Causal Bulk Viscous Cosmology

Bulk viscosity as Unified Dark Matter

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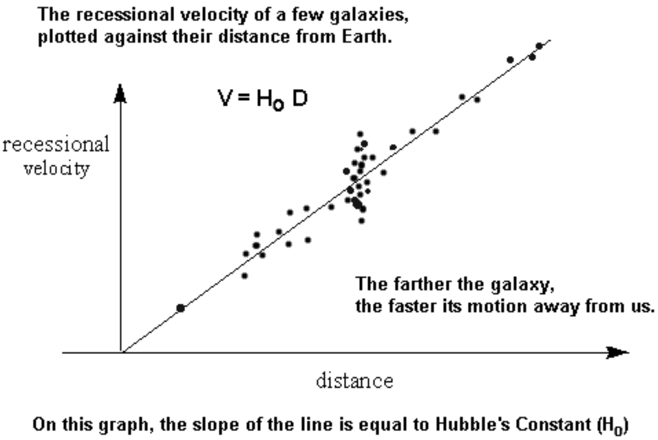
Summary

The Expanding Universe

Edwin Hubble – Proc. Nat. Acad. Sci. 15 (1929) 168-173



http://astrosun2.astro.cornell.edu/academics/courses//astro201/hubbles_law.htm

Fundamental Cosmological Probes

1. Cosmic Background Radiation

   A. A. Penzias and R. W. Wilson – Astrophys. J. 142 (1965) 419-421

   Lambda website: `http://lambda.gsfc.nasa.gov/`

2. Correlation in the relative position of large-scale structures

   M. Tegmark et al. – Phys. Rev. D69 (2004) 103501

   M. Tegmark's home page: `http://space.mit.edu/home/tegmark/`

3. Baryon Acoustic Oscillations

   D. Eisenstein et al. – Astrophys. J. 633 (2005) 560-574

   M. White's webpage: `http://astro.berkeley.edu/~mwhite/bao/`

4. Type Ia Supernovae

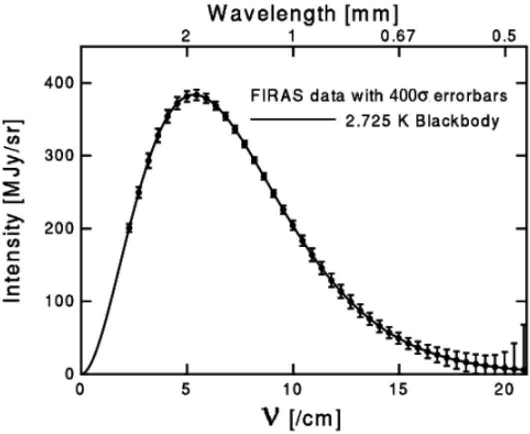
   A. G. Riess et al. – Astron. J. 116 (1998) 1009-1038

   S. Perlmutter et al. – Astrophys. J. 517 (1999) 565-586

   Supernova Cosmology Project: `http://www.supernova.lbl.gov/`

The Cosmic Background Radiation

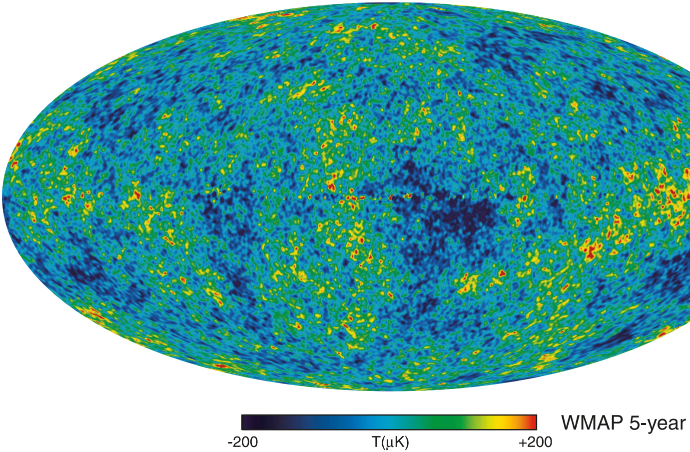
The black-body spectrum



http://lambda.gsfc.nasa.gov/product/cobe/firas_image.cfm

The Cosmic Background Radiation

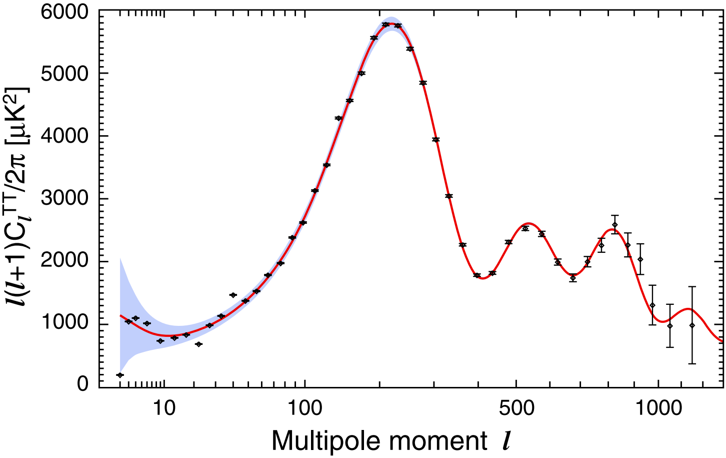
The temperature anisotropies



http://wmap.gsfc.nasa.gov/resources/featured_images_5yr_release.html

The Cosmic Background Radiation

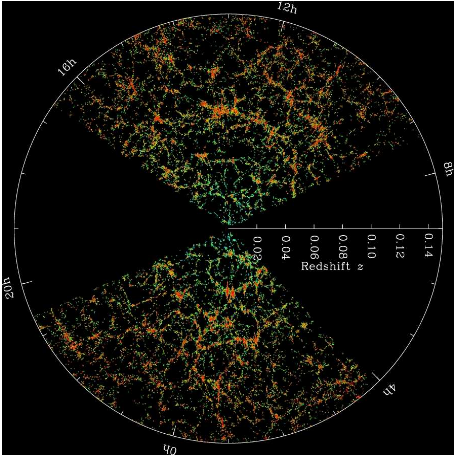
The angular power spectrum



D. Larson et al. – Fig. 1 – arXiv:1001.4635 [astro-ph.CO]

The Large-Scale Structures

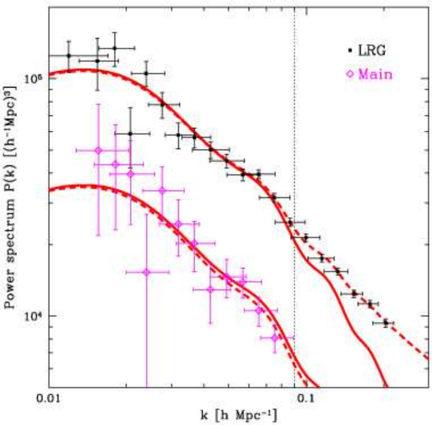
The Sloan Digital Sky Survey



http://www.sdss.org/

The Large-Scale Structures

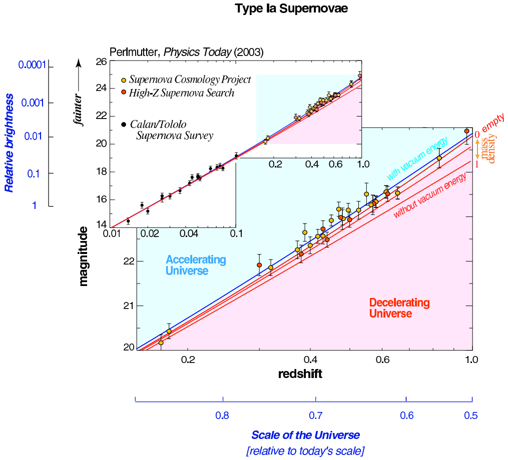
The matter power spectrum



http://space.mit.edu/home/tegmark/sdss.html

Type Ia Supernovae

The expansion of the Universe is accelerating



Relativistic Cosmology

Cosmological principle

- A. Friedmann – Z. Phys. A 10 (1922) 377386
- G. Lemaître – (1933) Gen. Rel. Grav. 29 (1997) 641-680
- H. P. Robertson – Rev. Mod. Phys. 5 (1933) 62-90
- A. G. Walker – Proc. Lon. Math. Soc. 2 42 (1937) 90127

Cosmological principle: The Universe is isotropic and homogeneous:

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2\right) \ ,$$

$$ds^2 = a^2(\eta)\left[d\eta^2 - \left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2\right)\right] \ ,$$

$a(t)$ is the scale factor, K is the spatial curvature.

The Stress-Energy Tensor

The physical meaning of its components

- L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*
- S. Weinberg, *Gravitation and Cosmology*
- B. F. Schutz, *A first course in General Relativity*
- V. Mukhanov, *Physical foundations of Cosmology*

Hydrodynamical description of the Universe matter content.
In the Momentarily-Comoving-Reference-Frame (MCRF):

1. $T^{00} = \rho$ is the energy density
2. $T^{0i} = T^{i0}$ is the energy flux (e.g. heat conduction)
3. $T^{ij} = T^{ji}$ represents the flux of i momentum across a j surface.

T^{ij} for $i \neq j$ describes forces parallel to the fluid interfaces, i.e. viscosity

The perfect fluid

Definition:
A perfect fluid possesses no heat conduction nor viscosity in the MCRF.
This implies $T^{i0} = 0$ and T^{ij} diagonal.

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p g_{\mu\nu} \ ,$$

where p is the total pressure and u_μ is the four-velocity of the fluid element.
Isotropy and homogeneity require:

1. The three eigenvalues of T^{ij} to be equal (p)
2. ρ and p to depend only on the time

Friedmann Equations

Description of the expansion rates of the Universe

Expansion rate:

$$\dot{a}^2 + K = \frac{8\pi G}{3}\rho a^2 \qquad \left(\dot{a} = \frac{da}{dt}\right) \ ,$$

Acceleration rate:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right) \ .$$

The Hubble parameter is $H := \dot{a}/a$. Or, in the conformal time, $\mathcal{H} := a'/a$ (with $a' = da/d\eta$).
Friedmann equations contain the energy conservation:

$$T^{\mu\nu}{}_{;\nu} = 0 \qquad \Rightarrow \qquad \dot{\rho} + 3H\left(\rho + p\right) = 0 \ .$$

Cosmological Perturbations

The gravitational potential Φ

Scalar perturbations the longitudinal gauge:

$$ds^2 = a(\eta)^2 \left[(1 + 2\Phi) - (1 - 2\Phi)\, \delta_{ij} dx^i dx^j \right] \ .$$

Perturbed Einstein equations (without shear):

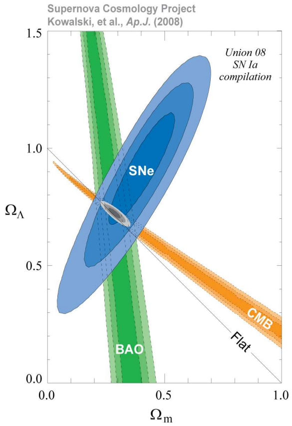
$$\Delta\Phi - 3\mathcal{H}\left(\Phi' + \mathcal{H}\Phi\right) = 4\pi G a^2 \delta\rho \ ,$$

$$\Delta\left(\Phi' + \mathcal{H}\Phi\right) + \left(\mathcal{H}^2 - \mathcal{H}'\right)\Theta = 0 \ ,$$

$$\Phi'' + 3\mathcal{H}\Phi' + \left(\mathcal{H}^2 + 2\mathcal{H}'\right)\Phi = 4\pi G a^2 \delta p \ .$$

General relation: $\delta p = c_s^2 \delta\rho + \tau\delta S$.
Adiabatic perturbations: $\delta S = 0$.

The ΛCDM Model



Basic equations ($\Omega := \rho/\rho_{\rm critical}$):

$$\frac{\mathcal{H}^2}{a^2 H_0^2} = \Omega_{\Lambda 0} + \frac{\Omega_{\rm m0}}{a^3} + \frac{\Omega_{\rm K0}}{a^2} \,.$$

$$\Phi'' + 3\mathcal{H}\Phi' + \left(\mathcal{H}^2 + 2\mathcal{H}'\right)\Phi = 0 \,.$$

Constraints coming from different probes:

- $\Omega_{\rm m0} = 0.285^{+0.020+0.010}_{-0.020-0.010}$
- $\Omega_{\rm K0} = -0.010^{+0.010+0.006}_{-0.011-0.004}$
- $w_0 = -1.001^{+0.069+0.080}_{-0.073-0.082}$

$\Omega_{\rm m0} \sim \Omega_{\Lambda 0}$: cosmic coincidence

Unification of Dark Matter and Dark Energy

Motivations

Problems:

- The nature of the cosmological constant (or Dark Energy, in general) and of Dark Matter
- The cosmic coincidence, or the cosmological constant problem.

  S. Weinberg – Rev. Mod. Phys. 61 (1989) 1-23

What if Dark Matter and Dark Energy were aspects of the same entity?

Motivation: we may gain new insight on Λ and shed new light on the cosmological scenario. Often, watching things from other points of view helps to better understand what is happening.

Unification via Bulk Viscosity

The real fluid

A real fluid contains non-equilibrium terms

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p g_{\mu\nu} + q_\mu u_\nu + q_\nu u_\mu + \pi_{\mu\nu} \ ,$$

viscosity, i.e. anisotropic stresses $\pi_{\mu\nu}$, and heat fluxes q_μ. They are not compatible with the cosmological principle, but the trace of $\pi_{\mu\nu}$, say Π, is
→ Bulk Viscous Cosmology

Murphy (1973), Belinsky, Nikomarov and Khalatnikov (1979), Pavon, Bafaluy and Jou (1991), Maartens (1995, 1996), Zimdahl (1996), Zimdahl and Fabris (2005),...

Now p plays the role of a total pressure

$$p = p_{\rm eq} + \Pi$$

sub-divided into the equilibrium contribution and the dissipative one.

Description of Bulk Viscosity

The thermodynamics of the irreversible process

C. Eckart – Phys. Rev. 58 (1940) 267-269, *ibid.* 919-924

In Eckart theory, the entropy S is linked to Π

$$Tn\dot{S} = -\theta\Pi \; ,$$

where the expansion factor is

$$\theta = \nabla_\mu u^\mu = 3H \; .$$

By virtue of the second principle of thermodynamics, Π has to be negative. The simplest choice in order to assure this is:

$$\Pi = -\theta\xi \; < 0 \; ,$$

where ξ is the bulk viscosity coefficient.

Negative pressure contribution → UDM via bulk viscosity

Unification Picture within Eckart Theory

B. Li and J. D. Barrow – Phys. Rev. D79 (2009) 103521

Assume $p_{\mathrm{eq}} = 0$ and $\Pi = -3\alpha H \rho^m$. The background expansion of the ΛCDM is nicely reproduced. But at the perturbative level: severe problems with the ISW effect.

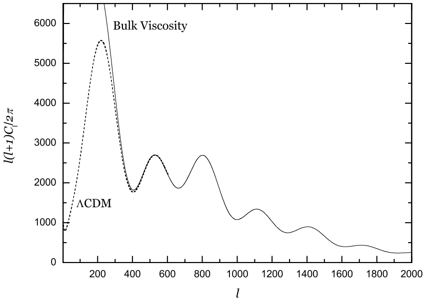


Fig. 5 – Here $m = -0.4$ and $\beta := \alpha H_0 \rho_0^{m-1} = 0.236$

Is the Problem within Eckart Theory?

W. Israel – Phys. Lett. A57 (1976) 107-110, Annals Phys. 100 (1976) 310-331

W. Israel and J. M. Stewart – Annals Phys. 118 (1979) 341-372

W. A. Hiscock and L. Lindblom – Annals Phys. 151 (1983) 466-496

Issues with Eckart theory:

1. Dissipative perturbations propagate at infinite speeds.
2. The equilibrium states in the theory are unstable.

Our question: would the results obtained by Li and Barrow change upon using a causal theory?

Israel-Stewart Transport Theory

W. Israel and J. M. Stewart – Annals Phys. 118 (1979) 341-372

Bulk viscosity evolution is governed by

$$\tau\dot{\Pi} + \Pi = -\theta\xi - \frac{1}{2}\tau\Pi\left[\theta + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right] \ ,$$

where τ is the relaxation time and T is the temperature. Note that the dot means

$$\dot{\Pi} := u^{\mu}\nabla_{\mu}\Pi \ ,$$

i.e. derivation along the fluid wordline.

Note that τ and ξ are, in general, not constant.

The Temperature Evolution

Given general $p = p(\rho, n)$ and $T = T(\rho, n)$, where n is the particle number density, Gibbs integrability condition implies

$$n\frac{\partial T}{\partial n} + (\rho + p_{\rm eq})\frac{\partial T}{\partial \rho} = T\frac{\partial p_{\rm eq}}{\partial \rho} \,.$$

Together with energy and particle number conservation it gives:

$$\frac{\dot{T}}{T} = -\theta\left[\frac{\partial p_{\rm eq}}{\partial \rho} + \frac{\Pi}{T}\frac{\partial T}{\partial \rho}\right] \,.$$

Assuming $T = T(\rho)$ and $p_{\rm eq} = p_{\rm eq}(\rho)$ we obtain

$$\frac{1}{T}\frac{dT}{d\rho} = \frac{c_{\rm s}^2}{\rho + p_{\rm eq}} \,, \qquad \Rightarrow \qquad \frac{\dot{T}}{T} = -\theta c_{\rm s}^2\left(1 + \frac{\Pi}{\rho + p_{\rm eq}}\right) \,,$$

where $c_{\rm s}^2 := dp_{\rm eq}/d\rho$ is the adiabatic speed of sound.

Propagation of Perturbations in a Viscous Medium

The bulk viscous speed of sound

W. A. Hiscock and L. Lindblom – Annals Phys. 151 (1983) 466-496

It can be proven that the speed of sound related to bulk viscous pressure perturbations has the form:

$$c_{\rm b}^2 = \frac{\xi}{(\rho + p_{\rm eq})\tau} ,$$

and that it sums with the adiabatic $c_{\rm s}^2$:

$$c_{\rm b}^2 + c_{\rm s}^2 \leq 1 \qquad \Rightarrow \qquad c_{\rm b}^2 \leq 1 - c_{\rm s}^2 .$$

Therefore, there is no complete freedom in the choice of ξ and τ.

Viscous Perturbations
The common part

Stress-energy tensor perturbations:

$$\delta T^0{}_0 = \delta\rho \ ,$$

$$\delta T^i{}_0 = \left(\rho + p_{\rm eq} + \Pi\right) v^i \ ,$$

$$\delta T^i{}_j = -\delta^i{}_j \left(\delta p_{\rm eq} + \delta\Pi\right) \ ,$$

where $v^i := a\delta u^i$. Einstein equations:

$$\Delta\Phi - 3\mathcal{H}\left(\Phi' + \mathcal{H}\Phi\right) = 4\pi G a^2 \delta\rho \ ,$$

$$\Delta\left(\Phi' + \mathcal{H}\Phi\right) + \left(\mathcal{H}^2 - \mathcal{H}'\right)\Theta = 0 \ ,$$

$$\Phi'' + 3\mathcal{H}\Phi' + \left(\mathcal{H}^2 + 2\mathcal{H}'\right)\Phi = 4\pi G a^2 \left(\delta p_{\rm eq} + \delta\Pi\right) \ ,$$

where we have considered the divergence of the $(0 - i)$ equation and defined $\Theta \equiv \partial_i v^i$.

Perturbations in Eckart Theory

Being $\Pi = -\theta\xi$, we simply have

$$\delta\Pi = -\delta\theta\xi - \theta\delta\xi \ .$$

The perturbations of the expansion scalar is

$$\delta\theta = \delta\left(\nabla_\mu u^\mu\right) \qquad \Rightarrow \qquad \delta\theta = \partial_\mu \delta u^\mu + \Gamma^\mu_{\rho\mu}\delta u^\rho + \delta\Gamma^\mu_{\rho\mu} u^\rho \ .$$

Working out we obtain

$$a\delta\theta = \partial_i v^i - 3\left(\Phi' + \mathcal{H}\Phi\right) \ ,$$

with $v^i \equiv a\delta u^i$.
Note how bulk viscosity mix up geometry with thermodynamics.

Perturbations in Israel-Stewart Theory

Write the transport equation in the following way:

$$\dot{\Pi} + \frac{1}{\tau}\Pi = -\theta\left[f(\rho) + \frac{\Pi}{2}g(\rho) + \frac{\Pi^2}{2}h(\rho)\right] ,$$

where $f(\rho) := \xi/\tau = c_{\mathrm{b}}^2(\rho + p_{\mathrm{eq}})$ and

$$g(\rho) := 1 + (\rho + p_{\mathrm{eq}})\frac{1}{f}\frac{df}{d\rho} + c_{\mathrm{s}}^2 , \qquad h(\rho) := \frac{1}{f}\frac{df}{d\rho} + \frac{c_{\mathrm{s}}^2}{\rho + p_{\mathrm{eq}}} .$$

A general perturbation yields

$$\frac{1}{a}\delta\Pi' + \delta\left(\frac{\Pi}{\tau}\right) = \frac{\Phi}{a}\Pi' - \delta\theta\left[f(\rho) + \frac{\Pi}{2}g(\rho) + \frac{\Pi^2}{2}h(\rho)\right]$$

$$-\theta\delta\rho\left[\frac{df(\rho)}{d\rho} + \frac{\Pi}{2}\frac{dg(\rho)}{d\rho} + \frac{\Pi^2}{2}\frac{dh(\rho)}{d\rho}\right] - \theta\delta\Pi\left[\frac{g(\rho)}{2} + \Pi h(\rho)\right] .$$

Assumptions for the Bulk Viscosity Parameters

Being the ΛCDM model the best description we have of our universe, it is reasonable to demand that the viscous fluid reproduce a similar background expansion.
We consider a fluid with $p_{\rm eq} = 0$ and $\Pi = -A$, with A constant.
The energy conservation equation reads

$$\dot{\rho} = -3H\left(\rho + p\right) = -3H\left(\rho - A\right) \ ,$$

and its general solution is

$$\rho = A + C_1 a^{-3} \ ,$$

where C_1 is an integration constant.
In Eckart theory we infer that $\xi = A/\theta$.
Key point: even if $\Pi = -\theta\xi = -A$ and A is a constant, $\delta\Pi \neq 0$!

ΛCDM Background for Israel-Stewart Theory

Inserting $\Pi = -A$ and $p_{\rm eq} = c_{\rm s}^2 = 0$ in the transport equation gives

$$\tau = \frac{A}{\theta}\left[f(\rho) - \frac{A}{2}\left(1 + \frac{\rho}{f(\rho)}\frac{df(\rho)}{d\rho}\right) + \frac{A^2}{2}\frac{1}{f(\rho)}\frac{df(\rho)}{d\rho}\right]^{-1} .$$

We investigate the ansatz $f = 1/\gamma$, with γ constant, for which

$$\tau = \frac{\gamma A}{\theta\left(1 - \frac{\gamma A}{2}\right)} , \qquad c_{\rm b}^2 = \frac{1}{\gamma\rho} .$$

In order to avoid causality issues, we must ask that

$$1 < \gamma A < 2 .$$

Bulk Viscosity Coefficients Perturbations

In Eckart theory, a formal perturbation of $\xi = A/\theta$ leads to

$$\delta\xi = -\frac{A}{6H^3}\delta\left(H^2\right) \ .$$

Note that $\delta\left(H^2\right) \propto \delta\rho$, no velocity perturbation here!

Using the $(0-0)$ Einstein equation to eliminate $\delta\rho$:

$$\delta\xi = -\frac{Aa}{9\mathcal{H}^3}\left[\Delta\Phi - 3\mathcal{H}\left(\Phi' + \mathcal{H}\Phi\right)\right] \ .$$

In Israel-Stewart theory, the perturbation $\delta\tau$ can be calculated in the same fashion as above, obtaining

$$\delta\tau = -\frac{Aa}{9\mathcal{H}^3\left(1 - \frac{\gamma A}{2}\right)}\left[\Delta\Phi - 3\mathcal{H}\left(\Phi' + \mathcal{H}\Phi\right)\right] \ .$$

Results for the Eckart Case

Equation for the gravitational potential

Using

$$\Theta = \frac{\Delta\left(\Phi' + \mathcal{H}\Phi\right)}{\mathcal{H}' - \mathcal{H}^2}\,, \qquad \delta\xi = -\frac{Aa}{9\mathcal{H}^3}\left[\Delta\Phi - 3\mathcal{H}\left(\Phi' + \mathcal{H}\Phi\right)\right]\,,$$

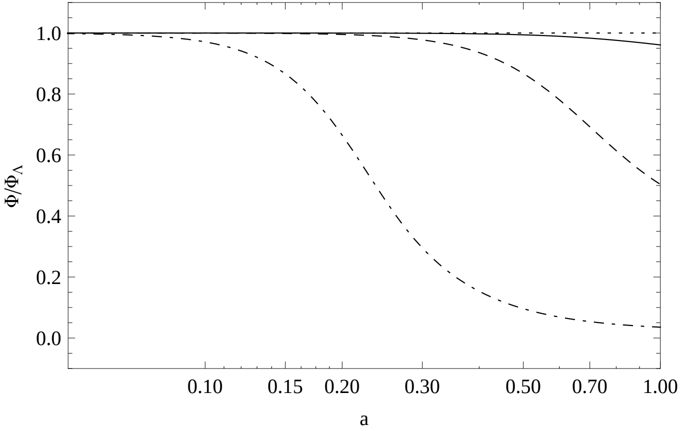
we obtain a closed second-order differential equation for the gravitational potential:

$$\ddot{\Phi} + \frac{4\mathcal{H} + \dot{\mathcal{H}}a}{\mathcal{H}a}\dot{\Phi} + \frac{\mathcal{H} + 2\dot{\mathcal{H}}a}{\mathcal{H}a^2}\Phi = -\frac{\Omega_A H_0^2 k^2}{2\mathcal{H}^4}\left(\Phi - \frac{\dot{\Phi}a + \Phi}{\frac{\dot{\mathcal{H}}}{\mathcal{H}}a - 1}\right)\,,$$

whose left-hand-side is identical, by construction, to the ΛCDM one. Note the k^2 in the right-hand-side term.

Results for the Eckart Case

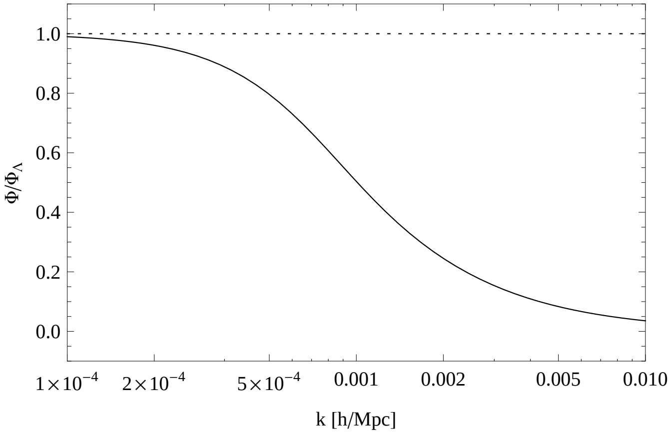
The gravitational potential evolution as a function of a



$k = 2 \cdot 10^{-4}, 10^{-3}, 10^{-2}\ h\ \mathrm{Mpc}^{-1}$ (solid, dashed, dot-dashed lines, respectively).

Results for the Eckart Case

The gravitational potential evolution as a function of k



Function of the wavenumber k for $a = 1$ fixed.

Results for Israel-Stewart Theory

Evolution equation for the gravitationl potential

From

$$\delta\dot{\Pi} = -\frac{3}{a\gamma A}\delta\Pi - \frac{\delta\theta}{\mathcal{H}\gamma}\left(1 - \frac{\gamma A}{2}\right) - \frac{3\theta}{a\gamma^2 A}\left(1 - \frac{\gamma A}{2}\right)^2 \delta\tau \ ,$$

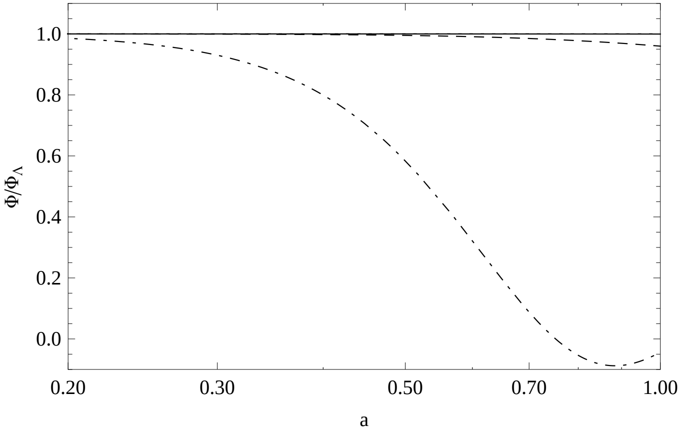
and the equation for $\delta\theta$ and $\delta\tau$ we get:

$$\ddot{\Phi} + \frac{4\mathcal{H} + \dot{\mathcal{H}}a}{\mathcal{H}a}\dot{\Phi} + \frac{\mathcal{H} + 2\dot{\mathcal{H}}a}{\mathcal{H}a^2}\Phi = \frac{3H_0^2\Omega_A}{2\mathcal{H}^2}\left(\frac{\delta\Pi}{A}\right) \ ,$$

$$\frac{d}{da}\left(\frac{\delta\Pi}{A}\right) = -\frac{3}{\gamma A a}\left(\frac{\delta\Pi}{A}\right) - \frac{(2-\gamma A)}{2\gamma A a}\frac{k^2}{\mathcal{H}^2}\left(\Phi - \frac{\dot{\Phi}a + \Phi}{\frac{\dot{\mathcal{H}}}{\mathcal{H}}a - 1}\right) \ .$$

Results for Israel-Stewart Theory

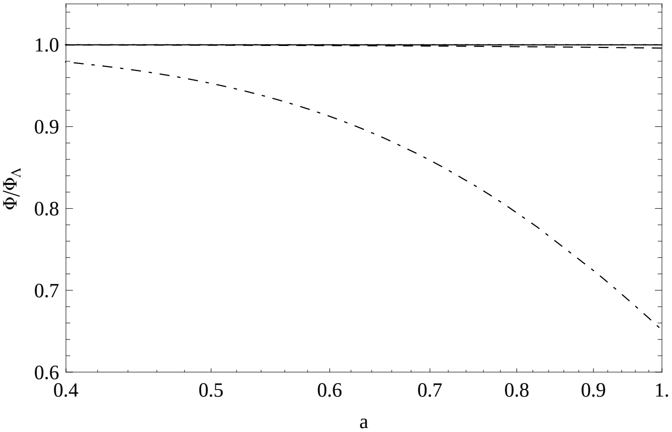
Gravitational potential evolution as a function of a and $\gamma A = 1.9$



$k = 0.0001, 0.001, 0.01\ h\ \text{Mpc}^{-1}$ (solid, dashed, dot-dashed, respectively).

Results for Israel-Stewart Theory

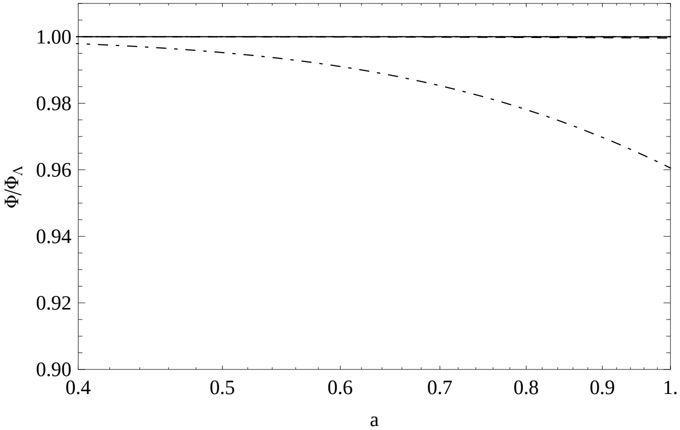
Gravitational potential evolution as a function of a and $\gamma A = 1.99$



$k = 0.0001, 0.001, 0.01\ h\ \text{Mpc}^{-1}$ (solid, dashed, dot-dashed, respectively).

Results for Israel-Stewart Theory

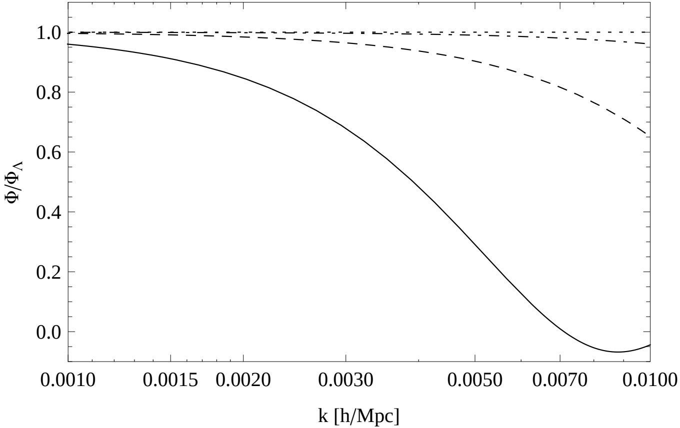
Gravitational potential evolution as a function of a and $\gamma A = 1.999$



$k = 0.0001, 0.001, 0.01\ h$ Mpc^{-1} (solid, dashed, dot-dashed, respectively).

Results for Israel-Stewart Theory

Gravitational potential evolution as a function of k



Evolution in function of k and for $a = 1$. Here also $\gamma A = 1.9, 1.99, 1.999$ (solid, dashed, dot-dashed, respectively).

Results for Israel-Stewart Theory

When $\gamma A \rightarrow 2$

For $\gamma A \rightarrow 2$ the situation is particularly intriguing. The ΛCDM model evolution of the gravitational potential is exactly reproduced.
However, this is a limiting case where both the relaxation time and ξ diverge. A very large τ characterises a so-called *frozen-in* non-equilibrium state.
For $\gamma A \rightarrow 2$ indeed we have

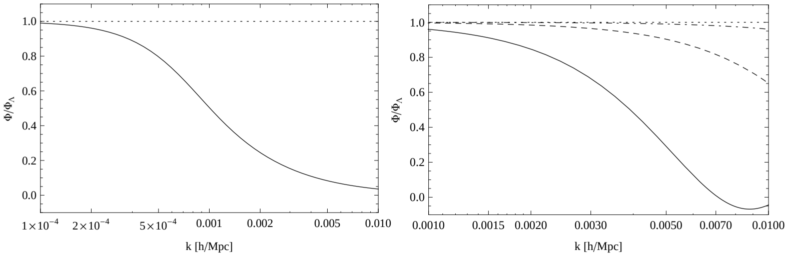
$$\left(\frac{\delta\Pi}{A}\right)^{\cdot} = -\frac{3}{2a}\left(\frac{\delta\Pi}{A}\right) ,$$

which gives

$$\delta\Pi = \delta\Pi(a_*)\left(\frac{a}{a_*}\right)^{-3/2} ,$$

which is a rapidly decaying source term for the gravitational potential equation.

Comparison between the two Approaches



Fixing a ΛCDM background expansion for Eckart theory leaves little hope for the model (see the work by Li and Barrow). On the other hand, Eckart theory is non-causal. It must be changed for e.g. the Israel-Stewart one, which seems to give more chances to the viscous unification picture.

Conclusions and Prospects

1. Up to now, our main conclusion is that Israel-Stewart (IS) theory may give better predictions than Eckart one. Moreover, IS theory should be the one to be used, being causal.
2. We have used special assumptions: $\tau = \gamma\xi$, $p_{\rm eq} = 0$ and $\Pi = -A$ with A constant. Though reasonable, they may be quite restrictive.
3. IS theory is a transport theory. Perhaps it is not fair to impose since the beginning an evolution for Π. We should try and find it from the transport equation itself, assuming ansatz's just for τ and ξ.
4. The results we find for $\gamma A \to 2$ seem to be promising. However, we have to deal with diverging τ, ξ. Is it possible to avoid such situation?