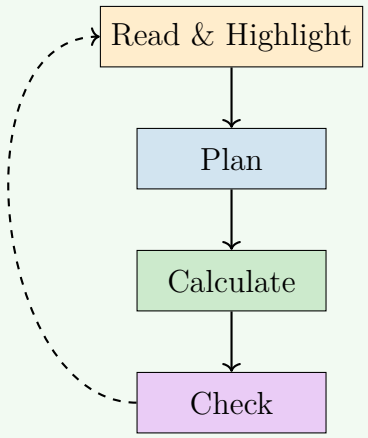
Learning Objectives: Introduction to Mathematics

- Develop basic ***problem-solving skills and strategies*** to complete everyday tasks (e.g., convert currencies, calculate taxes, and apply ratios).
- ***Interpret graphs***, analyze slopes, and calculate using graphs.
- Identify ***foundational mathematical rules*** (e.g., rounding rules, number domains, distinguish markup/margin, etc.) and use them to solve problems.

4-Step Problem Solving



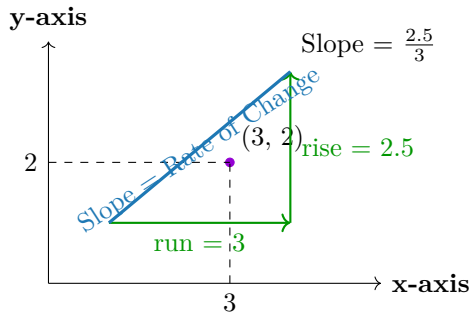
You can solve any question through:

- Goal identification
- Simple strategy selection
- Step-by-step logic
- Verification and reflection

Key Definitions

- **Exchange Rate:** An equation showing the value of one currency against another.
- **Ratio:** Comparison between two quantities of the same unit.
- **Natural Numbers ($\mathbb{N}$):** Positive whole numbers only.
- **Integers ($\mathbb{Z}$):** All whole numbers, including zero.
- **Real Numbers ($\mathbb{R}$):** All numbers that can be written as a simple fraction.
- **Profit Mark-Up:** Profit as a percentage of cost price.
- **Profit Margin:** Profit as a percentage of sale price.
- **Perimeter:** The sum (add up) of every side length of a shape.

Reading Graphs



- The x-axis is horizontal; y-axis is vertical
- A coordinate like (3, 2) means: right 3, up 2
- Slope = $\frac{\text{rise}}{\text{run}}$ = rate of change
- Steeper slope = faster change

Common Pitfalls

- **Rounding errors: round only at the final step**
- **Sign mistakes: especially in inequalities**
- **Wrong units: ensure consistency (e.g., hours vs minutes)**
- **Domain/range assumptions: check for restrictions**
- **Reading graphs: reading axes incorrectly**

Week 1 Study Plan (4 days)

Day 1 - 30 minutes

Read through the worked examples (pages 3-12), then read through the three assignments for this week. **Highlight any similarities** between questions in the worked examples and in the assignments - *this will help you later on in the week.* Starting next week, there will be an online revision quiz that will take about 10-15 minutes on Day 1.

Day 2 - 30 minutes

Read through the questions in Assignment 1 (page 13) for **5 minutes**, then set a timer for ***25 minutes*** and attempt the **3 questions into your copybook**. You can ***use the notes*** (i.e., the worked examples, etc.), your calculator and your formula book.

Day 3 - 30 minutes

Read through Assignment 2 (page 14-16) and **complete the exercises as you go** (the steps can be found in the plan boxes)

Day 4 - 30 minutes

Read through the questions in Assignment 3 (page 17-18) for **5 minutes**, then set a timer for ***25 minutes*** and attempt the 3 questions into your copybook. You can ***use the notes*** (i.e., the worked examples, etc.), your calculator and your formula book.

Worked Examples
Week 1: Introduction to Mathematics

Use these worked examples to **help you during your assignments.**

Key Principle: Introduction to Solving Problems
This Week's Focus: This week is an introduction to the mathematics course. This course is all about learning how to **identify, break down and solve problems** on our own. Let's start with some simple, every day problems! Begin to train your mind to meet every problem with the thought: *"How can I solve this one?"*

Question 1: Real-World Financial Skills

This is a great introduction to what this mathematics course will assess you on. These are important problems that everyone should be able to understand and solve. You will need to understand how to convert currency and calculate taxes at some point in your life!

2022 JC HL Paper (Question 1 - Suggested time: 10 minutes)

Part (a)

Jane buys a laptop online for \$699, plus a shipping cost of \$30.
The exchange rate is \$1 = €0.90.

Work out ***in euro the total cost*** *to Jane of buying the laptop online.*

Part (b)

Jane has a gross annual income of €56,000.
Jane pays income tax on her gross income at:

- 20% on the first €44,300
- 40% on the balance

(i) *Work out Jane's annual* ***income tax at each rate****:*

(ii) *Jane has annual* ***tax credits*** *of €3,300.*
Work out Jane's ***annual take-home pay****.*

What should you do to solve this (or any) question? Follow this process:

Read & Highlight	Plan	Calculate	Check
Highlight the **goal** of the problem	**Break down** problem	**Show every step** to get to the answer	**Verify**, does this make sense?

(a) **Jane buys a laptop for \$699 + \$30 shipping. Exchange rate: \$1 = €0.90. Find total cost in euros.**

PLAN: "First, find the **sum of the costs in dollars**. Then, **convert to euros** using exchange rate. Because *euros is worth more than dollars*, we know the *total cost in euros should be* ***smaller*** *than the total cost in dollars* (this can help us verify our answer!)."

CALC:

$$\text{Total Cost in Dollars} = 699 + 30 = 729$$
$$\text{Conversion to Euros} = 729 \times 0.90$$
$$\text{Total in Euros} = \boxed{656.10}$$

! **Why Multiply?** We **multiplied** the LHS of the conversion rate by **729** to go **from \$1 to \$729**, so we need to **do the same to the RHS**

CHECK: *"Verify by* ***going backwards:*** *€656.10 ÷ 0.90 = \$729. Matches!"*

(b) **Jane's income: €56,000. Tax: 20% on first €44,300, 40% on balance.**

! You don't always need to plan, sometimes calculations are very obvious.
Key Rule: As you read through your assignments/exams, if you can complete a calculation in *under 30 seconds*, **do it immediately**.

(i) **Tax some salary 20% and the rest at 40%:**

! **Common Mistake: Avoid double counting! The first €44,300 Jane earns gets taxed at 20%. From this point forward, anything else she earns is taxed at 40%.**

CALC:

$$20\%\ \text{tax} = 44,300 \times 0.20 = \boxed{8,860}$$
$$\text{Balance} = 56,000 - 44,300 = 11,700$$
$$40\%\ \text{tax} = 11,700 \times 0.40 = \boxed{4,680}$$

Jane paid €8,860 in the 20% band and €4,680 in the 40% band.

(ii) **Take-home pay with €3,300 tax credits:**

PLAN: "Credits reduce tax owed, not income! Sequence: ***Total tax*** (from **summing** tax paid at each band) $\rightarrow$ ***Tax due*** (by **subtracting** tax credits from tax due) $\rightarrow$ ***Take-home*** (by **subtracting** tax due from gross salary"

CALC:

$$\begin{aligned}\text{Total tax} &= 8,860 + 4,680 = 13,540\\ \text{Tax due} &= 13,540 - 3,300 = 10,240\\ \text{Take-home} &= 56,000 - 10,240 = \boxed{45,760}\end{aligned}$$

Jane's take-home salary is €45,760

▶ **Think Deeper:** What if credits were €15,000? Would tax due be negative? What would this mean in real life?

Question 2: Number Properties

Here's another question from the 2022 JC HL paper. This one explores the value of certain numbers - all you need to know is how to use a calculator to evaluate these numbers!

2022 JC HL Paper (Question 10 - 15 minutes)

Part (a)

Write the following four numbers in order, from the smallest to the biggest.

$$\frac{22}{7} \quad \pi \quad \sqrt{10} \quad 3.14$$

Part (b)

Put a tick (✓) in the correct box in each row of the table below to show whether each number is rational or irrational. Give a reason for each answer.

Number	Tick one box only		Reason
	Rational	Irrational	
$\sqrt{10}$			
π			
3.14			
$\frac{22}{7}$			

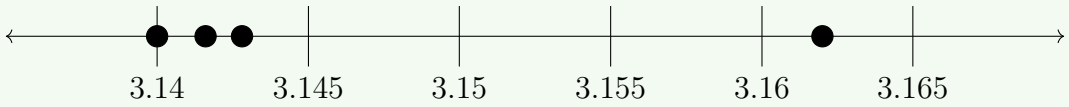
Part (c)

How many digits does the number 3.14×10^{100} have, when it is written out fully? Justify your answer.

(a) **Order from smallest**: $\frac{22}{7}$, π, $\sqrt{10}$, 3.14

CALC: *Approximate values:*

$$\pi \approx 3.1416,\ \sqrt{10} \approx 3.162,$$
$$\frac{22}{7} \approx 3.1428,\ 3.14 = 3.1400$$



3.14 π $\frac{22}{7}$ $\sqrt{10}$

Order: $\boxed{3.14 < \pi < \frac{22}{7} < \sqrt{10}}$

CHECK: **Subtract** the smallest number from the one next to it, you should get a **positive number**. Repeat this until you reach the largest number.

(b) **Classify as rational/irrational:**

! What is a **rational number**? It is a number that can be written as in the form of a **simple fraction**! (***Simple:*** nothing can divide into both the top and bottom of the fraction)

Number	Rational	Irrational	Reason
$\sqrt{10}$		✓	**Not perfect square** → Non-terminating, non-repeating decimal
3.14	✓		**Terminating decimal** → Can be written as $\frac{314}{100}$
π		✓	**Not perfect square** → Non-terminating, non-repeating decimal
$\frac{22}{7}$	✓		**Terminating decimal** → This is already in simple fraction form!

(c) **Digits in** 3.14×10^{100}**:**

CALC:

$$3.14 \times 10^{100} = 314 \times 10^{98}$$

$$\text{Digits} = 3 \text{ (from 314)} + 98 \text{ (zeros)} = \boxed{101}$$

CHECK: Does this make sense? Is this how **scientific notation** works? ***Yes!*** Since the original number has two digits to the right of the decimal (3.14), **shifting it 100 places results in 98 zeros after the 4.**

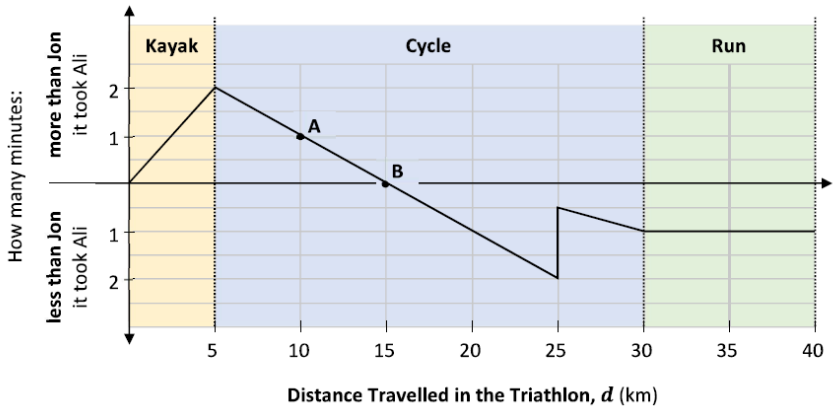
Question 3: Interpreting Graphs & Numerical Data

Key Strategies for Graph Problems

Read Axes	Key Points	Context
Identify what each axis represents	Look for intercepts, jumps, slopes	Relate to real-world meaning

2016 JC HL Paper 1 (Question 6 - 10 minutes)

Ali and Jon took part in a triathlon. In the triathlon they had to complete a 5 km kayak, then a 25 km cycle, and then a 10 km run.
The diagram below was drawn after both of them had finished the race. It shows how many minutes more than Jon (or less than Jon) it took Ali to travel d km in the triathlon, for $0 \leq d \leq 40$. For example, the point A shows that it took Ali 1 minute more than Jon to travel the first 10 km. In total, it took Ali 1 minute less than Jon to finish the triathlon.



2016 JC HL Paper 1 (Question 6 - 10 minutes)

Part (a)

Did Ali finish the kayak section ahead of Jon, behind Jon, or at the same time as Jon?

Ali finished the kayak section:

- ☐ ahead of Jon
- ☐ behind Jon
- ☐ at the same time as Jon

Part (b)

Ali had to stop briefly during the triathlon. Jon did not stop. State what distance Ali had travelled when he stopped, and for how long he was stopped.

Part (c)

What was happening to Jon and Ali at the point marked B on the diagram?

Part (d)

The table below shows the time it took Jon to complete each of the three sections in the triathlon, as well as his total time for the triathlon. Using the diagram, fill in the four missing times for Ali.

	Kayak	Cycle	Run	Total
Jon's time (minutes)	32	38	36	106
Ali's time (minutes)				

Part (e)

Jon and Ali also ran a 400 m race. Jon's average speed for the 400 m was 7.8 metres per second. It took Ali 2 seconds more than Jon to run the 400 m. Work out Ali's average speed for the 400 m race. Give your answer in metres per second, correct to 1 decimal place.

(a) **Did Ali finish the kayak section ahead of Jon, behind Jon, or at the same time as Jon?**

PLAN: "The ***y-axis*** tells us if **Jon** is **ahead** of Ali (if the graph is **above** the y-axis) or if **Ali** is **ahead** of Jon (if the graph is **below** the y-axis. The kayak section ends at 5 km. Look at the graph at d=5 km to see the **time difference** and **who is in the lead**."

CALC:
- At d=5 km, the graph is **at +2**.
- This means Ali took **two minutes more** than Jon for the first 5 km
- Therefore: Ali finished the kayak section 2 minutes **behind** Jon.

(b) **When and for how long did Ali stop?**

PLAN: "A stop would appear as a vertical jump in the graph since distance doesn't change but time increases. Find the vertical jump in Ali's graph, then measure it vertically to find out how long he stopped for (time is along the y-axis)."

CALC:
- There is a vertical jump at d=25 km
- The jump is 1.5 minutes upward
- **Distance Ali had travelled:** $\boxed{25 \text{ km}}$
- **Length of stop:** $\boxed{1.5 \text{ minutes}}$

! **Why vertical?** When Ali stops, he covers no distance but time passes. The graph shows time difference (along the vertical axis, going upwards) increasing without distance change (along the horizontal axis).

(c) **What was happening at point B?**

PLAN: Find point B on the graph. Use it's **x-coordinate** to find out what **distance (km)** into the race this point is at, then use it's **y-coordinate** to find out the **time difference (minutes)** between Jon and Ali.

CALC:

- Point B is where the graph crosses the x-axis. At point B (d=30 km), the graph crosses from positive to negative.
- This means: **After 15 km**, Ali and Jon's **time difference is zero** at that distance. They are beside each other at this point in the race. **Ali is about to overtake Jon** (because the graph is going from positive to negative).

CHECK: Double-check the y-axis, make sure that it is Ali who is overtaking Jon.

(d) **Fill in Ali's times**

PLAN: The graph shows the time difference between Ali and Jon. **Ali's time = Jon's time +** *difference* **at each section end**. For each section, we need to consider **how much the time difference has changed** (i.e., between the beginning and end of the run, there was no change, so it took them both the same amount of time).

CALC:

Kayak (5 km):	Difference $= +2$ min	$\rightarrow$	$32 + (2) = \boxed{34 \text{ mins}}$
Cycle (30 km):	Difference $= -1$ min	$\rightarrow$	$34 + 38 + (-3) = \boxed{71 \text{ mins}}$
Run (40 km):	Difference $= -1$ min	$\rightarrow$	$71 + 36 + (0) = \boxed{105 \text{ mins}}$

! Common Mistake: Adding the time difference value at each interval. For example, you may think to sum all of the time differences at the start of each race (-2 +1 +1 = 0), which clearly gives the wrong answer.

CHECK: The questions states "In total, it took Ali 1 minutes less to finish the marathon than Jon. Ali finished in 105 minutes, Jon finished in 106 minutes."

(e) **Ali's average speed for 400m race**

PLAN: "**Speed = Distance/Time**. Find Jon's time first, then Ali's time (2s longer), then Ali's speed."

CALC:

$$\text{Jon's time} = \frac{400}{7.8} \approx 51.282 \text{ seconds}$$
$$\text{Ali's time} = 51.282 + 2 = 53.282 \text{ seconds}$$
$$\text{Ali's speed} = \frac{400}{53.282} \approx 7.506 \rightarrow \boxed{7.5 \text{ mins}}$$

CHECK: "Verify calculation: 7.5 m/s $\times$ 53.282 s $\not\approx$ 400 m. Yes, 7.5 $\times$ 53.282 = 399.615 $\approx$ 400 m (rounding acceptable)."

▶ **Think Deeper:** Why did we round down to 7.5 instead of up to 7.6? The digit after 7.50 is 6, but we look at the thousandths place (6) which is $\geq$ 5, but since we're rounding to 1 decimal place, we look at hundredths place (0) which is $<$ 5 so we don't round up.

Question 1 - Time: 5 minutes (Q4 P1 2017 JC HL)

(a) A shop buys cartons of Fruitex from the UK. In December 2015, the exchange rate was €1 = £0.7241. The shop bought Fruitex for £380.
Find the price of the Fruitex in euro (€). Give your answer correct to the nearest cent.
(b) Fruitex and Juicy are each made from mixing fruit juice and water. In Fruitex, the ratio of fruit juice to water is 3:7.
20 litres of Fruitex is mixed with 40 litres of Juicy. In this 60-litre **mixture**, the ratio of fruit juice to water is 7:8.
(ii) Find the ratio of fruit juice to water in Juicy. Give your answer in its simplest form.

Question 2 - Time: 10 minutes (Q1 P1 2017 JC HL)

A person's Body Mass Index (BMI) is given by the following formula: $\textbf{BMI} = \frac{w}{h^2}$ where w is their weight in kg, and h is their height in metres.
(a)
(i) Geri is an athlete. Her weight is 77.5 kg and her height is 1.63 m.
Work out Geri's **BMI**. Give your answer correct to one decimal place.
(ii) Geri loses some weight during her training.
Her height stays the same. Her new BMI is 24.0.
Work out her new **weight**. Give your answer in kg, correct to one decimal place.
(b) Alex and Jo have the same weight. Alex is 10 cm **taller** than Jo. Is Alex's BMI is **greater than** Jo's or **less than** Jo's. Justify your answer fully.

Question 3 - Time: 10 minutes (Q7 P1 2018 JC HL)

Joonas has an unlimited supply of €5 notes and €2 coins.
(a) Fill in the table to show three different ways in which he can use these to make exactly €27. One way is already done.

	Number of €5 notes	Number of €2 coins
Way 1	1	11
Way 2		
Way 3		

(b) Explain how he could use his supply of €5 notes and €2 coins to make every whole number value of money greater than €3 (as simply as you can!).

Assignment 2
Targeted Error Analysis: Spot & Fix the Mistake

Identify common calculation traps in foundational problems. **Fix the errors** and reflect on **why they happen**.

Exercise 1: Measurement Precision (Q1 2023 JC HL)

Xena's car has a screen which shows the **temperature** in **degrees Celsius**, **correct to the *nearest whole number***. When Xena was going to work one day, the temperature ***on the screen*** was ***18°C***.

(a) Write down three possible values that the ***actual temperature*** could have been when Xena was going to work.
(b) Fill in the missing values in the inequality in t below to show the ***range of possible values of*** t when Xena was going to work.

$$\_\_\_\_ \leq t < \_\_\_\_$$

(c) If you were to graph the ***actual temperature***, t, over the course of a day, would it be more appropriate to graph $t \in \mathbb{N}$, $t \in \mathbb{Z}$, or $t \in \mathbb{R}$? Give a reason for your answer.

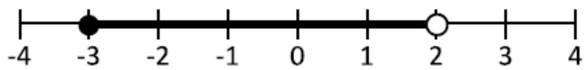
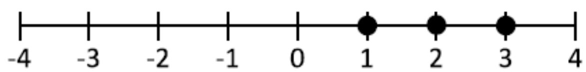
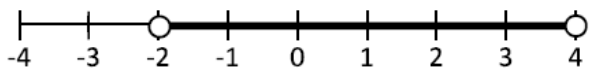
! Flawed Solution:
(a) (a) **Possible values: 17.5, 18, 18.5**
(b) (b) **Inequality:** $17.5 \leq t \leq 18.5$
(c) (c) **Graph type:** $t \in \mathbb{Z}$ ***Reason: Temperature can be positive or negative (or zero).***

PLAN Your Task:
a) **Spot any errors** and explain why they are incorrect.
b) **Correct** parts (a)-(c) with annotations (explain why this version is correct).

▶ **Think About It:** How could rounding errors occur? What rules come to mind when you think about rounding a number? Why are the rules this way?

Exercise 2: Number Line Analysis (Q11 P1 2018 JC HL)

Write down **an inequality in** x represented by **each of the number lines** shown below. Put a tick (✓) in the correct box in each case to show whether $x \in \mathbb{N}$, $x \in \mathbb{Z}$, or $x \in \mathbb{R}$. The first one is done.

Number line	**Inequality in *x***	**Domain** (Tick **one** box only in each case)
	$-3 \leq x < 2$	ℕ ☐ ℤ ☐ ℝ ☑
		ℕ ☐ ℤ ☐ ℝ ☐
		ℕ ☐ ℤ ☐ ℝ ☐
		ℕ ☐ ℤ ☐ ℝ ☐

! Flawed Solution:
Number Line 2: $x \in \mathbb{R}$, **because it can be negative.**
Number Line 3: $x \in \mathbb{Z}$, **because it doesn't include decimals, only whole numbers.**
Number Line 4: $x \in \mathbb{R}$, **because it can be negative.**

PLAN Your Task:

a) **Identify** any domain errors and any other **possible mistakes**.
b) **Explain** why each domain is right or wrong, and give the correct domain where necessary.
c) **Fix** any other mistakes by re-completing the question.

▶ **Think About It** What ***visual cues*** are there on the number lines? Is there a difference between a full and an empty circle? Why are some circles connected by a thick line, and some are separated? How can this help you understand what numbers to include in your inequality? ***Create a rule for number line → inequality conversion that works for you!***

Exercise 3: Profit Analysis (Q2 P1 2017 JC HL)

A sports shop buys t-shirts for €25 and sells them for €49.

(a)

(i) Find the ***mark up*** for the t-shirts (**profit** as a percentage of **cost price**).

(ii) Find the ***margin*** for the t-shirts (**profit** as a percentage of **selling price**). Give your answer correct to the nearest percent.

(b) The shop also sells runners, at a mark up of 50%. Find the margin for these runners. Give your answer correct to the nearest percent.

! Flawed Solution:

(a) (a)(i) Markup: $\frac{49-25}{25} \times 100 = 96\%$

(b) (a)(ii) Margin: $\frac{49-25}{49} \times 100 \approx 49\%$

(c) (b) Runners margin: $\frac{3}{2} \div 50\% = 75\%$

PLAN Your Task:

a) **Highlight** each value used and label it (e.g., cost, selling price, etc.).

b) **Draw out the formula**, like fraction diagrams, and fill in the values.

c) **Recalculate** to confirm answers.

▶ **Think About It:** "Why would students confuse **markup vs. margin**? Come up with a ***real-world analogy to distinguish them***. Is there a quick way to convert between one and the other?"

Assignment 3
Past Exam Questions

Finish a great week's work with some ***more challenging problems.***

Question 1 - Time: 5 minutes (Q7 P1 2019 JC HL)

(a) Describe each of the following sets. Be as specific as possible.

(i) The set of natural numbers, $\mathbb{N}$.

(ii) The set of integers, $\mathbb{Z}$.

(b) Graph the following inequality on a number line.

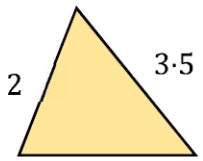
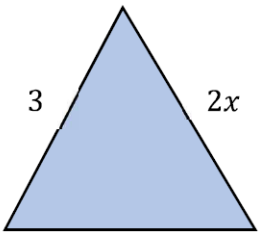
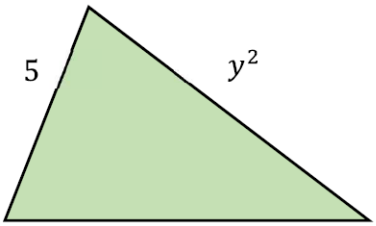
$$-3 < x \leq 2 \text{ , where } x \in \mathbb{R}$$

(c) Use algebra to solve the following inequality:

$$-7 < 8 - 3g \leq 11$$

Question 2 - Time: 10 minutes Q2 2022 JC HL)

The three triangles A, B, and C are shown below. The given lengths of the sides of each triangle are in centimetres, where x, $y \in \mathbb{R}$.

Triangle A	Triangle B	Triangle C
		

(a) The perimeter of Triangle A is 8 cm. Two of the sides have length 2 cm and 3.5 cm, respectively, as shown. Work out the length of the third side of Triangle A.

(b)

(i) Write down the perimeter of Triangle B, in terms of x.

(ii) The perimeter of Triangle B is 24 cm. Use this to work out the value of x.

(c) The perimeters of the three triangles A, B, and C form a linear sequence. Triangle C has the largest perimeter.

(i) The perimeter of Triangle C is k cm, where $k \in \mathbb{N}$. Find the value of k.

(ii) Hence work out the value of y, where $y \in \mathbb{N}$.

Question 3 - Time: 10 minutes (Q2 2025 JC HL)

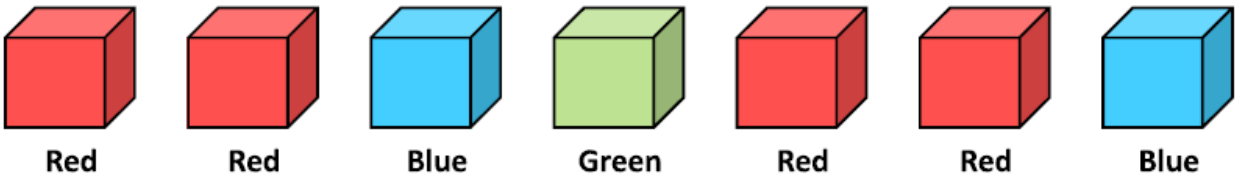
(a) Write down the next two terms in each of the following sequences. The type of each sequence is given.

(i) Linear: $1, 5, 9, \_\_\_\_\_, \_\_\_\_\_$

(ii) Tripling: $4, 12, 36, \_\_\_\_\_, \_\_\_\_\_$

(iii) Quadratic: $12, 19, 23, \_\_\_\_\_, \_\_\_\_\_$

(b) Amy makes a sequence using coloured blocks. The first 7 blocks in the sequence are shown below. The pattern repeats every 4 blocks.



(i) What colour is the 10th block in the sequence?

(ii) Amy picks one block at random from the first 25 blocks in the sequence. What is the probability that this block is **red**?