The effect of the cosmological constant on the bending of light

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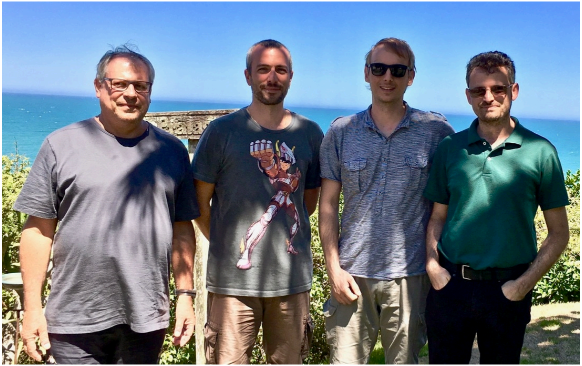
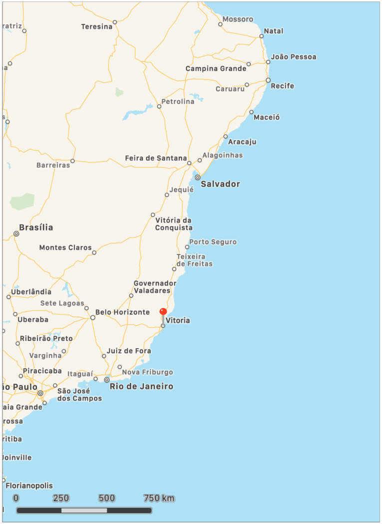


From left to right: Júlio C. Fabris, OFP, Valerio Marra, Davi C. Rodrigues.

Where is Vitória?



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Outline of the talk

Lensing and Λ

Criticisms

McVittie metric

Conclusions

References relevant for this talk

- G. McVittie, Mon. Not. Roy. Astron. Soc. 93, 325 (1933).
- J. Islam, Physics Letters A 97, 239 (1983).
- W. Rindler and M. Ishak, Phys. Rev. D76, 043006 (2007).
- M. Park, Phys. Rev. D78, 023014 (2008).
- T. Schucker, Gen. Rel. Grav. 41, 67 (2009).
- F. Simpson, J. A. Peacock, A. F. Heavens, MNRAS 402 (2010)
- T. Biressa and J. A. de Freitas Pacheco, Gen. Rel. Grav. 43, 2649 (2011)
- OFP, Phys. Rev. D93, 024020 (2016)
- L. M. Butcher, Phys. Rev. D94, 083011 (2016)
- OFP, Universe 2, 4, 25 (2016)
- OFP and Leonardo Giani, Phys. Rev. D95, 101301 (2017)

Local effects of Λ

Λ has an effect on orbital motion (Eddington, The mathematical theory of relativity, 1923, Sec. 45). In Eddington's original notation $\alpha \equiv \Lambda$:

$$ds^2 = -\gamma dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + \gamma dt^2 \,, \qquad \gamma = 1 - \frac{2m}{r} - \frac{\alpha r^2}{3} \,. \tag{1}$$

Orbit equation for a massive particle:

$$\frac{d^2u}{d\phi^2} + u = \frac{m}{h^2} + 3mu^2 - \frac{\alpha^2}{3h^2u^3} \,, \tag{2}$$

where $u \equiv 1/r$, h is the angular momentum $r^2 d\phi/ds = h$.
Additional motion of perihelion:

$$\frac{\delta\varpi}{\phi} = \frac{\alpha a^3}{2m}(1 - e^2)^3 \,, \tag{3}$$

where a semi-major axis and e eccentricity.

Local effects of Λ

Eddington states that (if $\alpha > 0$) $\sqrt{3/\alpha} > 10^{25}$ cm (the farthest structure known), so:

$$\alpha < 10^{-50}\ \mathrm{cm}^{-2} = 10^{-46}\ \mathrm{m}^{-2}\ , \tag{4}$$

impossible to detect. Islam (1983) finds

$$|\Lambda| < 10^{-42}\ \mathrm{m}^{-2}\ , \tag{5}$$

in order to avoid a detectable correction to Mercury's perihelion precession. For reference, the today measured (from Planck 2018, arXiv:1807.06209) value is:

$$\Lambda = (1.089 \pm 0.029) \times 10^{-52}\ \mathrm{m}^{-2}\ . \tag{6}$$

Local effects of Λ

Islam concludes his paper showing the trajectory equation for a photon:

$$\frac{d^2u}{d\phi^2} + u = 3Mu^2 \ , \tag{7}$$

with no extra contribution from Λ. Hence, he concludes that *we get the same result as before for bending of light near the sun.*

W. Rindler and M. Ishak [Phys. Rev. D76, 043006 (2007)] contest this conclusion: Λ does influence the bending of light via the metric, which has to be used in order to compute the bending angle.

We now review their argument.

Effect of Λ on the bending of light

W. Rindler and M. Ishak, Phys. Rev. D76, 043006 (2007).

Framework: Schwarzschild-de Sitter metric (sometimes also known as Kottler metric) [F. Kottler, Annalen der Physik 361, 401 (1918)].

$$ds^2 = \alpha(r)dt^2 - \alpha(r)^{-1}dr^2 - r^2 d\Omega^2 \ , \tag{8}$$

with

$$\alpha(r) \equiv 1 - \frac{2m}{r} - \frac{\Lambda r^2}{3} \ . \tag{9}$$

This metric has two horizons, $r \approx 2m$ and $r \approx \sqrt{3/\Lambda}$ (for $m \ll 1/\sqrt{\Lambda}$). It is not asymptotically flat.

Geometry of Kottler solution

Fig. 1 of W. Rindler and M. Ishak, Phys. Rev. D76, 043006 (2007).

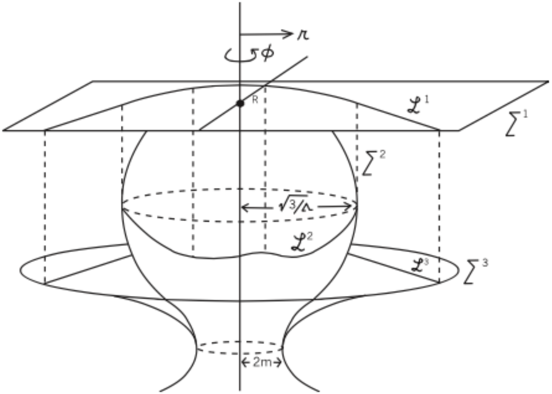


Fig. 1 Schwarzschild and Schwarzschild-de Sitter geometries. Σ^3 is the Flamm paraboloid representation of a central coordinate plane in Schwarzschild; Σ^2 is the corresponding surface in Schwarzschild-de Sitter; Σ^1 is an auxiliary plane with an r, ϕ graph, $\mathcal{L}^1$, of the orbit equation (6). The curves $\mathcal{L}^2$ and $\mathcal{L}^3$ are the vertical projections of $\mathcal{L}^1$ onto Σ^2 and Σ^3, and represent the true spatial curvature of the orbits.

Orbital equation for photons

For $\theta = \pi/2$ and at first-order in m/R:

$$\frac{1}{r} = \frac{\sin\phi}{R} + \frac{3m}{2R^2}\left(1 + \frac{1}{3}\cos 2\phi\right) , \tag{10}$$

where, for $\phi = \pi/2$:

$$\frac{1}{r_0} = \frac{1}{R} + \frac{m}{R^2} , \tag{11}$$

and r_0 is the closest approach distance and R is the distance of the zeroth-order solution (a straight line) from the centre.

The above equations hold true both for Schwarzschild and Schwarzschild-de Sitter metrics. No Λ appears.

Trajectory of photons

Fig. 2 of W. Rindler and M. Ishak, Phys. Rev. D76, 043006 (2007).

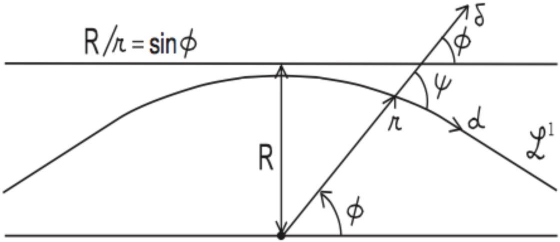


Fig. 2 The orbital map. This is a plane graph of the orbit equation (6) and coincides with Σ^1 in Figure 1. The one-sided deflection angle is $\psi - \phi \equiv \epsilon$.

Calculating the bending angle

In Schwarzschild metric, one can take the limit $r \to \infty$:

$$0 = \frac{\sin \phi_\infty}{R} + \frac{3m}{2R^2}\left(1 + \frac{1}{3}\cos 2\phi_\infty\right) , \tag{12}$$

so, for ϕ_∞ small:

$$\phi_\infty = -\frac{2m}{R} , \tag{13}$$

and, due to spherical symmetry, the bending angle is:

$$\delta = \frac{4m}{R} . \tag{14}$$

But we cannot take $r \to \infty$ in Schwarzschild-de Sitter space.

Results

The proposal is then to use:

$$\cos\psi = \frac{g_{ij}\delta^i d^j}{(g_{ij}\delta^i\delta^j)^{1/2}(g_{ij}d^i d^j)^{1/2}}\,, \tag{15}$$

where d is the vector tangent to the photon orbit and δ is the $\phi =$ constant direction. Here Λ comes into play:

$$\tan^2\psi = r^2\alpha(r)/(dr/d\phi)^2\,. \tag{16}$$

Using the equation for the trajectory, valid for $m/R \ll 1$, and considering $\Lambda r^2 \ll 1$:

$$\tan\psi = \tan\phi\left(1 - \frac{m}{r} - \frac{\Lambda r^2}{6}\right)\left(1 + \frac{m}{r} + \frac{2m}{R\sin\phi}\right)\,. \tag{17}$$

Recall that r and ϕ are not independent but linked by the trajectory equation (10).

Results

For $R \ll r \ll \sqrt{1/\Lambda}$, we can consider small angles and approximate the previous expression as:

$$\psi = \phi + \frac{2m}{R} - \frac{m\Lambda r^2}{3R} \ . \tag{18}$$

The total bending angle is:

$$\delta = 2(\psi - \phi) = \frac{4m}{R} - \frac{2m\Lambda r^2}{3R} \ , \tag{19}$$

and comes with an extra contribution containing Λ.
Note that for $m = 0$ we must recover the former subdominant terms of the expansion and we get:

$$\delta_{m=0} = -\frac{\Lambda R r}{6} \ , \tag{20}$$

i.e. we have bending of light even in absence of a lens.

Other analysis

Supporting the result of W. Rindler and M. Ishak, Phys. Rev. D76, 043006 (2007)

- T. Schucker, Gen. Rel. Grav. 41, 67 (2009). He uses a self-contained method, avoiding the lens equation. Analysing the lensing cluster of SDSS J1004+4112, he finds:

$$\Lambda = (2.1 \pm 1.5) \times 10^{-52}\ \mathrm{m}^{-2}\ ;$$

- T. Biressa and J. A. de Freitas Pacheco, Gen. Rel. Grav. 43, 2649 (2011). They find for the bending angle:

$$\delta \simeq \frac{4M}{b} - Mb\left(\frac{1}{r_S^2} + \frac{1}{r_{\mathrm{obs}}^2}\right) + \frac{2Mb\Lambda}{3} - \frac{b\Lambda}{6}(r_S + r_{\mathrm{obs}})\ ,$$

  and determine corrections of 2% in the mass estimates. Masses are slightly lower if a cored density profile is used and slightly higher if an isothermal density profile is adopted.

Criticisms

M. Park, Phys. Rev. D78, 023014 (2008)

The main criticism is that the results presented above are based on Kottler metric, which is static, and therefore does not take into account the relative motion of source, lens and observer.

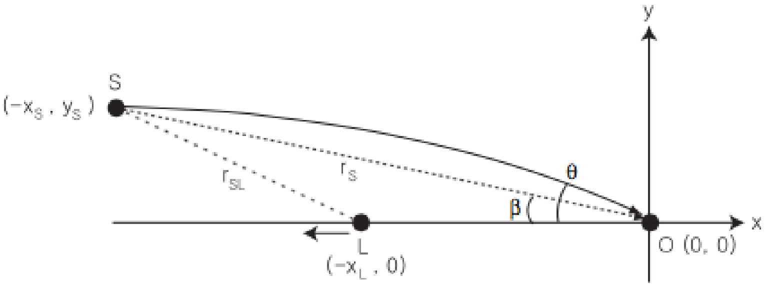


FIG. 1. Lensing schematics.

Result found by M. Park, Phys. Rev. D78, 023014 (2008)

M. Park found for the following lens equation:

$$\theta = \beta + \frac{2md_{\mathrm{SL}}}{\beta d_{\mathrm{S}} d_{\mathrm{L}}} \left[1 + \mathcal{O}(H^3) + \mathcal{O}(\beta^2)\right] + \mathcal{O}(m^2) \ , \tag{21}$$

where $H^2 = \Lambda/3$. Here $m \to 0$ implies no deflection, differently from the results based on Kottler metric, which assert that there should be a $\mathcal{O}(\Lambda) \sim \mathcal{O}(H^2)$ correction to the conventional lensing analysis, even for no mass.

M. Ishak, W. Rindler and J. Dossett, MNRAS, 403, 2152 (2010) questioned the final result of Park since other terms including $H^2 = \Lambda/3$ terms were apparently dropped out the calculation at some point, leading to the conclusion that Λ does not contribute to lensing except via the angular diameter distances.

McVittie metric

G. McVittie, Mon. Not. Roy. Astron. Soc. 93, 325 (1933)

$$ds^2 = -\left(\frac{1-\mu}{1+\mu}\right)^2 dt^2 + (1+\mu)^4 a(t)^2 (d\rho^2 + \rho^2 d\Omega^2) \;, \qquad (22)$$

where $a(t)$ is the scale factor and

$$\mu \equiv \frac{M}{2a(t)\rho} \;, \qquad (23)$$

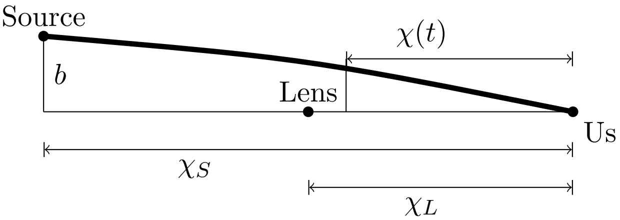
where M is the mass of the point-like lens. When $\mu \ll 1$, McVittie metric can be approximated as

$$ds^2 = -(1-4\mu)\,dt^2 + (1+4\mu)a(t)^2(d\rho^2 + \rho^2 d\Omega^2) \;, \qquad (24)$$

which is the usual perturbed FLRW metric in the Newtonian gauge; 2μ is the gravitational potential.

Scheme of lensing

OFP, Phys. Rev. D93, 024020 (2016)



The relation between χ and the background expansion is the usual one for the FLRW metric:

$$\frac{d\chi}{dt} = -\frac{1}{a} \,, \tag{25}$$

and the comoving distances of the source and of the lens, χ_S and χ_L respectively, do not change.

Null geodesics equations

For the transversal displacement l^i:

$$\frac{a}{p}\frac{1-\mu}{1+\mu}\frac{d}{d\chi}\left(\frac{p}{a}\frac{1+\mu}{1-\mu}\frac{dl^i}{d\chi}\right) = \frac{2(1-\mu)}{(1+\mu)^7}\delta^{il}\partial_l\mu$$

$$+2Ha\left[1+\frac{2\partial_t\mu}{(1+\mu)H}\right]\frac{dl^i}{d\chi}$$

$$-\frac{2}{1+\mu}\left(\delta^i_j\partial_k\mu+\delta^i_k\partial_j\mu-\delta_{jk}\delta^{il}\partial_l\mu\right)\frac{dl^j}{d\chi}\frac{dl^k}{d\chi}\ ,$$

The equation describing the evolution of the proper momentum p is the following:

$$\frac{1}{p}\frac{dp}{dt} = -H - \frac{2}{1+\mu}\partial_t\mu + 2\frac{P^i\partial_i\mu}{p(1+\mu)^2}\ . \qquad (26)$$

Approximations

We consider $\mu \ll 1$ and small displacements $l^i \ll \chi$.

$$\frac{d^2 l^i}{d\chi^2} = 4\partial_i \mu \ . \tag{27}$$

Using $\mu \equiv M/2a\rho$ in the equation above, one gets:

$$\frac{d^2 l}{d\chi^2} = -\frac{2Ml}{a(\chi)\left[(\chi-\chi_L)^2 + l^2\right]^{3/2}} \ . \tag{28}$$

With the following definitions:

$$x \equiv \chi/\chi_L \ , \qquad \alpha \equiv 2M/\chi_L \ , \qquad y \equiv l/\chi_L \ , \tag{29}$$

the displacement equation becomes:

$$\frac{d^2 y}{dx^2} = -\alpha \frac{y}{a(x)\left[(x-1)^2 + y^2\right]^{3/2}} \ . \tag{30}$$

Note that a vanishing α implies that $a(x)$ has no effect on the trajectory. This is good.

Solving the displacement equation (30)

Considering small α:

$$y = y^{(0)} + \alpha y^{(1)} + \alpha^2 y^{(2)} + \dots \;, \tag{31}$$

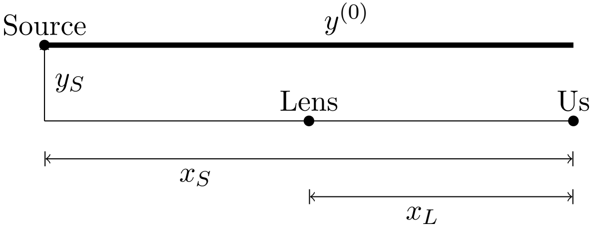
and initial conditions:

$$y(x_S) = y_S \;, \qquad y(0) = 0 \;, \tag{32}$$

the zero-order solution is a straight line:

$$y^{(0)} = C_1 x + C_2 \;. \tag{33}$$

Zeroth-order solution



We choose the two integration constants so that $y^{(0)} = y_S$, i.e. the trajectory is a straight, horizontal line.

First-order equation

The first-order equation is the following:

$$\frac{d^2y^{(1)}}{dx^2} = -\frac{y_S}{a(x)\left[(x-1)^2 + y_S^2\right]^{3/2}} \,, \tag{34}$$

for which we must choose the following initial conditions:

$$y^{(1)}(x_S) = 0 \,, \qquad y^{(1)}(0) = -y_S/\alpha \,. \tag{35}$$

For a constant Hubble factor $H = H_0$:

$$\chi = \int_0^z \frac{dz'}{H(z')} = \frac{z}{H_0} \equiv \frac{1}{H_0}\left(\frac{1}{a} - 1\right) \,, \tag{36}$$

so that:

$$\frac{d^2y^{(1)}}{dx^2} = -\frac{y_S\left(1 + H_0\chi_L x\right)}{\left[(x-1)^2 + y_S^2\right]^{3/2}} \,. \tag{37}$$

The bending angle

In the limit $y_S \ll 1$ the deviation angle

$$\delta \equiv \left.\frac{dy}{dx}\right|_{x=0} - \left.\frac{dy}{dx}\right|_{x=x_S} , \tag{38}$$

is the following:

$$\delta = \frac{2\alpha(1+\chi_L H_0)}{y_S} + \mathcal{O}(y_S) = \frac{4M(1+\chi_L H_0)}{b} + \mathcal{O}(b/\chi_L) \; . \tag{39}$$

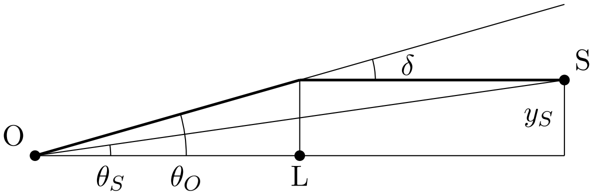
The mass seems to have been increased by a relative amount of $H_0\chi_L = z_L$, the redshift of the lens.

The lens equation

The lens equation (in the thin-lens approximation):

$$\theta_O D_S = \theta_S D_S + \delta D_{LS} \ , \tag{40}$$

with D's angular-diameter distances.



It becomes:

$$\theta_O - \theta_S = \frac{4M(1+z_L)}{b} \frac{D_{LS}}{D_S} \ . \tag{41}$$

The lens equation

Since b is a comoving transversal distance:

$$b = \chi_L \theta_O \ . \tag{42}$$

Moreover,

$$D_L = \frac{\chi_L}{1 + z_L} \ . \tag{43}$$

Therefore:

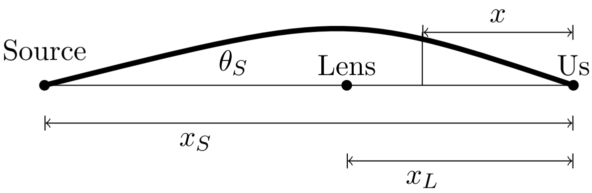
$$b = (1 + z_L) D_L \theta_O \ , \tag{44}$$

and in the lens equation the redshift factor cancels out, leaving us with:

$$\theta_O(\theta_O - \theta_S) = \frac{4M}{D_L} \frac{D_{LS}}{D_S} \ . \tag{45}$$

i.e. there is no difference from the standard case (with no cosmological expansion).

Einstein's ring systems



The zeroth-order trajectory is now:

$$y^{(0)} = \theta_S(x_S - x) \ , \tag{46}$$

where $\theta_S \ll 1$. In order for the trajectory to reach us, we must choose the initial condition $y^{(1)}(0) = -\theta_S x_S/\alpha$.

Einstein's radius

Computing again the deflection angle, we get:

$$\delta = \frac{4M(1+\chi_L H_0)}{\theta_S(\chi_S-\chi_L)} + \frac{\chi_L}{2(\chi_S-\chi_L)} + \mathcal{O}(\theta_S)\ . \tag{47}$$

Introducing the angular diameter distance one has:

$$\theta_S(\chi_S-\chi_L) = \theta_E\chi_L = \theta_E D_L(1+z_L)\ . \tag{48}$$

The redshift factor cancels out again, giving once more the usual formula for the lens equation and for the Einstein's radius.

Calculation for $H = H(t)$

OFP, Universe, 4, 25 (2016)

Sorry, but for some reason I changed notation from one paper to the other. So:

$$\chi \to x \,, \qquad x \to X \,, \qquad y \to Y \,. \tag{49}$$

Hence Eq. (30) becomes:

$$\frac{d^2Y}{dX^2} = -\alpha \frac{Y}{a(X)\left[(X-1)^2 + Y^2\right]^{3/2}} \,, \tag{50}$$

where $Y \equiv y/x_L$, $X \equiv x/x_L$ and $\alpha \equiv 2M/x_L$.

Calculation for $H = H(t)$

OFP, Universe, 4, 25 (2016)

Since $dY/dX = \tan\theta$ and $\tan\theta \sim \theta$, we can cast the above equation in the following form:

$$\frac{d\theta}{dX} = -\alpha \frac{Y}{a(X)\left[(X-1)^2 + Y^2\right]^{3/2}} \,. \tag{51}$$

We *define* the bending angle as follows:

$$\delta \equiv \int_{\theta_S}^{\theta_O} d\theta = -\alpha \int_{X_S}^{0} \frac{Y(X)dX}{a(X)\left[(X-1)^2 + Y(X)^2\right]^{3/2}} \,, \tag{52}$$

i.e. as the variation of the slope of the trajectory between the source and the observer.

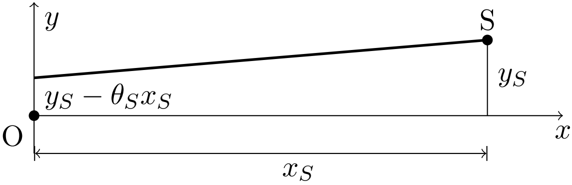
Calculation for $H = H(t)$

OFP, Universe, 4, 25 (2016)

The zeroth-order solution (i.e. the one for $\alpha = 0$) of Eq. (50) is a straight line in comoving coordinates:

$$y = \theta_S(x - x_S) + y_S \ , \tag{53}$$

where $\theta_S \ll 1$ is the slope of the trajectory and (x_S, y_S) are the comoving coordinates of the source.



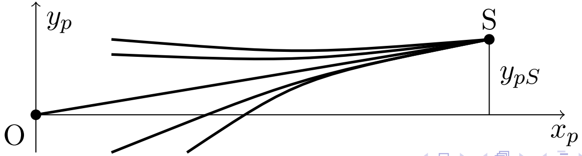
Why cosmology should affect the bending of light?

OFP, Universe, 4, 25 (2016)

For proper distances (multiply by the scale factor $a(x)$) we obtain:

$$y_p = \theta_S x_p + \frac{a(x_p)}{a_S}(y_{pS} - \theta_S x_{pS}) \,, \tag{54}$$

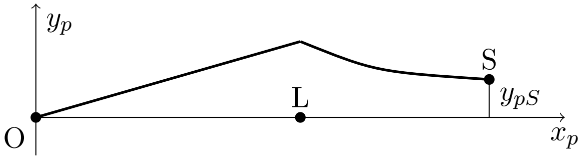
The above are not straight lines [also noticed in F. Simpson, J. A. Peacock, A. F. Heavens, MNRAS 402 (2010)]. They are bent because of the $a(x_p)$ factor on the right hand side, whose effect vanishes only for $y_{pS} = \theta_S x_{pS}$. The latter condition, when substituted in Eq. (53), represents the ray which gets to $y = 0$ when $x = 0$, i.e. the observed ray.



Why cosmology should affect the bending of light?

OFP, Universe, 4, 25 (2016)

The Hubble flow seems to bend away the trajectories such that we cannot detect any light. Let us speculate. If a cosmologically bent ray passes sufficiently close to a lens, then its trajectory could be bent back by the gravitational field of the lens, allowing us to detect it.



This "back-bending" seems to suggest that cosmology must somehow enter the gravitational lensing phenomenon.

Calculation for $H = H(t)$

OFP, Universe, 4, 25 (2016)

Now, write Eq.

$$d\theta = -\frac{\alpha dX}{a(X) Y_S^2 \left[1 + \frac{(X-1)^2}{Y_S^2}\right]^{3/2}} . \tag{55}$$

When $(X-1)^2 \gg Y_S^2$ the above integration, whatever function a might be of X, is $\mathcal{O}(Y_S)$. On the other hand, when $(X-1)^2 \ll Y_S^2$ the above integration is $\mathcal{O}(1/Y_S^2)$. Therefore, the main contribution comes from $X = 1$ and spans the interval $1 - Y_S < X < 1 + Y_S$. That is, most of the deflection takes place very close to the lens, as it happens for the case of the Schwarzschild metric. For this reason, we also approximate $a(X)$ with a_L, which is the scale factor when $x = x_L$.

Calculation for $H = H(t)$

OFP, Universe, 4, 25 (2016)

Therefore, we end up with the following bending angle:

$$\delta = -\frac{\alpha}{a_L Y_S^2}(-2Y_S) = \frac{2\alpha}{a_L Y_S} = \frac{4M}{a_L y_S} \ . \tag{56}$$

The above formula is general, valid for any Hubble factor. We recover the usual well-known formula:

$$\theta_O(\theta_O - \theta_S) = \frac{4M}{D_L}\frac{D_{LS}}{D_S} \ . \tag{57}$$

Therefore, we can conclude that cosmology does not modify the bending angle at the leading order of the expansion in powers of μ and θ. The cosmological "drift" discussed earlier for the zeroth-order solution is already taken into account when using angular-diameter distances so that the final result does not change.

Computing the next-to-leading term

OFP, Universe, 4, 25 (2016)

When $H = H_0 =$ constant, one can find an analytic expression for $a(x)$:

$$x = \int_a^1 \frac{da'}{H_0 a^2} = \frac{1}{H_0}\left(\frac{1}{a} - 1\right) = \frac{z}{H_0}\,, \tag{58}$$

where in the last equality we introduced the redshift. The scale factor as function of the comoving distance is thus:

$$\frac{1}{a(x)} = H_0 x + 1 = H_0 X x_L + 1 = z_L X + 1\,, \tag{59}$$

and the displacement equation becomes:

$$\frac{d\theta}{dX} = -\frac{\alpha Y_S (z_L X + 1)}{\left[(X-1)^2 + Y_S^2\right]^{3/2}}\,. \tag{60}$$

Computing the next-to-leading term

OFP, Universe, 4, 25 (2016)

This equation can be solved exactly and the bending angle is the following:

$$\delta = \frac{\alpha}{Y_S}\left[\frac{1+z_L+z_L Y_S^2}{\sqrt{1+Y_S^2}} + \frac{(X_S-1)(1+z_L)-z_L Y_S^2}{\sqrt{(X_S-1)^2+Y_S^2}}\right] . \quad (61)$$

Expanding this solution for a small impact parameter Y_S one gets:

$$\delta = \frac{2\alpha(1+z_L)}{Y_S}\left[1+Y_S^2\frac{2(z_L-1)+X_S[2+X_S(z_L-1)-4z_L]}{4(z_L+1)(X_S-1)^2}\right] ,(62)$$

where we have already truncated $\mathcal{O}(Y_S^4)$ terms and put in evidence the leading order contribution $2\alpha(1+z_L)/Y_S$.

Computing the next-to-leading term

OFP, Universe, 4, 25 (2016)

For the lens equation we have:

$$\theta_O(\theta_O - \theta_S) = \frac{4MD_{LS}}{D_L D_S}\left[1 + \theta_O^2 \frac{2z_L^2(z_L - 1) + z_S[2z_L + z_S(z_L - 1) - 4z_L^2]}{4(z_L + 1)(z_S - z_L)^2}\right] . \qquad (63)$$

Let's focus on Einstein ring systems, i.e. $\theta_S = 0$. We have in this case the mass estimate:

$$\frac{4MD_{LS}}{D_L D_S} = \theta_O^2\left[1 - \theta_O^2 \frac{2z_L^2(z_L - 1) + z_S[2z_L + z_S(z_L - 1) - 4z_L^2]}{4(z_L + 1)(z_S - z_L)^2}\right] . \qquad (64)$$

The next-to-leading order correction is $\mathcal{O}(\theta_O^4)$ and depends on the redshifts of the lens and of the source.

Computing the next-to-leading term

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Consider for example the Einstein ring Q0047-2808 of the CASTLES survey, for which $\theta_O = 2.7''$, $z_S = 3.60$ and $z_L = 0.48$. Substituting these numbers in Eq. (64), the correction on the mass estimate is therefore

$$\frac{4MD_{LS}}{\theta_O^2 D_L D_S} = 1 + 0.12\,\theta_O^2 = 1 + 2.03 \cdot 10^{-11}\,. \tag{65}$$

This is an extremely small correction which nonetheless depends on cosmology. Note that it is only one order of magnitude larger than the terms $\mathcal{O}(\mu^2)$ that we have neglected in our calculations.

Summary and conclusions

The debate on whether the cosmological constant, and by extension cosmology, affects the bending of light seems to be settled.

1. At the leading order (in the ratio m/R, in Λr^2 and in general in all the relevant dimensionless quantities) there is no effect of the cosmological expansion on the bending of light.
2. Cosmology however does affect the bending of light, since the redshifts of lens and source do enter in next-to-leading order contributions.

Thank you!

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