. Find its center and radius, and discuss its position relative to the origin.
4. **(Synthesis)** Given that a circle passes through $(0, 0)$, $(4, 0)$, and $(0, 6)$, find its equation. Discuss the geometric significance of the given points.

A3: Further Practice Questions

Part A: Core Skills & Fluency

1. Distance, Midpoint, and Slope

Calculate the distance, midpoint, and slope for each pair of points.

a) $A(1, 2)$ and $B(4, 6)$
b) $C(-3, 5)$ and $D(7, -1)$
c) $E(5, -2)$ and $F(-3, -2)$ *(Horizontal line)*
d) $G(4, -1)$ and $H(4, 5)$ *(Vertical line)*
e) $J(-1/2, 2)$ and $K(3/2, -3)$
f) $L(0, 0)$ and $M(5, 12)$

2. Formula Application and Manipulation

Use the core formulas to find missing information.

a) The midpoint of segment AB is $M(2, 5)$. If point A is $(-1, 3)$, find the coordinates of point B.
b) The slope between $(1, 4)$ and $(x, 8)$ is 2. Find the value of x.
c) Find the possible values of k if the distance between $(2, k)$ and $(5, 1)$ is 5 units.
d) Point $P(x, y)$ is equidistant from $A(1, 7)$ and $B(7, 1)$. Show that the relationship between x and y simplifies to $y = x$.

3. Equations of Straight Lines

Find the equation of the line for each condition. Write your answer in both slope-intercept ($y = mx + c$) and general form ($Ax + By + C = 0$).

a) Passing through $(2, -3)$ with a slope of $m = 4$.
b) Passing through $(1, 5)$ and $(7, -3)$.
c) Passing through $(-2, 5)$ and parallel to $3x - 4y + 12 = 0$.
d) Passing through $(3, -4)$ and perpendicular to $2x + y - 7 = 0$.
e) With an x-intercept of 5 and a y-intercept of -2.
f) A horizontal line passing through $(3, 7)$.
g) A vertical line passing through $(-4, 1)$.
h) Given the line $2x - 5y = 10$, find its slope, y-intercept, and x-intercept.

4. Systems of Linear Equations

Solve the following systems algebraically and interpret the geometric meaning of your solution.

a) *(Intersection Point)*

$$\begin{cases} 2x + 3y = 7 \\ x - 4y = -5 \end{cases}$$

b) *(Parallel Lines - No Solution)*

$$\begin{cases} y = 3x - 2 \\ 6x - 2y = 7 \end{cases}$$

c) *(Coincident Lines - Infinite Solutions)*

$$\begin{cases} y = 2x - 3 \\ 4x - 2y = 6 \end{cases}$$

d) *(Perpendicular Lines)*

$$\begin{cases} y = 2x + 1 \\ x + 2y = 12 \end{cases}$$

Part B: Application & Geometric Analysis

5. Triangle Analysis: A triangle has vertices at $A(-2, 1)$, $B(4, 7)$, and $C(6, -3)$.

a) Calculate the lengths of all three sides and classify the triangle as equilateral, isosceles, or scalene.
b) Calculate the slopes of all three sides and determine if the triangle is a right-angled triangle.
c) Find the area of the triangle. *(Hint: You can use the formula or enclose it in a rectangle.)*

6. Quadrilateral Properties: The vertices of a quadrilateral are $P(1, 1)$, $Q(5, 3)$, $R(3, 7)$, and $S(-1, 5)$.

a) Prove that this quadrilateral is a parallelogram by showing that opposite sides are parallel.
b) Prove that the diagonals bisect each other by finding their midpoints.
c) Is this parallelogram a rhombus? A rectangle? A square? Justify your answer using calculations for side lengths and slopes.

7. Collinearity of Points: Show that the points $P(-2, 5)$, $Q(2, 3)$, and $R(10, -1)$ are collinear (lie on the same straight line). Provide two different methods of proof (e.g., using slopes and using distances).

8. Medians and the Centroid: For the triangle with vertices $A(0, 6)$, $B(4, -2)$, and $C(-4, 0)$:

a) Find the coordinates of M, the midpoint of side BC.
b) Find the equation of the median from vertex A to the midpoint M.
c) The centroid is the point where the three medians intersect. It is located two-thirds of the way from the vertex to the midpoint of the opposite side. Calculate the coordinates of the centroid of triangle ABC.

C. Weekend Assessment – Past Exam Questions

Learning Focus: Applying learning to past exam questions under exam conditions.

Practical Advice: To tackle the questions in this assessment, consider the following tips:

- For function composition (Q1(a)), carefully substitute one function into the other and simplify.
- When working with logarithmic equations (Q1(b)), apply logarithm laws systematically and check domains.
- For circle geometry problems (Q2, Q3, Q4, Q5), identify centre and radius first, then apply relevant formulas.
- When finding tangents to circles (Q2(a)(ii), Q3(b)), use perpendicular relationships with radii.
- For circle intersection problems (Q5(a)(ii)), calculate distances between centres and compare with radii.
- When working with exponential growth (Q6), differentiate carefully and interpret rates in context.
- For geometric problems with circles (Q7), use coordinate geometry methods and distance formulas.

General Tips:

- **Show your work** - partial credit is available for correct methods
- **Check domains/restrictions** - especially in logarithmic and rational expressions
- **Label answers clearly** - especially when questions have multiple parts
- **Use appropriate precision** - note where decimal places are required
- **Verify your answers** - do they make sense in the context of the problem?

Question 1 (25 marks)
(a) $f(x) = 6x - 5$ and $g(x) = \frac{x+5}{6}$. Investigate if $f(g(x)) = g(f(x))$.
(b) The real variables y and x are related by $y = 5x^2$.
(i) The equation $y = 5x^2$ can be rewritten in the form $\log_5 y = a + b \log_5 x$. Find the value of a and the value of b.
(ii) Hence, or otherwise, find the real values of y for which

$$\log_5 y = 2 + \log_5 \left(\frac{126}{25} x - 1 \right).$$

Question 2 (30 marks)
(a) A circle s has the equation $(x-4)^2 + (y+2)^2 = 45$.
(i) Write down the centre and radius of the circle s.

$$\text{Centre} = (\quad , \quad)$$

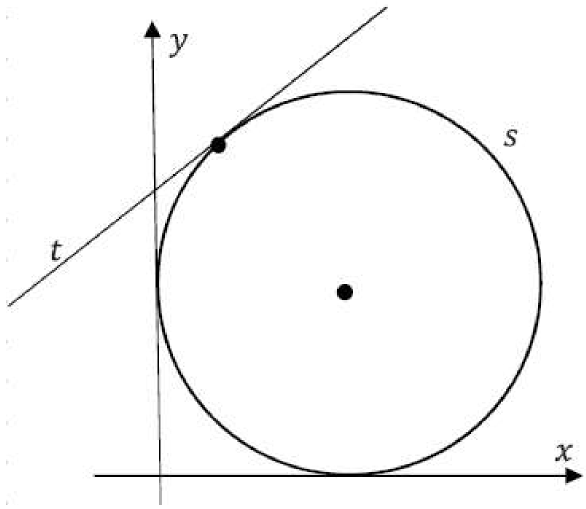
$$\text{Radius} = \_\_$$

(ii) Find the equation of the tangent to s at the point $(-2, -5)$. Write your answer in the form $y = mx + c$, where $m, c \in \mathbb{Z}$.
(b) The circle t has the following equation, where $k \in \mathbb{R}$ is a constant:

$$x^2 + y^2 + 28x - 46y + k = 0$$

The horizontal line $y = k$ is a tangent to the circle t. Find two possible values of k.

Question 3 (25 marks)
(a) The point $(-2, k)$ is on the circle $(x-2)^2 + (y-3)^2 = 65$. Find the two possible values of k, where $k \in \mathbb{Z}$.
(b) The circle s is in the first quadrant. It touches both the x-axis and the y-axis. The line $t : 3x - 4y + 6 = 0$ is a tangent to s as shown. Find the equation of s.



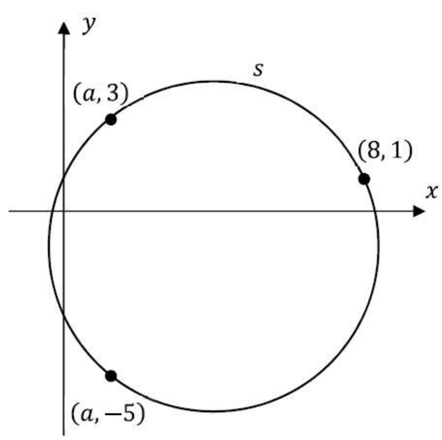
Question 4 (30 marks)
(a) The circle c has equation, where $h \in \mathbb{R}$:

$$(x-h)^2 + (y+3)^2 = 12$$

(i) Give the centre and radius of the circle c. Give your answer in terms of h, if appropriate.

Centre = (,) Radius = __

(ii) The perpendicular distance from the line $x - 4y + 7 = 0$ to the centre of the circle c is 5 units. Work out the two possible values of h. Give each answer in surf form.
(b) The circle s passes through the points $(8, 1)$, $(a, 3)$, and $(a, -5)$, as shown in the diagram on the right (not to scale), where $0 < a < 5$, $a \in \mathbb{R}$. The radius of the circle s is $\sqrt{20}$. Find the equation of the circle s.



Question 5 (30 marks)

(a) The circle s has equation:

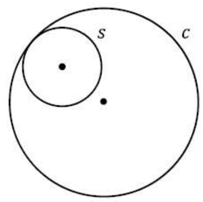
$$x^2 + y^2 + 4x - 6y + 5 = 0$$

(i) Write down the centre and radius of the circle s.

Centre = (,) Radius = __

(ii) Show that the circles s and c touch internally. The circle c has equation:

$$(x-2)^2 + (y+1)^2 = 72$$



(b) Another circle has its centre on the vertical line through the point (9, 0). The points (7, 10) and (12, 8) are on this circle. Find the equation of this circle. Note that your answer may contain non-integer values.

Question 6 (55 marks)
The number of bacteria in the early stages of a growing colony of bacteria can be approximated using the function:

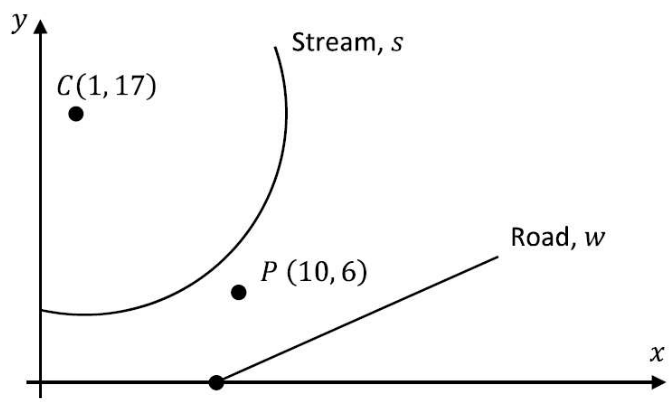
$$N(t) = 450e^{0.065t}$$

where t is the time, measured in hours, since the colony started to grow, and $N(t)$ is the number of bacteria in the colony at time t.
(a) (i) Find the number of bacteria in the colony after 4.5 hours. Give your answer correct to the nearest whole number.
(ii) Find the time, in hours, that it takes the colony to grow to 790 bacteria. Give your answer correct to 1 decimal place.
(b) Using the function $N(t) = 450e^{0.065t}$, find the average number of bacteria in the colony during the period from $t = 3$ to $t = 12$. Give your answer correct to the nearest whole number.
(c) Find the rate at which $N(t) = 450e^{0.065t}$ is changing when $t = 12$. Give your answer correct to one decimal place. Interpret this value in the context of the question.
(d) After k hours, the rate of increase of $N(t)$ is greater than 90 bacteria per hour. Find the least value of k, where $k \in \mathbb{N}$.
(e) The number of bacteria in the early stages of a different colony of bacteria can be approximated using the function:

$$P(t) = 220e^{0.17t}$$

where $P(t)$ is the number of bacteria and t is measured in hours. Assume that both colonies start growing at the same time. Find the time, to the nearest hour, at which the number of bacteria in both colonies will be equal.

Question 7 (50 marks)
Ameena, Petro, and Fiadh are taking part in an adventure race. The co-ordinate diagram below shows part of the course for this race. Each unit on the diagram represents 100 metres.



(a) The arc in the diagram represents part of a stream. The arc is part of a circle s with centre $C(1, 17)$ and radius 12.
(i) Write down the equation of the circle s.
(ii) Ameena is at the point $(a, 8)$ on the stream (circle s), where $a \in \mathbb{R}$, $a > 0$. Work out the value of a. Give your answer in surf form.
(iii) Petro is at the point $P(10, 6)$. Work out the shortest distance from the point P to the stream (circle s). Give your answer correct to the nearest metre. Remember that each unit on the diagram represents 100 metres.
(b) There is a straight path, l, that is not shown on the diagram. l is parallel to the y-axis, and is a tangent to the stream s in the first quadrant. Write down the equation of this path l. Remember that the radius of s is 12.
The line segment w represents a road, where w has the equation:

$$x - 3y = 9$$

for $0 \leq y \leq 8$.
(c) Find the co-ordinates of the point on the road w that is closest to the point $P(10, 6)$. It might be useful to find the equation of the line through P that is perpendicular to w.
(d) Fiadh is at the point $F(9, 0)$ on the road w. She travels 1200 m along the road w away from the point F, in the first quadrant, and then stops. Work out the co-ordinates of the point at which she stops. Give each value correct to 1 decimal place. Remember that each unit on the diagram represents 100 metres.

Self-Assessment

After completing the assessment:

- Grade your work honestly
- Identify areas needing improvement
- Scan and submit via Google Classroom
- Reflect on your performance in your weekly reflection

Another excellent week of work completed - ***well done!*** You are another step closer to *smashing your exams*, and another week closer to your summer holdiays!

Weekly Reflection Zone

What worked well this week?

What challenges did I face?

What surprised me the most this week?

Key mathematics concepts I want to review:

Goals for next week: