SKJ Education

LC HL PHYSICS

CORE PHYSICS PROGRAM: WEEK 4

GRAVITATIONAL FIELDS

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LC HL PHYSICS – CORE PROGRAM

Week 4: Gravitational Fields and Orbital Dynamics

Learning Objectives

- To explain and compare gravitational fields with electric and magnetic fields.
- To describe the concept of the gravitational field.
- To apply Kepler's Laws to analyse the motion of celestial bodies and orbital dynamics.

Key Terms - Week 4

- **Gravitational Field (g):** A vector field that represents the gravitational force per unit mass at a given point.
- **Comparison of Fields:**
  - **Gravitational Fields:** Always attractive, depend on mass.
  - **Electric Fields:** Can be attractive or repulsive, depend on charge.
  - **Magnetic Fields:** Can be attractive or repulsive, depend on moving charges or changing electric fields.
- **Kepler's Laws:**
  - **First Law (Law of Ellipses):** The orbit of a planet is an ellipse with the Sun at one focus.
  - **Second Law (Law of Equal Areas):** The line connecting the planet to the Sun sweeps out equal areas in equal times.
  - **Third Law (Law of Harmonies):** The square of the orbital period is proportional to the cube of the semi-major axis ($T^2 \propto r^3$).
- **Orbital Dynamics:** The study of the motion of celestial bodies, such as planets, moons, and satellites, under the influence of gravitational forces.
- **Gravitational Potential Energy:** The energy associated with the position of an object in a gravitational field, given by $U = -\frac{GMm}{r}$.
- **Orbital Energy:** The sum of the kinetic energy and gravitational potential energy of an object in orbit.

Weekly *Challenge*: Investigate the orbital dynamics of a planet or satellite in our solar system. Use Kepler's Laws to analyse its motion and calculate its orbital period, semi-major axis, or other relevant parameters. Share your findings on Google Classroom.

WEEK 4 STUDY PLAN

Day	Activities & Time Commitment	✓	Rating (1-10)
Monday	- Review Learning Objectives (5 min) - Rank your current ability (5 min) - Review Key Terms (10 min) - Complete Exercise A1 (15 min) - Watch Video (Exercise A2) (20 min) *Focus: PREPARATION*		
Tuesday	- Complete Exercise B (60 min) *Focus: QUESTIONING*		
Wednesday	- Reflect on content so far (what has been challenging?) (10 min) - Plan remaining study sessions (10 min) *Focus: PROCESSING*		
Thursday	- Complete Exercise C (30 min) - 1-hour online lesson (60 min) *Focus: ERROR ANALYSIS*		
Friday	- Complete Exercise D (45 min) *Focus: REVISION*		
Saturday	- Complete Exam Question Assessment (D) (60 min) *Focus: EXECUTION*		
Sunday	- Correct assessment (30 min) - Complete self-reflection (15 min) - Plan next week (15 min) *Focus: REFLECTION & RECHARGING*		

Study Tips for Success

- **Active Recall:** After studying, close your notes and write down **everything you remember**. Force your brain to grow.
- **Spaced Repetition:** Review concepts **multiple times** over several days.
- **Physics in Action:** Look for **real-world examples** of the concepts you're learning.
- **Ask Questions:** Don't hesitate to ask for help when concepts are unclear. Reach out via *Google Classroom* or email; *steven@skjeducation.com*.

B. The 60-Minute Deep Think

Think First, Calculate Second This exercise is built on a **Predict** → **Explain** → **Solve** → **Reflect** cycle. The marks are in the reasoning. Spend at least 15 minutes on Parts 1 & 2 before doing any major calculation. Draw all the diagrams yourself, labeling all key features.

Part 1: 15 mins - Conceptual Understanding

Scenario: The Heavy Astronaut Who Floats

An astronaut has a mass of 80 kg. On Earth's surface, she stands on a scale that reads 784 N (her weight). She then travels to the International Space Station (ISS), which orbits at an altitude of 400 km above Earth's surface. Once aboard, she floats freely through the cabin—completely "weightless." Objects released from her hand hover motionless beside her instead of falling.

A student watching footage of the ISS concludes: "The astronaut floats because there's no gravity in space. At that altitude, she's escaped Earth's gravitational pull."

But here's the puzzle: Earth's radius is about 6,370 km. The ISS orbits at 6,770 km from Earth's center—only about 6% farther than the surface. Could gravity really have dropped to zero over such a relatively small distance?

Diagram: Earth shown as a circle with radius $R_E = 6370$ km. The ISS orbit shown as a dashed circle at radius $r = 6770$ km (only slightly larger). An astronaut floating inside the ISS cabin. Arrows pointing toward Earth's center indicating gravitational field direction.

1. **Predict:** Before calculating anything, estimate: What fraction of her Earth-surface weight do you think the gravitational force on the astronaut is at ISS altitude? Is it (a) essentially zero, (b) about half, (c) about 90%, or (d) actually *larger* than on Earth? If gravity still pulls on her significantly, why does she float?
2. **Explain Your Reasoning:** What is the crucial difference between "weightlessness" (what the astronaut experiences) and "zero gravitational force" (what the student assumed)? What is the astronaut's motion relative to Earth, and how does this explain the floating?
3. **Connect:** The gravitational force follows an inverse-square law, just like the electric force between charges. If gravity hasn't vanished at ISS altitude, what role is it playing? What would happen to the ISS if gravity were suddenly "switched off"?

Instructions to Student: "Write your full answers to these questions before moving on. Resist the urge to look ahead—your initial reasoning, even if flawed, is valuable data for your learning."

Part 2: 20 mins - Strategic Deconstruction

Deconstruction

Step A: Diagrammatic Reasoning

Sketch four clear, labeled diagrams for this scenario:

1. A **gravitational field line diagram** showing Earth and the field lines radiating inward. Mark the position of the ISS orbit. Show that field lines are only slightly less dense at ISS altitude compared to the surface. Add a small vector at the ISS position showing $\vec{g}$ pointing toward Earth's center.
2. A **comparative diagram** showing side-by-side: (a) the gravitational field around Earth (mass M), and (b) the electric field around a negative point charge ($-Q$). Label the similarities: both point radially inward, both follow inverse-square laws, both have field strength decreasing with $1/r^2$.
3. A **free-body diagram** of the astronaut: (a) standing on Earth's surface (show weight W and normal force N), and (b) in orbit on the ISS (show *only* the gravitational force—what happened to the normal force?). This contrast is crucial.
4. An **orbital motion diagram** showing the ISS velocity vector (tangent to orbit) and acceleration vector (toward Earth's center). Label this as centripetal acceleration. Show that the astronaut inside the station has the *same* acceleration as the station itself.

Step B: Qualitative Principle

State the fundamental physics principles that explain this phenomenon. Your answer should address:

- The definition of gravitational field strength g and how it varies with distance from a spherical mass (Newton's Law of Gravitation)
- Why "weightlessness" is actually *free fall*—the astronaut and the station are both falling toward Earth at the same rate
- Why the astronaut doesn't crash into Earth despite constantly falling (the key insight Newton had about orbital motion)
- The analogy with a charged particle orbiting a fixed opposite charge—what provides the centripetal force in each case?

Explain why a scale on the ISS would read zero even though $F_g \neq 0$.

Step C: Quantitative Setup

Now, and only now, write down the relevant equations. Define all variables clearly.

1. Write Newton's Law of Gravitation for the force between Earth (mass M) and an object of mass m at distance r from Earth's center.
2. Write the expression for gravitational field strength $g(r)$ at distance r from Earth's center.
3. For a circular orbit, the gravitational force provides centripetal force. Write this equality and derive the orbital velocity v in terms of G, M, and r.
4. Write Kepler's Third Law relating orbital period T to orbital radius r. Show how it derives from the orbital velocity expression.
5. Write the analogous Coulomb's Law expression for the electric force, and note the structural similarity to Newton's Law of Gravitation. **Do not solve yet.**

Part 3: 15 mins - Precise Execution

Calculation

The Decisive Calculation:
Use the following values:

- Mass of Earth: $M = 5.97 \times 10^{24}$ kg
- Radius of Earth: $R_E = 6.37 \times 10^6$ m
- Altitude of ISS: $h = 400$ km $= 4.00 \times 10^5$ m
- Gravitational constant: $G = 6.67 \times 10^{-11}$ N $\cdot$ m^2/kg^2
- Astronaut's mass: $m = 80$ kg

Calculate:

1. The gravitational field strength g at Earth's surface ($r = R_E$).
2. The gravitational field strength g at ISS altitude ($r = R_E + h$). Express this as a percentage of surface gravity.
3. The gravitational force on the 80 kg astronaut at ISS altitude. Compare this to her weight on Earth's surface (784 N).
4. The orbital velocity of the ISS required for a stable circular orbit at this altitude.
5. The orbital period of the ISS. How many times does it orbit Earth per day?

Show all steps clearly and include units throughout.

The Check:
Once you have your numerical answers:

1. Confirm that gravity at ISS altitude is *not* negligible—the student's assumption was wrong. What percentage of her surface weight does the astronaut still experience as gravitational force?
2. Verify Kepler's Third Law: Calculate T^2/r^3 for the ISS orbit and confirm it equals $4\pi^2/(GM)$, which is a constant for all Earth satellites.
3. A geostationary satellite has $T = 24$ hours. Using Kepler's Third Law and your ISS result, calculate the altitude of a geostationary orbit. Why is it so much higher than the ISS?

Part 4: 10 mins - Reflection

Reflection & Integration

Reflection Prompts:

1. What was the key misconception this exercise was designed to expose? Many students believe "weightlessness in space" means "no gravity in space." How does this exercise demonstrate that this mental model is fundamentally flawed? What is the correct interpretation of weightlessness?
2. How did drawing the free-body diagrams (Part 2, Step A, diagram 3) clarify the difference between standing on Earth and orbiting in the ISS? Specifically, what force is present on Earth's surface but *absent* in orbit—and why does this explain the sensation of weightlessness?
3. **Field Comparison:** Complete this analogy table:

Property	**Gravitational Field**	**Electric Field**
Source of field	Mass M	Charge Q
Force law	$F =?$	$F =?$
Field strength	$g =?$	$E =?$
Direction	Always attractive	?
"Charge" that feels force	Mass m	Charge q

   What is the key difference between gravitational and electric fields regarding the direction of force?
4. **Exam Link:** Find a past paper question involving satellite motion or gravitational field strength at altitude. Identify how the question tests whether students understand: (a) the inverse-square dependence of g, and (b) the relationship between orbital radius and period (Kepler's Third Law). How is this typically structured?
5. **Extension Thinking:** The Moon orbits Earth at about 384,000 km from Earth's center with a period of 27.3 days. Without calculating, explain qualitatively why the Moon's orbital period is so much longer than the ISS's 90 minutes, even though both are "falling" around Earth under gravity.

The Takeaway:
Formulate a single, powerful "rule of thumb" or statement that encapsulates the lesson from this exercise. Your statement should capture the relationship between gravitational field strength, orbital motion, and the experience of weightlessness.
Example format: "An astronaut in orbit experiences weightlessness not because gravity is absent, but because [explain the actual reason]. The gravitational field [does what?], and this is what [causes what aspect of orbital motion?]."

Solutions for Part 3:

1. Gravitational field strength at Earth's surface:

$$g_{\text{surface}} = \frac{GM}{R_E^2} = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.37 \times 10^6)^2}$$
$$= \frac{3.98 \times 10^{14}}{4.06 \times 10^{13}} = 9.81 \text{ m/s}^2 \approx 9.8 \text{ m/s}^2 \checkmark$$

2. Gravitational field strength at ISS altitude:

$$r_{\text{ISS}} = R_E + h = 6.37 \times 10^6 + 4.00 \times 10^5 = 6.77 \times 10^6 \text{ m}$$
$$g_{\text{ISS}} = \frac{GM}{r_{\text{ISS}}^2} = \frac{3.98 \times 10^{14}}{(6.77 \times 10^6)^2}$$
$$= \frac{3.98 \times 10^{14}}{4.58 \times 10^{13}} = 8.69 \text{ m/s}^2$$

As a percentage of surface gravity:

$$\frac{g_{\text{ISS}}}{g_{\text{surface}}} = \frac{8.69}{9.81} = 88.6\% \approx \mathbf{89}\%$$

3. Gravitational force on astronaut at ISS altitude:

$$F_g = mg_{\text{ISS}} = (80)(8.69) = 695 \text{ N}$$

Comparison: This is 695/784 = 88.6% of her surface weight. **She still experiences nearly 90% of her Earth-surface gravitational force!**

4. Orbital velocity of ISS:

Setting gravitational force equal to centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} =$$
$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{3.98 \times 10^{14}}{6.77 \times 10^6}}$$
$$= \sqrt{5.88 \times 10^7} = 7,670 \text{ m/s} \approx \mathbf{7.7} \text{ km/s}$$

5. Orbital period of ISS:

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.77 \times 10^6)}{7670}$$
$$= \frac{4.25 \times 10^7}{7670} = 5,540 \text{ s} = \mathbf{92.3} \text{ minutes}$$

Orbits per day:

$$\frac{24 \times 60}{92.3} = \frac{1440}{92.3} \approx \mathbf{15.6} \text{ orbits per day}$$

Exercise D1: Revision of Foundational Skills - Work, Energy, and Power

1. Key Definitions and Concepts

a) Define the following terms and state their SI units:

   i) Work
   ii) Energy
   iii) Kinetic energy
   iv) Gravitational potential energy
   v) Elastic potential energy
   vi) Power
   vii) Efficiency
   viii) Joule

b) Explain the difference between:
   i) Energy and power
   ii) Kinetic energy and momentum
   iii) Gravitational potential energy and elastic potential energy

c) State the principle of conservation of energy in your own words.

d) Under what conditions is mechanical energy conserved? When is it not conserved?

e) Explain what is meant by a "conservative force" and give two examples.

f) What is meant by "work done against friction"? Where does this energy go?

g) Can work be negative? Explain with an example.

h) Can kinetic energy be negative? Can potential energy be negative? Explain.

2. Work Done by a Constant Force

a) State the equation for work done by a constant force and explain each term.

b) Calculate the work done in the following situations:

   i) A 50.0 N force moves an object 8.00 m in the direction of the force
   ii) A 120 N force acts at 30.0ř to the horizontal, moving an object 5.00 m horizontally
   iii) A 200 N force acts at 60.0ř to the direction of motion over 12.0 m
   iv) A 75.0 N force acts perpendicular to a displacement of 10.0 m

c) A person pushes a 40.0 kg crate 15.0 m across a floor with a horizontal force of 100 N. The friction force is 60.0 N. Calculate:
   i) Work done by the applied force
   ii) Work done by friction
   iii) Net work done on the crate

d) Explain why no work is done when:
   i) Carrying a heavy suitcase horizontally at constant velocity
   ii) A satellite orbits Earth in a circular path

e) A crane lifts a 500 kg load vertically through 20.0 m at constant velocity. Calculate the work done by:
   i) The crane
   ii) Gravity
   iii) The net work done

3. Kinetic Energy Calculations

a) State the equation for kinetic energy and verify that its SI unit is the joule.
b) Calculate the kinetic energy of:

   i) A 1200 kg car travelling at 25.0 m/s
   ii) A 0.145 kg baseball moving at 40.0 m/s
   iii) A 70.0 kg sprinter running at 10.0 m/s
   iv) A 2.50×10^5 kg aircraft flying at 250 m/s

c) A 1500 kg car accelerates from 10.0 m/s to 30.0 m/s. Calculate:
   i) The initial kinetic energy
   ii) The final kinetic energy
   iii) The change in kinetic energy
d) Show that the work-energy theorem states: $W_{net} = \Delta KE$.
e) A 5.00 kg object initially at rest has 400 J of work done on it. Calculate its final speed.
f) If the speed of an object doubles, by what factor does its kinetic energy change? Justify mathematically.

4. Gravitational Potential Energy

a) State the equation for gravitational potential energy near Earth's surface and explain each term.
b) Why is it necessary to define a reference point (zero level) for potential energy?
c) Calculate the gravitational potential energy of:

   i) A 2.00 kg book on a shelf 1.80 m above the floor
   ii) A 75.0 kg person at the top of a 50.0 m building
   iii) A 500 g ball held 2.50 m above a table that is 1.00 m above the floor (relative to the floor)
   iv) A 60.0 kg diver on a 10.0 m platform

d) A 4.00 kg object is raised from 2.00 m to 7.00 m above the ground. Calculate:
   i) The change in gravitational potential energy
   ii) The work done against gravity
e) Explain why the change in potential energy is independent of the path taken.
f) A 50.0 kg child slides down a frictionless slide from a height of 3.00 m. Calculate the potential energy lost.

5. Power and Efficiency

a) Define power and state two equations for calculating it.
b) Convert the following:

   i) 500 W to kW
   ii) 2.50 kW to W
   iii) 1.00 horsepower to watts (1 hp = 746 W)
   iv) 3.00 MW to W

c) Calculate the power in the following situations:
   i) A motor does 6000 J of work in 20.0 s
   ii) A crane lifts a 200 kg load through 15.0 m in 12.0 s
   iii) A car engine exerts a driving force of 3000 N while travelling at 20.0 m/s
   iv) A person runs up stairs, gaining 1500 J of potential energy in 5.00 s

d) Define efficiency and state its equation.

e) A motor has an efficiency of 85.0% and provides 5.10 kW of useful output. Calculate:
   i) The input power
   ii) The power wasted

f) An electric kettle is rated at 2.20 kW. If it is 90.0% efficient, how much energy is usefully transferred to the water in 3.00 minutes?

g) Explain why no machine can be 100% efficient.

6. Conservation of Energy - Basic Applications

a) A 2.00 kg ball is dropped from rest from a height of 10.0 m. Using energy conservation, calculate:
   i) Its speed just before hitting the ground
   ii) Its speed when it is 4.00 m above the ground

b) A 500 g ball is thrown vertically upward with an initial speed of 15.0 m/s. Using energy methods, find:
   i) The maximum height reached
   ii) The height at which its speed is 10.0 m/s

c) A roller coaster car of mass 600 kg starts from rest at the top of a 40.0 m hill. Assuming no friction, calculate its speed at:
   i) The bottom of the hill
   ii) A height of 15.0 m
   iii) A height of 25.0 m

d) Verify your answer to part (a)(i) using the UVATS equations. Comment on the two methods.

e) A 0.250 kg ball is thrown downward from a height of 20.0 m with an initial speed of 8.00 m/s. Find its speed just before impact using energy conservation.

Self-Assessment

After completing the assessment:

- Grade your work honestly
- Identify areas needing improvement
- Scan and submit via Google Classroom
- Reflect on your performance in your weekly reflection

Another excellent week of work completed - ***well done!*** You are another step closer to *smashing your exams*, and another week closer to your summer holdiays!

Weekly Reflection Zone

What worked well this week?

What challenges did I face?

What surprised me the most this week?

Key physics concepts I want to review:

Goals for next week: