The Equation of State of Matter and the Hubble Tension

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Outline

The cosmic distance ladder

In cosmology, measuring distances is essential (length $\Leftrightarrow$ geometry $\Leftrightarrow$ matter).

The cosmic distance ladder is a sequence of measuring techniques allowing us to determine distances up to the cosmological scales (hundreds of millions of light-years).

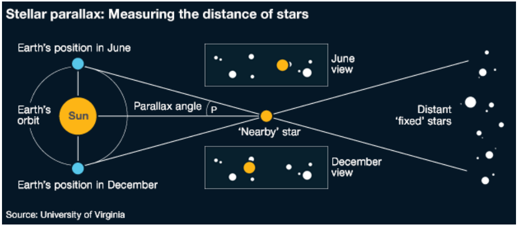
It is called "ladder" because different techniques work only within some ranges of distances that overlap, allowing thus calibration.

So, step by step, we can reach the cosmological scales.

The parallax

The first step is the parallax, which is a direct method of determining distances by trigonometry (it is the same as *triangulation* used on Earth).

Stars that are not too far from us will be seen to move with respect to the background of fixed stars.



The parsec

Let θ be the parallax angle. If $\theta \ll 1$, the distance to the star is:

$$d_{\rm parallax} = \frac{1\ \mathsf{AU}}{\theta}\,, \tag{1}$$

where $1\ \mathsf{AU} = 1.5 \times 10^{11}$ m is the astronomical unit.

If $\theta = 1$ seconds of arc, we define the **parsec**:

$$d_{\rm parallax} = \frac{6.48 \times 10^5\ \mathsf{AU}}{\pi} \approx 3.09 \times 10^{16}\ \mathsf{m} \equiv 1\ \mathsf{pc}\,, \tag{2}$$

The ESA *Gaia* spacecraft has been able to measure distances of stars up to 100 pc.

The Milky Way is about 30 kpc in diameter.

Standard Candles

In order to measure larger distances (particularly, of objects outside the Milky Way), we need to rely on indirect methods.

Standard candles are sources whose luminosity is known and whose distance can therefore be obtained via a measurement of their flux:

$$d \propto \sqrt{L/(4\pi F)}\,. \tag{3}$$

An extra proportionality factor has to be taken into account for the largest distances, when the expansion of the universe (the so-called Hubble flow) dominates the peculiar motions.

Cepheid Variables

Cepheid variables are pulsating stars, whose period of pulsation is related to their luminosity:

$$M_X = a + b\,(\log_{10} P - 1)\,. \qquad (4)$$

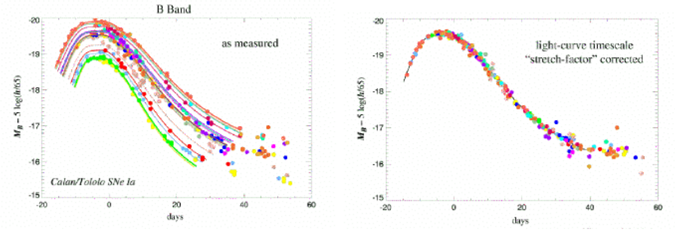
This period-luminosity relationship can be calibrated if we can determine the distance of a Cepheid variable through parallax.

Then, we can use this relationship to determine the luminosity (and so the distance) of Cepheids which are too far to use parallax.

Type Ia supernovae

A type Ia supernova explosion occurs when a white dwarf accretes material from a companion until it exceeds the Chandrasekhar limit.

Key feature: The faster the light curve decays from its peak, the fainter the absolute magnitude at the peak.



Type Ia supernovae as standard candles

Type Ia supernovae are standardizable candles thanks to **Phillips's relation**:

$$M_{\mathrm{max}} = a + b\,\Delta m_{15}(B)\,, \tag{5}$$

where a and b are parameters to be fitted (same for all supernovae!) and $\Delta m_{15}(B)$ is the variation of the apparent magnitude in the B band 15 days after the peak.

Calibration: know beforehand the values of M_{max} and $\Delta m_{15}(B)$ for many type Ia supernovae to find a and b.

So, we do another step on the cosmic distance ladder and, since type Ia supernovae are very bright, we are able to go very far (hundreds of Mpc, in fact) in the realm of cosmology.

The Hubble-Lemaître law

In 1929, Hubble re-examines the problem of the drift motion of the Solar System with respect to distant nebulae:

$$Kr + X\cos\alpha\cos\delta + Y\sin\alpha\cos\delta + Z\sin\delta = v\ . \qquad (6)$$

K is now known as Hubble's constant H_0;

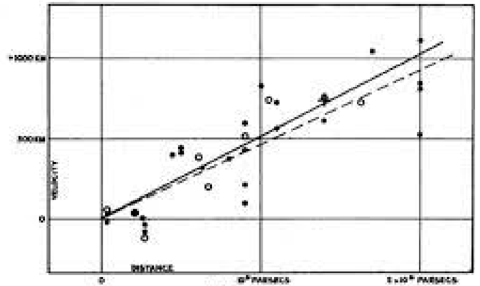


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

The Hubble-Lemaître law

For sufficiently large distances, we can neglect the peculiar relative motion:

$$v = H_0 r \ . \tag{7}$$

Hubble's original result:

$$H_0 = (465 \pm 50) \ \mathrm{km\ s^{-1}\ Mpc^{-1}} \ . \tag{8}$$

A previous investigation (alas, forgotten) by Lundmark in 1925:

$$X \cos\alpha \cos\delta + Y \sin\alpha \cos\delta + Z \sin\delta + k + lr + mr^2 - v = 0 \ . \tag{9}$$

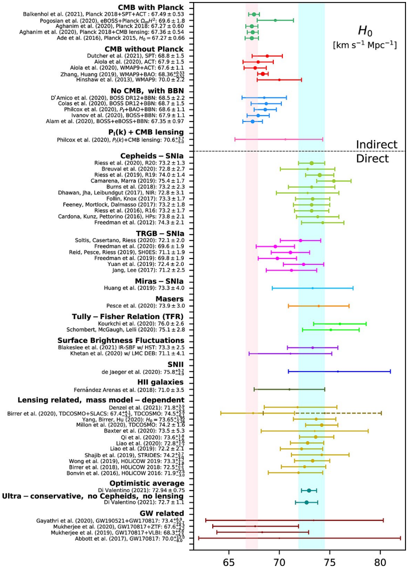
yielded $l \approx 10000 \ \mathrm{km\ s^{-1}\ Mpc^{-1}}$. Lemaître also provided in 1927 the estimate of $625 \ \mathrm{km\ s^{-1}\ Mpc^{-1}}$.

Local determination of H_0

- 3 rungs parallax-cepheids-SNeIa: $H_0 = 73.04 \pm 1.04$ km s^{-1} Mpc^{-1} (SH0ES collaboration - 2112.04510);
- Change the second rung (Chicago-Carnegie Hubble Program - 2408.06153): $H_0 = 68.81 \pm 1.79$ (stat) ± 1.32 (sys) for the TRGB, and $H_0 = 67.80 \pm 2.17$ (stat) ± 1.64 (sys) for the JAGB method.
- Non-distance ladder measurements based on gravitational lensing: $H_0 = 73.3 \pm 1.8$ (H0LiCOW collaboration - 1907.04869);
- Non-distance ladder measurements based on type II supernovae: $H_0 = 74.9 \pm 1.9$ km s^{-1} Mpc^{-1} (Vogl et al. - 2411.04968).

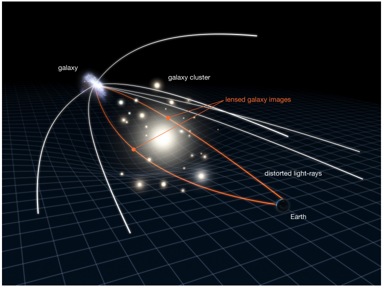
Local determination of H_0

Di Valentino et al. - 2103.01183



Determination of H_0 via strong lensing time delays

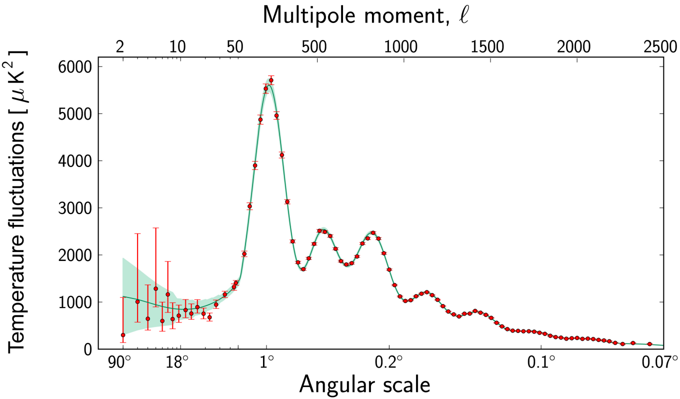
The arrival time of each of the multiple images of quasars depends on different distances traveled, and hence on H_0. Refsdal's idea (Refsdal, 1964).



Credits: NASA and ESA

H_0 from the CMB

We can actually (statistically) measure the angular size of $r_\star$ in the CMB sky (it is the first peak position). It is $\theta_\star$ (or θ_s), another of the six free parameters fitted to the CMB data.



https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_Power_Spectrum

How is H_0 measured from CMB?

Benchwork: FLRW geometry $ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$ (flat for simplicity here; taking into account spatial curvature does not solve the Hubble tension).

Expansion rate: $H(z) = H_0 E(z)$, with $E(z)$ depending on the matter content.
Comoving distance to a source at a redshift z:

$$\chi = \int_0^z \frac{dz'}{H(z')} = \frac{1}{H_0} \int_0^z \frac{dz'}{E(z')} \ . \tag{10}$$

Luminosity distance: $D_L(z) = (1+z)\chi$.

Angular diameter distance: $D_A(z) = \chi/(1+z)$.

H_0 FROM THE CMB

Use the definition of the (comoving) angular-diameter distance:

$$\int_0^{z_\star} \frac{dz'}{H(z')} = D_A(z_\star) = r_\star/\theta_\star \; . \tag{11}$$

Here, $H(z)$ contains the contribution of dark energy (to which the CMB spectrum is practically insensitive). For the ΛCDM:

$$H(z)^2 = H_0^2[\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda] \; . \tag{12}$$

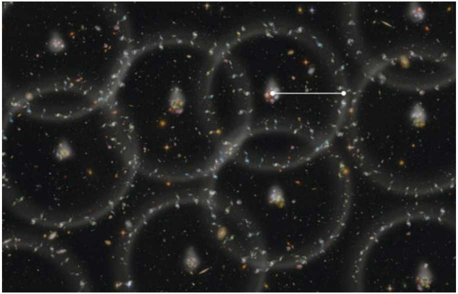
So, adjust $\Omega_\Lambda H_0^2$ to match $D_A(z_\star) = r_\star/\theta_\star$. Then H_0 is determined.

Using Planck 2018 data, in particular $100\theta_\star = 1.04110 \pm 0.00031$ and $z_\star = 1089.92 \pm 0.25$, one gets $r_\star = 144.43 \pm 0.26$ Mpc and $H_0 = 67.36 \pm 0.54$ km s^{-1} Mpc^{-1} (all at 68% CL).

Late-times model-independent constraints

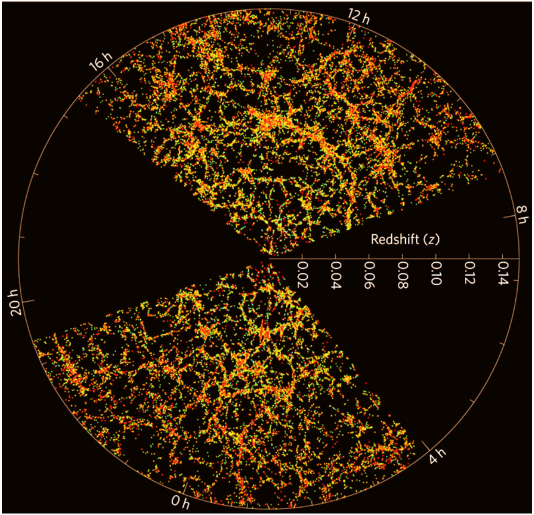
BAO and uncalibrated SNIa

A lot happens from z_* (roughly 0.4 Myr after the Big Bang) to $z = 0$ (today, roughly 14 Gyr after the Big Bang). We need extra constraints. Baryon acoustic oscillations (BAO) help us because their physics depends on $r_\star^{\mathrm{drag}} \approx r_\star$ (since photons and baryons were tightly coupled, but photons were many more).



https://astro.ucla.edu/~wright/BAO-cosmology.html

Galaxy distribution

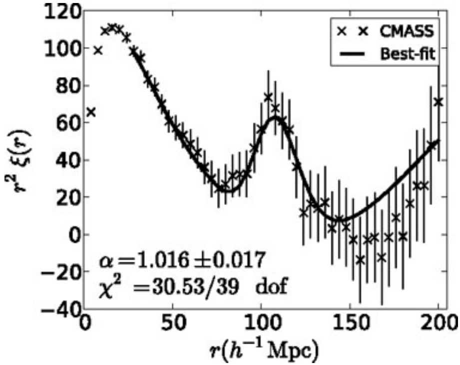


Credits: SDSS collaboration

BAO

The BAO angular and radial features also depend on $r_\star$ (they are also statistically determined):

$$r_\star^{\text{drag}} = \theta_{BAO} D_A(z_{BAO}) \,, \qquad H(z_{BAO}) r_\star^{\text{drag}} = \Delta z_{BAO} \,. \tag{13}$$



https://ned.ipac.caltech.edu/level5/March14/Percival/Percival4.html

BAO and uncalibrated SNeIa constraints

BAO and uncalibrated SNeIa ("Hubble flow" SNeIa, very far, high z) allow constraining $H_0 r_\star^{\mathrm{drag}}$ at different redshifts ($\ll z_\star$) $\rightarrow H_0 = 67.66 \pm 0.42$ (using Planck 2018, to determine $r_\star^{\mathrm{drag}}$).

One can use this "inverse" cosmic ladder approach (from CMB down in redshift to SNIa) to calibrate SNeIa M_B (absolute magnitude). This is then found incompatible with the same M_B determined from the usual "direct" distance ladder (parallax-Cepheids-SNeIa) (Camarena and Marra - 2101.08641).

Model-independent constraints and CMB

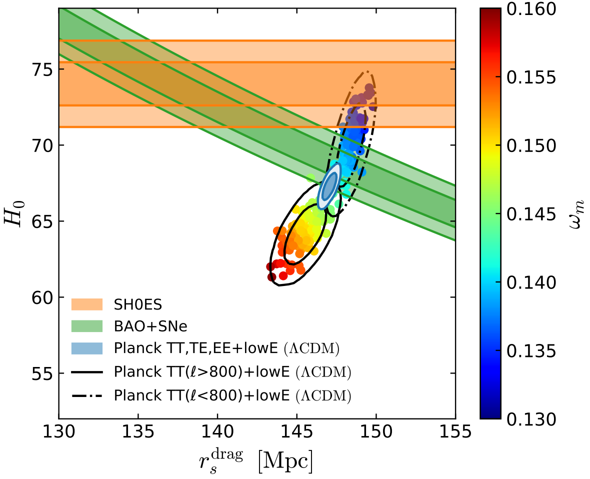


Fig. 1 of (Knox and Millea - 1908.03663)

Example of concordance

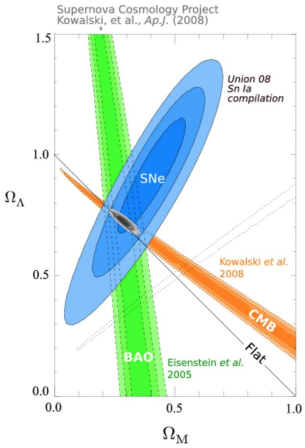


Figure 1: Empirical constraints on Ω_Λ and Ω_M cosmological parameters. Contours at 68.3%, 95.4%, and 99.7% confidence level (1, 2 and 3-σ) obtained from CMB, BAO, and the Union SN set (coloured regions), as well as their combination (grayscale region, assuming $w = -1$)

Cosmoverse - https://cosmoversetensions.eu/

Cost (European Cooperation in Science and Technology) Action

Ways to ease the tension

Proposed solutions

100 pages of proposals: (Di Valentino et al. - 2103.01183).

General approach. Work on:

$$H_0 = \frac{\theta_\star}{r_\star} \int_0^{z_\star} \frac{dz}{E(z)} \, . \tag{14}$$

Only $\theta_\star$ is directly measured here (one of the 6 parameters that fit the CMB data). So, work on the rest:

- Reduce $r_\star \rightarrow$ modify the early-times history of the universe (e.g. increase $N_{\rm eff}$);
- Modify $E(z)$, i.e. the late-times history of the universe (e.g. $w < -1$).

Late-times solutions

New cosmological physics

- Phantom dark energy (Di Valentino et al. - 2005.12587);
- Interacting dark energy (Di Valentino et al. - 1908.04281);
- Decaying dark matter (Pandey et al. - 1902.10636);
- Running vacuum (Gómez-Valent et al. - 2305.15774);
- Bulk viscosity (Normann and Brevik - 2107.13533)
- ...

Typically don't work because of constraints from BAO, SNIa and Cosmic chronometers.

Late-times solutions

New local physics

New Physics affecting the local calibration:

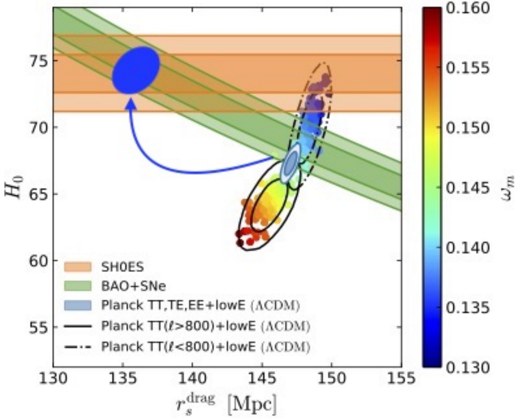
- Screened fifth forces (Desmond and Sakstein - 1907.03778);
- Late-times transition in $G_{\rm eff}$ (Marra and Perivolaropoulos - 2102.06012);
- Chameleon dark energy (Cai et al. - 2102.02020)

Still, the tension with H0LiCOW remains.

Another intriguing possibility is that the usual FLRW framework might need to be changed (Camarena et al. - 2207.09995). For example, drop the Copernican principle and use ΛLTB instead of FLRW.

Early-times solutions

The main idea is to reduce $r_\star$ (somehow).



Adapted from (Knox and Millea, 2020)

Early-times solutions

Work on:

$$r_\star = \int_{z_\star}^{\infty} \frac{c_\mathrm{s}(z)dz}{H(z)} \ . \tag{15}$$

- In order to alleviate the Hubble tension, $r_\star \approx 138$ Mpc is sufficient, i.e. a value smaller than the *Planck* best fit of about 4%;
- The value $r_\star \approx 144$ Mpc is obtained already integrating up to $z \simeq 4 \times 10^5$. This suggests that there is no need to modify the cosmological physics for larger redshifts.

The only relevant physical cosmological phenomenon that occurs between $z \simeq 4 \times 10^5$ and $z_\star$ is recombination.

Early dark energy

E.g. some primordial scalar field (many references). Issues:

- Too *ad-hoc*, i.e. choose the “right” potential, the “right” initial conditions;
- It must eventually disappear (then, also *ad hoc*);
- Increases the S_8 tension (tension between how much matter we have and how much it clusters, basically);

In general (Hill et al. - 2003.07355) any new physics before recombination (meaning, a new matter component doing the job of easing the Hubble tension) unbalances the dark matter content worsening the S_8 tension.

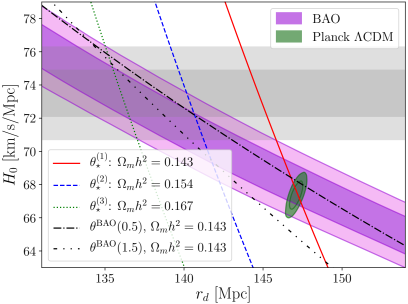
(Poulin et al. - 1811.04083)

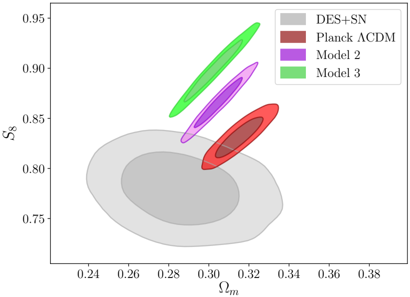
$$\Omega_\varphi = \frac{2\Omega_\varphi(a_c)}{(a/a_c)^{3(w_n+1)} + 1} \,. \tag{16}$$

Seen as a scalar field, this is initially frozen, and then it dilutes faster than matter.

General problem for early-times solutions

Worsening of the S_8 tension





From (Jedamzik et al. - 2010.04158)

The S_8 tension

The quantity σ_8 is the rms density fluctuation averaged on spheres of 8 h^{-1} Mpc. The combination:

$$S_8 = \sigma_8(\Omega_m/0.3)^{1/2}\,, \tag{17}$$

is typically used as the informative quantity in the ΛCDM model. From the Kilo-Degree Survey (KiDS)-1000 and from the Dark Energy Survey (DES) Year 3 analysis of the cosmic shear one gets, for a flat ΛCDM model (2305.17173):

$$S_8 = 0.790^{+0.018}_{-0.014}\,, \tag{18}$$

whereas from the *Planck* collaboration ΛCDM parameters we have:

$$S_8 = 0.828 \pm 0.016 \text{ (TTTEEE)}\,. \tag{19}$$

Matter and its equation of state

Matter (CDM and baryons) is modeled in cosmology as pressureless dust:

$$w \equiv \frac{P}{\rho} = 0\,. \tag{20}$$

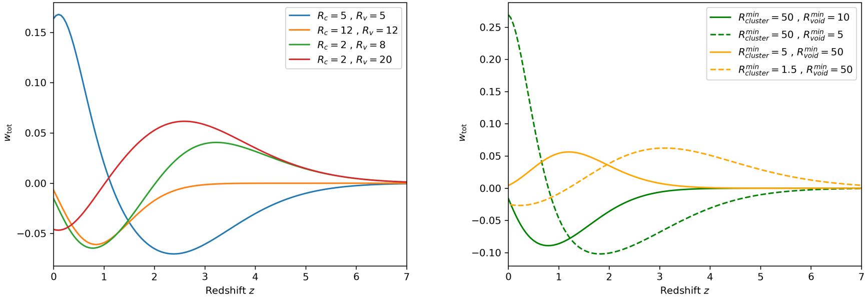
It is a key ingredient of the cosmological recipe, effectively describing, at present, about one-third of the whole content of the universe.

On the other hand, dust at early times, before recombination, is substantially different from dust at late times, when structures are formed and the universe becomes highly inhomogeneous at small scales.

Backreaction from structures

The structure formation process (small scales) “reacts back” on large scales (Giani, Von Marttens, Camilleri - 2410.15295).

This entails a non-zero equation of state for matter, $p = \varepsilon\rho$. This might solve the Hubble tension and the S_8 tension, if $\varepsilon > 0$.



H_0 FROM THE CMB

$$r_\star = \theta_\star \int_0^{z_\star} \frac{dz}{H(z)} \,, \tag{21}$$

where $\theta_\star$ is the acoustic angular scale, $100\theta_\star = 1.04110 \pm 0.00031$; $z_\star$ is the recombination redshift, $z_\star = 1089.92 \pm 0.25$;

$$r_\star = \int_{z_\star}^{\infty} \frac{c_\mathrm{s}(z)dz}{H(z)} \,, \tag{22}$$

is the sound horizon, where $c_\mathrm{s}(z)$ is the speed of sound of the baryon-photon fluid:

$$c_\mathrm{s}(z) = \frac{1}{\sqrt{3[1+R(z)]}} \,, \qquad R(z) = \frac{3\Omega_\mathrm{b} h^2}{4\Omega_\gamma h^2} \frac{1}{1+z} \,, \tag{23}$$

where Ω_b and Ω_γ are the baryon and photon density parameters, respectively.

H_0 from the CMB

Use the definition of the (comoving) angular-diameter distance:

$$\int_0^{z_\star} \frac{dz'}{H(z')} = D_A(z_\star) = r_\star/\theta_\star \ . \tag{24}$$

Here, $H(z)$ contains the contribution of dark energy (to which the CMB spectrum is practically insensitive). For the ΛCDM:

$$H(z)^2 = H_0^2[\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda] \ . \tag{25}$$

So, adjust $\Omega_\Lambda H_0^2$ to match $D_A(z_\star) = r_\star/\theta_\star$. Then H_0 is determined.

Using Planck 2018 data, one gets $r_\star = 144.43 \pm 0.26$ Mpc and $H_0 = 67.36 \pm 0.54$ km s^{-1} Mpc^{-1} (all at 68% CL).

Nonzero pressure matter

Assume a change in the equation of state of matter after recombination:

$$\int_0^{z_t} \frac{dz}{\sqrt{\Omega_{\rm m} h^2 (1+z)^{3+\varepsilon}(1+z_t)^{-\varepsilon} + \Omega_\Lambda h^2 + \Omega_{\rm r} h^2 (1+z)^4}}\,, \tag{26}$$

i.e., a constant change in the equation of state $w = 0 \to w = \varepsilon$ up to redshift z_t. From z_t to $z_\star$ the model is the usual ΛCDM one, with $\varepsilon = 0$. The combination:

$$\Omega_{\rm m} h^2 (1+z)^{3+\varepsilon}(1+z_t)^{-\varepsilon}\,, \tag{27}$$

is introduced in order not to modify the abundance of matter at z_t. In this way, pre-recombination cosmology is the standard one. With such a correction, the actual present time density parameter is:

$$\Omega_{\rm m}(1+z_t)^{-\varepsilon}\,. \tag{28}$$

New determination of H_0

The Hubble constant is then determined by:

$$h = \sqrt{\Omega_{\mathrm{m}}(1+z_t)^{-\varepsilon} h^2 + \Omega_\Lambda h^2 + \Omega_{\mathrm{r}} h^2} \,. \qquad (29)$$

For example, by choosing:

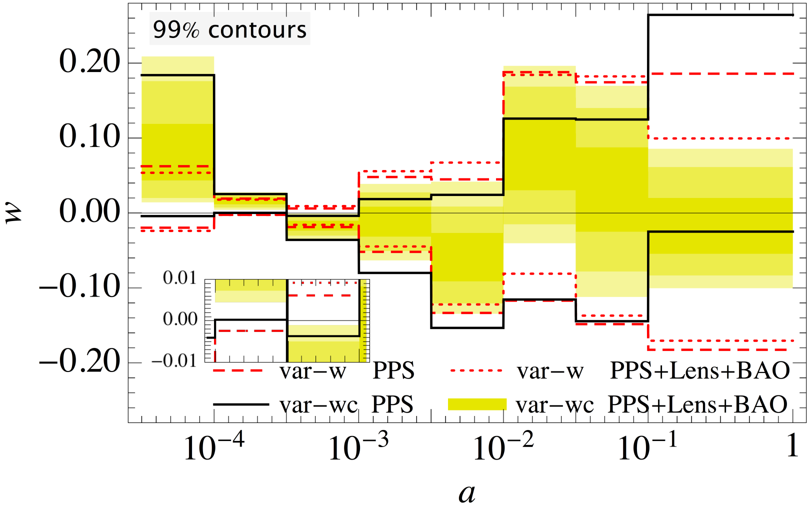
$$z_t = z_\star \,, \qquad \varepsilon = 0.003 \,, \qquad (30)$$

we find:

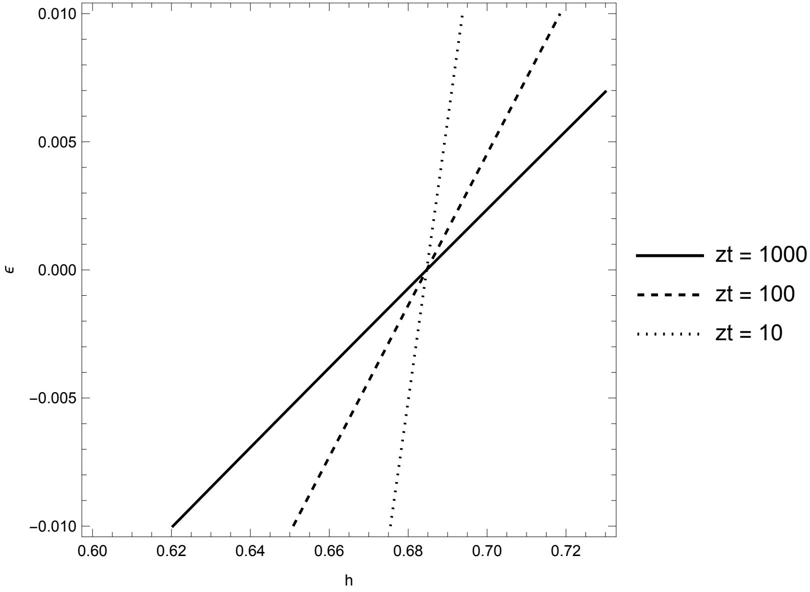
$$\Omega_\Lambda h^2 = 0.36 \,, \qquad H_0 = 70.45 \text{ km/s/Mpc} \,. \qquad (31)$$

The tension is then eased if not removed.
Note that, according to Ilić et al. (2004.09572), the correction $|\varepsilon| < 0.01$ is consistent with observation. So, a small positive pressure seems to help solve the Hubble tension.

Ilić et al. (2004.09572) findings



How much pressure do we need?



What happens to S_8 with a nonzero matter EoS

In general, for most extensions of the ΛCDM model in which DE is dynamical it is not trivial to extend the definition $S_8 = \sigma_8(\Omega_m/0.3)^{1/2}$ because one has to carefully identify that portion of the cosmic fluid which clusters.

In our present case this is not an issue, because DE is still described by Λ and DM has simply a nonvanishing equation of state, so that:

$$S_8 \to S_8(1 + z_t)^{-\varepsilon/2} \,. \tag{32}$$

Therefore, a positive ε not only "goes in the right direction" to solve the Hubble tension, but also to solve the one on S_8.

Evolution of the gravitational potential

For $z_t = z_\star$ and $\varepsilon = 0.003$ one gets a corrective factor of 0.99, certainly unable to relieve the S_8 tension. On the other hand, σ_8 also changes.

Let us consider the equation ruling the evolution of the gravitational potential, on a perturbed FLRW background:

$$\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi = -4\pi G a^2 \delta P \,. \tag{33}$$

Here, the prime denotes derivation with respect to the conformal time η; the conformal Hubble parameter is $\mathcal{H} = a'/a$; δP is the pressure perturbation; finally, only one gravitational potential is being considered since we are assuming zero anisotropic stress.

Solution for Φ with non zero ε

In the case with a tiny $w = \varepsilon$ and zero pressure perturbation δP (zero effective speed of sound):

$$a \propto \eta^{2(1-3\varepsilon)}\,, \quad \mathcal{H} = \frac{2(1-3\varepsilon)}{\eta}\,, \quad \Phi'' + \frac{6(1-3\varepsilon)}{\eta}\Phi' - \frac{12\varepsilon}{\eta^2}\Phi = 0\,, \quad (34)$$

where only $\mathcal{O}(\varepsilon)$ corrections have been considered.
Looking for a $\Phi \propto \eta^p$ solution, one can find:

$$\Phi \propto \eta^p\,, \qquad p = -\frac{21}{5}\varepsilon\,. \quad (35)$$

So, with this correction, the potential at η_0 is diminished with respect to the initial value of a factor:

$$\alpha \equiv \frac{\Phi_0}{\Phi_t} = \left(\frac{\eta_0}{\eta_t}\right)^{-\frac{21}{5}\varepsilon} = \left(\frac{a_0}{a_t}\right)^{-\frac{21}{10}\varepsilon} = (1+z_t)^{-\frac{21}{10}\varepsilon}\,. \quad (36)$$

Solution of the tension

In the standard case $w = 0$, dust, $\alpha = 1$. So, if $w = \varepsilon$ is positive, we see that we have a reduction in the gravitational potential, which corresponds to a reduction of δ and in turn of σ_8.

In particular, we have:

$$z_t = z_\star \,, \qquad w = 0.003 \,, \qquad \alpha = 0.96 \,. \tag{37}$$

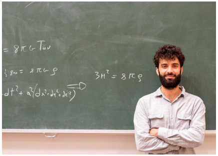
So, combining the two corrections found, we have:

$$S_8^{(\varepsilon)} = S_8 (1+z_t)^{-\varepsilon/2} (1+z_t)^{-\frac{21}{10}\varepsilon} \approx 0.95 S_8 = 0.784 \,, \tag{38}$$

which solves the S_8 tension. Note that it is quite possible that the tension on S_8 requires a non-linear solution (Preston, Amon, and Efstathiou - 2305.09827).

Evolution of the idea

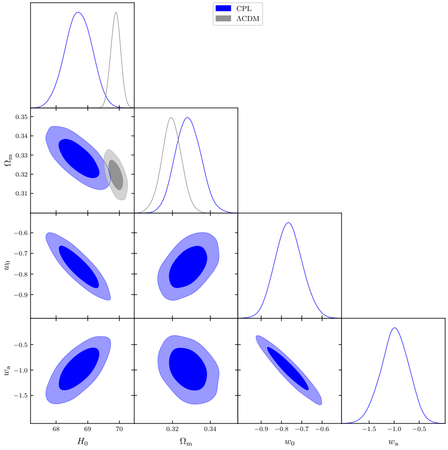
2505.08467 with L. Giani and R. Von Marttens

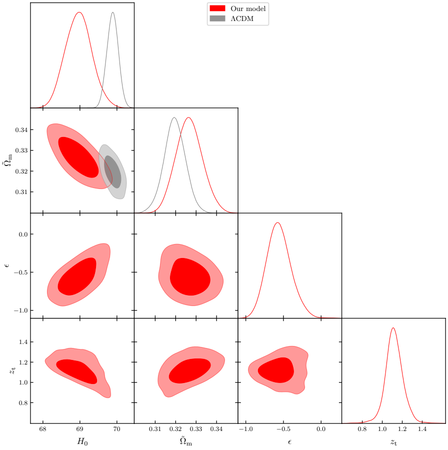


Unfortunately, a statistical analysis (using data from DESI DR2 BAO, DESY5 SnIa and CMB distance prior data) seems to favor negative values of ε. The model is a competitive alternative to the CPL parametrization of DE. See a similar idea (Chen and Loeb - 2505.02645).

Evolution of the idea

2505.08467 with L. Giani and R. Von Marttens





Conclusions and Perspectives

- Is there actually a Hubble tension?
- A possible solution to the Hubble tension should take into account and be compatible with all datasets;
- A possible solution should not worsen the S_8 tension, but preferably solve it as well;
- A possible solution should be well-motivated and not *ad hoc*; it has to fit the whole cosmological picture;
- Early-times solutions of the Hubble tension must fit into the very well understood pre-recombination physics, without spoiling it. Is this possible?
- Perhaps there is need for late-times solutions;
- Perhaps there is no **the** solution, but a combination of small corrections (both at early- and late-times) will solve the tension.