# The Cosmic Microwave Background radiation and the Integrated Sachs-Wolfe effect

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# Outline

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- G. Lemaître, *A homogeneous Universe of constant mass and growing radius accounting for the radial velocity of extragalactic nebulae*, Annales Soc. Sci. Brux. Ser. I Sci. Math. Astron. Phys. A **47** (1927) 49;
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- R. K. Sachs and A. M. Wolfe, *Perturbations of a cosmological model and angular variations of the microwave background*, Astrophys. J. **147** (1967) 73 [Gen. Rel. Grav. **39** (2007) 1929];
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- D. Bertacca and N. Bartolo, *ISW effect in Unified Dark Matter Scalar Field Cosmologies: An analytical approach*, JCAP **0711** (2007) 026 [arXiv:0707.4247 [astro-ph]];

# Reference web sites

- LAMBDA:
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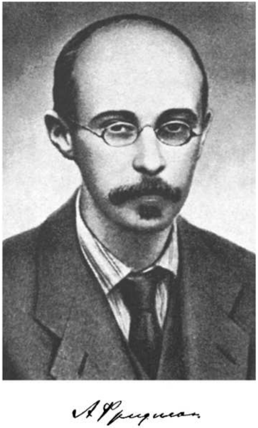
- Planck:
  http://www.esa.int/SPECIALS/Planck/index.html;
- Wayne Hu's home page:
  http://background.uchicago.edu/~whu/;

# Cosmology



# The Standard FLRW cosmology

## The FLRW metric

*Friedmann, 1922, Lemaître 1927, Robertson 1929, Walker 1935*

Cosmological principle: spatial isotropy and homogeneity

$$ds^2 = dt^2 - a(t)^2\gamma_{ij}dx^i dx^j = a(\eta)^2\left(d\eta^2 - \gamma_{ij}dx^i dx^j\right) \ ,$$

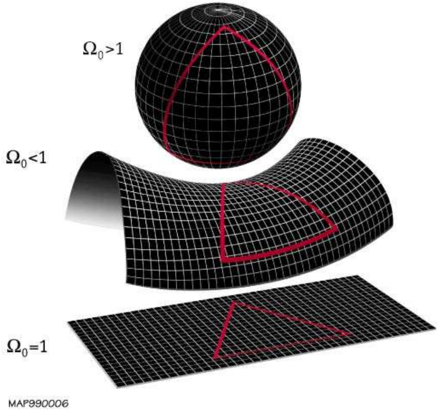
a(t) scale factor,

$$\gamma_{ij} = \delta_{ij}\frac{1}{\left(1+\frac{1}{4}Kr^2\right)^2} \ ,$$

metric of the spatial hyper-surfaces (constant time $t$).

Closed, flat or open ($K = 1, 0, -1$).

# The spatial FLRW metric



# The evolution equations

Einstein's equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ with FLRW metric:

Friedmann equation

$$H^2 = \frac{8\pi G}{3} T^0{}_0 - \frac{K}{a^2} \;,$$

Raychaudhuri equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} T \;,$$

$H = \dot{a}/a$ Hubble parameter, $T^\mu{}_\nu$ energy momentum tensor.

Hubble constant: $H(t = t_0) = H_0$.
$H_0 \approx 70$ (km/s)/Mpc $\Rightarrow 1/H_0 \approx 13$ Gyr and $c/H_0 \approx 4000$ Mpc.

# Energy content of the universe

Which energy-momentum tensor? A possibility is the perfect fluid one:

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p g_{\mu\nu} \ ,$$

where $g_{\mu\nu}$ is the FLRW metric.
Compatibly with the cosmological principle: $\rho = \rho(t)$ and $p = p(t)$.
Friedmann and Raychaudhuri equations become

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \ , \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \ .$$

When $\rho + 3p < 0$, i.e. $p < -\rho/3$, the expansion of the universe accelerates $\rightarrow$ Dark energy.

# Some fluid solutions

Energy conservation equation $\nabla_\nu T^{0\nu} = 0$ :

$$\dot{\rho} = -3\frac{\dot{a}}{a}\rho\,(1 + w) \ .$$

For $w$ constant:

$$\rho = \rho_0 a^{-3(1+w)} \ .$$

| $w \equiv p/\rho$ | Fluid type | $a(t)$ |
|---|---|---|
| 0 | Dust (non-relativistic matter) | $\propto t^{2/3}$ |
| 1/3 | Radiation (relativistic matter) | $\propto t^{1/2}$ |
| -1 | Cosmological constant | $\propto e^{H_0 t}$ |

Dust: Baryons (standard model particles), Cold dark matter, ...
Radiation: Photons, Neutrinos, Warm dark matter, ...

# The dark universe

Observation indicates the existence of two unknown components making up the 96% of the universe energy content:

Dark energy $\rightarrow$ Accelerated expansion, ...
Dark matter $\rightarrow$ Structure formation, Gravitational lensing phenomena, Galaxy rotation curves, ...

Dark energy candidates: cosmological constant, quintessence, quartessence, *k*-essence, geometrical effects, modified theories of gravity, ...
Dark matter candidates: axions, neutralinos, quartessence, geometrical effects, modified theories of gravity, ...

# Cosmological perturbation theory

*Lifshitz, 1946*

$\delta g_{\mu\nu}$ small perturbation of FLRW metric $g_{\mu\nu}$:

$$ds^2 = (g_{\mu\nu} + \delta g_{\mu\nu})\, dx^\mu dx^\nu \ .$$

Perturbed Einstein's equations:

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} \ ,$$

computed for the perturbed metric $\delta g_{\mu\nu}$.

Issues: Choice of a reference frame, Fictitious perturbations, ...
Techniques: Scalar-Vector-Tensor decomposition, Fourier transformation, Gauge-invariant variables, ...

# Evolution of perturbations and Jeans theory

$\delta \equiv \delta\rho/\rho$, density contrast. Newtonian approximation:

$$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\Delta\delta - 4\pi G\rho\delta = 0 \; .$$

Fourier transform:

$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} + \left(\frac{k^2 c_s^2}{a^2} - 4\pi G\rho\right)\delta_{\mathbf{k}} = 0 \; .$$

Challenge between pressure and gravity, Jeans length:

$$\lambda_J = \frac{2\pi a}{k_J} = c_s\sqrt{\frac{\pi}{G\rho_0}} \; ,$$

$\lambda > \lambda_J$, collapse. $\lambda < \lambda_J$, oscillations.

Note the importance of the speed of sound $c_s$.

# Matter and radiation perturbations

For matter, $c_{\rm s} = 0$, therefore $\lambda_{\rm J} = 0 \to$ collapse at all scales.
For radiation, $c_{\rm s} = 1/\sqrt{3}$, therefore $\lambda_{\rm J} \neq 0 \to$ oscillations on small scales.

For matter, $a \propto t^{2/3}$, so $H = 2/(3t)$ and

$$\delta_{\mathbf{k}} = C_1\ t^{-1} + C_2\ t^{2/3} \approx C_2\ t^{2/3} \propto a\ .$$

For radiation or any other fluid with a non-vanishing slowly varying speed of sound:

$$\delta_{\mathbf{k}} = \frac{1}{\sqrt{c_{\rm s} a}} \exp\left(\pm ik \int \frac{c_{\rm s} dt}{a}\right)\ ,$$

for scales $\lambda \ll \lambda_{\rm J}$.

# The standard cosmological model: ΛCDM

Define the density parameter as

$$\Omega_X = \frac{\rho_X}{\rho_{\rm cr0}} \ ,$$

where the present critical density is

$$\rho_{cr0} = \frac{3H_0^2}{8\pi G} = 1.9h^2 \times 10^{-29} \text{g cm}^{-3} \ .$$

$\approx$ 10 protons per $m^3$ or $\approx$ 100 solar masses per $kpc^3$.

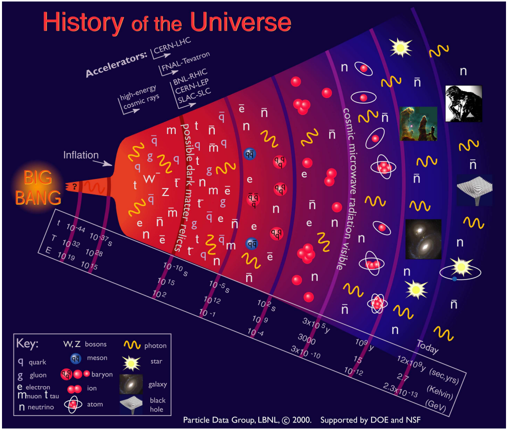
$$\frac{H^2}{H_0^2} = \Omega_\Lambda + \frac{\Omega_{\rm m}}{a^3} + \frac{\Omega_{\rm r}}{a^4} + \frac{\Omega_K}{a^2} \ .$$

Note $1 = \Omega_\Lambda + \Omega_{\rm m} + \Omega_{\rm r} + \Omega_K \approx 0.73 + 0.27 + 10^{-4} + 10^{-2}$ .

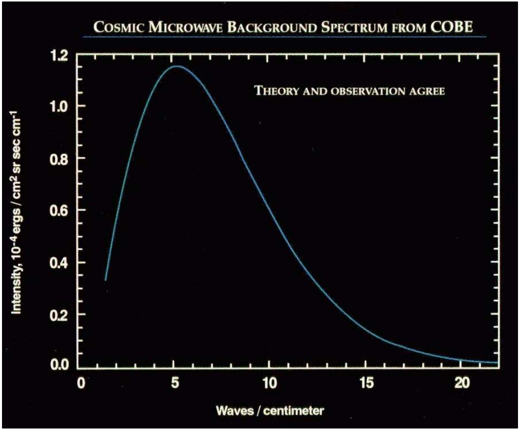
# The Cosmic Background radiation

- Penzias and Wilson, 1964;
- Isotropic;
- Black-body spectrum with temperature $T \approx 2.726$ K;
- Red-shifted photons that have (almost) freely streamed from an epoch when the universe became transparent for the first time to radiation (decoupling);
- Support the Hot Big Bang theory;
- There are anisotropies in the temperature. the seeds of the cosmic structures;
- Hundreds of experiments concerning CMB. Satellites: COBE, WMAP, Planck;

# History of the universe



# The COBE FIRAS spectrum



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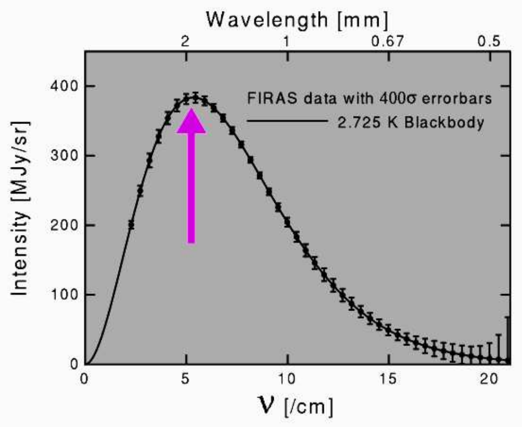
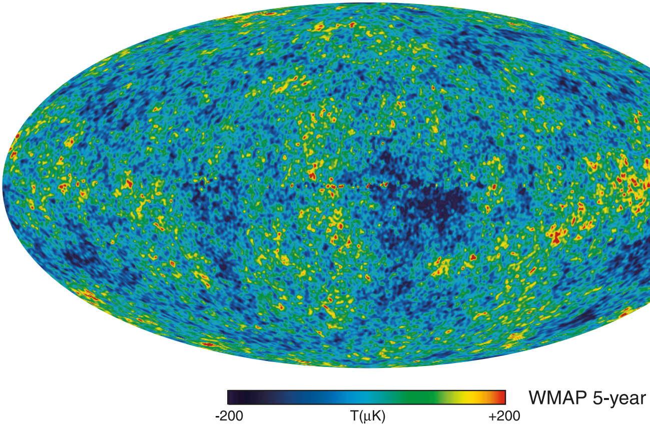
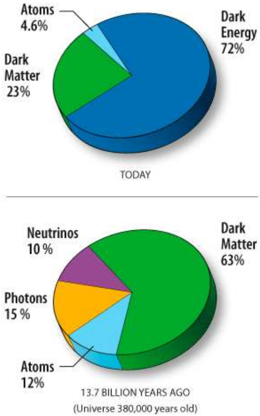



-200
T(μK)
+200
WMAP 5-year

# The WMAP5 results



# CMB Physics

## Simplistic scenario

- Before $z_* \simeq 1000$ (recombination) the universe was hot enough to ionise hydrogen and dense enough to tightly link electrons to photons via Compton scattering, i.e. the Hubble radius was much larger than the mean free path for Compton scattering;
- In their turn, electrons were linked to protons via Coulomb interaction, so baryons and radiation were dynamically tightened in a single photon-baryon fluid (tight coupling approximation, baryon-photon fluid);
- This mixture was also gravitationally influenced by the presence of dark matter, which was not coupled, by its very definition, to photons;

# Physics in the simplistic scenario

- Physics of the baryon-photon mixture at the decoupling era, when the two components untie. We can naively look at the decoupling as a snapshot of the entire universe at $z_*$, a portrait which propagates to us.
- CMB photons propagating to us are not immune from other interactions. In particular, they are subject to the potential wells formed by dark matter and the structures within.

# The Angular power spectrum

Photon and temperature perturbations

$$\rho_\gamma \propto T^4 \Rightarrow \delta\rho_\gamma/\rho_\gamma = 4\delta T/T \equiv 4\Theta \ .$$

Expand the relative temperature fluctuations in spherical harmonics

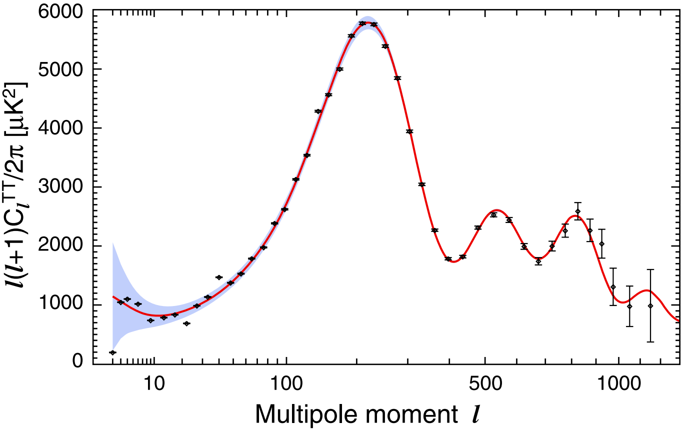
$$\Theta(\theta,\phi) = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta,\phi) \ .$$

For a Gaussian distribution $\langle \bar{a}_{lm} a_{l'm'} \rangle = \delta_{ll'}\delta_{mm'}C_l$ the correlation function

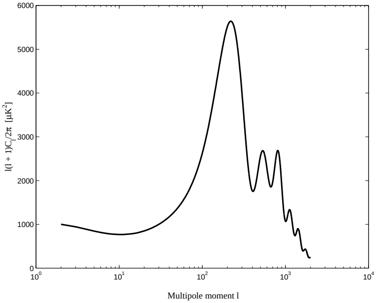
$$\langle \Theta(\mathbf{n})\Theta(\mathbf{n'}) \rangle = \frac{1}{4\pi}\sum_{l=2}^{\infty}(2l+1)C_l\mathcal{P}_l(\cos\theta) \ .$$

$\mathcal{P}_l$ Legendre polynomials.

# The CMB angular power spectrum



# The CMB angular power spectrum




Features:

- Growth for small $l$: late times ISW effect
- Peaks: Acoustic oscillations
- Different peaks height: Baryon Drag
- Global fall-off: Silk Damping

# Physics of the features

- Dark energy drives the acceleration of the universe $\rightarrow$ Late times ISW effect;
- The baryon-photon fluid has non-zero speed of sound. Thus, on small scales, its homogeneities oscillate $\rightarrow$ Acoustic oscillations;
- Baryons tend to agglomerate and their mass thwarts the oscillations in the baryon-photon fluid $\rightarrow$ Different peaks height;
- The tight-coupling approximation fails on small scales $\rightarrow$ Global fall-off;

# Multipole decomposition

Consider the multipole decomposition

$$\Theta(\theta, \eta, k) = \sum_{l=0}^{\infty} (-1)^l \Theta_l(\eta, k) P_l(\cos\theta) \; .$$

Relation between $C_l$ and $\Theta_l$

$$\frac{2l+1}{4\pi} C_l = \frac{1}{2\pi^2} \int_0^{\infty} \frac{dk}{k} k^3 \frac{|\Theta_l(\eta, k)|^2}{2l+1} \; .$$

# General equations I: Collisional brightness equation

Boltzmann equation with a collisional part given by Thomson scattering (neglecting polarization)

$$\dot{\Theta} + ik\mu\left(\Theta + \Psi\right) = -\dot{\Phi} + \dot{\tau}\left[\Theta_0 - \Theta - \frac{1}{10}\Theta_2\mathcal{P}_2(\mu) - i\mu V_b\right] ,$$

with

- $\mu = \cos\theta$;
- $\dot{\tau} = x_e n_e \sigma_T a$ differential optical depth to Thomson scattering;
- $P_2$ quadrupole moment of energy distribution;
- $V_b$ baryons' velocity;
- $\Psi$ and $\Phi$ gravitational potentials.

# General equations II: Continuity and Euler equations

Equations for baryons in the total matter rest frame.
Continuity equation:

$$\dot{\Delta}_b = -k(V_b - \Theta_1) + \frac{3}{4}\dot{\Delta}_\gamma \ ,$$

and Euler equation

$$\dot{V}_b = -\frac{\dot{a}}{a}V_b + k\Psi + \dot{\tau}(\Theta_1 - V_b)/R \ ,$$

where

$$R = 3\rho_b/4\rho_\gamma = 3.0 \cdot 10^4(1+z)^{-1}\Omega_b h^2 \ .$$

$\Omega_b h^2 \approx 0.01 - 0.02$.

# Tight coupling limit: baryon-photon fluid

$\dot{\tau} \gg 1 \Rightarrow V_b = \Theta_1$ and $\Theta_l = 0$ for $l \geq 2$.
Fluctuations are then described by

$$\ddot{\Theta}_0 + \frac{\dot{a}}{a}\frac{R}{1+R}\dot{\Theta}_0 + k^2 c_s^2 \Theta_0 = F(\eta) \ ,$$

where

$$F(\eta) = -\ddot{\Phi} - \frac{\dot{a}}{a}\frac{R}{1+R}\dot{\Phi} - \frac{k^2}{3}\Psi$$

and $c_s^2 = 1/3(1+R)$ .
Suppose $R \ll 1$ and a constant driving force $F = -k^2\Psi/3$

$$\ddot{\Theta}_0 + \frac{k^2}{3}\left(\Theta_0 + \Psi\right) = 0 \ .$$

# Acoustic oscillations

With adiabatic initial conditions, $\Theta_0 \neq 0$ and $\dot{\Theta}_0 = 0$, the solution of the equation is

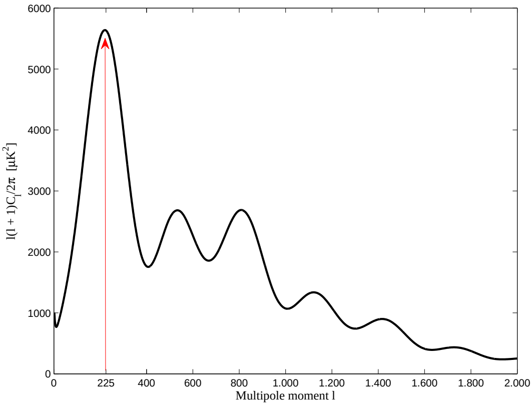
$$\Theta_0 + \Psi = \frac{\Psi}{3}\cos\left(\frac{k}{\sqrt{3}}\eta\right) \ .$$

The effective temperature is $\Theta_0 + \Psi$ (Sachs-Wolfe effect) and it oscillates between $-\Psi/3$ and $\Psi/3$.
The peaks are located at

$$k_n = \frac{n\pi\sqrt{3}}{\eta_*} \ .$$

For a flat Universe, the first peak is located at $l = 225$.

# Acoustic oscillations



# Baryon drag I

The baryons "drag" the baryon-photon fluid into the potential wells, making them deeper

Suppose $R$ constant but not negligible

$$\ddot{\Theta}_0 + \frac{k^2}{3}\left(\frac{\Theta_0}{1+R} + \Psi\right) = 0$$

The solution becomes

$$\Theta_0 + \Psi = \frac{1}{3}\Psi(1+3R)\cos\left(\frac{k}{\sqrt{3(1+R)}}\eta\right) - R\Psi$$

The bottom of the well is now at $-\frac{\Psi}{3}(1+6R)$ and therefore peaks and throats are no more symmetric

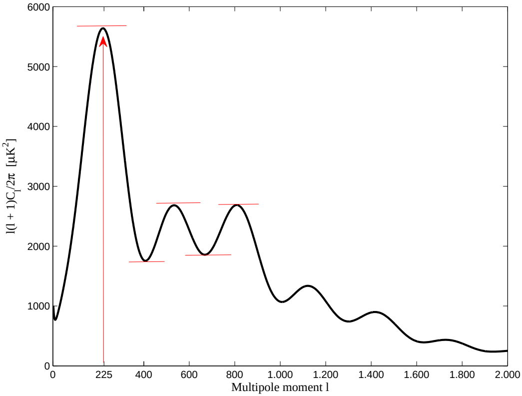
# The baryon drag II

If we do not neglect the damping term

$$\ddot{\Theta}_0 + \frac{\dot{a}}{a}\frac{R}{1+R}\dot{\Theta}_0 + \frac{k^2}{3}\left(\frac{\Theta_0}{1+R} + \Psi\right) = 0$$

The amplitude of the oscillations decays as $(1+R)^{-1/4}$

# The baryon drag effect



# Diffusion Damping

*Silk, 1968*

Tight coupling is not prefect: photons have a non zero mean free path $\lambda_D$ in the baryon-photon fluid $\Rightarrow$ hot photons and cold photons are mixed for $\lambda < \lambda_D$ $\Rightarrow$ suppression of the correlation

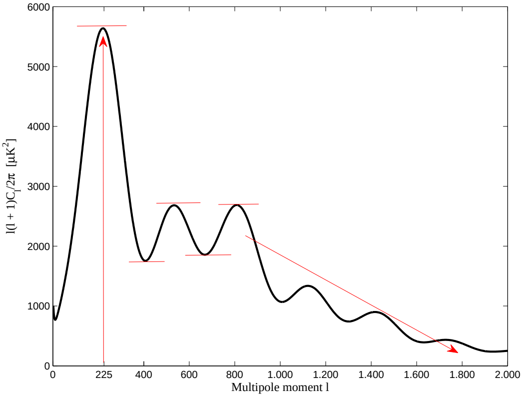
$$(\Theta_0 + \Psi) \to (\Theta_0 + \Psi)e^{-(k/k_D)^2}$$

The diffusion scale is given by

$$k_D^{-1}(\eta) = \frac{1}{6}\int_0^\eta d\eta \frac{1}{\dot{\tau}} \frac{R^2 + 4(1+R)/5}{(1+R)^2}$$

Diffusion scale (Silk scale) $\lambda_D \lesssim 3\ Mpc \Rightarrow l \gtrsim 800$

# The Silk Damping



# The ISW effect

*Sachs and Wolfe, 1967*

The gravitational potentials occurring in the driving force

$$F(\eta) = -\ddot{\Phi} - \frac{\dot{a}}{a}\frac{R}{1+R}\dot{\Phi} - \frac{k^2}{3}\Psi$$

are constant only in the matter dominated era $\Rightarrow$ the potential wells which photons pass through have a time-independent shape $\Rightarrow$ no Integrated Sachs-Wolfe effect $\Rightarrow$ Sachs-Wolfe plateau

Another form of energy dominates $\Rightarrow$ the potential wells change their shape $\Rightarrow$ one has to integrate along the photon pattern $\Rightarrow$ Integrated Sachs-Wolfe effect

- Radiation $\rightarrow$ Early times ISW effect
- Dark energy $\rightarrow$ Late times ISW effect

# The early times and late times ISW effect

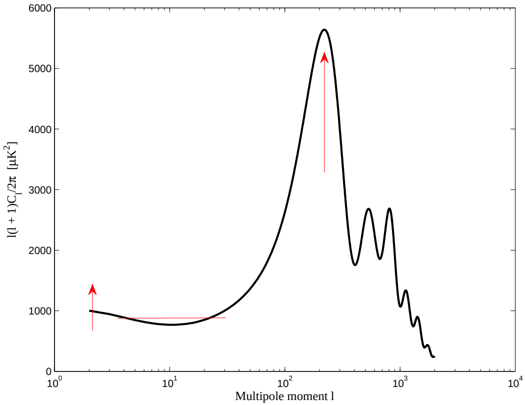
Early times ISW effect

The potential decays on large scales (small $l$'s) $\Rightarrow$ photons experience a kick $\Rightarrow$ height of the peak in the angular power spectrum

Late times ISW effect

Again the potential decays. The wells are swept out by the acceleration, but this happens on larger scales $\Rightarrow$ growth of the angular power spectrum for very small $l$'s

# The ISW effect



# The ISW effect

## An analytic approach

*Bertacca and Bartolo, 2007*

One fluid model with adiabatic perturbations $\Rightarrow \Phi = -\Psi$

Evolution equation for the potential

$$u'' - c_s^2 \Delta u - \frac{\theta''}{\theta} u = 0$$

where $u \equiv \frac{\Phi}{\sqrt{\rho+p}}$ and $\theta = \frac{1}{a\sqrt{1+w}}$ The ISW effect contribution to the angular power spectrum

$$\frac{2l+1}{4\pi} C_l = \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} k^3 \frac{|\Theta_l(\eta_0, k)|^2}{2l+1}$$

with

$$\frac{\Theta_l(\eta_0, k)}{2l+1} = 2 \int_{\eta_*}^{\eta_0} \Phi'(\tau, k) j_l[k(\eta_0 - \tau)] d\tau$$

# The effect of the speed of sound on the $C_l$'s

The sound velocity plays a fundamental role:

$$C_l \propto 4c_s^4(\eta_0)(l+1/2)\int_{\frac{l+1/2}{\eta_0-\eta_{1/3}}}^{\infty}\frac{dk}{k}k^{n_S-1}\cos^2(D_0 k)\;.$$

It makes the angular power spectrum to grow as $l^3$ until $l \approx 25$ and therefore sensibly reduces the peak-to-plateau ratio.

$l \approx 25$ comes from $l \approx k_{eq}(\eta_0 - \eta_{1/3})$ and $1/k_{eq}$ Hubble scale at equivalence. Meszaros effect.

$D_0 = \int_{\eta_{1/3}}^{\eta_0} c_s(\tau)d\tau$ sound horizon.

Strong constraint on cosmological models!! $c_s^2 k_{eq}^2 < |\theta''/\theta|$.

# The generalized Chaplygin gas

*Kamenshchik, Moschella and Pasquier, 2001*

A fluid with an equation of state

$$p = -A\rho^{-\alpha} \, ,$$

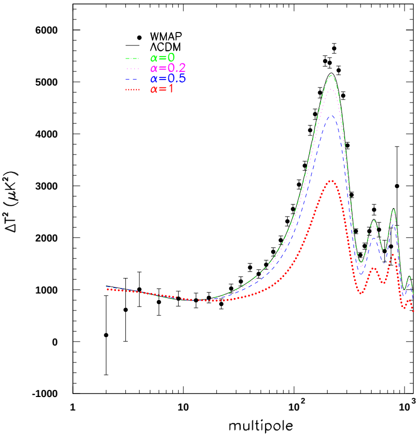
which interpolates the dust-dominated era and the cosmological-dominated era:

$$\rho = \left(A + \frac{B}{a^{3(\alpha+1)}}\right)^{\frac{1}{\alpha+1}} \, .$$

It contains all-in-one the properties of dark matter and dark energy.

- Mild constraints from Supernovae Ia;
- Severe constraints from large scale structure observation;
- Severe constraints from CMB;

# The generalized Chaplygin gas



# The ISW constraints on the generalized Chaplygin gas

The sound velocity in the gCg model is

$$c_s^2 = \frac{\alpha}{1 + \frac{B}{A} a^{-3(\alpha+1)}} .$$

Its value at present time is

$$c_s^2 = \bar{A}\alpha ,$$

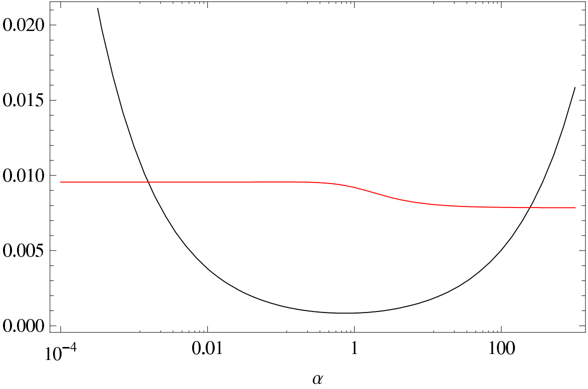
with $\bar{A} \approx 0.7$.

*OFP, 2009*

Asking for $c_s^2 k_{eq}^2 < |\theta''/\theta|$ we find the constraint $\alpha < 10^{-4}$.

However, giving up the conservative limit $\alpha < 1$, another constraint can be found: $\alpha > 350$.

# The ISW effect and the generalized Chaplygin gas




$k_{\rm J}$ (black curve) and $k_{\rm eq}$ (red "quasi-horizontal" line) as functions of $\alpha$. $k_{\rm J}$ is evaluated at $z = 0$ and $z_{\rm tr} = 0.79$, while $k_{\rm eq}$ is evaluated at $z_{\rm eq} = 3176$. The wavenumbers are in units $h$ Mpc$^{-1}$.

# Explicit calculation of the ISW effect for large $\alpha$

For the generalized Chaplygin gas model with large $\alpha$, one can find

$$\frac{l(l+1)C_l^{\mathrm{ISW}}}{2\pi D} = \frac{8l(l+1)}{(1+z_{\mathrm{tr}})^2}\int_0^\infty \frac{dk}{k}\left[\int_0^{z_{\mathrm{tr}}} dz\, \mathrm{j}_l(kz)\right]^2 .$$

After manipulations we obtain

$$\frac{l(l+1)C_l^{\mathrm{ISW}}}{2\pi D} = \frac{2\sqrt{\pi}\, z_{\mathrm{tr}}^2}{(1+z_{\mathrm{tr}})^2}\frac{\Gamma(l+1)}{\Gamma(l+3/2)}\, {}_3\mathrm{F}_2\left(l, -\frac{1}{2}, \frac{l+1}{2}; l+\frac{3}{2}, \frac{l+3}{2}; 1\right) ,$$

which has asymptotic behaviour

$$\frac{l(l+1)C_l^{\mathrm{ISW}}}{2\pi D} \sim \frac{2\pi z_{\mathrm{tr}}^2}{(1+z_{\mathrm{tr}})^2}\frac{1}{l} .$$

# Conclusions

- CMB observation is a fundamental tool for cosmology (we are in the precision era);
- CMB physics is very rich and encompasses microscopic to macroscopic scales;
- The Integrated Sachs-Wolfe effect is a fundamental tool for discriminating among dark energy models;

Research:

- Cross-correlation ISW-LSS;
- Rees-Sciama effect (second order ISW);
- Sunyaev-Zeldovich effect;
- Non-gaussianities in the CMB fluctuations;

Tools:

- CAMB (Code for Anisotropies in the Cosmic Background);
- CosmoMC

Grazie a tutti per l'attenzione!