Review of the cosmological constant problem

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Block Course
July 3, 2019

Outline

Introduction

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Introduction

Some Reviews

- *S. Weinberg, The cosmological constant problem, Reviews of Modern Physics, Vol. 61, No. 1, January 1989;*
- *S. Carroll, The cosmological constant, Living Rev. Relativity, 4, (2001), 1*;
- *J. Polchinski, The Cosmological Constant and the String Landscape, hep-th/0603249;*
- *J. Martin, Everything you always wanted to know about the cosmological constant problem (but were afraid to ask), C. R. Physique 13 (2012) 566665*;
- *C. P. Burgess, The Cosmological Constant Problem: Why it's hard to get Dark Energy from Micro-physics, arXiv:1309.4133 [hep-th];*
- *A. Padilla, Lectures on the Cosmological Constant Problem, arXiv:1502.05296v1 [hep-th].*

General Relativity

Einstein-Hilbert action plus a cosmological constant term:

$$S = \frac{c^4}{16\pi G_N}\int \mathrm{d}^4x\sqrt{-g}\left(R - 2\Lambda\right) + S_{\mathrm{matter}}\left[g_{\mu\nu}, \Psi\right] \;. \qquad (1)$$

Field equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G_N}{c^4}T_{\mu\nu} \;. \qquad (2)$$

Λ was introduced by Einstein in order to model a static universe (*A. Einstein, Sitzungs. König. Preuss. Akad. (1917) 142-152*). It has been rejected afterwards being the expansion of the universe observationally established. It revives today as the most successful model for the accelerated expansion of the universe.

General Relativity and the cosmological constant from a mathematical viewpoint

Lovelock's theorems:

- *D. Lovelock, The Einstein Tensor and Its Generalizations, Journal of Mathematical Physics (1971) 12 (3) 498501*;
- *D. Lovelock, The Four-Dimensionality of Space and the Einstein Tensor, Journal of Mathematical Physics (1972) 13 (6) 874876*.

Given field equations in vacuum:

$$A^{\mu\nu} = 0\ , \tag{3}$$

and the following hypothesis:

Lovelock's theorem

Hypothesis and thesis

1. $A^{\mu\nu} = A^{\nu\mu}$ (symmetry)
2. $A^{\mu\nu} = A^{\mu\nu}(g_{\mu\nu}, g_{\mu\nu,\rho}, g_{\mu\nu,\rho\sigma})$
3. $\nabla_\mu A^{\mu\nu} = 0$ (divergencelessness, ∇_μ is the covariant derivative)
4. $A^{\mu\nu}$ is linear in the second derivative of the metric.

Then

$$A^{\mu\nu} = aG^{\mu\nu} + bg^{\mu\nu} \; , \tag{4}$$

where a and b are arbitrary constants and:

$$G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R \; , \tag{5}$$

is the Einstein tensor. The theorem holds in any dimensions.

(In the second paper Lovelock shows that the symmetry condition is superfluous in 4 dimensions)

Lovelock's theorem

Lagrangian density

The Lagrangian density associated to $A^{\mu\nu} = aG^{\mu\nu} + bg^{\mu\nu}$ is

$$L = \sqrt{-g}\sum_{p=1}^{m-1} a_p \delta^{\alpha_1}_{[\beta_1} \ldots \delta^{\alpha_{2p}}_{\beta_{2p}]} \Pi^{2p-1}_{r=1} R_{\alpha_r \alpha_{r+1}}{}^{\beta_r \beta_{r+1}} + a_0\sqrt{-g}\;, \quad (6)$$

with $m = n/2$ if the number of dimensions n is even,
$m = (n+1)/2$ if n is odd. For $n = 4$:

$$L = \sqrt{-g}a_1\delta^{\alpha_1}_{[\beta_1}\delta^{\alpha_2}_{\beta_2]} R_{\alpha_1\alpha_2}{}^{\beta_1\beta_2} + a_0\sqrt{-g} = \sqrt{-g}(a_1 R + a_0)\;, \quad (7)$$

i.e. the Einstein-Hilbert action plus a cosmological term.

Resurrection of the cosmological constant

Cosmology

Friedmann-Lemaître-Robertson-Walker metric (with flat spatial hypersurfaces):

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \; , \tag{8}$$

Friedmann equations with Λ:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G_N}{3}\rho_{\rm tot} + \frac{\Lambda}{3} \; , \tag{9}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho_{\rm tot} + 3p_{\rm tot}) + \frac{\Lambda}{3} \; . \tag{10}$$

The cosmological constant works as antigravity and behaves as a perfect fluid with equation of state:

$$w_\Lambda \equiv \frac{p_\Lambda}{\rho_\Lambda} = -1 \; . \tag{11}$$

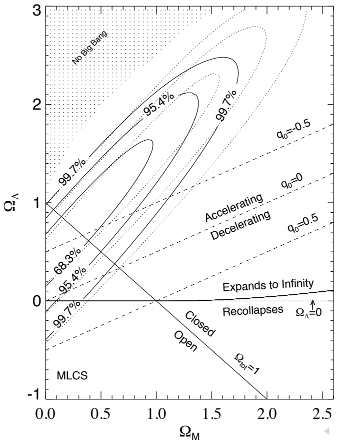
Accelerated expansion of the universe

Type Ia supernovae are standard candles which allowed to extend the cosmic distance ladder to large redshifts ($z \sim 1$) and from which the accelerated expansion of the universe was discovered.

- *A. G. Riess et al. [Supernova Search Team], Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant , Astron. J.* **116** *(1998) 1009 [astro-ph/9805201]*
- *S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Measurements of Omega and Lambda from 42 High-Redshift Supernovae , Astrophys. J.* **517** *(1999) 565 [astro-ph/9812133]*

Accelerated expansion of the universe

From Riess' paper



Accelerated expansion of the universe

From Riess' paper

Table 8: Cosmological Results

						no constraint*			$\Omega_{tot} \equiv 1$	$\Omega_\Lambda \equiv 0$	$\Omega_M \equiv 0.2$
Method (high-z SNe)	H_0	Ω_M	Ω_Λ	χ^2_ν	t_0	$p(\Omega_\Lambda \geq 0)$	$p(q_0 \leq 0)$	q_0	Ω_M	Ω_M	Ω_Λ
†MLCS+Snap.(15)	⋯	⋯	⋯	1.19	⋯	99.7%(3.0σ)	99.5%(2.8σ)	-0.98±0.40	0.28±0.10	-0.34±0.21	0.65 ± 0.22
†ΔM_{15}+Snap.(15)	⋯	⋯	⋯	1.03	⋯	>99.9%(4.0σ)	>99.9%(3.9σ)	-1.34±0.40	0.17±0.09	-0.48±0.19	0.84 ± 0.18
MLCS+Snap.+97ck(16)	⋯	$0.24^{+0.56}_{-0.24}$	$0.72^{+0.72}_{-0.48}$	1.17	⋯	99.5%(2.8σ)	99.3%(2.7σ)	-0.75±0.32	0.24±0.10	-0.35±0.18	0.66 ± 0.21
ΔM_{15}+Snap.+97ck(16)	⋯	$0.80^{+0.40}_{-0.48}$	$1.56^{+0.52}_{-0.70}$	1.04	⋯	>99.9%(3.9σ)	>99.9%(3.8σ)	-1.14±0.30	0.21±0.09	-0.41±0.17	0.80 ± 0.19
MLCS(9)	65.2±1.3‡	⋯	⋯	1.19	$13.6^{+1.0}_{-0.8}$	99.6%(2.9σ)	99.4%(2.4σ)	-0.92±0.42	0.28±0.10	-0.38±0.22	0.68 ± 0.24
ΔM_{15}(9)	63.8±1.3‡	⋯	⋯	1.05	$14.8^{+1.0}_{-0.8}$	>99.9%(3.9σ)	>99.9%(3.8σ)	-1.38±0.46	0.16±0.09	-0.52±0.20	0.88 ± 0.19
MLCS+97ck(10)	65.2±1.3‡	$0.00^{+0.60}_{-0.00}$	$0.48^{+0.72}_{-0.24}$	1.17	$14.2^{+1.3}_{-1.0}$	99.5%(2.8σ)	99.3%(2.7σ)	-0.74±0.32	0.24±0.10	-0.38±0.19	0.68 ± 0.22
ΔM_{15}+97ck(10)	63.7±1.3‡	$0.72^{+0.44}_{-0.56}$	$1.48^{+0.56}_{-0.68}$	1.04	$15.1^{+1.1}_{-0.9}$	>99.9%(3.8σ)	> 99.9%(3.7σ)	-1.11±0.32	0.20±0.09	-0.44±0.18	0.84 ± 0.20
Snap.(6)	63.4±2.7‡	⋯	⋯	1.30	⋯	89.1%(1.6σ)	78.9%(1.3σ)	-0.70±0.80	0.40±0.50	0.06 ± 0.70	0.44 ± 0.60

*$\Omega_M \geq 0$

†Complete set of spectroscopic SNe Ia.

‡This uncertainty reflects only the statistical error from the variance of SNe Ia in the Hubble flow.
It does not include any contribution from the (much larger) SN Ia absolute magnitude error.

Latest Supernovae type Ia data

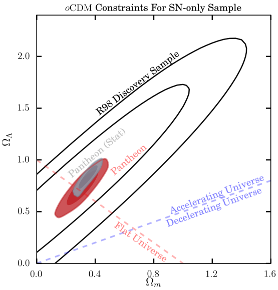
Pantheon Sample: *D. M. Scolnic **et al.**, Astrophys. J.* **859** *(2018) no.2, 101 [arXiv:1710.00845 [astro-ph.CO]].*

Table 8.

Analysis	Model	w	Ω_m	Ω_Λ
SN-stat	ΛCDM		0.284 ± 0.012	0.716 ± 0.01
SN-stat	oCDM		0.348 ± 0.040	0.827 ± 0.06
SN-stat	wCDM	-1.251 ± 0.144	0.350 ± 0.035	
SN	ΛCDM		0.298 ± 0.022	0.702 ± 0.02
SN	oCDM		0.319 ± 0.070	0.733 ± 0.11
SN	wCDM	-1.090 ± 0.220	0.316 ± 0.072	

Notes: Cosmological constraints for the SN-only sample with and without systematic uncertainties. Values are given for three separate cosmological models: ΛCDM, oCDM and wCDM.

dataset mainly to be in-line with general community reproducibility. We still use the binned distances to generate the systematic covariance matrix, which is used as a 2d 40-bin interpolation grid to create a covariance matrix for the full SN dataset. Diagonal uncertainties from the individual distances can be added together with the



Again from the Pantheon paper

Table 11.

Sample	Ω_m	Ω_Λ	Ω_K	H_0
CMB+BAO	0.310 ± 0.008	0.689 ± 0.008	0.001 ± 0.003	67.900 ± 0.747
CMB+H0	0.266 ± 0.014	0.723 ± 0.012	0.010 ± 0.003	73.205 ± 1.788
CMB+BAO+H0	0.303 ± 0.007	0.694 ± 0.007	0.003 ± 0.002	68.723 ± 0.675
SN+CMB	0.299 ± 0.024	0.698 ± 0.019	0.003 ± 0.006	69.192 ± 2.815
SN+CMB+BAO	0.309 ± 0.007	0.690 ± 0.007	0.001 ± 0.002	67.985 ± 0.699
SN+CMB+H0	0.274 ± 0.012	0.717 ± 0.011	0.009 ± 0.003	72.236 ± 1.572
SN+CMB+BAO+H0	0.303 ± 0.007	0.695 ± 0.007	0.003 ± 0.002	68.745 ± 0.684

Notes: Cosmological constraints from different combinations of probes when assuming the oCDM model.

Table 12.

Sample	w	Ω_m	H_0
CMB+BAO	-0.991 ± 0.074	0.312 ± 0.013	67.508 ± 1.633
CMB+H0	-1.188 ± 0.062	0.265 ± 0.013	73.332 ± 1.729
CMB+BAO+H0	-1.119 ± 0.068	0.289 ± 0.011	70.539 ± 1.425
SN+CMB	-1.026 ± 0.041	0.307 ± 0.012	68.183 ± 1.114
SN+CMB+BAO	-1.014 ± 0.040	0.307 ± 0.008	68.027 ± 0.859
SN+CMB+H0	-1.056 ± 0.038	0.293 ± 0.010	69.618 ± 0.969
SN+CMB+BAO+H0	-1.047 ± 0.038	0.299 ± 0.007	69.013 ± 0.791

Notes: Cosmological constraints from different combinations of probes when assuming the wCDM model. The value of $w = -1$ corresponds to the cosmological constant hypothesis.

The value of Λ from cosmology

So, there is strong observational evidence of Λ driving the accelerated expansion of the universe. The amount of Λ energy density in the universe is about 70% percent, hence:

$$\rho_\Lambda \approx \Omega_\Lambda \rho_{\rm cr} \approx 10^{-47}\ \mathrm{GeV}^4 \approx 10^{-52}\ \mathrm{m}^{-2}\ . \qquad (12)$$

What is the problem with Λ?

- Huge discrepancy with the predictions coming from quantum field theory (old cosmological constant problem);
- Why ρ_Λ has the above tiny value? (new cosmological constant problem).

The first question was tackled already by Zel'dovich and Sakharov (*Y. B. Zeldovich, JETP letters 6 (1967), 316-317*; *A. D. Sakharov, Dokl. Akad. Nauk SSSR (1967) 177,70-71*).

Vacuum

Vacuum fluctuations

Vacuum fluctuations, or quantum zero point fluctuations, are considered to exist because of:

- The Lamb shift (*W. E. Lamb, R. C. Retherford, Physical Review. 72 (1947) (3): 241243*);
- The Casimir effect (*H. B. G. Casimir, D. Polder, Physical Review. 73 (1948) (4): 360372*),

(although see *R. L. Jaffe, Casimir effect and the quantum vacuum, PRD 72, 021301(R) (2005)*)

Our concern is however to understand whether and how this energy gravitates. The standard approach is semi-classical gravity, i.e.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu} + 8\pi G_N \langle T_{\mu\nu} \rangle \;, \tag{13}$$

where

$$\langle T_{\mu\nu} \rangle \equiv \langle 0 | T_{\mu\nu} | 0 \rangle \;. \tag{14}$$

Vacuum fluctuations

In Minkowski space we have that

$$\langle T_{\mu\nu} \rangle \propto \eta_{\mu\nu} \; , \tag{15}$$

hence by the equivalence principle, in curved space one has:

$$\langle T_{\mu\nu} \rangle = -\rho_{\rm vac}(x) g_{\mu\nu} \; , \tag{16}$$

and because of Bianchi identities $\rho_{\rm vac}$ has to be a constant. So, the field equations become:

$$G_{\mu\nu} + \Lambda_{\rm eff} g_{\mu\nu} = 8\pi G_N T_{\mu\nu} \; , \tag{17}$$

with

$$\Lambda_{\rm eff} = \Lambda + 8\pi G_N \rho_{\rm vac} \; , \tag{18}$$

the effective cosmological constant (whose measured value is 10^{-47} GeV^4).

Classical contributions

These come from fields which settle at the minimum of the potential to which they are subject. Consider a simple example:

$$T_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - g_{\mu\nu} \left[\frac{1}{2} g^{\rho\sigma} \partial_\rho \Phi \partial_\sigma \Phi + V(\Phi) \right] . \tag{19}$$

If the field rolls down to a minimum of its potential:

$$\langle T_{\mu\nu} \rangle = -V(\Phi_{\rm min}) g_{\mu\nu} . \tag{20}$$

If we can set $V(\Phi_{\rm min}) = 0$ there is no contribution to $\Lambda_{\rm eff}$. Phase transitions are therefore quite problematic because the position of the minimum is shifted and $V(\Phi_{\rm min}) = 0$ can be realised only before or after the phase transition.

Electroweak phase transition

Realistic cases are the electroweak phase transition and the QCD transition. In the electroweak case (after the transition) we have the potential ($\lambda \simeq 0.1$ is a coupling):

$$V(H) = -\frac{\lambda v^4}{4} + \frac{1}{2}\lambda v^2 H^2 + \frac{\lambda}{2}\frac{v}{\sqrt{2}}H^3 + \frac{\lambda}{16}H^4 \;, \qquad (21)$$

with $m_H^2 = \lambda v^2$ being the Higgs mass and $v = \langle H \rangle$. Then:

$$\rho_{\rm vac} = -\frac{1}{4}m_H^2 v^2 \;, \qquad v^2 = \frac{\sqrt{2}}{4G_F^2} \;, \qquad (22)$$

lead to:

$$\rho_{\rm vac} = -\frac{\sqrt{2}}{16}\frac{m_H^2}{G_F^2} \approx -1.2 \times 10^8 \text{ GeV}^4 \;. \qquad (23)$$

$G_F \simeq 1.16 \times 10^{-5}$ GeV^{-2} is Fermi's constant and $m_H \approx 125$ GeV.

Electroweak phase transition

Higgs Potential (plot taken from Martin's review)

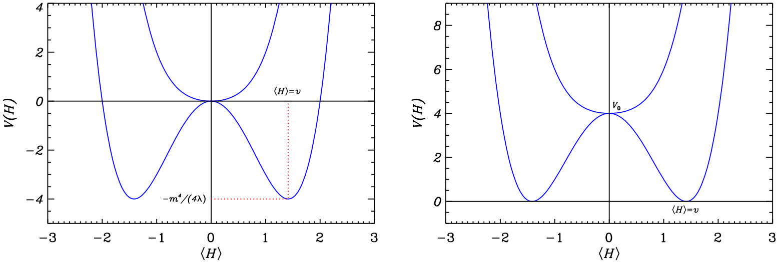


Fig. 2. Effective potential of the Higgs boson before and after the electroweak phase transition. The left panel corresponds to a situation where the vacuum energy vanishes at high temperature. As a consequence $\rho_{\rm vac}$ is negative at temperature smaller than the critical temperature. This is the situation treated in the text where the quantity $-m^4/(4\lambda)$ is explicitly calculated. On the right panel, the off-set parameter V_0 is chosen such that the vacuum energy is zero after the transition. As a consequence, it does not vanish at high temperatures.

Quantum-mechanical contribution to $\Lambda_{\rm eff}$

Simple example of a scalar field

Let us see the calculation of the zero-point energy-momentum tensor for a massive non-interacting scalar field:

$$\Phi(x) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2\omega_k}}\left(a_k e^{ik^\mu x_\mu} + a_k^\dagger e^{-ik^\mu x_\mu}\right) , \tag{24}$$

with

$$\omega_k \equiv k^0 = \sqrt{k^2 + m^2} . \tag{25}$$

The energy-momentum tensor is:

$$\langle T_{\mu\nu}\rangle = \int \frac{d^3k}{(2\pi)^3 2k^0} k_\mu k_\nu . \tag{26}$$

Quantum-mechanical contribution to $\Lambda_{\rm eff}$

Putting a cutoff

Consider a cutoff $M \to \infty$. Then:

$$\langle\rho\rangle = \frac{1}{4\pi^2}\int_0^M dk k^2\sqrt{k^2+m^2} =$$
$$\frac{M^4}{16\pi^2}\left[\sqrt{1+\frac{m^2}{M^2}}\left(1+\frac{m^2}{2M^2}\right) - \frac{m^4}{2M^4}\ln\left(\frac{M}{m}+\frac{M}{m}\sqrt{1+\frac{m^2}{M^2}}\right)\right]$$
$$= \frac{M^4}{16\pi^2}\left(1+\frac{m^2}{M^2}+\ldots\right) \; , \quad (27)$$

Quantum-mechanical contribution to $\Lambda_{\rm eff}$

Putting a cutoff

and also for the pressure:

$$\langle p \rangle = \frac{1}{3}\frac{1}{4\pi^2}\int_0^M dk \frac{k^4}{\sqrt{k^2+m^2}} =$$

$$\frac{1}{3}\frac{M^4}{16\pi^2}\left[\sqrt{1+\frac{m^2}{M^2}}\left(1-\frac{3m^2}{2M^2}\right)+\frac{3m^4}{2M^4}\ln\left(\frac{M}{m}+\frac{M}{m}\sqrt{1+\frac{m^2}{M^2}}\right)\right]$$

$$= \frac{1}{3}\frac{M^4}{16\pi^2}\left(1-\frac{m^2}{M^2}+\dots\right) . \quad (28)$$

At the leading order ($m/M \to 0$) $\langle p \rangle = \langle \rho \rangle / 3$, as radiation does, so something is going wrong. Indeed, putting a cutoff spoils Lorentz invariance. Considering just the logarithmic terms instead gives the expected behaviour $\langle p \rangle = -\langle \rho \rangle$.

Quantum-mechanical contributions

Dimensional regularisation

Using dimensional regularisation one gets:

$$\langle\rho\rangle = \frac{\mu^{4-d}}{(2\pi)^{d-1}}\frac{1}{2}\int_0^\infty dk k^{d-2} d^{d-2}\Omega\omega_k$$
$$= \frac{\mu^4}{2(4\pi)^{d-1}}\frac{\Gamma(-d/2)}{\Gamma(-1/2)}\left(\frac{m}{\mu}\right)^d \,, \qquad (29)$$

with μ an arbitrary scale. Similarly

$$\langle p\rangle = \frac{\mu^4}{4(4\pi)^{d-1}}\frac{\Gamma(-d/2)}{\Gamma(1/2)}\left(\frac{m}{\mu}\right)^d \,, \qquad (30)$$

Now $\langle p\rangle = -\langle\rho\rangle$ as expected.

Quantum-mechanical contributions

Renormalisation

Considering $d = 4 - \epsilon$ one can easily investigate the pole structure of the Gamma function and see that:

$$\langle\rho\rangle = -\frac{m^4}{64\pi^2}\left[\frac{2}{\epsilon} + \frac{3}{2} - \gamma - \ln\left(\frac{m^2}{4\pi\mu^2}\right)\right] + \dots \, . \tag{31}$$

By subtracting the divergent term we finally have:

$$\langle\rho\rangle = \frac{m^4}{64\pi^2}\ln\left(\frac{m^2}{\mu^2}\right) \, . \tag{32}$$

In general one can show the same result for any free field, provided a minus sign for the fermionic ones. Hence:

$$\langle\rho\rangle = \frac{1}{64\pi^2}\sum_n (-1)^{2S_n} g_n m_n^4 \ln\left(\frac{m_n^2}{\mu^2}\right) \, . \tag{33}$$

Quantum-mechanical contributions

Pauli Sum Rules

Pauli already observed in 1951 (ETH lectures) that even using a UV cutoff the correct result is obtained if the following conditions are met:

$$\sum_n (-1)^{2S_n} g_n = 0\;, \sum_n (-1)^{2S_n} g_n m_n^2 = 0\;, \sum_n (-1)^{2S_n} g_n m_n^4 = 0\;. \tag{34}$$

Visser shows how these conditions provide a bridge between the finiteness of the zero-point energy and Lorentz invariance. He also speculates on the consequences of taking these relations to be valid non-perturbatively, leading to the necessity of physics beyond the standard model (*M. Visser, Phys. Lett. B* **791** *(2019) 43 [arXiv:1808.04583 [hep-th]]*).

The value of the cosmological constant and the problem

Summing up all the contributions considered so far, we have:

$$\rho_{\rm vac} = \frac{1}{64\pi^2}\sum_n (-1)^{2S_n} g_n m_n^4 \ln\left(\frac{m_n^2}{\mu^2}\right) + \rho_\Lambda + \rho_{\rm vac}^{\rm EW} + \rho_{\rm vac}^{\rm QCD} + \ldots \, . \tag{35}$$

We don't know μ, so we could simply fine tune it and ρ_B in order to give the observed result. The problem is that such a fine-tuning is needed at each loop order, since the coupling constants are not all smalls in the Standard Model. In other words, perturbation theory doesn't work here. Just taking the electroweak scale:

$$\rho_{\rm vac}^{\rm EW} \approx -1.2 \times 10^8 \text{ GeV}^4 \, . \tag{36}$$

This has the wrong sign and it is in modulus way larger than the observed value

Issues in the calculations

The result:

$$\langle \rho \rangle = \frac{1}{64\pi^2} \sum_n (-1)^{2S_n} g_n m_n^4 \ln\left(\frac{m_n^2}{\mu^2}\right) \,, \tag{37}$$

will still hold if:

- we take into account interactions?
- we consider nonzero spacetime curvature?

In Martin's review it is shown that the answer is yes to both the questions. For the first, the case $\lambda\Phi^4/4!$ (a self interaction scalar field) is considered and the Gaussian effective potential approach (which is non-perturbative).

Attempted solutions

Example of a small cosmological constant

J. Holland and S. Hollands, Class. Quantum Grav. 31 (2014) 125006

Here the authors solve exactly a 2D toy model (Gross-Neveu):

$$\mathcal{L} = N\left[i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + \frac{g^2}{2}(\bar{\psi}\psi)^2\right] , \tag{38}$$

for the zero-point energy momentum tensor, finding:

$$\langle\theta_{\mu\nu}\rangle = -\frac{1}{4\pi\ell^2}e^{-2\pi/g^2}\eta_{\mu\nu} + \mathcal{O}(1/N) . \tag{39}$$

The speculation is whether a similar result could hold also for the Standard Model, giving an exponential suppression.

Self-adjusting fields and Weinberg's no-go theorem

If there is some field ϕ such that:

$$\Box\phi \propto T^{\mu}{}_{\mu} \propto R \,, \tag{40}$$

and that it evolves to an equilibrium value ϕ_0 such that $T^{\mu}{}_{\mu}(\phi_0) = 0$. Then $R = 0$ and the Minkowski solution can be enforced. In some sense the field "adjusts" itself to the huge zero-point energy, eating it up.
Weinberg's no-go theorem states that this is impossible without a fine-tuning (see his review).

Unimodular gravity

In unimodular gravity one demands that $-g = 1$ and it is not dynamical. The field equations then become:

$$R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R = 8\pi G_N \left(T_{\mu\nu} - \frac{1}{4}g_{\mu\nu}T \right) \; . \qquad (41)$$

Since the zero-point contribution is a trace, here it does not enter. Taking Bianchi's identities, one has:

$$\nabla_\mu \left(R + 8\pi G_N T \right) = 0 \quad \Rightarrow \quad R + 8\pi G_N T = 4\Lambda \; , \qquad (42)$$

and so we rewrite the field equations as:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu} \; . \qquad (43)$$

Unimodular gravity

Unimodular gravity seems to work perfectly. So what are the problems with that? According to Weinberg: *In my view, the key question in deciding whether this is a plausible classical theory of gravitation is whether it can be obtained as the classical limit of any physically satisfactory quantum theory of gravitation.*

According to *A. Padilla and I. D. Saltas, Eur. Phys. J. C* **75** *(2015) no.11, 561, [arXiv:1409.3573 [gr-qc]]* unimodular gravity is equivalent to GR (so the CCP still exists, it is moved to the Lagrangian multiplier that enforces $-g = 1$).

Opposite view by *C. Barceló, R. Carballo-Rubio and L. J. Garay, Annals of Physics 398 (2018) 9-23* address the above-mentioned citation by Weinberg, in the positive sense.

Sequestering mechanism

N. Kaloper and A. Padilla, Phys. Rev. Lett. 112 (2014) 091304 [arXiv:1309.6562 [hep-th]]

Sequestering action:

$$S = \sigma\left(\frac{\Lambda}{\lambda^4\mu^4}\right) + \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \lambda^4\mathcal{L}_m(\lambda^{-2}g^{\mu\nu},\Psi) - \Lambda\right] . \tag{44}$$

Variation:

$$\frac{1}{\lambda^4\mu^4}\sigma'\left(\frac{\Lambda}{\lambda^4\mu^4}\right) = \int d^4x\sqrt{-g}\,, \tag{45}$$

$$\frac{4\Lambda}{\lambda^4\mu^4}\sigma'\left(\frac{\Lambda}{\lambda^4\mu^4}\right) = \int d^4x\sqrt{-g}\lambda^4\tilde{T}^\alpha_\alpha\,, \tag{46}$$

$$M_P^2 G^\mu{}_\nu = -\Lambda\delta^\mu{}_\nu + \lambda^4\tilde{T}^\mu{}_\nu\,, \tag{47}$$

with $\lambda^4\tilde{T}_{\mu\nu} = T_{\mu\nu}$.

Sequestering mechanism

N. Kaloper and A. Padilla, Phys. Rev. Lett. 112 (2014) 091304 [arXiv:1309.6562 [hep-th]]

The first condition implies that the spacetime volume is finite. Combining the previous equations, one gets:

$$\Lambda = \frac{1}{4}\frac{\int d^4x\sqrt{-g}\,T^\alpha_\alpha}{\int d^4x\sqrt{-g}}\;, \tag{48}$$

and the field equations become:

$$M_P^2 G^\mu{}_\nu = T^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu\frac{\int d^4x\sqrt{-g}\,T^\alpha_\alpha}{\int d^4x\sqrt{-g}}\;, \tag{49}$$

and the zero-point contribution drops out, similarly to what happens in the unimodular gravity case.

Other interesting tentatives

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