SYDNEY GRAMMAR SCHOOL

NAME

MATHS MASTER

CANDIDATE NUMBER

2025 Trial Examination

Form VI Mathematics Advanced

Thursday 14th August, 2025

8:40am

General Instructions

- Reading time — 10 minutes
- Working time — 3 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.
- A loose reference sheet is provided separate to this paper.
- Carefully remove the central staple.

Thirty Four Questions — 100 Marks

Section I (10 marks) Questions 1 – 10

- This section is multiple-choice. Each question is worth 1 mark.
- Record your answers on the provided answer sheet.

Section II (90 marks) Questions 11 – 34

- Relevant mathematical reasoning and calculations are required.
- Answer the questions in this paper in the spaces provided.

Collection

- Your name and master should only be written on this page.
- Write your candidate number on this page, on the start of the separate section and on the multiple choice sheet.

Checklist

- Reference sheet
- Multiple-choice answer sheet
- Candidature: 102 pupils

Writer: YH

Section I

Questions in this section are multiple-choice.
Record the single best answer for each question on the provided answer sheet.

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1. What is the gradient of a line perpendicular to the line $y = 2x + 1$?

   (A) 2

   (B) -2

   (C) $\dfrac{1}{2}$

   (D) $-\dfrac{1}{2}$

2. Which of the following is the natural domain of the function $f(x) = \dfrac{1}{\sqrt{x-3}}$?

   (A) $x > 3$

   (B) $x < 3$

   (C) $x \geq 3$

   (D) $x \leq 3$

3. Which of the following is the range of $y = 1 - e^x$?

   (A) $y < 1$

   (B) $y \leq 1$

   (C) $0 < y < 1$

   (D) $0 \leq y \leq 1$

4. How many solutions does the equation $x^2 = \cos x$ have?

   (A) 0

   (B) 1

   (C) 2

   (D) 3

5. The function $y = f(x)$ is dilated vertically by a factor of 3. Which of the following corresponds to the function of this transformation?

   (A) $y = f(3x)$

   (B) $y = 3f(x)$

   (C) $y = \frac{1}{3}f(x)$

   (D) $y = f\left(\frac{x}{3}\right)$

6. A particle is moving along the x-axis. At a certain time, the particle's acceleration is opposite in direction to its velocity. Which of the following MUST be true at that time?

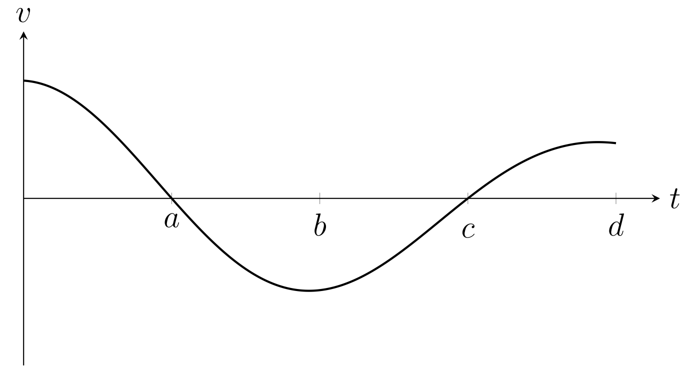
   (A) the particle's speed at that time is increasing.

   (B) the particle's speed at that time is decreasing.

   (C) the particle's velocity at that time is increasing.

   (D) the particle's velocity at that time is decreasing.

7. The velocity-time graph of a moving particle is given below. At what time does the particle reach its minimum displacement within the time frame $0 \leq t \leq d$?



(A) $t = a$

(B) $t = b$

(C) $t = c$

(D) $t = d$

8. Suppose $h(x) = [f(3x)]^2$. Which of the following is the correct expression for $h'(x)$?

(A) $2f(3x)$

(B) $2f'(3x)f(3x)$

(C) $6f(3x)$

(D) $6f'(3x)f(3x)$

9. Which of the following is the solution to $x^2 \leq x$?

(A) $0 \leq x \leq 1$

(B) $x \leq 0$

(C) $x \geq 1$

(D) There are no solutions.

10. Which of the following definite integrals gives the largest value?

(A) $\displaystyle\int_{-1}^{1} e^x \, dx$

(B) $\displaystyle\int_{-1}^{1} \frac{e^{x^2}}{2} \, dx$

(C) $\displaystyle\int_{-1}^{1} \sin x^3 \, dx$

(D) $\displaystyle\int_{-1}^{1} \sqrt{1-x^2} \, dx$

End of Section I

The paper continues in the next section

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CANDIDATE NUMBER

Section II

Part A

QUESTION ELEVEN (2 marks)

Solve $x - \dfrac{6}{x} = 1$. **2**

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QUESTION TWELVE (5 marks)

Find the derivative of the following:

(a) $y = 2x^3 + \dfrac{1}{x}$ **2**

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Question 12 continues on the next page.

QUESTION TWELVE (continued)

Find the derivative of the following:

(b) $y = 5e^{3x}$ **1**

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(c) $y = \tan 2x$ **1**

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(d) $y = \ln(3x - 1)$ **1**

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QUESTION THIRTEEN (3 marks)

Find the equation of the tangent to $y = \sqrt{2x+1}$ at $x = 4$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers and $a > 0$. $\boxed{3}$

QUESTION FOURTEEN (5 marks)

The probability distribution of a random variable X is given in the table below.

x	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.25	0.3	k

(a) Find $P(0 < X \leq 3)$. **1**

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(b) Find the value of k. **1**

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(c) Find $E(X)$. **1**

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(d) Find $E(X^2)$ **1**

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(e) Find $\mathrm{Var}(X)$. **1**

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QUESTION FIFTEEN (2 marks)

A table of future value interest factors for an annuity of \$1 is shown below.

Period \ Rate	1%	2%	4%	8%
5	5.101	5.204	5.416	5.867
10	10.462	10.950	12.006	14.487
20	22.019	24.297	29.778	45.762
40	48.886	60.402	95.026	259.057

(a) Ben contributes \$500 every year for 10 years to an annuity paying 8% per annum, compounded annually. Find how much his investment is worth at the end of 10 years. Give your answer correct to the nearest dollar. **1**

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(b) Charlie contributes \$500 every three months for 10 years to an annuity paying 8% per annum, compounded quarterly. Find how much his investment is worth after 10 years. Give your answer correct to the nearest dollar. **1**

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QUESTION SIXTEEN (2 marks)

Find the values of x such that the following limiting sum exists. **2**

$$1 + (1 - 2x) + (1 - 2x)^2 + (1 - 2x)^3 + \cdots .$$

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QUESTION SEVENTEEN (3 marks)

A team has a $\frac{1}{3}$ chance of winning any given game. If they played 3 games, find the probability of them winning:

(a) at least one game. **1**

(b) exactly one game. **2**

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If you use this space, clearly indicate which question you are answering.

CANDIDATE NUMBER

Section II

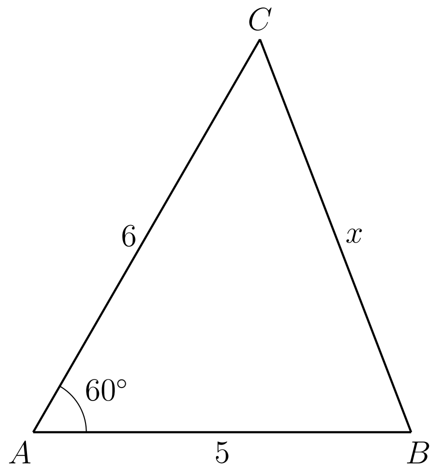
Part B

QUESTION EIGHTEEN (2 marks)

Find the value of the following arithmetic series: **2**

$$1 + 5 + 9 + \cdots + 2025.$$

QUESTION NINETEEN (3 marks)



The diagram above show $\triangle ABC$ with $AB = 5\,\text{cm}$, $BC = x\,\text{cm}$, $AC = 6\,\text{cm}$ and $\angle CAB = 60°$.

(a) Find the area of $\triangle ABC$ correct to two decimal places. **1**

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(b) Find the exact value of x. **2**

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QUESTION TWENTY (2 marks)

Consider the equation $y = 2\sin 3t + 4$.

(a) State its period. **1**

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(b) State its range. **1**

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QUESTION TWENTY ONE (6 marks)

Consider the graph of the curve given by $y = \dfrac{1}{27}x^3 - x + 2$.

(a) Find the coordinates of its stationary points and determine their nature. $\boxed{3}$

Question 21 continues on the next page.

(b) Find any points of inflection. **2**

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(c) Given that $x = -6$ is an x-intercept, sketch the graph of the curve. Clearly label any stationary points, points of inflection and intercepts with the coordinate axes. **1**

QUESTION TWENTY TWO (4 marks)

Find:

(a) $\displaystyle\int \left(\frac{1}{x^2} - \cos 3x\right)\, dx$ $\boxed{\mathbf{2}}$

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(b) $\displaystyle\int (1 - 2x)^5 \, dx$ $\boxed{\mathbf{2}}$

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CANDIDATE NUMBER

Section II

Part C

QUESTION TWENTY THREE (5 marks)

Alex takes out a loan at an interest rate of 5% per annum, compounded annually. At the end of each year, interest is charged and then Alex makes a repayment.

(a) Suppose Alex borrows \$10 000 and he pays off the loan in 10 years with regular annual repayments of $\$M$. Let A_n be the amount Alex still owes by the end of the n^{th} year.

(i) Find A_1 and hence A_2 in terms of M. **1**

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Question 23 continues on the next page.

(ii) Find A_n in terms of M and n. **1**

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(iii) Find M correct to two decimal places. **2**

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(b) If Alex only wants to repay \$350 each year in order to pay off his loan in 20 years, what is the maximum amount he would be able to borrow under the same interest rate? Give your answer correct to two decimal places. **1**

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QUESTION TWENTY FOUR (3 marks)

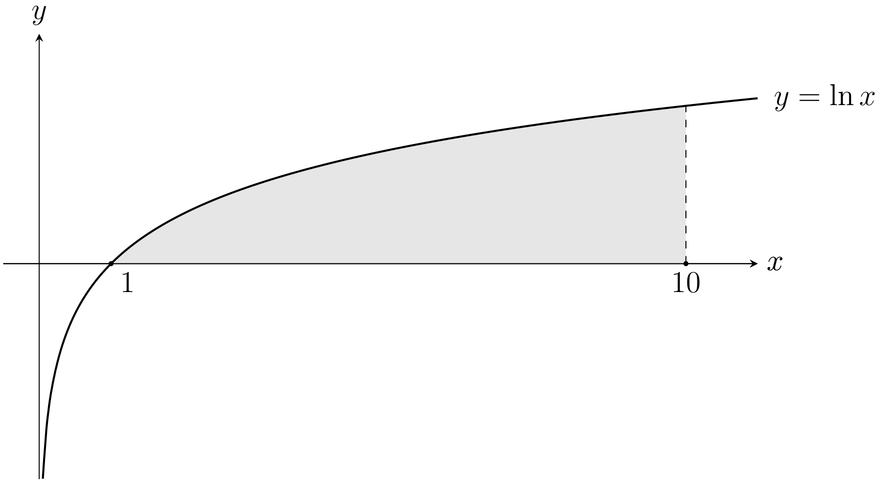
Suppose $f(x) = e^x(x+1)$.

(a) Find the first derivative of $f(x)$. $\boxed{1}$

(b) Hence, find the second derivative of $f(x)$. $\boxed{1}$

(c) By identifying a pattern, write down an expression for the n^{th} derivative of $f(x)$, where n is a positive integer. $\boxed{1}$

QUESTION TWENTY FIVE (4 marks)



The region bounded by $y = \ln x$ and the x-axis between $x = 1$ and $x = 10$ is shaded in the diagram above.

(a) By using the trapezoidal rule with 4 function values, find an approximation for the area of the shaded region. Give your answer correct to two decimal places. **2**

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(b) By considering an appropriate region bounded by $y = \ln x$ and the y-axis, find the exact area of the shaded region. **2**

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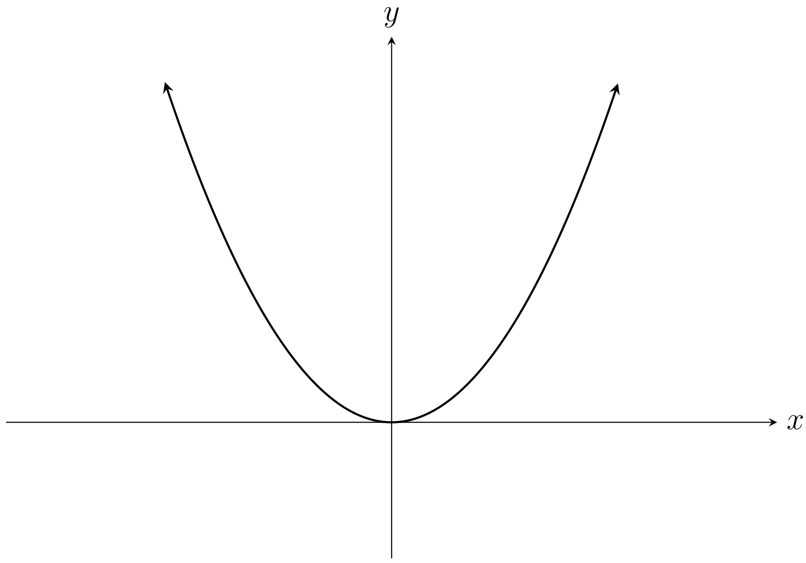
QUESTION TWENTY SIX (3 marks)

Find the area bounded between the graphs of $y = \sqrt{x-1}$ and $y = x - 1$. **3**

QUESTION TWENTY SEVEN (2 marks)



The diagram above shows the graph of a parabola $y = f(x)$, whose vertex is at the origin. Let k be some positive number.

(a) State coordinates of the vertex of $y = f(x) + k$ in terms of k. $\boxed{1}$

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(b) Write down the solution to $f(x) > f(x - k)$ in terms of k. $\boxed{1}$

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Extra writing space (Use this space only for questions in Part C)

If you use this space, clearly indicate which question you are answering.

CANDIDATE NUMBER

Section II

Part D

QUESTION TWENTY EIGHT (6 marks)

The mass of a decaying radioactive substance, in kg, at time t years is given by $M = 2e^{-kt}$, where k is a positive constant.

(a) State the initial mass of the radioactive substance. **1**

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(b) Find the half-life of the radioactive substance. Give your answer as an exact value in terms of k. **1**

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(c) Find $\dfrac{dM}{dt}$. **1**

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(d) A biomaterial is also decaying simultaneously, but at every instance in time its rate of decay is twice as fast as the radioactive substance. Find an expression for the mass of the biomaterial, in terms of k and t, given that its initial mass is 3 kg. **3**

QUESTION TWENTY NINE (6 marks)

A particle is moving along the x-axis. Its velocity in metres per second, at time t seconds, is given by

$$v = \frac{2}{t+1} - 1.$$

(a) Find the time at which the particle is stationary. **1**

(b) Find the acceleration of the particle when it is stationary. **2**

Question 29 continues on the next page.

(c) The total distance travelled by the particle in the first two seconds is $\ln b$ metres, for some constant $b > 0$. Find the value of b. **3**

QUESTION THIRTY (3 marks)

(a) Find $\dfrac{d}{dx}\, x \sin x$. $\boxed{1}$

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(b) Hence, find $\displaystyle\int x \cos x \, dx$. $\boxed{1}$

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(c) Hence, find $\displaystyle\int_0^{\pi} x \cos x \, dx$. $\boxed{1}$

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QUESTION THIRTY ONE (3 marks)

In early planetary models, planets orbit the sun in circular orbits. Suppose a planet completes its orbit of radius r around the sun in time T, while moving at a constant speed v.

(a) Express v in terms of r and T by using: **1**

$$\text{constant speed} = \frac{\text{distance travelled}}{\text{time}}.$$

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Question 31 continues on the next page.

(b) By examining astronomical data made in the 1600's, Kepler noticed that r^3 is always directly proportional to T^2 for all orbits, that is, $r^3 = kT^2$, for some fixed positive constant k. By finding an appropriate derivative, determine whether planets further from the sun move faster or slower. **2**

Extra writing space (Use this space only for questions in Part D)

If you use this space, clearly indicate which question you are answering.

CANDIDATE NUMBER

Section II

Part E

QUESTION THIRTY TWO (4 marks)

A patient is undergoing a treatment involving repeated injections of M milliliters of medicine, separated by regular time intervals of t hours. Let A_n be the amount of medicine in the body immediately after the n^{th} injection, so $A_1 = M$. While in the body, the medicine decays exponentially such that the amount left in the body right before the $(n+1)^{\text{th}}$ injection is given by $A_n e^{-t}$.

(a) Find an expression for A_2, and show that $A_3 = M(1 + e^{-t} + e^{-2t})$. **1**

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Question 32 continues on the next page.

(b) The patient is safe from side effects as long as the amount of medication in the body stays below $2M$ milliliters. Find the minimum value of t, to nearest minute, such that the patient is always safe as this treatment continues indefinitely. **3**

The paper continues on the next page.

QUESTION THIRTY THREE (5 marks)

The Grammar timetable follows a six day cycle, with the days labelled A to F. In a certain year, the first school day is Monday and is an A day. Part of the timetable is shown in the table below, with weeks 8 to 35 omitted. For simplicity we assume the entire school year consists of 40 weeks.

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1	A	B	C	D	E
Week 2	F	A	B	C	D
Week 3	E	F	A	B	C
Week 4	D	E	F	A	B
Week 5	C	D	E	F	A
Week 6	B	C	D	E	F
Week 7	A	B	C	D	E
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Week 36	B	C	D	E	F
Week 37	A	B	C	D	E
Week 38	F	A	B	C	D
Week 39	E	F	A	B	C
Week 40	D	E	F	A	B

(a) Find the probability that a randomly selected school day is an A day. **1**

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(b) When a school day is randomly selected, find the probability that the day is an A day, given that it is a Monday. Hence state, with justification, whether these two events are independent. **2**

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Question 33 continues on the next page.

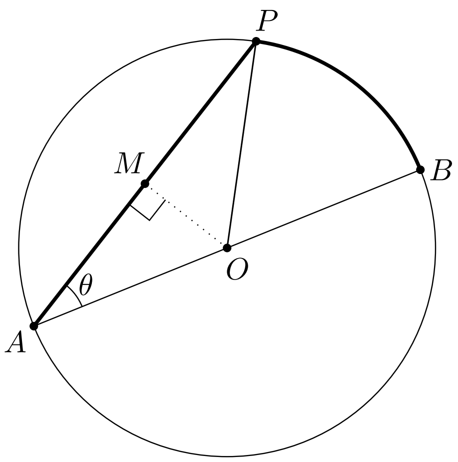
(c) Suppose there are: **2**

- 0 periods of maths on A days.
- 1 period of maths on B days.
- 2 periods of maths on each of D, E and F days.
- 3 periods of maths on C days.

Let X be the number of maths periods on a randomly chosen Monday. Find $E(X)$.

The paper continues on the next page.

QUESTION THIRTY FOUR (7 marks)



Alice wants to move from point A to point B, which are two points on the circumference of a circular pond with centre O and radius R, such that AB is a diameter. She can do so by swimming directly across the pond, or run the arc AB. Alternatively, she also has the option of first swimming in a straight line to a chosen point P on the circumference, then running arc PB.

Let $\angle PAB = \theta$, where $0 \leq \theta \leq \dfrac{\pi}{2}$. Alice swims and runs at constants speeds v_1 and v_2 respectively, where $0 < v_1 < v_2$. Let t be the time it takes for her to reach B.

(a) If M is the midpoint of AP then $\angle OMA = \dfrac{\pi}{2}$. (Do NOT show this) $\boxed{1}$

Find AM in terms of R and θ.

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Question 34 continues on the next page.

(b) Find an expression, in terms of v_1 and v_2, for the angle θ which <u>maximises</u> t. **4**

Question 34 continues on the next page.

(c) Find the condition on $\dfrac{v_2}{v_1}$ so that the minimum value of t is achieved by only running. **2**

END OF PAPER

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If you use this space, clearly indicate which question you are answering.

SYDNEY GRAMMAR SCHOOL

CANDIDATE NUMBER

2025 **Trial Examination**

Form VI

Mathematics Advanced

Thursday 14th August, 2025

Instructions

- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ◯ B ◯ C ◯ D ◯

Question Two

A ◯ B ◯ C ◯ D ◯

Question Three

A ◯ B ◯ C ◯ D ◯

Question Four

A ◯ B ◯ C ◯ D ◯

Question Five

A ◯ B ◯ C ◯ D ◯

Question Six

A ◯ B ◯ C ◯ D ◯

Question Seven

A ◯ B ◯ C ◯ D ◯

Question Eight

A ◯ B ◯ C ◯ D ◯

Question Nine

A ◯ B ◯ C ◯ D ◯

Question Ten

A ◯ B ◯ C ◯ D ◯

2025 Form VI Mathematics Advanced Trial Solutions

Question	1	2	3	4	5	6	7	8	9	10
Answer	D	A	A	C	B	B	C	D	A	A

Question 1 (1 mark)

$-\frac{1}{2} \times 2 = -1$

$\therefore D$ (✓)

Question 2 (1 mark)

$x - 3 > 0$

$x > 3$

$\therefore A$ (✓)

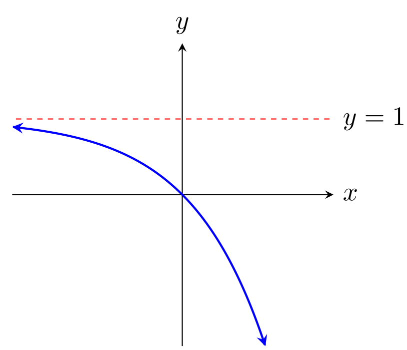
Question 3 (1 mark)

$e^x > 0$

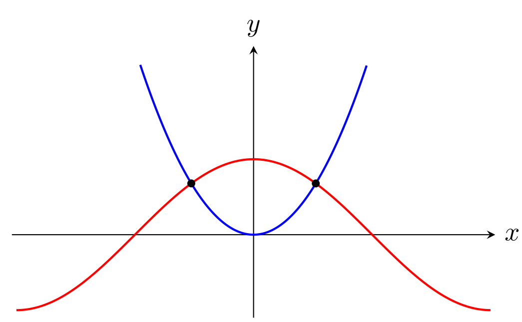
$\therefore 1 - e^x < 1$

$\therefore A$ (✓)

Result can also be seen graphically:



Question 4 (1 mark)



There are two intersections between the graphs $\therefore C$ (✓)

Question 5 (1 mark)

Replacing y with $\dfrac{y}{3}$ gives $\dfrac{y}{3} = f(x)$

$\therefore\ y = 3f(x)$

$\therefore\ B$ $(\checkmark)$

Question 6 (1 mark)

If $v = 1$ and $a = -2$, velocity is decreasing and speed is decreasing

If $v = -1$ and $a = 2$, velocity is increasing and speed is decreasing

$\therefore\ B$ $(\checkmark)$

Question 7 (1 mark)

At $t = b$, the particle returns to its starting position after traveling to the right,

after which its moves to the left until $t = c$, which gives its minimum displacement

$\therefore\ C$ $(\checkmark)$

Question 8 (1 mark)

$h(x) = [f(3x)]^2$

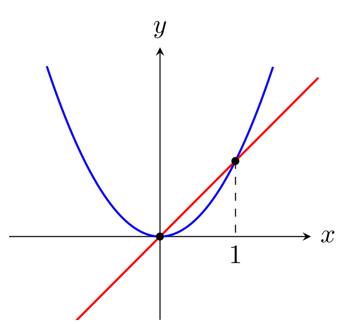
$h'(x) = 2f(3x) \times \dfrac{d}{dx} f(3x)$

$h'(x) = 2f(3x) \times f'(3x) \times 3$

$h'(x) = 6xf'(3x)\, f(3x)$

$\therefore\ D$ $(\checkmark)$

Question 9 (1 mark)



The parabola is below the line between $x = 0$ and $x = 1 \therefore A$ $(\checkmark)$

Question 10 (1 mark)

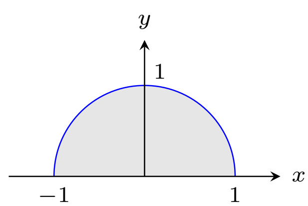
$f(x) = \sin x^3$ is an odd function so $\displaystyle\int_{-1}^{1} \sin x^3 \, dx = 0$

$$\int_{-1}^{1} e^x \, dx = \Big[e^x\Big]_{-1}^{1} = e - \frac{1}{e} = 2.35\cdots$$

$f(x) = \dfrac{e^{x^2}}{2}$ is an even function, so $\displaystyle\int_{-1}^{1} \frac{e^{x^2}}{2} \, dx = \int_{0}^{1} e^{x^2} \, dx$

$\displaystyle\int_{0}^{1} e^{x^2} \, dx \le \int_{0}^{1} e^{x} \, dx$ as $x^2 \le x$ for $0 \le x \le 1$

$$\therefore \int_{-1}^{1} \frac{e^{x^2}}{2} \, dx \le \Big[e^x\Big]_{0}^{1} = e - 1 = 1.71\cdots$$



$$\int_{-1}^{1} \sqrt{1 - x^2} \, dx = \frac{1}{2}\pi(1)^2 = \frac{\pi}{2} = 1.57\cdots$$

$\therefore A$ (✓)

Question 11 (2 marks)

$x - \dfrac{6}{x} = 1$

$x^2 - x - 6 = 0$ (✓)

$(x-3)(x+2) = 0$

$x = 3, \ x = -2$ (✓)

Question 12 (5 marks)

(a) [2]

$y = 2x^3 + \dfrac{1}{x}$

$y = 2x^3 + x^{-1}$

$y' = 6x^2 - x^{-2}$ (✓✓)

$y' = 6x^2 - \dfrac{1}{x^2}$

(b) [1]

$y' = 15e^{3x}$ (✓)

(c) [1]

$y' = 2\sec^2 2x$ (✓)

(d) [1]

$y' = \dfrac{3}{3x-1}$ (✓)

Question 13 (3 marks)

$y = \sqrt{2x+1}$

$y = (2x+1)^{1/2}$

$y' = \dfrac{1}{2}(2x+1)^{-1/2} \times 2$

$y' = \dfrac{1}{\sqrt{2x+1}}$ (✓)

$y'(4) = \dfrac{1}{3}$ and $y(4) = 3$ (✓)

$y - 3 = \dfrac{1}{3}(x-4)$

$3y - 9 = x - 4$

$x - 3y + 5 = 0$ (✓)

Question 14 (5 marks)

(a) [1]

$0.2 + 0.25 + 0.3 = 0.75$ (✓)

(b) [1]

$0.75 + 0.1 + k = 1$

$k = 0.15$ (✓)

(c) [1]

$E(X) = 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.25 + 3 \times 0.3 + 4 \times 0.15 = 2.2$ (✓)

(d) [1]

$E(X^2) = 0^2 \times 0.1 + 1^2 \times 0.2 + 2^2 \times 0.25 + 3^2 \times 0.3 + 4^2 \times 0.15 = 6.3$ (✓)

(e) [1]

$\text{Var}(X) = E(X^2) - \mu^2$

$\text{Var}(X) = 6.3 - 2.2^2 = 1.46$ (✓)

Question 15 (2 marks)

(a) [1]

$500 \times 14.487 = \$7244$ (nearest dollar) (✓)

(b) [1]

$500 \times 60.402 = \$30201$ (✓)

Question 16 (2 marks)

$-1 < 1 - 2x < 1$ (✓)

$-2 < -2x < 0$

$0 < x < 1$ (✓)

Question 17 (3 marks)

(a) [1]

$1 - \left(\frac{2}{3}\right)^3 = \frac{19}{27}$ (✓)

(b) [2]

$3 \times \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 = \frac{4}{9}$ (✓✓)

Q17(b) comments

1 mark for $\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$

Question 18 (2 marks)

$2025 = 1 + (n-1) \times 4$

$2024 = 4(n-1)$

$506 = n - 1$

$n = 507$ (✓)

$\frac{507}{2}(1 + 2025) = 513591$ (✓)

Question 19 (2 marks)

(a) [1]

$A = \frac{1}{2} \times 5 \times 6 \times \sin 60 = 12.99\,\text{cm}^2$ (✓)

(b) [2]

$x^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \cos 60$ (✓)

$x = \sqrt{31}$ (✓)

Question 20 (2 marks)

(a) [1]

$$T = \frac{2\pi}{3}$$ (✓)

(b) [1]

$2 \leq y \leq 6$ (✓)

Question 21 (6 marks)

(a) [3]

$$y = \frac{1}{27}x^3 - x + 2$$

$$y' = \frac{1}{9}x^2 - 1$$ (✓)

$y' = 0$ when $\frac{1}{9}x^2 - 1 = 0$

$y' = 0$ when $x^2 = 9$

$y' = 0$ when $x = -3$, $x = 3$

stationary points are at $(-3, 4)$ and $(3, 0)$

$$y'' = \frac{2}{9}x$$

$y''(-3) = -\frac{2}{3} < 0$, therefore $(-3, 4)$ is a local maximum turning point

$y''(3) = \frac{2}{3} > 0$, therefore $(3, 0)$ is a local minimum turning point (✓✓)

Q21(a) comments
1 mark for finding y' deduct 1 mark for each subsequent error in finding and classifying the two stationary points

(b) [2]

$y'' = 0$ when $x = 0$

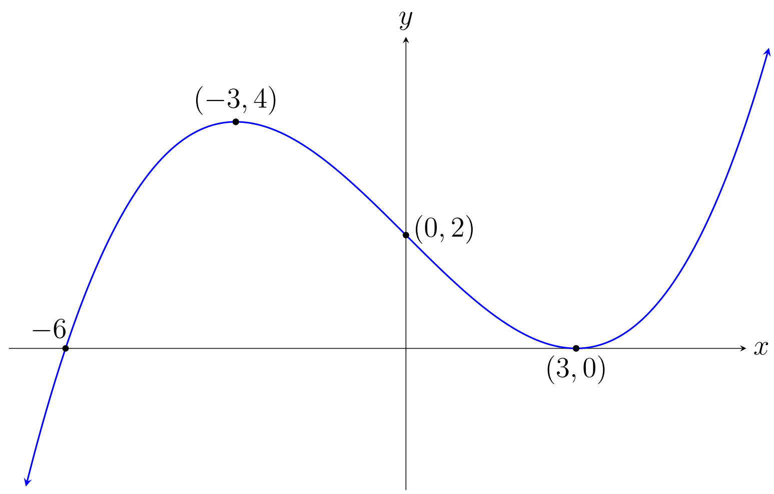
therefore there is a potential point of inflexion at $(0, 2)$ (✓)

$y''(-1) = -\frac{2}{9} < 0$

$y''(1) = \frac{2}{9} > 0$

therefore there is a concavity change about $x = 0$, $\therefore (0, 2)$ is a point of inflexion (✓)

(c) [1]



(✓)

Question 22 (4 marks)

(a) [2]

$$\int \frac{1}{x^2} - \cos 3x \, dx$$

$$= \int x^{-2} - \cos 3x \, dx$$

$$= -x^{-1} - \frac{1}{3}\sin 3x + C$$ (✓✓)

$$= -\frac{1}{x} - \frac{1}{3}\sin 3x + C$$

Q22(a) comments

Don't penalise $+C$

(b) [2]

$$\int (1-2x)^5 \ dx$$

$$= \frac{(1-2x)^6}{6 \times -2}$$

$$= -\frac{1}{12}(1-2x)^6 + C$$ (✓✓)

Q22(b) comments

1 for $\dfrac{(1-2x)^6}{6}$

Question 23 (5 marks)

(a) i. [1]

$A_1 = 10000(1.05) - M$

$A_2 = A_1(1.05) - M$

$A_2 = [10000(1.05) - M](1.05) - M$ (✓)

$A_2 = 10000(1.05)^2 - M(1.05) - M$

ii. [1]

$A_2 = 10000(1.05)^2 - M(1 + 1.05)$

$A_n = 10000(1.05)^n - M(1 + 1.05 + 1.05^2 + 1.05^3 + \cdots + 1.05^{n-1})$ (✓)

iii. [2]

$A_n = 10000(1.05)^n - M \times \frac{1.05^n - 1}{0.05}$ (✓)

$A_n = 10000(1.05)^n - 20M(1.05^n - 1)$

$A_n = 0$ when $n = 10$

$0 = 10000(1.05)^{10} - 20M(1.05^{10} - 1)$

$M = \frac{10000(1.05)^{10}}{20(1.05^{10} - 1)} = \$1295.05$ (✓)

(b) [1]

Let B be the maximum he can borrow. In part(a)(iii) we found:

$A_n = 10000(1.05)^n - 20M(1.05^n - 1)$

We replace 10000 with B, M with 350 and sub $n = 20$, we get:

$0 = B(1.05)^{20} - 20(350)(1.05^{20} - 1)$

$B = \frac{20(350)(1.05^{20} - 1)}{1.05^{20}} = \$4361.77$ (✓)

Question 24 (3 marks)

(a) [1]

$u = e^x \qquad v = x + 1$

$u' = e^x \qquad v' = 1$

$$\frac{d}{dx}\, e^x(x+1) = e^x(x+1) + e^x \qquad (\checkmark)$$

(b) [1]

$$\frac{d^2}{dx^2}\, e^x(x+1) = e^x(x+1) + e^x + e^x \qquad (\checkmark)$$

$$\frac{d^2}{dx^2}\, e^x(x+1) = e^x(x+1) + 2e^x$$

$$\frac{d^2}{dx^2}\, e^x(x+1) = e^x(x+3)$$

(c) [1]

$$\frac{d^n}{dx^n}\, e^x(x+1) = e^x(x+n+1) \qquad (\checkmark)$$

Question 25 (4 marks)

(a) [2]

$$h = \frac{10-1}{3} = 3 \qquad (\checkmark)$$

$$A \approx \frac{3}{2}\left[f(1) + 2f(4) + 2f(7) + f(10)\right]$$

$$A \approx \frac{3}{2}\left[\ln 1 + 2\ln 4 + 2\ln 7 + \ln 10\right] = 13.45\,(2\,\mathrm{dp}) \qquad (\checkmark)$$

(b) [2]

$x = e^y$

$$A = 10\ln 10 - \int_0^{\ln 10} e^y\, dy \qquad (\checkmark)$$

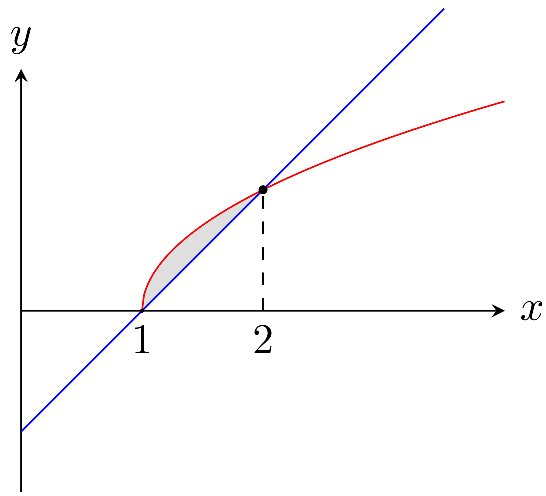
$$A = 10\ln 10 - \Big[e^y\Big]_0^{\ln 10}$$

$$A = 10\ln 10 - (10 - 1)$$

$$A = 10\ln 10 - 9 \qquad (\checkmark)$$

Q25(b) comments
1 mark for $10\ln 10$ or $x = e^y$

Question 26 (3 marks)



$\sqrt{x-1} = x - 1$

$x - 1 = x^2 - 2x + 1$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1\,,\ x = 2$ $(\checkmark)$

$A = \int_1^2 \sqrt{x-1} - (x-1)\,dx$ $(\checkmark)$

$A = \int_1^2 (x-1)^{1/2} - x + 1\,dx$

$A = \left[\frac{2}{3}(x-1)^{3/2} - \frac{x^2}{2} + x\right]_1^2$

$A = \frac{2}{3} - 2 + 2 - \left(-\frac{1}{2} + 1\right)$

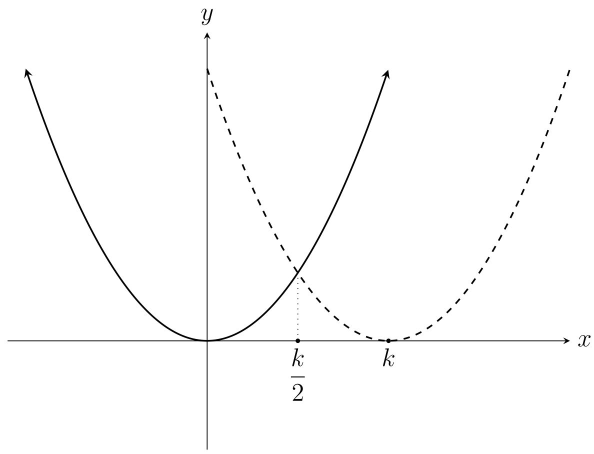
$A = \frac{1}{6}$ $(\checkmark)$

Question 27 (2 marks)

(a) [1]

$(0, k)$ (✓)

(b) [1]



$f(x) > f(x-k)$ when $x > \dfrac{k}{2}$ (✓)

Question 28 (6 marks)

(a) [1]

2 kg (✓)

(b) [1]

$1 = 2e^{-kt}$

$e^{kt} = 2$

$kt = \ln 2$

$t = \dfrac{1}{k}\ln 2$ (✓)

(c) [1]

$\dfrac{dM}{dt} = -2ke^{-kt}$ (✓)

(d) [3]

$\dfrac{dB}{dt} = -4ke^{-kt}$ (✓)

$B = \displaystyle\int -4ke^{-kt}\,dt$

$B = 4e^{-kt} + C$ (✓)

when $t = 0,\ B = 3$

$3 = 4 + C\ \therefore C = -1$

$\therefore B = 4e^{-kt} - 1$ (✓)

Question 29 (6 marks)

(a) [1]

$$\frac{2}{t+1} - 1 = 0$$

$$\frac{2}{t+1} = 1$$

$$2 = t + 1$$

$$t = 1\,\text{second} \qquad (\checkmark)$$

(b) [2]

$$v = 2(t+1)^{-1} - 1$$

$$a = -2(t+1)^{-2} \qquad (\checkmark)$$

$$a = -\frac{2}{(t+1)^2}$$

$$a(1) = -\frac{1}{2}\,\text{ms}^{-2} \qquad (\checkmark)$$

(c) [3]

$$\int_0^1 \frac{2}{t+1} - 1\,dt$$

$$= \Big[2\ln|t+1| - t\Big]_0^1$$

$$= 2\ln 2 - 1 \qquad (\checkmark)$$

As $a < 0$ for all t, and $v = 0$ when $t = 1$, the velocity will be negative after $t = 1$

so the distance travelled in the next second will be given by:

$$\Big[t - 2\ln|t+1|\Big]_1^2$$

$$= 2 - 2\ln 3 - (1 - 2\ln 2)$$

$$= 1 - 2\ln 3 + 2\ln 2 \qquad (\checkmark)$$

total distance is given by:

$$2\ln 2 - 1 + 1 - 2\ln 3 + 2\ln 2$$

$$= 4\ln 2 - 2\ln 3$$

$$= \ln 2^4 - \ln 3^2$$

$$= \ln\left(\frac{16}{9}\right) \qquad (\checkmark)$$

$$\therefore b = \frac{16}{9}$$

Question 30 (3 marks)

(a) [1]

$u = x \qquad v = \sin x$

$u' = 1 \qquad v' = \cos x$

$\frac{d}{dx} x \sin x = \sin x + x \cos x$ (✓)

(b) [1]

$$\int \sin x + x \cos x \, dx = x \sin x$$

$$\int \sin x \, dx + \int x \cos x \, dx = x \sin x$$

$$-\cos x + \int x \cos x \, dx = x \sin x$$

$$\int x \cos x \, dx = x \sin x + \cos x + C$$ (✓)

(c) [1]

$$\int_0^{\pi} x \cos x \, dx = \Big[x \sin x + \cos x \Big]_0^{\pi} = -2$$ (✓)

Question 31 (3 marks)

(a) [1]

$$v = \frac{2\pi r}{T}$$ (✓)

(b) [2]

$r^3 = kT^2$

$$\frac{1}{T^2} = \frac{k}{r^3}$$

$$\frac{1}{T} = \frac{\sqrt{k}}{r^{3/2}}$$

Substituting this into part (a) we get:

$v = 2\pi\sqrt{k} r^{-1/2}$ (✓)

$$\frac{dv}{dr} = -\pi\sqrt{k} r^{-3/2}$$

$$\frac{dv}{dr} = -\frac{\pi\sqrt{k}}{r\sqrt{r}}$$

$\therefore \frac{dv}{dr}$ is always negative for $r > 0$

$\therefore v(r)$ is a decreasing function for $r > 0$, therefore planets further away move slower (✓)

Q31(b) comments

Right conclusion based on correct derivative for last mark

Question 32 (4 marks)

(a) [1]

$A_2 = A_1 e^{-t} + M$

$A_2 = Me^{-t} + M$

$A_3 = A_2 e^{-t} + M$

$A_3 = (Me^{-t} + M)e^{-t} + M$

$A_3 = M + Me^{-t} + Me^{-2t}$ (✓)

$A_3 = M(1 + e^{-t} + e^{-2t})$

(b) [3]

Since A_n is an increasing function in n, we need to consider A_∞ for treatment to always be safe

A_∞ exists as $0 < e^{-t} < 1$ for $t \geq 0$

$A_\infty = M(1 + e^{-t} + e^{-2t} + e^{-3t} + \cdots)$

$A_\infty = M \times \dfrac{1}{1 - e^{-t}}$

$A_\infty = \dfrac{M}{1 - e^{-t}}$ (✓)

We require $A_\infty < 2M$

$\dfrac{M}{1 - e^{-t}} < 2M$

$\dfrac{1}{1 - e^{-t}} < 2$

$\dfrac{e^t}{e^t - 1} < 2$

$e^t < 2e^t - 2$

$e^t > 2$ (✓)

$t > \ln 2$ hours

$\ln 2 \times 60 = 41.588\cdots$

$t > 41.588\cdots$ mins

$t = 42$ mins (✓)

Question 33 (5 marks)

(a) [1]

There is an A day every week, except weeks 6, 12, 18, 24, 30, 36

$\therefore P(\mathrm{A}) = \dfrac{34}{200} = \dfrac{17}{100} = 17\%$ (✓)

(b) [2]

$P(\mathrm{A} \,|\, \mathrm{Monday}) = \dfrac{|\mathrm{A} \cap \mathrm{Monday}|}{|\mathrm{Mondays}|} = \dfrac{7}{40} = 17.5\%$ (✓)

Since this is not equal to the answer in part (i), these events are dependent (✓)

(c) [2]

$P(\mathrm{B} \,|\, \mathrm{Monday}) = P(\mathrm{C} \,|\, \mathrm{Monday}) = \dfrac{6}{40} = \dfrac{3}{20} = 15\%$ (✓)

$P(\mathrm{A} \,|\, \mathrm{Monday}) = P(\mathrm{D} \,|\, \mathrm{Monday}) = P(\mathrm{E} \,|\, \mathrm{Monday}) = P(\mathrm{F} \,|\, \mathrm{Monday}) = \dfrac{7}{40}$

$E(X) = 0 \times \dfrac{7}{40} + 1 \times \dfrac{6}{40} + 2 \times \dfrac{7}{40} \times 3 + 3 \times \dfrac{6}{40} = 1.65$ (✓)

Question 34 (7 marks)

(a) [1]

$$\cos\theta = \frac{AM}{R}$$

$$AM = R\cos\theta \qquad (\checkmark)$$

(b) [4]

$$t = \frac{2R\cos\theta}{v_1} + \frac{2R\theta}{v_2} \qquad (\checkmark)$$

$$\frac{dt}{d\theta} = -\frac{2R}{v_1}\sin\theta + \frac{2R}{v_2} \qquad (\checkmark)$$

$\dfrac{dt}{d\theta} = 0$ when:

$$\frac{2R}{v_1}\sin\theta = \frac{2R}{v_2}$$

$$\sin\theta = \frac{v_1}{v_2}$$

$$\theta = \sin^{-1}\left(\frac{v_1}{v_2}\right) \qquad (\checkmark)$$

$$\frac{d^2t}{d\theta^2} = -\frac{2R}{v_1}\cos\theta$$

Since $\dfrac{d^2t}{d\theta^2} \leq 0$ in the entire domain, $\theta = \sin^{-1}\left(\dfrac{v_1}{v_2}\right)$ is a local maximum. $(\checkmark)$

As this is the only turning point between the end points, the global maximum cannot occur at the end points, thus $\theta = \sin^{-1}\left(\dfrac{v_1}{v_2}\right)$ gives the global maximum.

(c) [2]

The minimum must occur at the end points $\theta = 0$ or $\theta = \dfrac{\pi}{2}$

When $\theta = 0$, $t = \dfrac{2R}{v_1}$

When $\theta = \dfrac{\pi}{2}$, $t = \dfrac{\pi R}{v_2}$ $(\checkmark)$

For the minimum to occur by only running, we require $\dfrac{\pi R}{v_2} < \dfrac{2R}{v_1}$

So $\dfrac{v_2}{v_1} > \dfrac{\pi}{2}$ $(\checkmark)$