LENTEAMENTO GRAVITACIONAL

INVERNO ASTROFÍSICO 2022
DOMINGOS MARTINS

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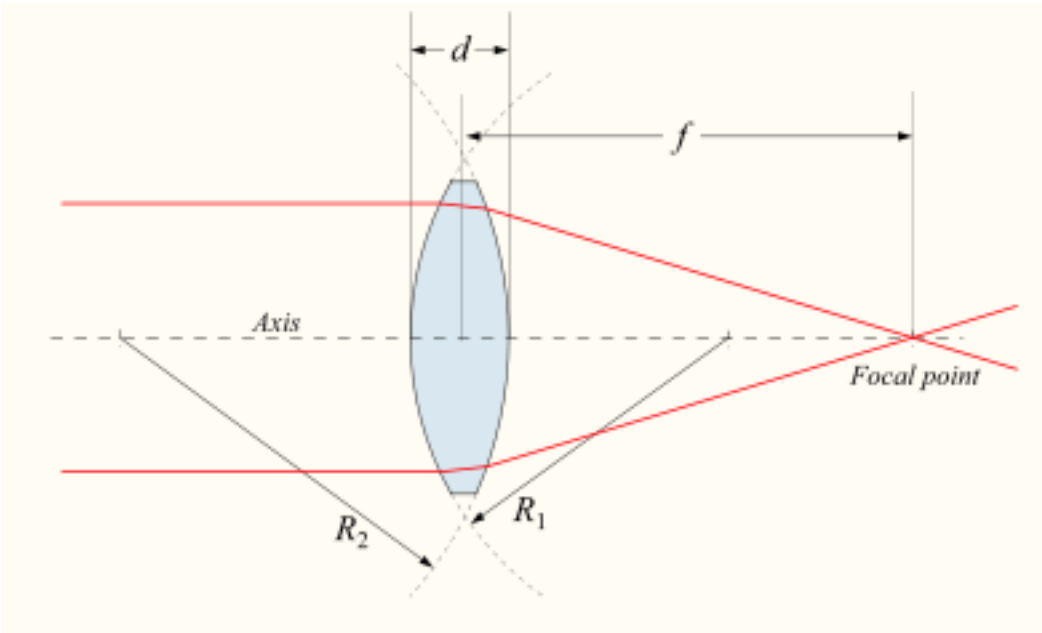
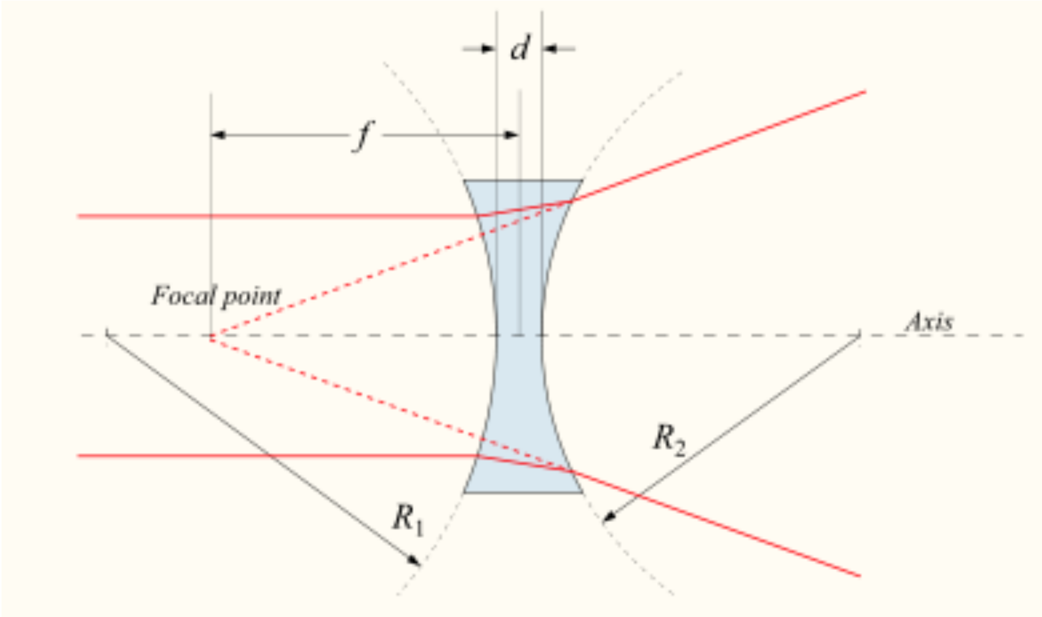
Plan of the talk

- (Material) lenses
- How Newton and Einstein bend light
- Strong lensing and weak lensing

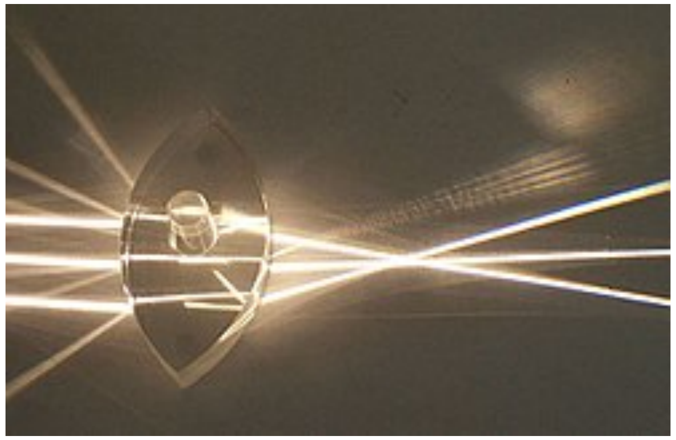
Material Lenses



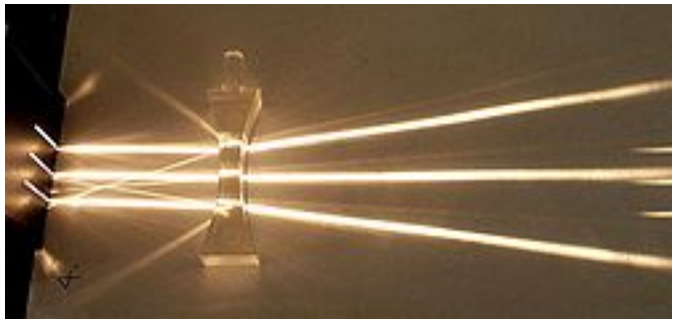
Positive (converging) lens



Negative (diverging) lens



Light is bent when passing through different media (Snell's laws, Fermat's principle, Maxwell's equations)



https://en.wikipedia.org/wiki/Lens_(optics)

Is gravity able to bend light?

Soldner was the first (in 1801) to study the bending of light (by the Moon and the Sun) in Newtonian theory.

Einstein derived (almost) the same result when he proposed the Principle of Equivalence (1908).

He predicted a 0.83" deflection by the gravitational field of the Sun.

At that time General Relativity was not ready yet, so the prediction was based on Newton's theory.

Newtonian bending of light

Recalling the solution of the 2-body problem:

$$\frac{1}{r} = \frac{GM\mu^2}{L^2}\left[1 + \sqrt{1 + \frac{2EL^2}{(GM\mu)^2\mu}}\cos(\varphi - \varphi_0)\right] .$$

The energy and the angular momentum are proportional to the mass, so let's define the energy and angular momentum per unit mass, ε and ℓ:

$$\frac{1}{r} = \frac{GM}{\ell^2}\left[1 + \sqrt{1 + \frac{2\varepsilon\ell^2}{(GM)^2}}\cos(\varphi - \varphi_0)\right] .$$

In this formula one can take $\mu \to 0$, thereby describing a photon.

4. *Über den Einfluß der Schwerkraft auf die Ausbreitung des Lichtes; von A. Einstein.*

Die Frage, ob die Ausbreitung des Lichtes durch die Schwere beinflußt wird, habe ich schon an einer vor 3 Jahren erschienenen Abhandlung zu beantworten gesucht.[1] Ich komme auf dies Thema wieder zurück, weil mich meine damalige Darstellung des Gegenstandes nicht befriedigt, noch mehr aber, weil ich nun nachträglich einsehe, daß eine der wichtigsten Konsequenzen jener Betrachtung der experimentellen Prüfung zugänglich ist. Es ergibt sich nämlich, daß Lichtstrahlen, die in der Nähe der Sonne vorbeigehen, durch das Gravitationsfeld derselben nach der vorzubringenden Theorie eine Ablenkung erfahren, so daß eine scheinbare Vergrößerung des Winkelabstandes eines nahe an der Sonne erscheinenden Fixsternes von dieser im Betrage von fast einer Bogensekunde eintritt.

Es haben sich bei der Durchführung der Überlegungen auch noch weitere Resultate ergeben, die sich auf die Gravitation beziehen. Da aber die Darlegung der ganzen Betrachtung ziemlich unübersichtlich würde, sollen im folgenden nur einige ganz elementare Überlegungen gegeben werden, aus denen man sich bequem über die Voraussetzungen und den Gedankengang der Theorie orientieren kann. Die hier abgeleiteten Beziehungen sind, auch wenn die theoretische Grundin S_1 befindlichen gleich beschaffenen Uhr gemessen wird. Wir ersetzen γh durch das Schwerepotential Φ von S_2 in bezug auf S_1 als Nullpunkt und nehmen an, daß unsere für das *homogene* Gravitationsfeld abgeleitete Beziehung auch für anders gestaltete Felder gelte; es ist dann

$$\nu_1 = \nu_2 \left(1 + \frac{\Phi}{c^2}\right) \cdot \qquad \text{(2a)}$$

Dies (nach unserer Ableitung in erster Näherung gültige) Resultat gestattet zunächst folgende Anwendung. Es sei ν_0 die Schwingungszahl eines elementaren Lichterzeugers, gemessen mit einer an demselben Orte gemessenen Uhr U. Diese Schwingungszahl ist dann unabhängig davon, wo der Lichterzeuger samt der Uhr aufgestellt wird. Wir wollen uns beide etwa an der Sonnenoberfläche angeordnet denken (dort befindet sich unser S_2). Von dem dort emittierten Lichte gelangt ein Teil zur Erde (S_1), wo wir mit einer Uhr U von genau gleicher Beschaffenheit als der soeben genannten die Frequenz ν des ankommenden Lichtes messen Dann ist nach (2a)

$$\nu = \nu_0 \left(1 + \frac{\Phi}{c^2}\right),$$

wobei Φ die (negative) Gravitationspotentialdifferenz zwischen Sonnenoberfläche und Erde bedeutet. Nach unserer Auffassung

Tangente legen, wobei c_1 bzw. c_2 die Lichtgeschwindigkeit in den Punkten P_1 bzw. P_2 bedeutet. Der Krümmungswinkel des Lichtstrahles auf dem Wege $c\,dt$ ist also

$$\frac{(c_1 - c_2)\,dt}{1} = -\frac{\partial c}{\partial n'}\,dt,$$

falls wir den Krümmungswinkel positiv rechnen, wenn der Lichtstrahl nach der Seite der wachsenden n' hin gekrümmt

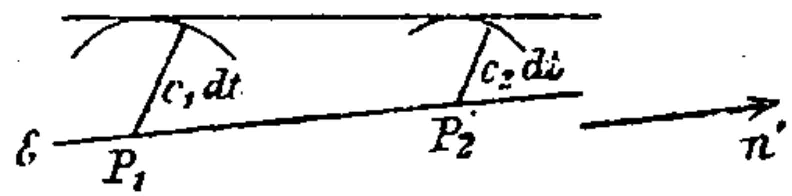


Fig. 2.

wird. Der Krümmungswinkel pro Wegeinheit des Lichtstrahles ist also

$$-\frac{1}{c}\frac{\partial c}{\partial n'}$$

oder nach (3) gleich

$$-\frac{1}{c^2}\frac{\partial \Phi}{\partial n'}.$$

Endlich erhalten wir für die Ablenkung α, welche ein Lichtstrahl auf einem beliebigen Wege (s) nach der Seite n' erleidet, den Ausdruck

$$\alpha = -\frac{1}{c^2}\int \frac{\partial \Phi}{\partial n'}\,ds. \tag{4}$$

Dasselbe Resultat hätten wir erhalten können durch unmittelbare Betrachtung der Fortpflanzung eines Lichtstrahles i körper zugewandten Seite von der Größe

$$\alpha = \frac{1}{c^2} \int\limits_{\vartheta=-\frac{\pi}{2}}^{\vartheta=+\frac{\pi}{2}} \frac{k\,M}{r^2} \cos\vartheta \cdot ds = \frac{2\,k\,M}{c^2\,\Delta},$$

wobei k die Gravitationskonstante, M die Masse des Himmelskörpers, Δ den Abstand des Lichtstrahles vom Mittelpunkt des Himmelskörpers bedeutet. *Ein an der Sonne vorbeigehender Lichtstrahl erlitte demnach eine Ablenkung vom Betrage $4 \cdot 10^{-6}$ = 0,83 Bogensekunden.* Um diesen Betrag erscheint die Winkeldistanz des Sternes vom Sonnenmittelpunkt durch die Krümmung des Strahles vergrößert. Da die Fixsterne der der Sonne zugewandten Himmelspartien bei totalen Sonnenfinsternissen sichtbar werden, ist diese Konsequenz der Theorie mit der Erfahrung vergleichbar. Beim Planeten Jupiter erreicht die zu erwartende Verschiebung etwa $^1/_{100}$ des angegebenen Betrages. Es wäre dringend zu wünschen, daß sich Astronomen der hier aufgerollten Frage annähmen, auch wenn die im vorigen gegebenen Überlegungen ungenügend fundiert oder gar abenteuerlich erscheinen sollten. Denn abgesehen von jeder Theorie muß man sich fragen, ob mit den heutigen Mitteln ein Einfluß

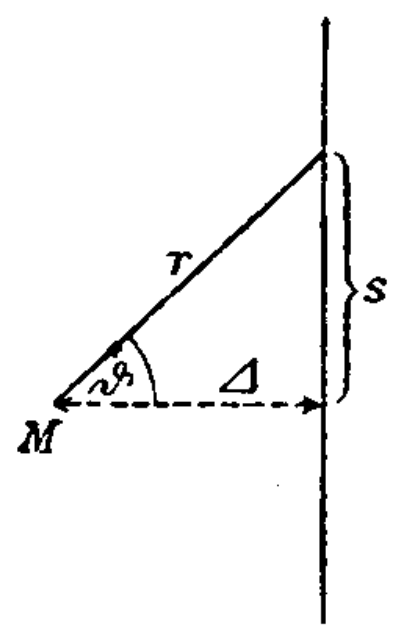


Fig. 3.

PUBLICATIONS
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Vol. XXXV. San Francisco, California, August, 1923 No. 206

HISTORICAL NOTE ON THE PROBLEM OF LIGHT DEFLECTION IN THE SUN'S GRAVITATIONAL FIELD

By Robert Trumpler

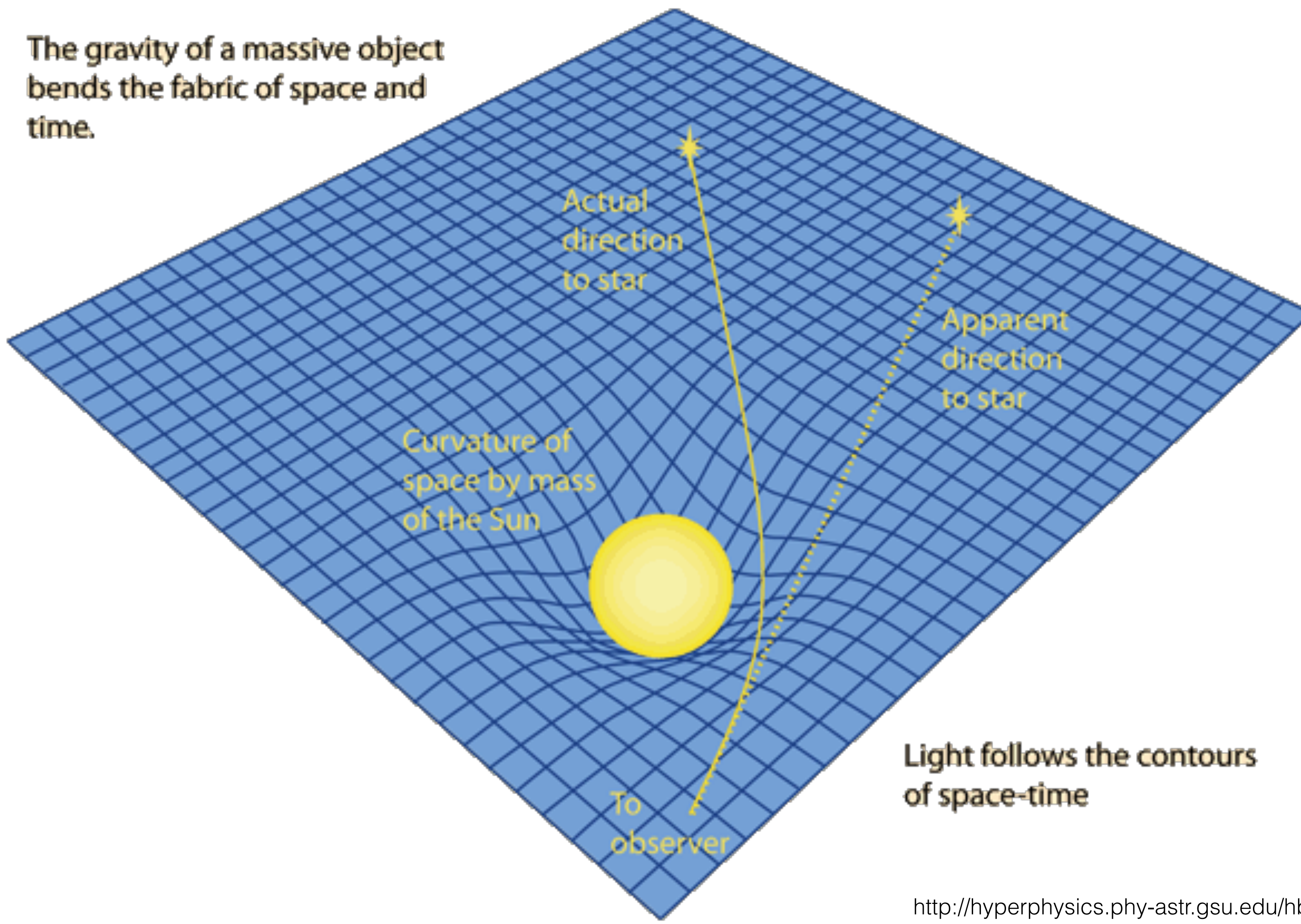
A series of articles recently published by Professor T. J. J. See, U. S. Navy,[1] gives a quite incorrect impression of the relation of J. Soldner's and of Einstein's work in connection with the deflection of light in the Sun's gravitational field. It therefore seems desirable to make a short statement of the history of this problem.

In 1801 Soldner[2] calculated the deflection of light according to:

1. The corpuscular theory of light (light consisting of material particles which are subject to gravitation), and
2. Newton's law of gravitation.

The problem was simply that of determining the hyperbolic orbit of a small mass traveling with the speed of light under the influence of the gravitation of a celestial body. Considering a ray of light just touching the surface of the attracting body, Soldner worked out the well known solution of the problem of

General Relativity



http://hyperphysics.phy-astr.gsu.edu/hbase/Relativ/grel.html

Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} + \Lambda \, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Geodesics

Trajectories of test-particles (they feel the geometry but not determine it). Zero 4-acceleration:

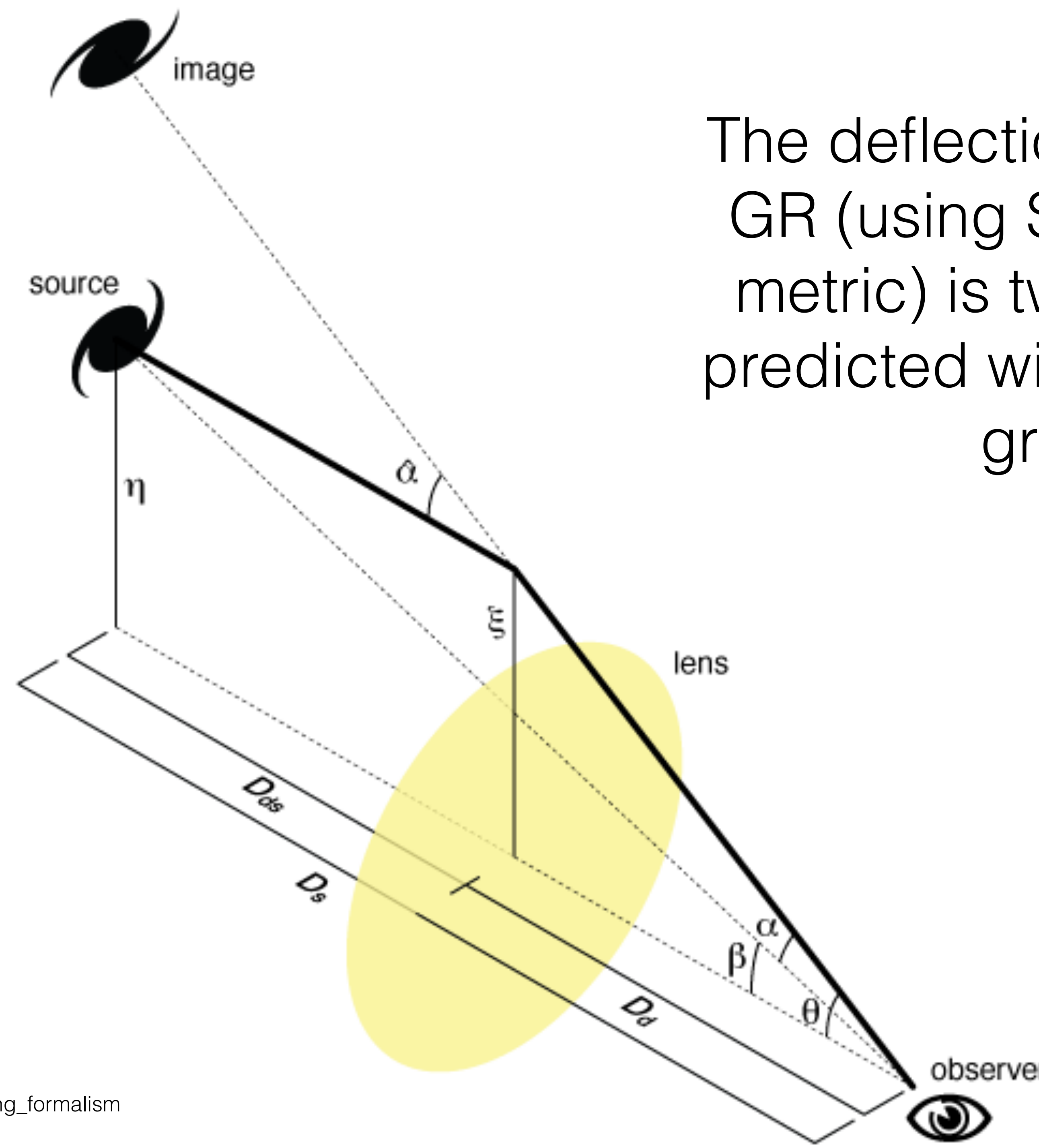
$$\frac{d^2x^\lambda}{dt^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt} = 0$$

The stationary path between two points

TABLE 8.1 Extremal Proper time $\delta \int d\tau = 0$ and Equations of Motion

	Variational Principle	Equation of Motion
Particle in flat spacetime	$\delta \int (-\eta_{\alpha\beta} dx^\alpha dx^\beta)^{1/2} = 0$	$\frac{d^2x^\alpha}{d\tau^2} = 0$
Geometric Newtonian	$\delta \int \left[(1 + 2\Phi/c^2)(cdt)^2 - (1 - 2\Phi/c^2)(dx^2 + dy^2 + dz^2)\right]^{1/2} = 0$	$\frac{d^2x^i}{dt^2} = -\frac{\partial\Phi}{\partial x^i}$
	(to leading order in $1/c^2$)	(to leading order in $1/c^2$)
General metric	$\delta \int (-g_{\alpha\beta}\, dx^\alpha dx^\beta)^{1/2} = 0$	$\frac{d^2x^\alpha}{d\tau^2} = -\Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau}$

Gravitational Lensing



The deflection predicted in GR (using Schwarzschild metric) is twice the value predicted within Newtonian gravity.

$$\hat{\alpha} = \frac{4GM}{c^2 b}$$

https://en.wikipedia.org/wiki/Gravitational_lensing_formalism

Why the factor 2?

GR takes into account the curvature of space, whereas Newton theory does not.

IX. *A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations made at the Total Eclipse of May* 29, 1919.

By Sir F. W. DYSON, *F.R.S., Astronomer Royal, Prof.* A. S. EDDINGTON, *F.R.S., and Mr.* C. DAVIDSON.

(*Communicated by the Joint Permanent Eclipse Committee.*)

Received October 30,—Read November 6, 1919.

[PLATE 1.]

CONTENTS.

I. PURPOSE OF THE EXPEDITIONS.

1. THE purpose of the expeditions was to determine what effect, if any, is produced by a gravitational field on the path of a ray of light traversing it. Apart from possible surprises, there appeared to be three alternatives, which it was especially desired to discriminate between—

(1) The path is uninfluenced by gravitation.

(2) The energy or mass of light is subject to gravitation in the same way as ordinary matter. If the law of gravitation is strictly the Newtonian law, this leads to an apparent displacement of a star close to the sun's limb amounting to $0''\cdot87$ outwards.

(3) The course of a ray of light is in accordance with EINSTEIN's generalised relativity theory. This leads to an apparent displacement of a star at the limb amounting to $1''\cdot75$ outwards.

In either of the last two cases the displacement is inversely proportional to the distance of the star from the sun's centre, the displacement under (3) being just double the displacement under (2).

It may be noted that both (2) and (3) agree in supposing that light is subject to gravitation in precisely the same way as ordinary matter. The difference is that, whereas (2) assumes the Newtonian law, (3) assumes EINSTEIN's new law of gravitation. The slight

3	+0·16	+0·30	−0·13	−0·32	0·00	−0·34	−0·20	+0·33	+0·10	−0·03
6	+0·14	−0·09	−0·54	−0·13	−0·16	+0·07	−0·27	−0·39	−0·08	−0·17
10	—	—	+0·73	+0·27	—	—	—	+0·07	+0·35	—

The average y-residual is $\pm 0''\cdot 22$, which gives a probable error for y of $\pm 0''\cdot 21$. It is satisfactory that this agrees so nearly with the probable error ($\pm 0''\cdot 22$) of the check plates, showing that the images are of about the same degree of difficulty and therefore presumably comparable. The probable error of x is $\pm 0''\cdot 25$, but we are not so much concerned with this.

The weight of the determination of $\delta\kappa$ is about 3 (strictly 3·23 for Plate X and 2·87 for Plate W). The probable error of κ is therefore $\pm 0''\cdot 12$, which corresponds to a probable error of $\pm 0''\cdot 38$ in the final values of the deflection.

As the four determinations involve only two eclipse plates and are not wholly independent, and further small accidental errors may arise through inaccurate determination of the orientation, the probable error of our mean result will be about $\pm 0''\cdot 25$. There is further the error of $\pm 0''\cdot 14$ affecting all four results equally, arising from the determination of scale. Taking this into account, and including the small correction $-0''\cdot 04$ previously mentioned, our result may be written

$$1''\cdot 61 \pm 0''\cdot 30.$$

It will be seen that the error deduced in this way from the residuals is considerably larger than at first seemed likely from the accordance of the four results. Nevertheless the accuracy seems sufficient to give a fairly trustworthy confirmation of EINSTEIN'S theory, and to render the half-deflection at least very improbable.

38. It remains to consider the question of systematic error. The results obtained with a similar instrument at Sobral are considered to be largely vitiated by systematic

* The residuals refer to the theoretical deflection $1''\cdot 75$, not the deduced deflections.

photograph fainter stars, and these will probably be at a greater distance from the sun.

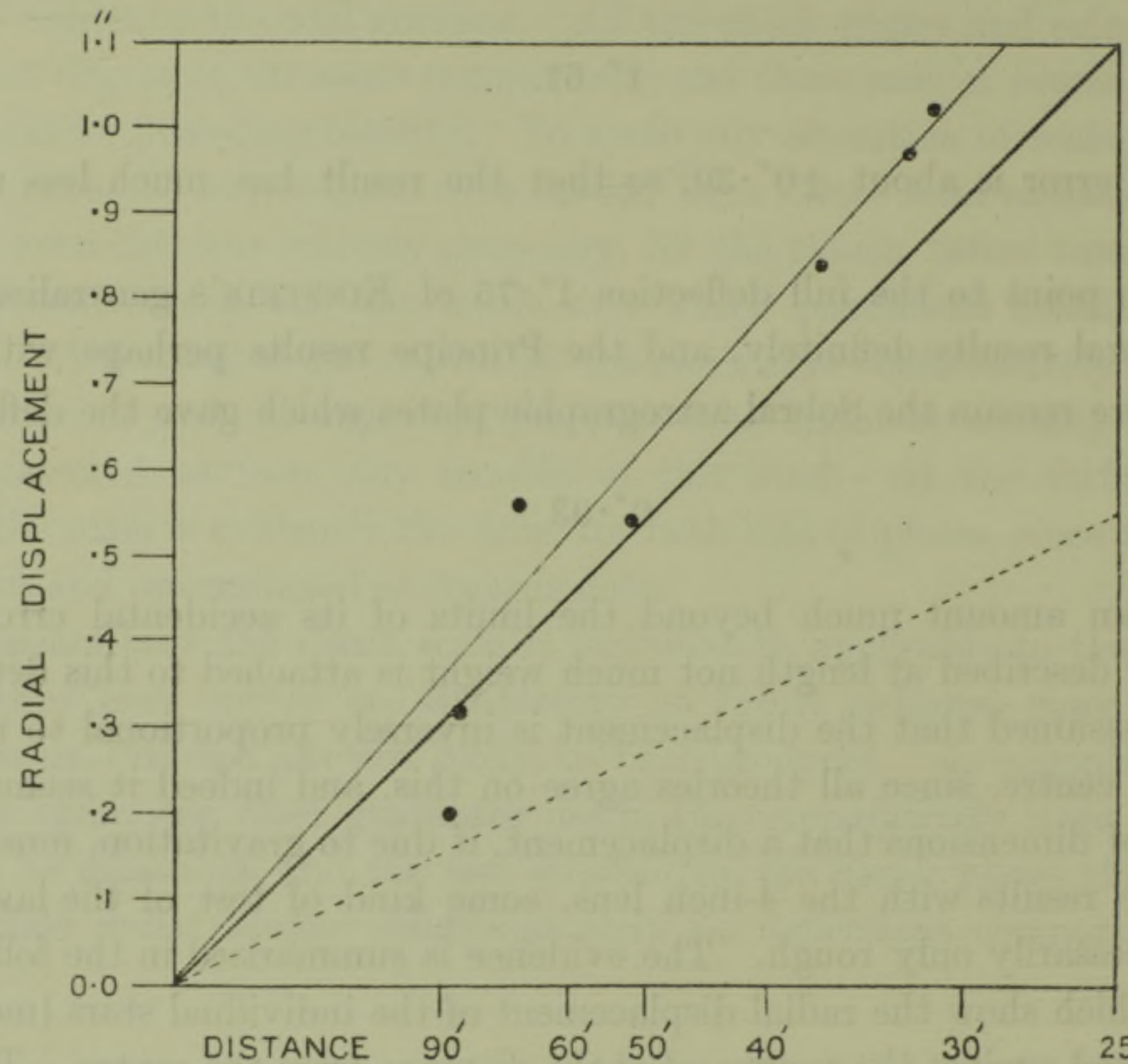


Diagram 2.

This *can* be done with such telescopes as the astrographic with the object-glass stopped down to 8 inches, if photographs of the same high quality are obtained as in regular stellar work. It will probably be best to discard the use of cœlostat mirrors. These are of great convenience for photographs of the corona and spectroscopic observations, but for work of precision of the high order required, it is undesirable to introduce complications, which can be avoided, into the optical train. It would seem that some form of equatorial mounting (such as that employed in the Eclipse Expeditions of the Lick Observatory) is desirable.

complications, which can be avoided, into the optical train. It would seem that some form of equatorial mounting (such as that employed in the Eclipse Expeditions of the Lick Observatory) is desirable.

In conclusion, it is a pleasure to record the great assistance given to the Expeditions from many quarters. Reference has been made in the course of the paper to some of these. Especial thanks are due to the Brazilian Government for the hospitality and facilities accorded to the observers in Sobral. They were made guests of the

Government, who provided them with transport, accommodation and labour. Dr. MORIZE, Director of the Rio Observatory, acting on behalf of the Brazilian Government, made most complete arrangements for the Expedition, and in this way contributed materially to its success.

On behalf of the Principe Expedition, special thanks are due to Sr. JERONYMO CARNEIRO, who most hospitably entertained the observers and provided for all their requirements, and to Sr. ATALAYA, whose help and friendship were of the greatest service to the observers in their isolated station.

We gratefully acknowledge the loan for more than six months of the astrographic object-glass of the Oxford University Observatory. We are also indebted to Mr. BELLAMY for the check plates he obtained in January and February.

Today, after more than 100 years since Einstein's 1908 and 1911 papers (and more than 200 years since Soldner's paper) gravitational lensing is an essential tool for unveiling the secrets of our universe.

Gravitational lensing is particularly useful for probing the mass of the lens and its distribution of matter. It can also be used to determine the Hubble constant (though there are methods which are more precise).

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ON THE MASSES OF NEBULAE AND OF CLUSTERS OF NEBULAE

F. ZWICKY

ABSTRACT

Present estimates of the masses of nebulae are based on observations of the *luminosities* and *internal rotations* of nebulae. It is shown that both these methods are unreliable; that from the observed luminosities of extragalactic systems only lower limits for the values of their masses can be obtained (sec. i), and that from internal rotations alone no determination of the masses of nebulae is possible (sec. ii). The observed internal motions of nebulae can be understood on the basis of a simple mechanical model, some properties of which are discussed. The essential feature is a central core whose internal *viscosity* due to the gravitational interactions of its component masses is so high as to cause it to rotate like a solid body.

In sections iii, iv, and v three new methods for the determination of nebular masses are discussed, each of which makes use of a different fundamental principle of physics.

Method iii is based on the *virial theorem* of classical mechanics. The application of this theorem to the Coma cluster leads to a minimum value $\overline{M} = 4.5 \times 10^{10} M_{\odot}$ for the average mass of its member nebulae.

Method iv calls for the observation among nebulae of certain *gravitational lens* effects.

Section v gives a generalization of the principles of ordinary *statistical mechanics* to the whole system of nebulae, which suggests a new and powerful method which ultimately should enable us to determine the masses of all types of nebulae. This method is very flexible and is capable of many modes of application. It is proposed, in particular, to investigate the distribution of nebulae in individual great clusters.

As a first step toward the realization of the proposed program, the Coma cluster of nebulae was photographed with the new 18-inch Schmidt telescope on Mount Palomar. Counts of nebulae brighter than about $m = 16.7$ given in section vi lead to the gratifying result that the distribution of nebulae in the Coma cluster is very similar to the distribution of luminosity in globular nebulae, which, according to Hubble's investigations, coincides closely with the theoretically determined distribution of matter in isothermal gravitational gas spheres. The high central condensation of the Coma cluster, the very gradual decrease of the number of nebulae per unit volume at great distances from its center, and the hitherto unexpected enormous extension of this cluster become here apparent for the first time. These results also suggest that the

units of mass which constitute a nebula. The average square velocity (21) might be derived from the shape of the spectral lines in the light from nebulae. Unfortunately, the practical determination of such shapes is at present exceedingly difficult, if not impossible. In addition the spectral lines in the light of nebulae are doubtless of complex origin, and the interpretation even of well-known shapes of lines is by no means an easy task.

IV. NEBULAE AS GRAVITATIONAL LENSES

As I have shown previously,[6] the probability of the overlapping of images of nebulae is considerable. The gravitational fields of a number of "foreground" nebulae may therefore be expected to deflect the

light coming to us from certain background nebulae. The observation of such gravitational lens effects promises to furnish us with the simplest and most accurate determination of nebular masses. No thorough search for these effects has as yet been undertaken. It would seem, perhaps, that if the masses of field nebulae were, on the average, as great as the masses of cluster nebulae obtained in section iii, gravitational lens effects among nebulae should have been long since discovered. Until many plates of rich nebular fields taken under excellent conditions of seeing have been carefully examined it would be dangerous, however, to draw any definite conclusions.

The mathematical analysis of the formation of images of distant nebulae through the action of the gravitational fields of nearer nebulae will be given in detail in an article to be published in the *Helvetica physica acta*.

1964MNRAS.128..307R

ON THE POSSIBILITY OF DETERMINING HUBBLE'S PARAMETER AND THE MASSES OF GALAXIES FROM THE GRAVITATIONAL LENS EFFECT*

Sjur Refsdal

(Communicated by H. Bondi)

(Received 1964 January 27)

Summary

The gravitational lens effect is applied to a supernova lying far behind and close to the line of sight through a distant galaxy. The light from the supernova may follow two different paths to the observer, and the difference Δt in the time of light travel for these two paths can amount to a couple of months or more, and may be measurable. It is shown that Hubble's parameter and the mass of the galaxy can be expressed by Δt, the red-shifts of the supernova and the galaxy, the luminosities of the supernova "images" and the angle between them. The possibility of observing the phenomenon is discussed.

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1. *Introduction.*—In 1937 Zwicky suggested that a galaxy, due to the gravitational deflection of light, may act as a gravitational lens. He considered the case of a galaxy A lying far behind and close to the line of sight through a distant galaxy B. If the line of sight through the centre of B goes through A, the "image" of A will be a ring around B, otherwise two separated "images" appear, on opposite sides of B. The phenomenon has later been discussed by Zwicky (1957) and Klimov (1963), and they both conclude that the possibility of observing the phenomenon should be good. In the present paper the case of a supernova lying behind a galaxy is considered. Two "images" of the supernova may then be seen, and we will show that from one such "double image" observation, Hubble's parameter and the mass of the deflecting galaxy can be determined. The possiblity of observing such a "double image" will be discussed.

2. *Determination of Hubble's parameter and the masses of galaxies.*—We consider a supernova S lying far behind and close to the line of sight through a distant galaxy, B, which will then act as a gravitational lens. For simplicity, we assume

1. The deflecting galaxy is spherically symmetric.
2. The red-shifts of S and B are small.

We can then apply the results previously obtained in the case of a star acting as a gravitational lens (Refsdal 1964). Using the same notation, we have

$$\alpha = \sqrt{\alpha_0^2 + \beta^2} \approx \alpha_0\left(1 + \frac{1}{2}\frac{\beta^2}{\alpha_0^2}\right) \approx \alpha_0 \qquad (1)$$

0957+561 A, B: twin quasistellar objects or gravitational lens?

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0957+561 A, B are two QSOs of mag 17 with 5.7 arc s separation at redshift 1.405. Their spectra leave little doubt that they are associated. Difficulties arise in describing them as two distinct objects and the possibility that they are two images of the same object formed by a gravitational lens is discussed.

SPECTROSCOPIC observations have been in progress for several years on QSO candidates using a survey of radio sources made at 966 MHz with the MkIA telescope at Jodrell Bank. Many of the identifications have been published by Cohen *et al.*[1] with interferometric positions accurate to ~2 arc s and a further list has been prepared by Porcas *et al.*[2]. The latter list consists of sources that were either too extended or too confused for accurate interferometric positions to be measured, and these were observed with the pencil-beam of the 300 ft telescope at NRAO, Green Bank at λ 6 cm and λ 11 cm. This gave positions with typical accuracy 5–10 arc s and the identifications are estimated as ~80% reliable.

The list of Porcas *et al.* includes the source 0957+561 which has within its field a close pair of blue stellar objects, separated by ~6 arc s, which are suggested as candidate identifications. Their positions and red and blue magnitudes, m_R and m_B, estimated from the Palomar Observatory Sky Survey (POSS) [...] be of lower accuracy than normal, but they are very nearly equal and object A is definitely bluer than object B. The mean position of the two objects is 17 arc s from the radio position, so the identification is necessarily tentative.

Observations

The two objects 0957+561 A, B were observed on 29 March 1979 at the 2.1 m telescope of the Kitt Peak National Observatory (KPNO) using the intensified image dissector scanner (IIDS). Sky subtraction was used with circular apertures separated by 99.4 arc s. Some observational parameters are given in Table 2. The spectral range was divided into 1,024 data bins, each bin 3.5 Å wide, and the spectral resolution was 16 Å. After 20-min integration on each object it was clear that both were QSOs with almost identical spectra and redshifts of ~1.40 on the basis of strong emission lines identified as C IV λ1549 and C III] λ1909. Further observations were made on 29 March and on subsequent nights as detailed in Table 2. By offsetting to observe empty sky a few arc seconds from one object on both 29 and 30 March it was confirmed that any contamination of the spectrum of one object by light from the other was negligible.

Table 1 Positions and magnitudes of 0957+561 A, B

Object	RA	Dec (1950.0)	M_R	M_B
0957+561A	09 57 57.3	+56 08 22.9	17.0	16.7

On 1 April the spectral range was altered slightly by tilting the grating to cover the anticipated redshifted wavelength of Mg II λ2798 which was just beyond the limiting wavelength on previous nights.

The spectra obtained on 1 April are shown in Fig. 2. Data on observed spectral lines are given in Table 3. These were taken from the spectra using the interactive picture processing system (IPPS) which makes a linear interpolation between two selected continuum points and calculates the centroid and equivalent width of the emission above the interpolated line. Data from all three nights were used in compiling Table 3; that on 1 April had double the signal-to-noise ratio of the other two nights and was weighted accordingly. The O IV] λ1402 line is outside the spectral range of Fig. 2 but was present in data taken on the other two nights. Although we believe that Mg II λ2798 is detected in the data of Fig. 2 for 0957+561B, and He II λ1640 is also detected taking into account all three nights' data, the low signal-to-noise ratio and poorly defined continuum prevent us deriving useful observed wavelengths or equivalent widths.

The data on the C IV λ1549 and C III] λ1909 lines are much more accurate than those on the other lines and we believe the r.m.s. errors in the observed wavelengths of the centroids of these lines are not greater than 3 Å while the r.m.s. errors in the equivalent widths are estimated to be 7 Å. Within the limits of observational error, the corresponding lines in each object are identical in observed wavelength and equivalent width. For each object there is a difference in the redshift derived from the C IV and C III] lines which is significantly greater than the combined r.m.s. error in each. This may be associated with the problem of giving a precise meaning to the redshift of a broad line of somewhat irregular shape. The mean values of the redshift from the C IV and C III] emission lines are 1.4054 for A and 1.4047 for B, the difference being within the errors of measurement.

Although no attempt was made to carry out accurate spectrophotometry, some characteristics of the continua seem fairly well defined. Below about 5,300 Å they appear to have identical shapes, with QSO A brighter than B by 0.35 mag. Above 5,300 Å, however, the flux from B rises more steeply than that from A and they are equal at ~6,500 Å. These results are consistent with the magnitude estimates of Table 1.

The pair of QSOs provides unusual opportunity to investigate the origin of absorption lines in QSO spectra, a matter which is still in dispute. Accordingly, spectra having a resolution of about

both QSOs. Even in the low resolution IIDS spectrum of QSO A there is clear evidence for Fe II λ2383 and Mg II λ2798 absorption. Fe II λλ2600 and 2344 are possibly also present. Weak and possibly real absorption lines also appear in the image tube spectrum at λ3536.1 and λ3835.1 of QSO A. The features at λ3835.1 and λ3844.0 have a separation close to that of the Mg II doublet (at redshift 0.372). However, λ3844 is already identified with Fe II λ1608 in the 1.390 system so that the evidence for Mg II at 0.372 is not convincing. In QSO B, the

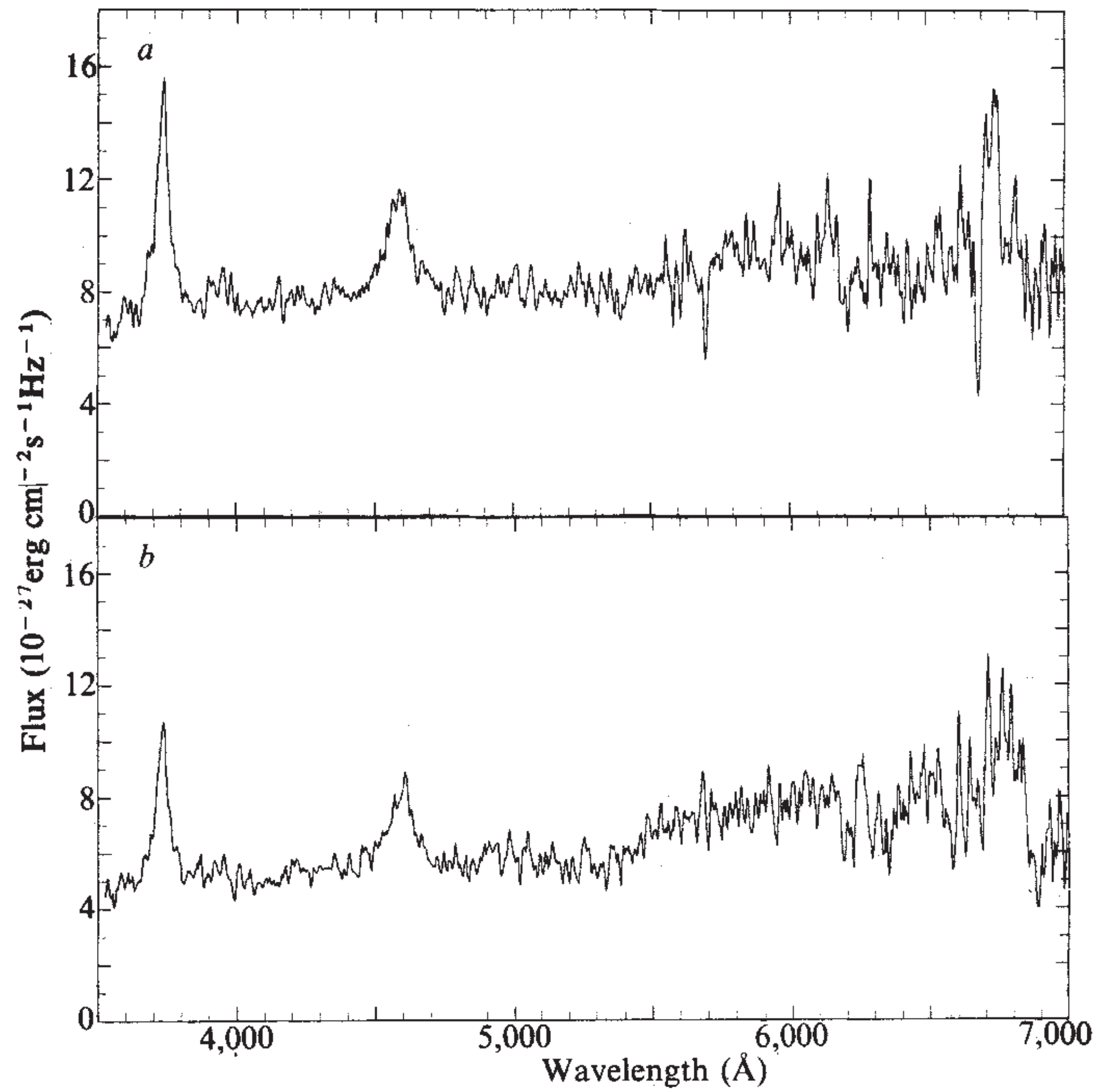
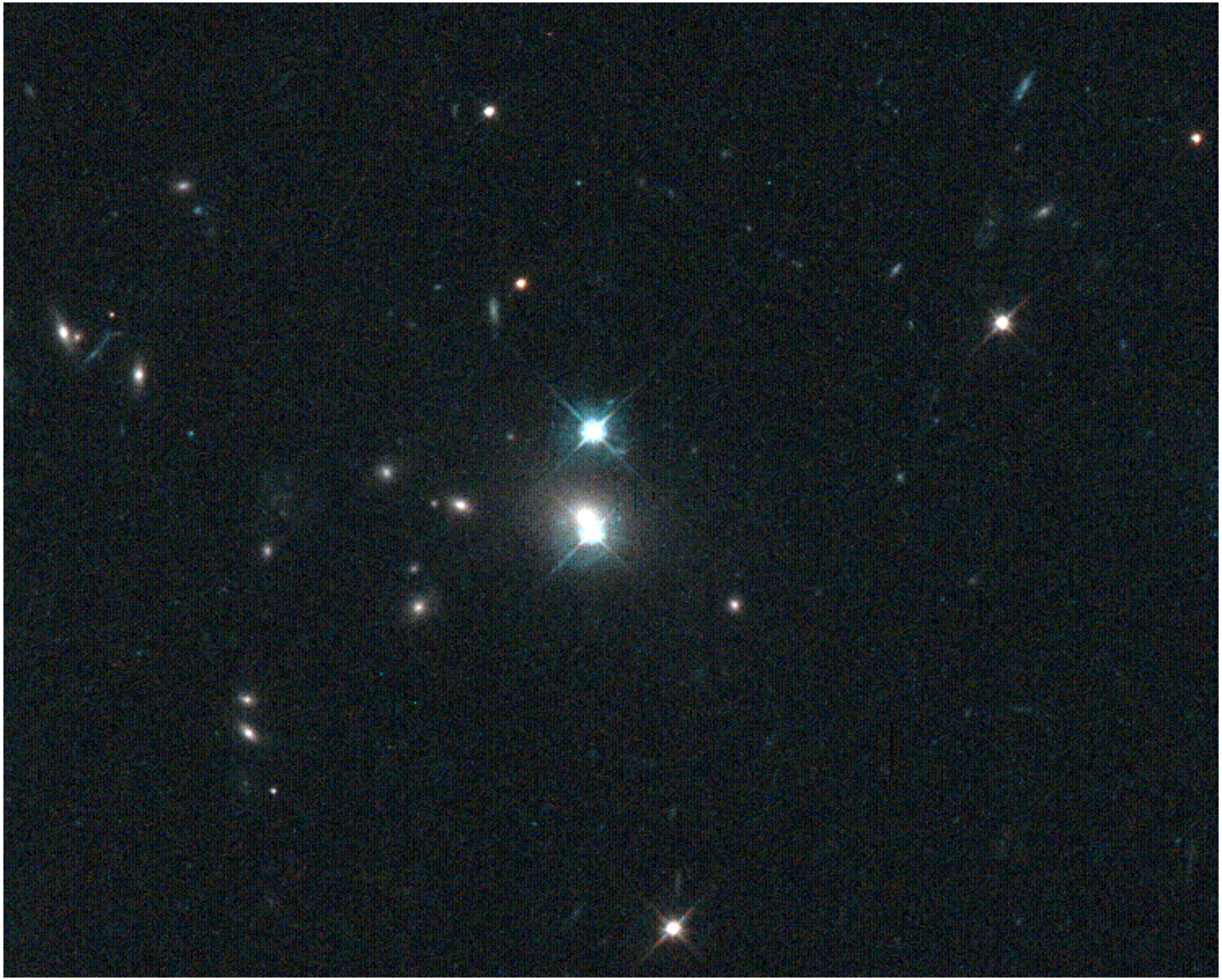
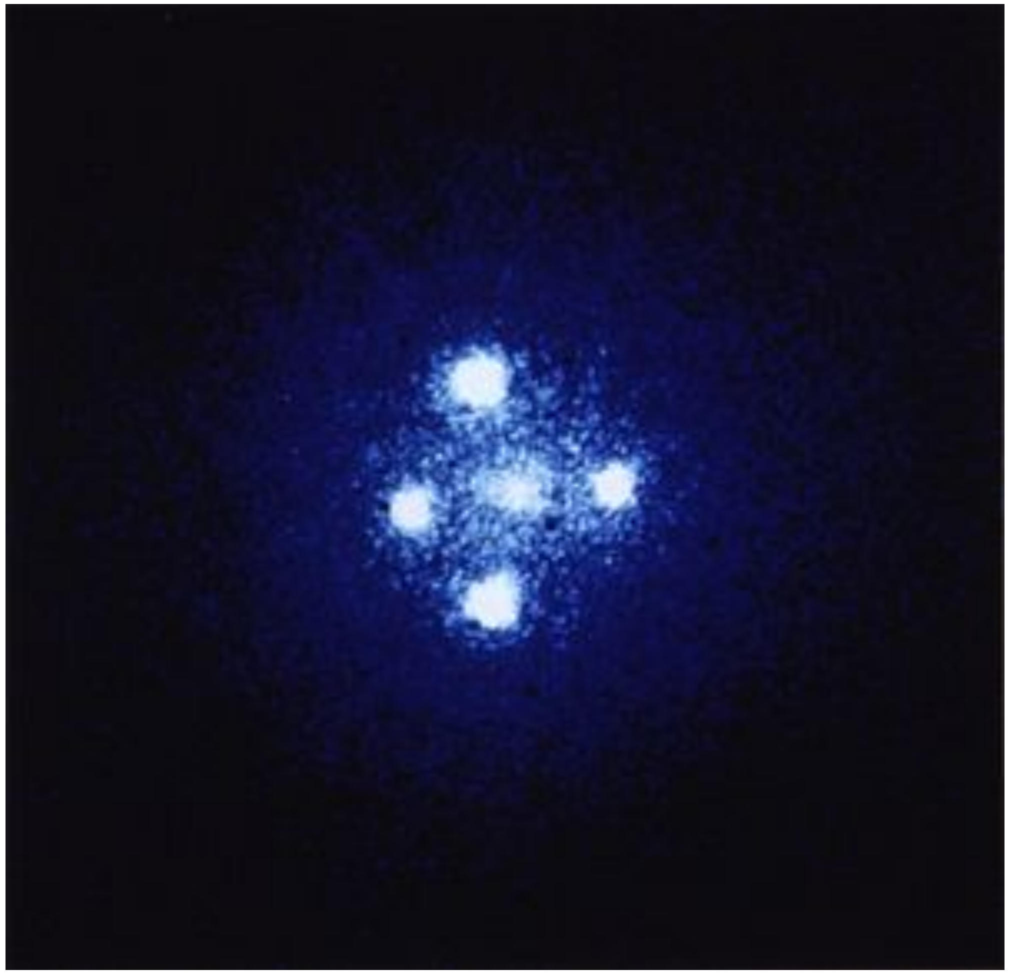


Fig. 2 IIDS scans of 0957+561 A(*a*) and B(*b*). The data are smoothed over 10 Å and the spectral resolution is 16 Å.



http://pages.astronomy.ua.edu/keel/agn/q0957.html

When multiple images are formed due to a gravitational lens (typically a galaxy), we call this phenomenon **Strong Gravitational Lensing**.



The **Einstein Cross** (**Q2237+030** or **QSO 2237+0305**), 4 images of a quasar behind ZW 2237+030 (discovered in 2006).

"Smiley" or "Cheshire Cat" (SDSS J1038+4849) imaged with HST (discovered in 2008)

Time delay

The goal is to measure individual time delays with an accuracy below 3%, in order to determine the Hubble constant.

https://cosmograil.epfl.ch/

For now, for HE 0435−1223:

H_0 = 71.9 + 2.4/-3.0 km/s/Mpc (Bonvin et al., 2016)

For comparison, from Planck (CMB observation):

H_0 = 67.74 ± 0.46 km/s/Mpc (Planck Collaboration, 2015)

Challenging because: 1) Huge angular resolution in observation is needed and 2) A reliable model of the lens is needed.

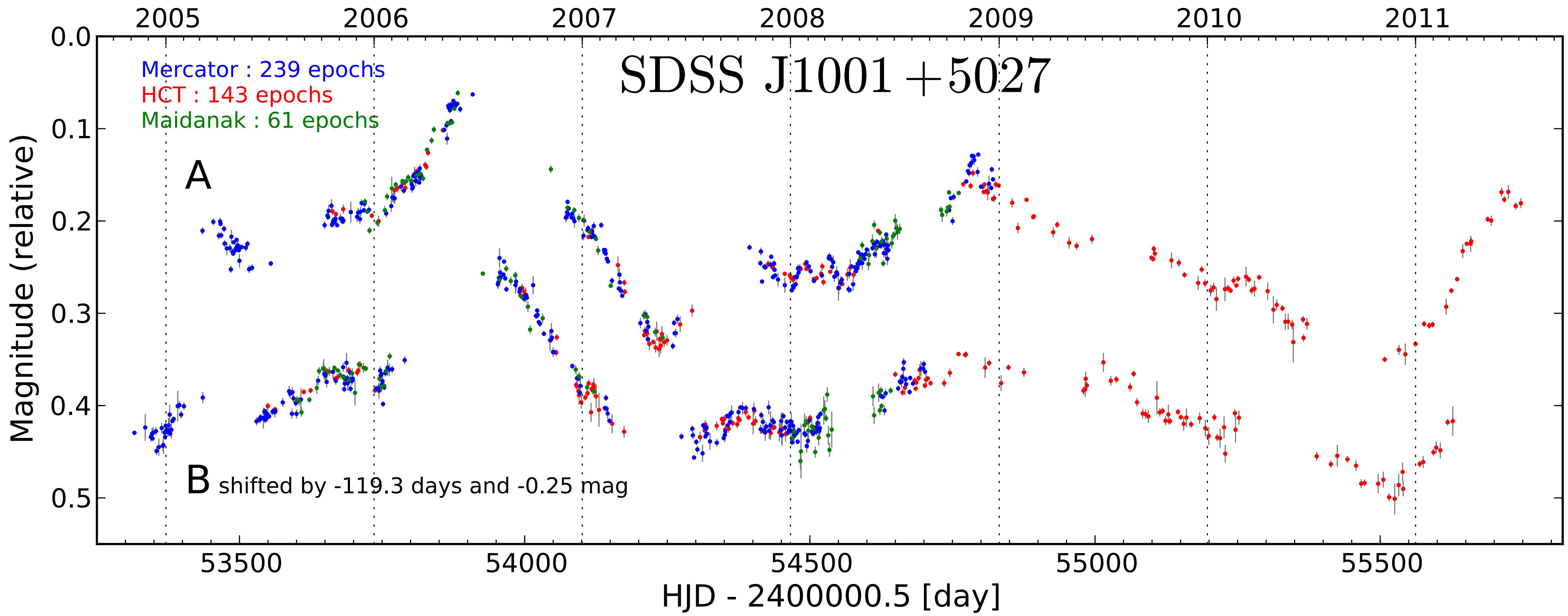


Fig. 4. R-band light curves of the quasars images A and B in SDSS J1001+5027 from March 2005 to July 2011. The 1σ photometric error bars are also shown. For display purpose, the curve of quasar image B is shown shifted in time by the measured time delay (see text). The light curves are available in tabular form from the CDS and the COSMOGRAIL website.

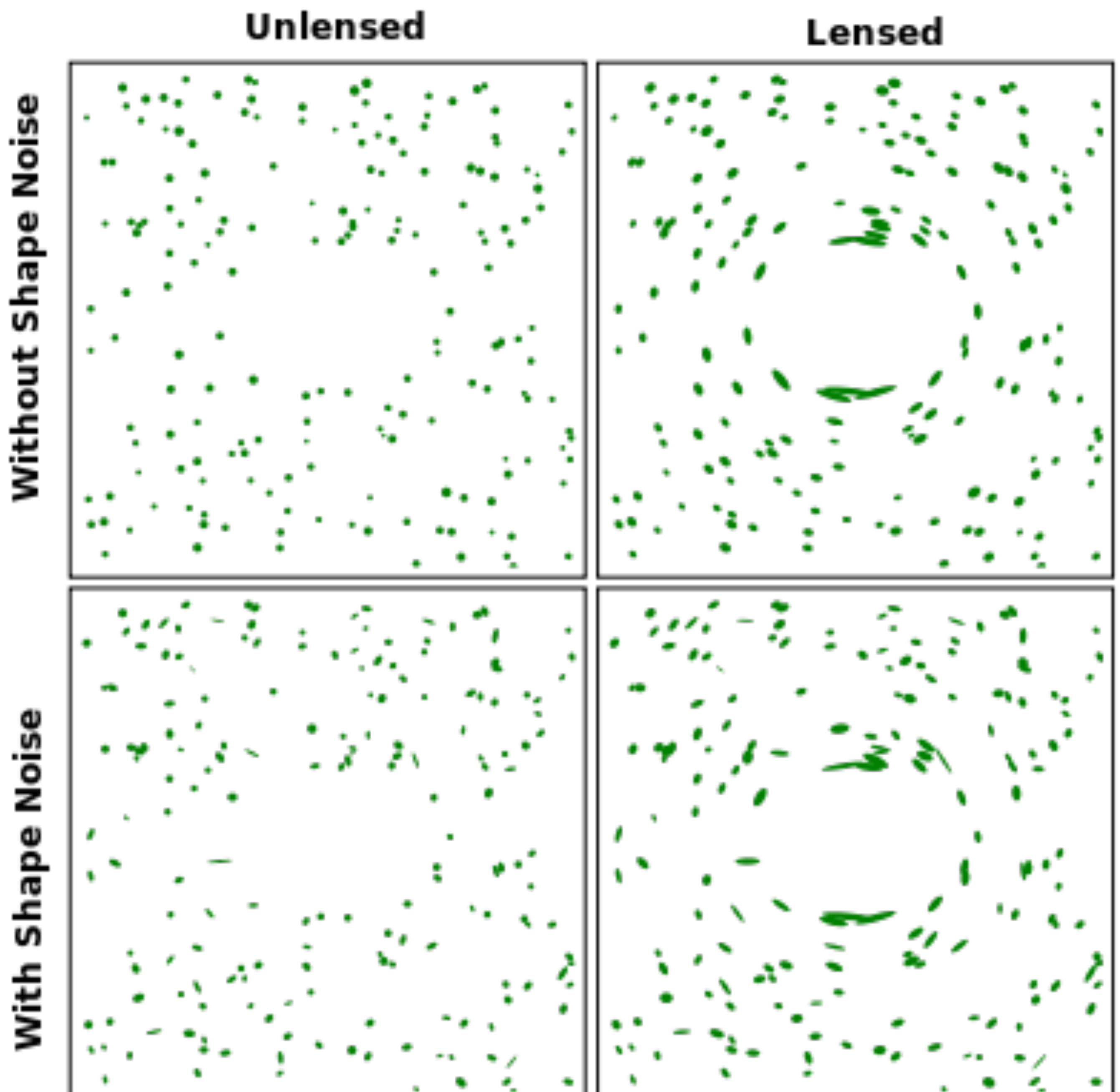
flux. We find that one has to subtract from curve B about 20% of its median flux to obtain an almost stationary magnitude shift of about 0.66 mag between the light curves. As this contamination would be several times larger than the entire flux of galaxy G1, we conclude that plausible errors of our light models for G1 cannot be responsible for the observed discrepancy between the light curves.

3. A new time-delay estimator

Although an unambiguous approximation of the time delay of SDSS J1001+5027 can be made by eye, accurately measuring its value is not trivial, and is made more difficult by the extrinsic variability between the light curves. Even more obvious features of the data, such as the sampling gaps due to non-visibility periods of the targets, could easily bias the results from a time-delay measurement technique. The impact of these effects on the quality of the time-delay inference clearly differs for each individual quasar lensing system and dataset. To check for potential systematic errors, we feel that a wise approach is to employ several numerical methods based on different fundamental principles.

In the present section we introduce a new time-delay estimation method, based on minimizing residuals of a high-pass filtered difference light curve between the quasar images.

Much more common than the formations of multiple images or Einstein's rings are **distortions** of background objects by foreground lenses (galaxies and clusters of galaxies). This is called **Weak Gravitational Lensing**.



How Weak Lensing works

The background field of galaxies is distorted by the lens.

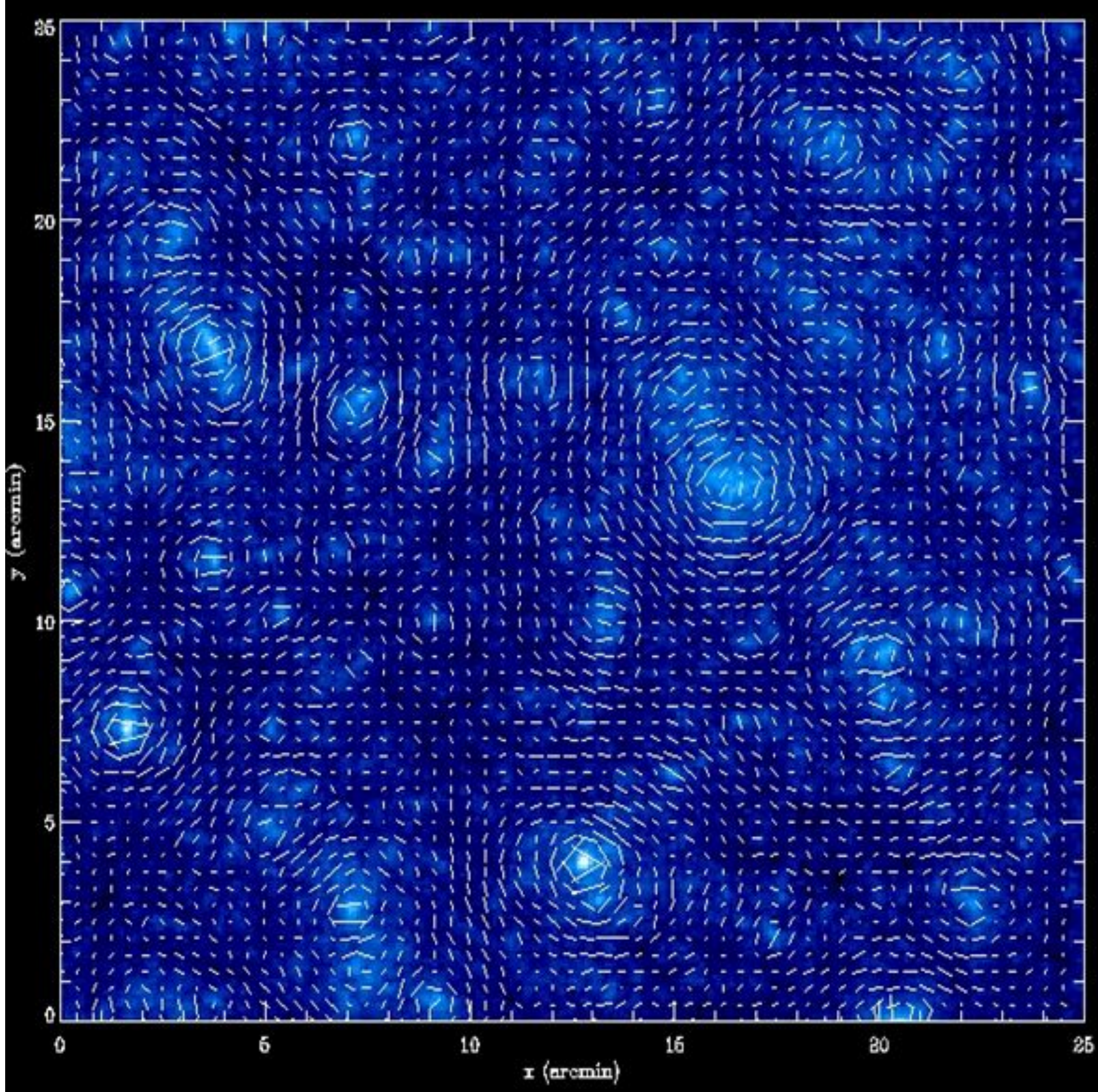
The lens produces a shear field, which depends on the second spatial derivative of the gravitational potential (it is a tidal field) integrated along the line of sight.

The ellipticities of the background galaxies (their distortions) allow to map the shear field and determine the gravitational potential and the mass distribution of the lens.

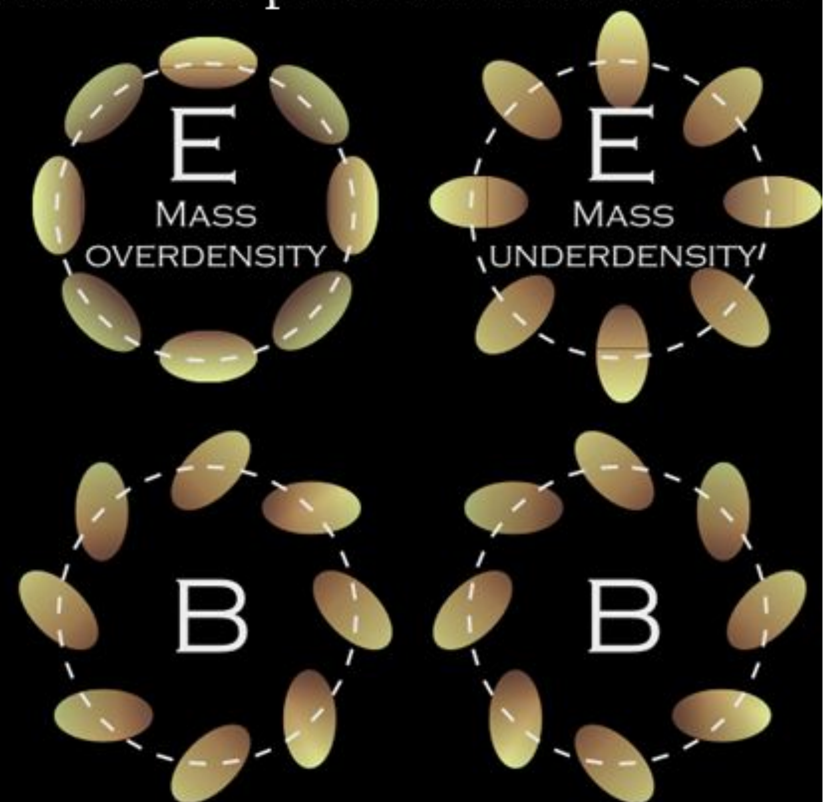
Differently from Strong Lensing, the signal from one galaxy is useless for Weak Lensing. We need many many galaxies (this makes Weak Lensing a statistics-based test).

Difficulty: Galaxies might have ellipticities because of intrinsic reasons (**shape noise**) or be aligned (**intrinsic alignment**) not because of lensing. The shape noise is controlled by averaging over many galaxies.

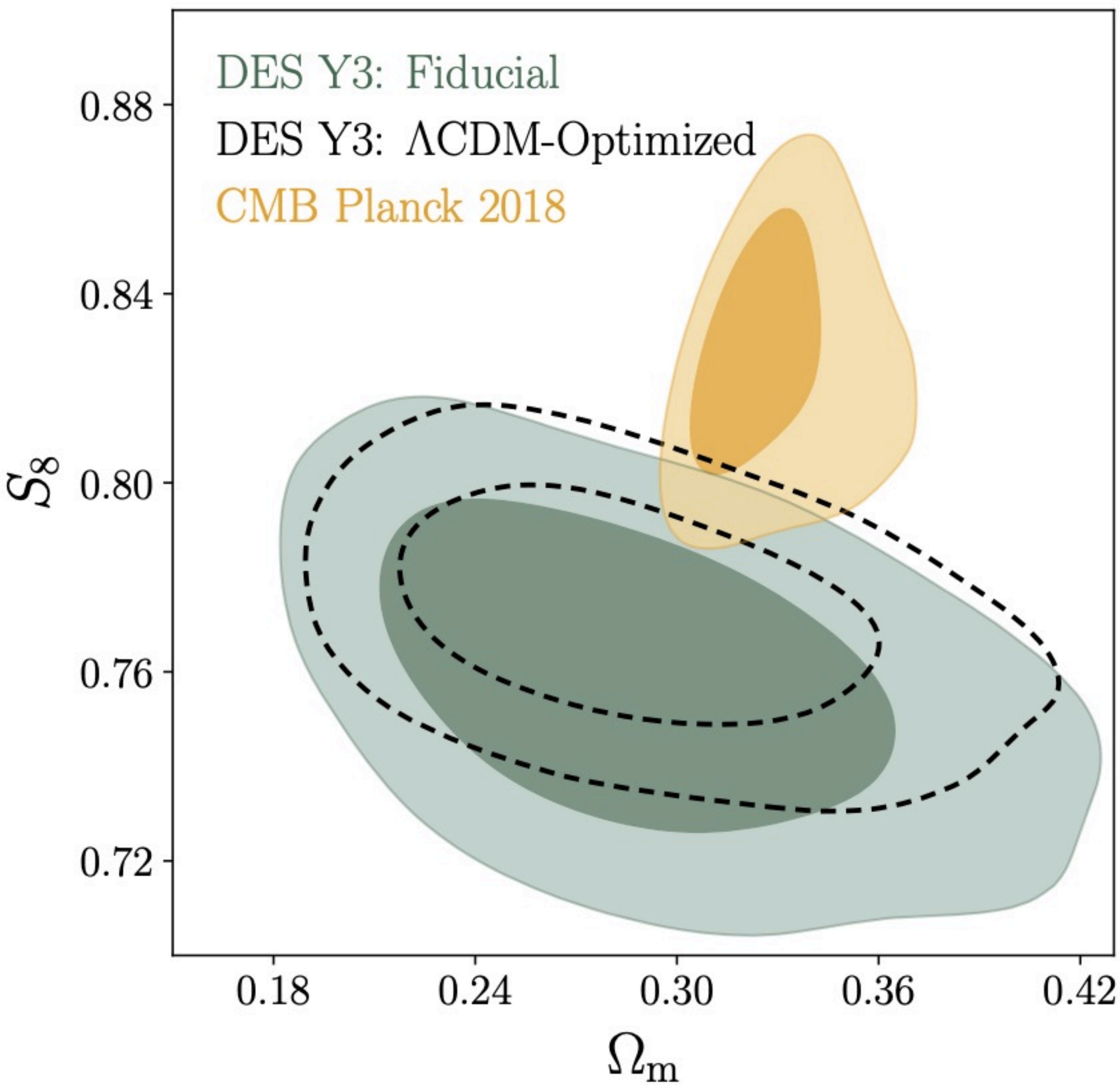
Weak gravitational lensing: patterns in a shear field



Clusters are patterns in a shear field:



Current Observation



https://www.darkenergysurvey.org/des-year-3-cosmology-results-papers/

Future (Stage IV) Projects

- J-PAS (Javalambre Physics of the Accelerating Universe Astrophysical Survey). Telescope. Brazil-Spain collaboration.

- EUCLID. Satellite (ESA). Launch planned in 2022?

- LSST (Large Synoptic Survey Telescope -> Vera C. Rubin Observatory). Location: Cerro Pachón, Chile. Ten-year survey starting in 2023?

- WFIRST (Wide Field Infrared Survey Telescope -> Nancy Grace Roman Space telescope). Satellite (NASA). Launch planned in mid 2027?

Thank You!