SKJ Education

LC HL MATHS

FOUNDATION PROGRAM: WEEK 1

ALGEBRA 101

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LC HL MATHEMATICS – FOUNDATION PROGRAM

Week 1: Algebra 101

Learning Objectives

- **1.1:** To manipulate algebraic expressions confidently using the rules of indices.
- **1.2:** To accurately factorise and expand expressions, including quadratic forms.
- **1.3:** To solve linear and quadratic equations using appropriate methods.
- **1.4:** To rearrange formulae to change the subject of an equation.
- **1.5:** To understand and apply the concept of algebraic fractions.

Key Terms - Week 1

- **Algebra:** The branch of mathematics that uses symbols and letters to represent numbers and quantities in formulae and equations.
- **Laws of Indices:** Rules for manipulating powers, e.g., $a^m \times a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, $a^{-n} = \frac{1}{a^n}$.
- **Factorisation:** The process of breaking down an expression into a product of simpler factors (e.g., $x^2 - 5x + 6 = (x-2)(x-3)$).
- **Quadratic Equation:** An equation of the form $ax^2 + bx + c = 0$, which can be solved by factorisation, completing the square, or using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.
- **Subject of a Formula:** The variable expressed in terms of the other variables in a formula (e.g., in $y = mx + c$, y is the subject).
- **Algebraic Fraction:** A fraction where the numerator and/or denominator are algebraic expressions (e.g., $\frac{x+1}{x^2-4}$).

Weekly *Challenge*: Find a real-world scenario that can be modelled by a **linear equation** (like calculating phone bill costs) and another by a **quadratic equation** (like the path of a thrown ball). Write down the equations and ***explain what each variable represents***.

WEEK 1 STUDY PLAN

Day	Activities & Time Commitment	✓	Rating (1-10)
Monday	- Review Learning Objectives (5 min) - Rank your current ability (5 min) - Review Key Terms (10 min) - Watch video (10-20 min) *Focus: PREPARATION*		
Tuesday	- Complete Exercises A1 & A2 (60 min) *Focus: EXPLORING*		
Wednesday	- Complete Exercise A3 (30 min) - Correct Exercises A1-3 and reattempt difficult questions (45 min) *Focus: PROCESSING*		
Thursday	- 1-hour online lesson (60 min) *Focus: QUESTIONING*		
Friday	- Complete Exercise B (40 min) *Focus: ERROR ANALYSIS*		
Saturday	- Complete Exam Question Assessment (C) (60 min) *Focus: EXECUTION*		
Sunday	- Correct assessment (30 min) - Complete self-reflection (15 min) - Plan next week (15 min) *Focus: REFLECTION & RECHARGING*		

Study Tips for Success

- **Active Recall:** After studying, close your notes and write down **everything you remember**. Force your brain to grow.
- **Spaced Repetition:** Review concepts **multiple times** over several days.
- **Mathematics in Action:** Look for **real-world examples** of the concepts you're learning.
- **Ask Questions:** Don't hesitate to ask for help when concepts are unclear. Reach out via *Google Classroom* or email; *steven@skjeducation.com*.
- **Celebrate Progress:** **Acknowledge your improvements**, no matter how small.

A1. Proficiency Drills

Learning Focus: Foundational concepts of **quadratic equation properties**, **absolute value equations and inequalities**, and **polynomial algebra**.

Part 1: Quadratic Equations - Decoding Their Nature

Key Concepts

Standard Quadratic Form: $ax^2 + bx + c = 0$, where $a \neq 0$.
The Discriminant (Δ): $\Delta = b^2 - 4ac$. This value dictates the nature of the roots (solutions).

- If $\Delta > 0$: Two distinct real roots. The graph crosses the x-axis at two points.
- If $\Delta = 0$: One real (repeated) root. The graph touches the x-axis at exactly one point (a tangent).
- If $\Delta < 0$: No real roots (two complex conjugate roots). The graph does not cross the x-axis.

Relationships Between Roots (α, β): For $ax^2 + bx + c = 0$:

- Sum of roots: $\alpha + \beta = -\frac{b}{a}$
- Product of roots: $\alpha\beta = \frac{c}{a}$

Task #1: Complete the table below for each quadratic equation.

Equation	Δ	Number of Real Roots	Sum of Roots	Product of Roots
$x^2 - 6x + 9 = 0$				
$x^2 + 3x - 10 = 0$				
$2x^2 + x + 1 = 0$				
$3x^2 + px + 12 = 0$	0			

Bonus Task: Draw the graph of each equation. Use this graph to **find the roots of these equations**, where possible.

Part 2: Absolute Value - Defining Ranges

Essential Knowledge

Definition: The absolute value $|A|$ of a real number A is its non-negative value, representing its distance from zero on the number line.
Solving Absolute Value Equations and Inequalities:

- **Equation:** $|A| = B \iff A = B$ or $A = -B$ (where $B \geq 0$).
- **Inequality (Less Than):** $|A| \leq B \iff -B \leq A \leq B$ (where $B \geq 0$).
- **Inequality (Greater Than):** $|A| \geq B \iff A \leq -B$ or $A \geq B$ (where $B \geq 0$).

Task #2: Solve the following equations and inequalities for $x \in \mathbb{R}$. Show your work.

1. $|4x - 7| = 9$
2. $|x + 6| - 2 \geq 0$
3. $|8 - 3x| < 10$
4. $|x^2 - 10| = 6$

Bonus Task: Draw the graph of each function inside the absolute value (modulus) operators. Use this graph to **illustrate your answer.**

Part 3: Polynomial Algebra - Expansion and Factors

Algebraic Vocabulary

- **Binomial Expansion:** The formula for expanding $(a+b)^n$. The first few terms are $\binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots$.
- **Factor Theorem:** If $(x - k)$ is a factor of a polynomial $f(x)$, then $f(k) = 0$. This means k is a root of the polynomial.
- **Polynomial Multiplication:** Each term in the first polynomial is multiplied by each term in the second polynomial, then like terms are combined.

Task #3: Consider the polynomial function $g(x) = (x + 3)(x^2 + qx - 4)$.

Polynomial Analysis

Question	**Your Answer**
a) Expand $g(x)$ into the form $Ax^3 + Bx^2 + Cx + D$.	
b) If the coefficient of x in $g(x)$ is 1, find the value of q.	
c) Using the value of q from part (b), verify that $(x+3)$ is indeed a factor of $g(x)$ using the Factor Theorem.	

Answers:

- **Task #1:**

Eq.	Δ	Real Roots	Sum	Product
1	0	One	6	9
2	49	Two	-3	-10
3	-7	Zero	-1/2	1/2
4	0	One	-12	4

- **Task #2:**
  1. $x = 4$ or $x = -1/2$
  2. $x \leq -8$ or $x \geq -4$
  3. $-2/3 < x < 6$
  4. $x = \pm 4$ or $x = \pm 2$
- **Task #3:** a) $x^3 + (q+3)x^2 + (3q-4)x - 12$, b) $q = 5/3$, c) $g(-3) = 0$

A2. Worked Example & Questions

Learning Focus: Warm up for advanced problem solving by applying the **equating coefficients method** for polynomial factorisation and solving **rational equations with extraneous solutions**.

Scenario

A polynomial $Q(x) = x^3 - 2x - 3r$ has a quadratic factor $P(x) = x^2 - px + 1$. Given that p and r are real numbers and $p < 0$.
Question: Find the values of p and r.

Solution Framework:

1. **Decode & Define:** "A quadratic polynomial $P(x)$ is a factor of a cubic polynomial $Q(x)$ implies that $Q(x) = P(x) \times L(x)$, where $L(x)$ is a linear polynomial. Let $L(x) = (x + q)$ since $x^3/x^2 = x$."

2. **Plan:**
   - Set up the equation: $x^3 - 2x - 3r = (x^2 - px + 1)(x + q)$.
   - Expand the right-hand side of the equation.
   - Equate the coefficients of corresponding powers of x on both sides.
   - Form a system of linear equations from the equated coefficients and solve for p, q, r.
   - Apply the constraint $p < 0$ to select the correct value for p.

3. **Execute:**
   - **Expansion:** $(x^2 - px + 1)(x + q) = x^3 + qx^2 - px^2 - pqx + x + q = x^3 + (q - p)x^2 + (1 - pq)x + q$
   - **Equating Coefficients (for $x^3 - 0x^2 - 2x - 3r$):**
     - x^2 term: $q - p = 0 \implies q = p$ (Equation 1)
     - x term: $1 - pq = -2$ (Equation 2)
     - Constant term: $q = -3r$ (Equation 3)
   - **Solve for p:** Substitute (Equation 1) into (Equation 2): $1 - p(p) = -2$ $1 - p^2 = -2$ $p^2 = 3 \implies p = \pm\sqrt{3}$ Given $p < 0$, select $p = -\sqrt{3}$.
   - **Solve for q:** Using Equation 1, $q = p = -\sqrt{3}$.
   - **Solve for r:** Using Equation 3, $q = -3r \implies -\sqrt{3} = -3r \implies r = \frac{\sqrt{3}}{3}$.

4. **Evaluate:** "The values found are $p = -\sqrt{3}$ and $r = \frac{\sqrt{3}}{3}$. The condition $p < 0$ is satisfied. The method of equating coefficients systematically determined all unknown values by transforming the polynomial factorization into a system of algebraic equations."

Test Yourself

1. **(Knowledge)** Define the "Equating Coefficients" method when used to factor polynomials or verify polynomial identities. Why is it an effective approach?
2. **(Application)** Given that $(x-2)$ is a factor of $f(x) = x^3 + kx^2 - 3x - 10$, use the Factor Theorem to find the value of k.
3. **(Analysis)** Consider the equation $\frac{3}{x-1} + \frac{x}{2} = \frac{3x-2}{x-1}$.
   a) Solve the equation for x.
   b) Identify any extraneous solutions. Explain why these solutions arise and how they are handled.
4. **(Synthesis)** A quadratic function $h(x) = x^2 + ax + b$ has roots that are -1 and 7. Another quadratic function $g(x) = 2x^2 - 6x + c$ has exactly one real root. Find the values of a, b, and c.

Challenge Questions

Now, move on to Exercise A3 - attempt as many questions as quickly as possible in the time remaining in your study session. Pay close attention to the **conditions** and **mathematical properties** required for each solution. You can do these questions **open book**, but think of it like an exam; try to ***maximise your marks*** in whatever time you have left!

Exercise A3: Core Skills & Fluency

1. Simplifying Algebraic Expressions

a) $\frac{6x^3y}{9x^2y^2} \cdot \frac{15y^3}{10x^2}$
b) $\frac{x^2-4}{x^2-2x} \div \frac{x+2}{x}$
c) $(x+3)^2 - (x-2)(x+4)$
d) $\frac{3}{x+2} + \frac{5}{x-1}$
e) $\frac{x^2-9}{x^2+5x+6}$
f) $\frac{2x^2-x-1}{x^2-1}$
g) $\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}$
h) $(2a^{-2}b)^3 \cdot 3a^5b^{-2}$

2. Solving Equations

a) $3x - 7 = 5x + 9$
b) $5(2x-1) = 3(x+4)$
c) $\frac{2x-5}{x-3} = \frac{x}{x-3}$ *(Check for extraneous solutions!)*
d) $\frac{4}{x-1} + 2 = \frac{10}{x-1}$
e) $x^2 - 5x - 24 = 0$
f) $2x^2 + 7x = 4$
g) $3x^2 - 4x - 5 = 0$ *(Use the quadratic formula)*
h) $\sqrt{x+7} = x - 5$

3. Solving Inequalities

a) $-5 \leq 2x - 1 < 7$
b) $x^2 - 3x - 10 > 0$
c) $\frac{3x+4}{x-2} > 2$
d) $|2x - 3| \leq 5$

For each solution, represent it on a number line.

4. Factorising Polynomials Completely

a) $x^3 + 6x^2 + 9x$
b) $81x^4 - 16$
c) $2x^3 + 3x^2 - 2x - 3$ *(Factor by grouping)*
d) $x^3 - 27$
e) $5x^2 - 17x + 6$
f) $x^3 + x^2 - 10x + 8$ *(Hint: Test integer factors of 8)*

5. Polynomial Division and Theorems

a) Divide $x^3 - 3x^2 + 4x - 8$ by $x - 2$ using long division.
b) The polynomial $f(x) = 3x^4 - 8x^3 + 5x^2 - 7x + 4$ is divided by $x + 2$. Use the Remainder Theorem to find the remainder.
c) Find the value of k such that $x - 4$ is a factor of $P(x) = x^3 - 5x^2 + kx + 8$.

Exercise A3: Application & Reasoning

6. Evaluating Functions: Evaluate $f(x) = 2x^2 + 5x - 7$ when:

a) $x = -3$ b) $x = 0$ c) $x = 2a$

7. Projectile Motion: A ball is thrown upward, and its height $h(t)$, in meters, is given by the function $h(t) = -5t^2 + 20t + 15$, where t is time in seconds.

- Find the height of the ball at $t = 2$ seconds.
- Determine the time at which the ball reaches its maximum height and the value of that maximum height.
- Interpret your results in the context of the problem.

8. Function Properties: Define $g(x) = x^3 - 4x^2 + 6x - 4$. Prove that $g(x)$ is neither an even nor an odd function.

9. Profit Maximization: A company's profit $P(x)$, in thousands of dollars, is given by $P(x) = -2x^2 + 12x - 5$, where x represents the number of products sold in thousands.

- Find the number of products the company must sell to maximize profit.
- Determine the maximum profit.
- Interpret the results in the context of the company's operations.

10. Detailed Inequality Analysis: Solve the inequality and explain your reasoning in detail:

$$\frac{x^2 - 4x + 3}{x - 1} > 0$$

Simplify the expression first. Include a graph and describe how any critical points or discontinuities impact the solution.

11. Geometry and Polynomial Roots: The volume V of a rectangular box is given by $V(x) = x^3 - 6x^2 + 11x - 6$.

- Find the roots of $V(x)$ by solving $V(x) = 0$.
- Explain how these roots can represent the dimensions (length, width, height) of the box. What are the dimensions if $x = 3$?

12. Reconstructing Polynomials: A polynomial $f(x) = 4x^3 - 10x^2 + 7x - 2$ is divided by $x - 2$.

- Perform the division to find the quotient $Q(x)$ and remainder R.
- Use your result to write $f(x)$ in the form $Q(x) \cdot (x - 2) + R$.
- Explain the relationship between the polynomial, divisor, quotient, and remainder in your own words.

13. Rational Function Analysis: Given the function $f(x) = \frac{x+3}{x-2}$, find:

- The domain and range of $f(x)$.
- The vertical and horizontal asymptotes of $f(x)$.
- Sketch the graph of $f(x)$, labelling all intercepts and asymptotes.

B. Calculation Error Analysis: Forensic Mathematics

Learning Focus: Developing **critical analytical skills** by identifying and correcting common mathematical **misconceptions** and **procedural errors** in algebra.

Analysis Protocol

1. **Locate the Error:** Pinpoint the specific step or statement that is incorrect.
2. **Diagnose the Error:** Classify the error type: **Procedural Error** (miscalculation, incorrect formula application), **Conceptual Error** (misunderstanding a definition or principle), or **Omission Error** (incomplete conditions or overlooked restrictions).
3. **Explain the Misconception:** Articulate the underlying flawed reasoning or missing knowledge demonstrated by the error.
4. **Correct the Solution:** Provide the complete, accurate, and step-by-step mathematical solution.
5. **Metacognitive Reflection:** "This error (e.g., misinterpreting discriminant conditions) is subtle because the algebra might seem correct initially. What is one personal strategy I can adopt to ensure I never overlook a crucial detail like this under exam pressure (e.g., always double-checking discriminant inequalities)?"

Forensic Mathematics Task

Your job is to find the **flaw in the thinking**, not just the right answer. Explain ***why*** each statement/calculation is wrong and **correct them**.

Error Analysis Exercises

Exercise 1: Rational Equation Solving
Question: Solve for x: $\frac{x}{2} + 3 = 5$
Incorrect Calculation: $x + 3 = 10 \implies x = 7$

Flawed Thinking

Error Analysis:

Correct Approach

Correction:

Exercise 2: Absolute Value Inequality

Question: Solve for x: $|x-3| \geq -2$

Incorrect Calculation: $-(-2) \leq x-3 \leq -2$ $2 \leq x-3 \leq -2$ $5 \leq x \leq 1$

Flawed Thinking	Correct Approach
Error Analysis:	**Correction:**

Exercise 3: Quadratic Discriminant

Question: Find the value of p for which $x^2 - 4x + (p+1) = 0$ has no real roots.

Incorrect Calculation: $\Delta = (-4)^2 - 4(1)(p+1) = 16 - 4p - 4 = 12 - 4p$

For no real roots, $\Delta > 0$. $12 - 4p > 0 \implies 12 > 4p \implies p < 3$.

Flawed Thinking	Correct Approach
Error Analysis:	**Correction:**

Exercise 4: Binomial Expansion Term

Question: Find the third term in the binomial expansion of $(3x-1)^5$ in descending powers of x.

Incorrect Calculation: Third term corresponds to $k = 2$.

$\binom{5}{2}(3x)^3(-1)^2 = 10 \cdot 3x^3 \cdot 1 = 30x^3$.

Flawed Thinking	Correct Approach
Error Analysis:	**Correction:**

Exercise 5: Algebraic Expansion

Question: Expand $(2x-3)^2$.

Incorrect Calculation: $(2x-3)^2 = (2x)^2 - (3)^2 = 4x^2 - 9$.

Flawed Thinking	Correct Approach
Error Analysis:	**Correction:**

Exercise 6: Rational Expression Simplification

Question: Simplify $\frac{x^2-4}{x^2-2x}$.

Incorrect Calculation: $\frac{x^2-4}{x^2-2x} = \frac{x^2}{x^2} - \frac{4}{2x} = 1 - \frac{2}{x}$.

Flawed Thinking	Correct Approach
Error Analysis:	**Correction:**

Exercise 7: Probability with Dependent Events

Question: A bag contains 4 red and 6 blue marbles. Two marbles are drawn without replacement. What is the probability that both are red?

Incorrect Calculation: $\text{P(both red)} = \frac{4}{10} \times \frac{4}{10} = \frac{16}{100} = \frac{4}{25}$.

Flawed Thinking	Correct Approach
Error Analysis:	**Correction:**

Bonus Exercise

If you have finished these exercises in under 40 minutes, go and **apply your error analysis skills to *your work*** from earlier in the week!

C. Weekend Assessment – Past Exam Questions

Learning Focus: Applying learning to past exam questions under exam conditions.

Q1: Remember $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r}b^r$. Start with highest power on a.
Discriminant $= 0$ gives $b^2 - 4ac = 0$.

Q2: If $(x+3)$ is factor, then $f(-3) = 0$. Use $|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|}$. Discriminant < 0.

Q3: Multiply out fully before comparing coefficients.
Find common denominator and check restrictions.

Q4: $|a| = b \Rightarrow a = b$ or $a = -b$.
Use remainder theorem when one polynomial divides another.

Q5: "Boy and girl in any order" means count both BG and GB.
Use $\binom{n}{r}$ for choosing questions without regard to order.

Q6: Multiply along branches, add probabilities of outcomes. $P(A|B) = \frac{P(A \cap B)}{P(B)}$.
Think systematically about how to count unique pairings.

Q7: Domain restriction: argument of ln must be > 0.
Calculate key points accurately for smooth curves.
Find derivative, set to zero for maximum/minimum.

General Tips:

- **Show your work** - partial credit is available for correct methods
- **Check domains/restrictions** - especially in rational equations/logarithms
- **Label answers clearly** - especially when questions have multiple parts
- **Use appropriate precision** - note where decimal places are required
- **Verify your answers** - do they make sense in the context of the problem?

Question 1 (30 marks)
(a) Write down, in descending powers of p, the first 3 terms in the binomial expansion of:

$$(2p+3)^7$$

Give each term in its simplest form. For example, the first term should be of the form ap^7, where a is a constant.
(b) $h(x)$ is the following function of x, where m and r are positive constants and $x \in \mathbb{R}$:

$$h(x) = 6mx^2 - 4rx + 54m$$

(i) The equation $h(x) = 0$ has exactly one solution. Use this to show that $r = 9m$.
(ii) Use the fact that $r = 9m$ to find the value of x.

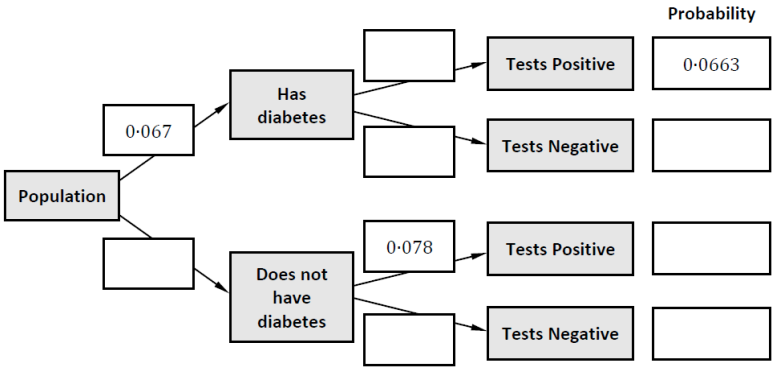
Question 2 (25 marks)
(a) $f(x) = x^2 + 5x + p$ where $x \in \mathbb{R}$, $-3 \leq p \leq 8$, and $p \in \mathbb{Z}$.
(i) Find the value of p for which $x + 3$ is a factor of $f(x)$.
(ii) Find the value of p for which $f(x)$ has roots which differ by 3.
(iii) Find the two values of p for which the graph of $f(x)$ will not cross the x-axis.
(b) Find the range of values of x for which $|2x + 5| - 1 \leq 0$, where $x \in \mathbb{R}$.

Question 3 (25 marks)
(a) In the expansion of $(2x + 1)(x^2 + px + 4)$, where $p \in \mathbb{N}$, the coefficient of x is twice the coefficient of x^2. Find the value of p.
(b) Solve the equation $\frac{3}{2x+1} + \frac{2}{5} = \frac{2}{3x-1}$ where $x \neq -\frac{1}{2}, \frac{1}{3}$, and $x \in \mathbb{R}$.

Question 4 (30 marks)
(a) Find the two values of $m \in \mathbb{R}$ for which $|5 + 3m| = 11$.
(b) For the real numbers h, j, and k: $\frac{1}{h} = \frac{k}{j+k}$. Express k in terms of h and j.
(c) $x^2 - px + 1$ is a factor of $x^3 - 2x - 3r$, where $p, r \in \mathbb{R}$ and $p < 0$. Find the value of p and the value of r.

Question 5 (25 marks)
(a) A class consists of 12 boys and 8 girls.
(i) Two students are selected at random from the class. What is the probability that the two students selected will be a boy and a girl in any order?
(ii) Four students are selected, one at a time, at random from the class. What is the probability that the first three students selected will be boys and the fourth will be a girl?
(b) An examination paper is made up of two sections, Section A consisting of 7 questions and Section B consisting of 8 questions. The paper contains the following instruction: "From section A you must answer question 1 and any three other questions. From Section B you must also answer any four questions." Find how many different combinations of questions may be answered if a candidate follows this instruction.

Question 6 (50 marks)
In parts (a) and (b) of this question, give answers correct to 4 decimal places, where relevant.
(a) Assume that 6-7% of people in Ireland have diabetes. For a particular test for diabetes, each person tests either positive or negative. The probability that someone who has diabetes gets a correct positive result is 99%. However, the probability that someone who does not have diabetes gets an incorrect positive result is 7.8%. One person is picked at random from the people in Ireland, and is tested for diabetes using this test. Use the information above to complete the tree diagram below, by:
(i) writing the proportion associated with each branch of the tree diagram into the appropriate box and
(ii) working out the probability for each outcome and writing it in the appropriate box in the final column.



Some values are already filled in. They have been given correct to 4 decimal places, where relevant.
Use the probabilities from the tree diagram to answer parts (a)(iii) and (a)(iv).
(iii) Find the probability that the person picked at random tests positive for diabetes.
(iv) The person picked at random tests positive for diabetes, using this test. Find the probability that they actually have diabetes.
(b) 5 people are picked at random from the people in Ireland. Assuming that 6-7% of people in Ireland have diabetes, work out the probability that 2 or more of these 5 people have diabetes.
(c) 20 people take part in a clinical trial. 10 of them will be picked to be in group A. The remaining 10 people will be in group B.
(i) How many different combinations of 10 people can be picked to be in group A?
(ii) Each person in group A is now paired with a person in group B, making 10 pairs. How many different sets of 10 pairs can be made?
(d) In a different clinical trial, 24 people are split into two groups, X and Y. 8 people are in group X and 16 people are in group Y. Each person in group X is paired with two people in group Y, so that everyone in group Y is in exactly one of these pairings. How many different sets of such pairings can be made? Give your answer in the form $a \times 10^n$ where $1 \leq a < 10, n \in \mathbb{N}$, and a is correct to 3 decimal places.

Question 7 (55 marks)

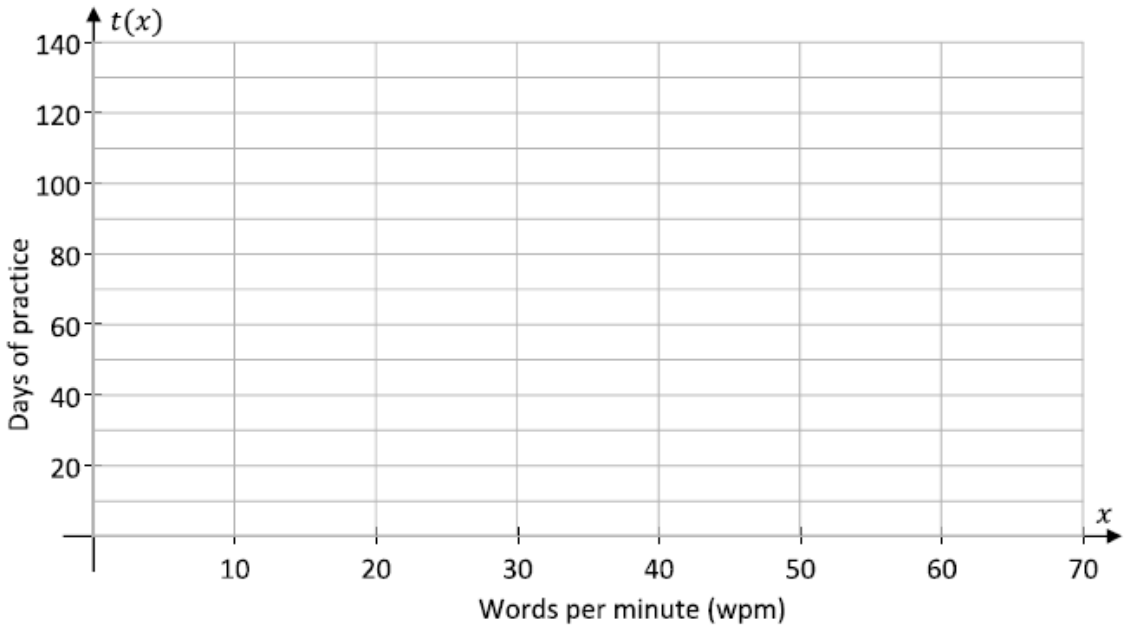
The time, in days of practice, it takes Jack to learn to type x words per minute (wpm) can be modelled by the function: $t(x) = k\left[\ln\left(1 - \frac{x}{80}\right)\right]$, where $0 \leq x \leq 70$, $x \in \mathbb{R}$, and k is a constant.

(a) Based on the function $t(x)$, Jack can learn to type 35 wpm in 35:96 days. Write the function above in terms of k and hence show that $k = -62.5$, correct to 1 decimal place.

(b) Find the number of wpm that Jack can learn to type with 100 days of practice. Give your answer correct to the nearest whole number.

(c) Complete the table below, correct to the nearest whole number and hence draw the graph of $t(x)$ for $0 \leq x \leq 70$, $x \in \mathbb{R}$ on the following page.

x (wpm)	0	10	20	30	40	50	60	70
$t(x)$ (days)								



(d) A simpler function that could also be used to model the number of days needed to attain x wpm is $p(x) = 1.5x$. Draw, on the diagram above, the graph of $p(x)$ for $0 \leq x \leq 70$, $x \in \mathbb{R}$.

(e) Let $h(x) = p(x) - t(x)$.

(i) Use your graphs above to estimate the solution to $h(x) = 0$ for $x > 0$.

(ii) Use calculus to find the maximum value of $h(x)$ for $0 \leq x \leq 70$, $x \in \mathbb{R}$. Give your answer correct to the nearest whole number.

Self-Assessment

After completing the assessment:

- Grade your work honestly
- Identify areas needing improvement
- Scan and submit via Google Classroom
- Reflect on your performance in your weekly reflection

Another excellent week of work completed - ***well done!*** You are another step closer to *smashing your exams*, and another week closer to your summer holdiays!

Weekly Reflection Zone

What worked well this week?

What challenges did I face?

What surprised me the most this week?

Key mathematics concepts I want to review:

Goals for next week: