The Generative Math Framework

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Classroom Snapshot

Gathered in a semicircle before a wall of whiteboards, the learners simplify expressions that stretch their earlier discussion of addition as creating and combining like terms. The prompts include mixed numbers written in both fraction and decimal form, designed to surface the underlying structure and potential misconceptions. Having already completed one expression as a group, Ellie, a fourth grader, volunteers to simplify the next. She begins, "First, we need to make like terms." Immediately, Amiya, a fifth grader, counters: "I disagree." Kevin, the teacher, echoes her. Luke, a third grader, then adds, "I disagree because you already have like terms," before walking to the board to illustrate his thinking.

What happened next reveals the distinctive culture of this learning community. Rather than being passive recipients of knowledge, learners of this multi-grade cohort were actively assessing and challenging one another's ideas. Ellie's use of "we" in her opening statement frames her contribution as an invitation to collective reasoning rather than a performance of correctness. Amiya's disagreement was not personal but a recognition of her responsibility to test her peer's conjecture. Luke's interjection shows his willingness to build on that exchange with evidence. Kevin's role was not to provide the answer but to press learners to clarify, extend, and connect their reasoning, positioning himself as a learner among learners.

This dynamic interaction reflects a departure from the transactional routines of traditional classrooms. Here, learners embraced struggle as part of sense-making, while Kevin guides the process through questions and persistence rather than solutions. Ellie, for instance, is supported in returning to prior knowledge and refining her approach, while the community witnessed what productive struggle looks like and how to sustain it. The episode illustrates how mathematics in

this setting is a generative act: learners consider and build understanding with existing or threading knowledge structures, achieve understanding as a collaborative effort, critique conjectures with care, and view both mistakes and persistence as essential to the work of doing mathematics.

The Framework

The Generative Math Framework is designed to make such learning moments the norm. At its core are three intertwined aims: collaborating to construct understanding of ideas, developing crucial learning skills through discourse and shared reasoning, and regarding struggle as essential to sense-making. These aims are not pursued in isolation; they work in concert, shaping both the design of the learning experiences and the culture of the classroom.

To bring these aims to life, the framework is premised on five key design features. Time is treated as a resource, allowing students sustained and unrushed opportunities to grapple with ideas. Sequencing is deliberate, with prompts crafted to surface connections and reveal the structure of mathematical concepts. Critical discourse is normalized, encouraging learners to voice disagreement, critique, and reasoning as a shared endeavor. Thinking is made visible through discourse, anchoring and advancing the discourse. Finally, lean coaching by the teacher ensures that guidance comes in the form of carefully posed questions, positioning the teacher as a co-learner in the community. Together, these features create the conditions in which mathematical thinking can thrive as a generative act.

The Generative Math Framework is designed to move mathematics learning beyond the transactional routines of traditional classrooms. Rather than positioning students as passive consumers of information, the framework cultivates communities of learners who generate mathematics together. Within these communities, every idea is valued because it carries the potential to advance or redirect the group's collective understanding. Learners make conjectures, pause to analyze and test them, and then refine or reject them through discourse. In this way, conjectures fuel wonder and inquiry, becoming the raw material from which deeper mathematical understanding emerges. So, mathematics content is not something to be

delivered; instead, it is something to be built together.

Central to the framework's design are three elements: Thought Exercise, Concept Study, and Studio. Each element provides unique opportunities for learners to grapple with ideas, articulate their thinking, and engage in collective sense-making. These elements require time, typically a minimum of twenty-five to thirty minutes each, because meaningful knowledge-building, productive struggle, and iterative refinement cannot be rushed. Tasks within these elements are deliberately challenging and structured to make thinking visible. Learners are expected not only to explore mathematical ideas but also to grow in their ability to communicate their reasoning, question assumptions, and offer evidence in support of or against an idea. This commitment to time and depth stands in sharp contrast to the brevity and efficiency emphasized in more traditional models that follow the routine of warm-ups, homework review, and then the "I do–we do–you do" lesson structure.

The teacher's role within the Generative Math Framework is equally distinctive. Teachers act as facilitators of mathematics learning, simultaneously learners and guides, holding the central concept in focus while monitoring how students navigate mathematical terrain. Their work is not to control the path but to optimize the learning within it, mediating the flow of ideas so that the community continues to advance in their ability to learn together. To do this, teachers employ several supportive practices. They engage in ongoing, active assessment to understand how learners are constructing knowledge and to encourage contributions even when uncertain. They use questioning not simply to guide toward correct answers but to surface reasoning, test validity, and spark further exploration. They allow and mediate struggle, helping learners cope with uncertainty and build resilience rather than avoiding difficulty. And they provide deliberate, explicit feedback aimed at strengthening both individual growth and the community's capacity to build knowledge together.

The Generative Math Framework redefines the mathematics classroom as a knowledge-building community. Learners are positioned as active agents whose ideas are indispensable to the group's progress. Teachers, meanwhile, orchestrate conditions that make generative learning possible, designing for depth, pressing for communication, and helping to develop habits of

inquiry and reflection. Together, these commitments result in deep conceptual understanding and the intellectual dispositions needed for learners to thrive as mathematicians.

The Elements

The framework is composed of three interdependent elements that together establish a cadence of exploration, development, and application. Each element is intentionally designed to deepen mathematical understanding and shape the habits of inquiry and collaboration that are key to a generative math learning experience.

Typically, the Generative Math Framework begins with a Thought Exercise**.** Thought Exercises provide learners with an opportunity to collectively grapple with a problem of significant complexity. They also create optimal conditions for intellectual risk-taking and math talk, i.e., sharing ideas with attempts at sound mathematical justification. The complexity of these problems presses learners to consider a broad range of mathematical understandings as they work toward solutions. Because the grappling occurs in a community context, learners' thinking becomes immediately visible and fuels discussion. With the teacher acting as a thoughtful, temperate mediator during Thought Exercises, members of the community come to recognize the strengths in each individual's contributions. Learners choose where they want to begin and which approaches to pursue. When offering ideas, they are less concerned with being correct and more concerned with contributing something that might prove useful. They recognize that any contribution - question, conjecture, justification, refutation, or suggested strategy - can prompt critical discussion. The purpose is to arrive at validity through consensus and mathematical reasonableness. When misconceptions surface, the community addresses them purposefully and in the moment..

There are six different Thought Exercises designed or selected for the Generative Math Framework: 100 Chart, Number Line, Analyzing Algorithms, Number Study, Quantitative Comparisons, and Translating Expressions and Equations. These exercises rotate across the academic year. Typically, a given Thought Exercise is explored over several weeks, allowing

learners to develop facility with its structure and content. Some Thought Exercises take place entirely within the whole community. Others begin with partner work, and then, the entire community reconvenes to critically examine the thinking from specific partnerships.

A Concept Study typically follows the Thought Exercise portion of the Math Block. Like Thought Exercises, Concept Studies are robust community discussions, but they have a more acute focus. Their purpose is for learners to attain an understanding of a specific idea or larger concept. Because of this precision, Concept Studies are the most teacher-moderated component of the Generative Math Framework. Their design involves a carefully sequenced set of expressions or other prompts accompanied by intentional questions. This combination is meant to guide learners from familiar mathematical understandings into new territory or offer a fresh vantage point from which to view familiar ideas. In other words, Concept Studies press learners to make cognitive leaps that ultimately lead to new or deeper mathematical understanding. They are not mere lessons aimed at covering a single standard or objective; rather, they intentionally pull on multiple threads, important underlying principles of mathematics referred to as connective ideas**.** By deliberately weaving these connective ideas together, Concept Studies help learners recognize the true substance or structure of a concept and make better sense of it.

The third element of the Generative Math Framework is Studio. Studio challenges learners to solve problems of various levels of rigor and presses them to apply their understanding of a broad range of concepts. Typically, learners work in partnerships, although there are occasional moments when learners work independently and confer with the teacher or peers as needed. When working in partnerships, learners complete problems at large vertical whiteboards. To ensure the work is genuinely collaborative - defined as *creating together* - learners follow the mantra: "One Marker. Two Minds." One learner acts as the scribe, recording only the partnership's collective thinking. The recorded work reflects the pair's consensus and is organized in a sequence of steps that reveals a coherent, mathematically sound approach. Because the work is displayed publicly on whiteboards, learners' thinking is visible and open to scrutiny from both the teacher and peers. This affords opportunities for questioning, feedback, and real-time assessment.

This immediate and meaningful assessment allows learners to field questions and receive feedback on their approaches. When learners demonstrate thinking that conveys a certain level of sophistication, they are not simply commended but receive commentary that highlights the merits of their approach. Likewise, when work reveals a misstep or misconception, learners are questioned about their reasoning, resulting in a conversation that helps to bring clarity. These *lean coaching* techniques give learners just enough information to retrace their steps or reconstruct an idea before the teacher steps away, allowing them to continue working with new insights.

In addition to assessing and deepening mathematical understanding, Studio offers a unique opportunity to observe and assess collaboration. By carefully observing how learners interact, teachers can provide targeted commentary on effective practices. When struggle emerges, teachers help partnerships develop specific goals and offer coaching. The collegial nature of Studio cultivates the essential skill of collaboration, reinforcing learners' ability to actively listen, make sense of others' ideas, and offer support or refutation that is grounded in sound mathematical reasoning.

Together, these three elements - Thought Exercises, Concept Studies, and Studio - create a coherent and intentional structure for the Generative Math Framework. Each plays a distinct but interconnected role: Thought Exercises cultivate curiosity and surface a wide range of ideas; Concept Studies focus and extend ideas into deeper conceptual understanding; and Studio provides a space for application, reflection, and collaboration. The framework develops learners' mathematical thinking, promotes a culture of shared inquiry, and builds both the cognitive and interpersonal capacities essential for rich, authentic mathematics learning.

Conclusion

The Generative Math Framework offers a deliberate alternative to conventional instructional models. By positioning learning as a community endeavor in which discourse leads to a collective construction of knowledge, it shifts learners' roles from passive recipients to active participants in the creation of mathematics. Its design emphasizes time, struggle, and

collaboration as essential conditions for authentic learning.

For educators, the framework is not a set of prescribed routines but an architecture for cultivating classrooms where mathematics is experienced as a generative act. Teachers who embrace this model step into the role of facilitators, listening, probing, and coaching learners as they navigate ideas together. The aim is not only deeper mathematical understanding, but also the development of learners who see themselves as capable contributors to a collective intellectual endeavor.

In short, the Generative Math Framework offers a vision of mathematics classrooms as communities of inquiry, i.e., where knowledge is built, refined, and expanded together and where the habits of questioning, reasoning, and collaboration endure well beyond any single experience.