2024 Form IV Yearly Solutions

1. (a) i. [1]

   $x = 3$ or $x = -6$ (✓)

   ii. [1]

   $x = \pm 4$ (✓)

   (b) [1]

   x^2 (✓)

   (c) [1]

   $\frac{10}{x^3}$ (✓)

   (d) [1]

   1.34×10^5 (3 significant figures) (✓✓)

   Q1(d) comments

   1 mark for 1.34 (don't penalise rounding anywhere else)

   1 mark for 10^5

   (e) [1]

   $2x + \frac{x}{2} = 1$

   $4x + x = 2$

   $5x = 2 \therefore x = \frac{2}{5}$ (✓)

   (f) i. [1]

   $(a + 5)(a - 5)$ (✓)

   ii. [2]

   $(3x - 2)(2x + 1)$ (✓)

   Q1(f)(ii) comments

   1 mark for $3x$ and $2x$

   (g) i. [1]

   5 (✓)

   ii. [1]

   -11 (✓)

2. (a) [2]

$y - 2 = 4(x - 1)$ (✓)

$y = 4x - 2$ (✓)

(b) [1]

$\log_3 x = 5$

$x = 3^5 = 243$ (✓)

(c) i. [1]

$P(2) = 2^3 - 2 + 5 = 11$ (✓)

ii. [1]

$P(x) + Q(x) = x^3 - x + 5 + x + 1 = x^3 + 6$ (✓)

(d) [1]

$x = \dfrac{1 \pm \sqrt{1 - 4(2)(-2)}}{2(2)} = \dfrac{1 \pm \sqrt{17}}{4}$ (✓)

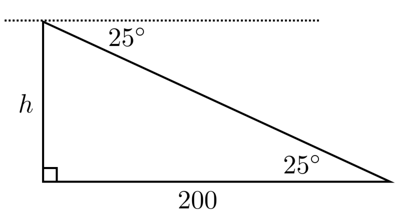
(e) i. [1]

$3 - 1 = 2$ (✓)

ii. [1]

3 (✓)

(f) i. [1]



(✓)

Q2(f)(i) comments

Either position for the 25° is ok

ii. [1]

$\tan 25^\circ = \dfrac{h}{200}$

$h = 200 \tan 25^\circ = 93\,\text{m}$ (nearest metre) (✓)

Q2(f)(ii) comments

Do not penalise units anywhere in this paper

(g) i. [1]

$$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$ (✓)

ii. [1]

$$= 1 - P(\text{all heads})$$

$$= 1 - \frac{1}{8} = \frac{7}{8}$$ (✓)

3. (a) i. [1]

$$\bar{x} = \frac{1+2+5+8}{4} = 4$$ (✓)

ii. [1]

$$\sigma = \sqrt{\frac{(1-4)^2 + (2-4)^2 + (5-4)^2 + (8-4)^2}{4}} = \frac{\sqrt{30}}{2} = 2.74\,(2\,\text{dp})$$ (✓)

(b) [1]

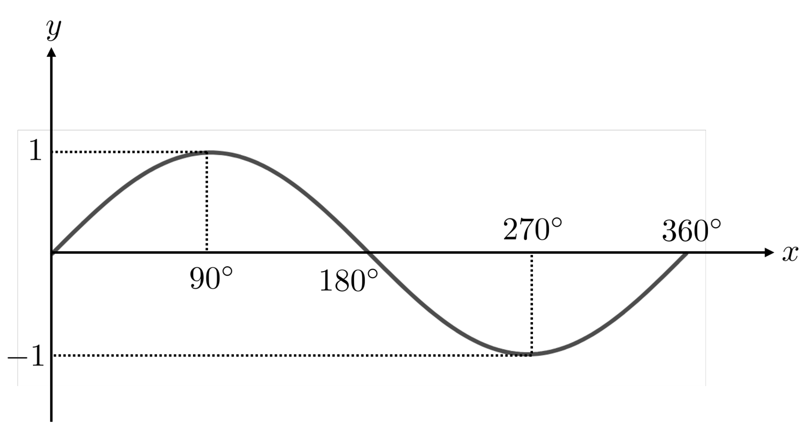
$$2^x = \frac{1}{4}$$

$$2^x = 2^{-2} \quad \therefore x = -2$$ (✓)

(c) [1]

$$r = P(2) = 2^3 - 5(2)^2 + 3 = -9$$

(d) i. [2]



(✓✓)

Q3(d)(i) comments

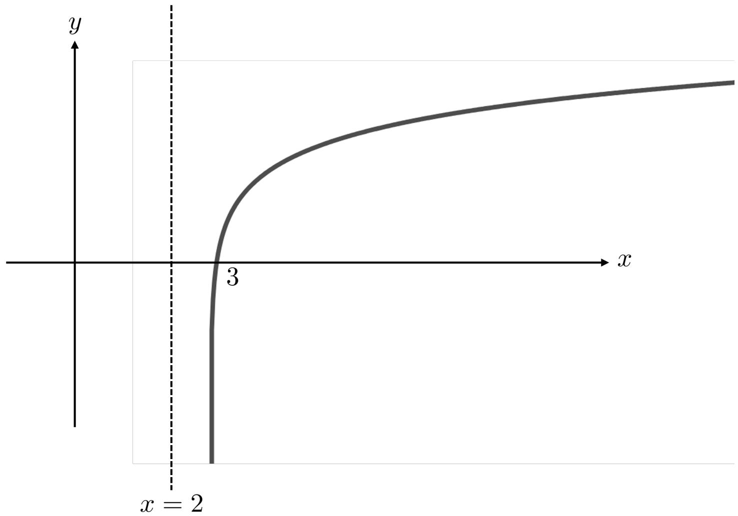
1 mark for both amplitude labels and correct sine curve

1 mark for all four x values

ii. [1]

$$\theta = 90^\circ$$ (✓)

(e) [2]



(✓✓)

Q3(e) comments

1 mark for log curve

1 mark for vertical asymptote and intercept

(f) i. [1]

$\log_2 xy = \log_2 x + \log_2 y = a + b$ (✓)

ii. [1]

$\log_2 \left(\frac{x^2}{y}\right) = 2\log_2 x - \log_2 y = 2a - b$ (✓)

iii. [1]

$\log_y x = \frac{\log_2 x}{\log_2 y} = \frac{a}{b}$ (✓)

4. (a) [2]

$\theta = 60°,\ 300°$ (✓✓)

(b) i. [1]

$x = \frac{0+6}{2} = 3$ (✓)

ii. [1]

When $x = 3$, $y = 3(-3) = -9$ (✓)

$V = (3, -9)$

(c) [3]

$$\frac{\sin\theta}{5} = \frac{\sin 40^\circ}{4} \quad (\checkmark)$$

$$\sin^{-1}\left(\frac{5\sin 40^\circ}{4}\right) = 53^\circ \ \text{(nearest degree)} \quad (\checkmark)$$

$$\theta = 180^\circ - \sin^{-1}\left(\frac{5\sin 40^\circ}{4}\right) = 127^\circ \ \text{(nearest degree)} \quad (\checkmark)$$

(d) [2]

$$30 = 800 \times 10^{-t} \quad (\checkmark)$$

$$10^t = \frac{80}{3}$$

$$t = \log_{10}\left(\frac{80}{3}\right) \text{ hours} \quad (\checkmark)$$

(e) i. [1]

$$\frac{6}{36} = \frac{1}{6} \quad (\checkmark)$$

ii. [1]

$$1 - \frac{1}{6} = \frac{5}{6} \quad (\checkmark)$$

iii. [1]

$$\frac{1}{2} \times \frac{5}{6} = \frac{5}{12} \quad (\checkmark)$$

5. (a) i. [1]

$$A = \frac{1}{2} \times 2 \times 3 \times \sin 120^\circ = \frac{3\sqrt{3}}{2} \quad (\checkmark)$$

ii. [2]

$$BC^2 = 3^2 + 2^2 - 2 \times 2 \times 3 \times \cos 120^\circ \quad (\checkmark)$$

$$BC = \sqrt{19} \quad (\checkmark)$$

iii. [1]

$$\frac{1}{2} \times BC \times h = \frac{3\sqrt{3}}{2}$$

$$h = \frac{3\sqrt{3}}{\sqrt{19}} \quad (\checkmark)$$

$$h = \frac{3\sqrt{57}}{19}$$

(b) [2]

$y - 2 = a(x-3)^2$

When $x = 0$, $y = -7$

$-9 = a(-3)^2 \quad \therefore a = -1$

$y - 2 = -(x-3)^2$ (✓✓)

Q5(b) comments

1 mark for accounting for either vextex coordinate

1 mark for accounting for both vertex coordinates, the concavity and y intercept with the negative sign in front

Accept $y - 2 = -(x-3)^2$ or $y = -(x-3)^2 + 2$

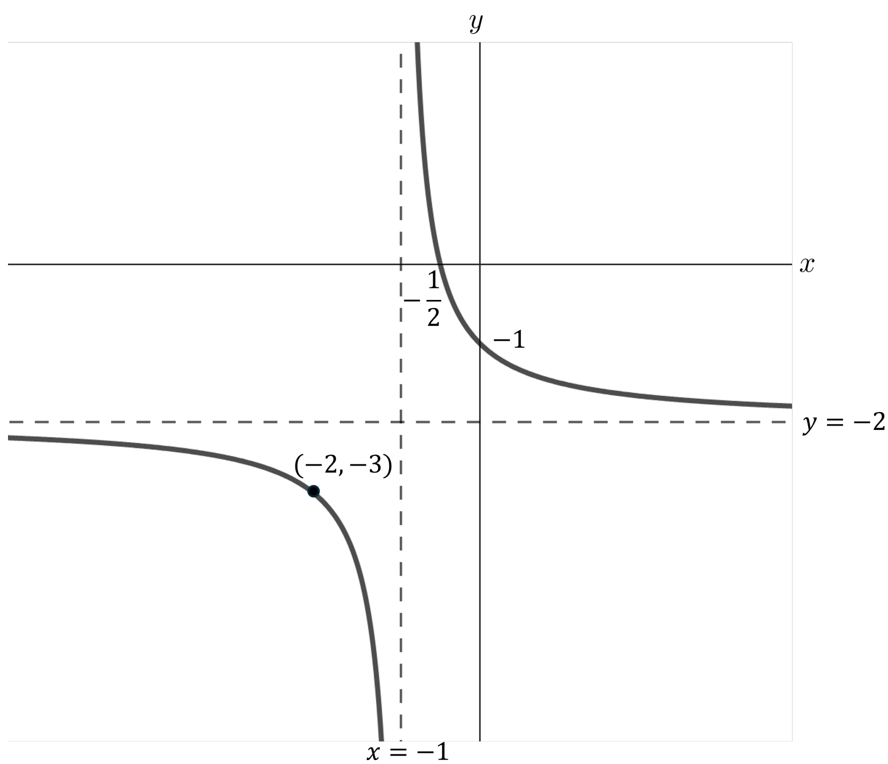
(c) i. [1]

$(0\,,\,-1)$ (✓)

ii. [1]

$\left(-\frac{1}{2}\,,\,0\right)$ (✓)

iii. [2]



(✓✓)

Q5(c)(iii) comments

1 mark for correct curve and intercepts

1 mark for asymptotes

$(-2, -3)$ optional

(d) [2]

$$x^2 - 4x + y^2 + 6y + 2 = 0$$

$$x^2 - 4x + 4 + y^2 + 6y + 9 = -2 + 4 + 9$$

$$(x-2)^2 + (y+3)^2 = 11 \qquad (\checkmark)$$

centre at $(2, -3)$ and radius is $\sqrt{11}$ $(\checkmark)$

6. (a) [2]

$$\alpha = \tan^{-1}(0.3)$$

$$\theta = 180 - \alpha = 163^\circ \text{ (nearest degree)} \qquad (\checkmark)$$

$$\theta = 360 - \alpha = 343^\circ \text{ (nearest degree)} \qquad (\checkmark)$$

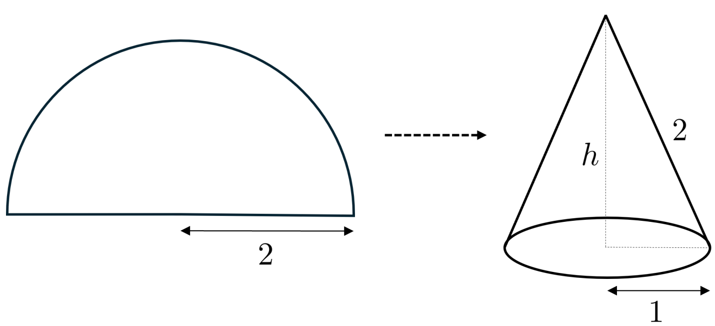
(b) [2]

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{1^2 + 1^2} = \sqrt{2} \qquad (\checkmark)$$

$$\tan\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) = 35^\circ \text{ (nearest degree)} \qquad (\checkmark)$$

(c)



i. [1]

$s = 2\,\text{cm}$ $(\checkmark)$

ii. [1]

$$\frac{1}{2} \times 2\pi(2) = 2\pi r \ \therefore\ r = 1\,\text{cm}$$

$$2\pi = 2\pi r \ \therefore\ r = 1\,\text{cm} \qquad (\checkmark)$$

iii. [2]

$$h = \sqrt{2^2 - 1^2} = \sqrt{3} \qquad (\checkmark)$$

$$V = \frac{1}{3}\pi(1)^2(\sqrt{3}) = \frac{\sqrt{3}}{3}\pi\,\text{cm}^3 \qquad (\checkmark)$$

(d) [2]

$$2\log_3 x + \log_3\left(\frac{6}{x}\right) = 4$$

$$\log_3\left(x^2 \times \frac{6}{x}\right) = 4$$

$$\log_3 6x = 4 \qquad (\checkmark)$$

$$6x = 3^4 \ \therefore x = \frac{27}{2} \qquad (\checkmark)$$

(e) [2]

$$32 > Q_3 + \frac{3}{2} \times IQR$$

$$32 > k + \frac{3}{2}(k-1) \qquad (\checkmark)$$

$$64 > 2k + 3k - 3$$

$$5k < 67$$

$$k < 13.4$$

Therefore the maximum integer value of k is 13 $(\checkmark)$

7. (a) [2]

$$\sin 3\theta = \frac{\sqrt{3}}{2}$$

$$0^\circ \leq 3\theta \leq 540^\circ$$

$$3\theta = 60^\circ,\ 120^\circ,\ 420^\circ,\ 480^\circ$$

$$\theta = 20^\circ,\ 40^\circ,\ 140^\circ,\ 160^\circ \qquad (\checkmark\checkmark)$$

Q7(a) comments

1 mark for any two correct solutions, or having 120° difference between both pairs of solutions

(b) i. [1]

$x - 2$ (✓)

ii. [2]

$$
\begin{array}{r|l}
 & 4x^2 + 8x - 5 \\ \hline
x - 2 & 4x^3 + 0 - 21x + 10 \\
 & -(4x^3 - 8x^2 \qquad\quad) \\ \hline
 & \qquad 8x^2 - 21x + 10 \\
 & \quad -(8x^2 - 16x \qquad) \\ \hline
 & \qquad\qquad -5x + 10 \\
 & \qquad -(-5x + 10) \\ \hline
 & \qquad\qquad\qquad 0
\end{array}
$$

(✓)

$P(x) = (x-2)(4x^2 + 8x - 5)$

$P(x) = (x-2)(2x-1)(2x+5)$ (✓)

Q7(b)(ii) comments

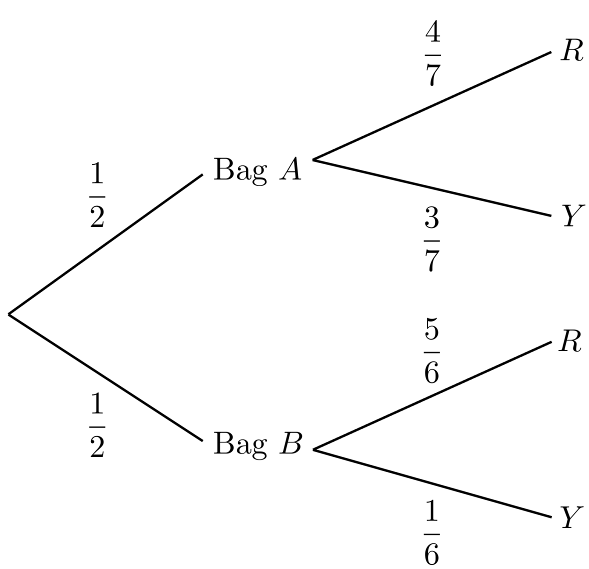
1 mark for $4x^3 - 8x^2$

1 mark for getting and factorising the quadratic factor

iii. [1]

$x = -\frac{5}{2},\ \frac{1}{2},\ 2$ (✓)

(c) i. [1]



(✓)

ii. [2]

$$P(A|R) = \frac{P(A \cap R)}{P(R)} = \frac{\frac{1}{2} \times \frac{4}{7}}{\frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{5}{6}} = \frac{24}{59} \qquad (\checkmark\checkmark)$$

Q7(c)(ii) comments

1 mark for any appearance of $\frac{1}{2} \times \frac{4}{7}$

iii. [1]

11 red balls can be added to bag A to make the bags identical in terms of the ratio of red to yellow balls, making the probabilities independent of which bag we choose.

$(\checkmark)$

(d) i. [1]

$y = 3^x + k$

$9 = 3^{-3} + k \ \therefore \ k = \frac{242}{27}$

move up by $\frac{242}{27}$ $(\checkmark)$

ii. [1]

$y = 3^{x-k}$

$9 = 3^{-3-k}$

$-3 - k = \log_3 9$

$-3 - k = 2 \ \therefore \ k = -5$

move left by 5 $(\checkmark)$

8. (a) [2]

$mx + 1 = \frac{1}{x}$

$mx^2 + x - 1 = 0$

$\Delta = 1^2 - 4(m)(-1) = 1 + 4m$ $(\checkmark)$

$\Delta < 0$ when $1 + 4m < 0$

$m < -\frac{1}{4}$ $(\checkmark)$

(b) i. [2]

Using the cosine rule on $\triangle POR$:

$$\cos\theta = \frac{7^2+5^2-3^2}{2\times5\times7}$$

$$\cos\theta = \frac{13}{14} \qquad (\checkmark)$$

$$\theta = \cos^{-1}\left(\frac{13}{14}\right) = 21^\circ\, 47\text{''} \text{ (nearest minute)} \qquad (\checkmark)$$

ii. [2]

Using the sine rule on $\triangle POQ$:

$$\frac{PQ}{\sin\theta} = \frac{5+RQ}{\sin 60^\circ} \qquad (\checkmark)$$

If $\cos\theta = \dfrac{13}{14}$ then $\sin\theta = \dfrac{3\sqrt{3}}{14}$

$$\frac{\sqrt{3}}{2}PQ = \frac{3\sqrt{3}}{14}(5+RQ)$$

$$7PQ = 3(5+RQ)$$

$$7PQ - 3RQ = 15 \qquad (\checkmark)$$

(c) [3]

$$4^x - 2^{x+2} - 12 = 0$$

$$(2^2)^x - 2^2\times 2^x - 12 = 0$$

$$(2^x)^2 - 4\times 2^x - 12 = 0 \qquad (\checkmark)$$

$$(2^x-6)(2^x+2) = 0 \qquad (\checkmark)$$

As $2^x > 0$, $2^x = 6$ is the only solution

$$\therefore\ (2^x)^3 = 6^3 \quad \therefore\ 8^x = 6^3 = 216 \qquad (\checkmark)$$

(d) [3]

$P(x) = a(x+1)^3 - b(x-1)$

$P(1) = 16$

$16 = a(2)^3 + 0 \therefore a = 2$ (✓)

$P(x) = 2(x+1)^3 - b(x-1)$

$P(0) = 0$

$0 = 2 - b(-1) \therefore b = -2$ (✓)

$P(x) = 2(x+1)^3 + 2(x-1)$

$P(-2) = -8$ (✓)

9. (a) [3]

$$\sigma^2 = \frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2 + \cdots + (x_n-\bar{x})^2}{n}$$

$n\sigma^2 = (x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2 + \cdots + (x_n-\bar{x})^2$

$n\sigma^2 = x_1^2 - 2x_1\bar{x} + \bar{x}^2 + x_2^2 - 2x_2\bar{x} + \bar{x}^2 + x_3^2 - 2x_3\bar{x} + \bar{x}^2 + \cdots + x_n^2 - 2x_n\bar{x} + \bar{x}^2$ (✓)

$n\sigma^2 = x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2 - 2\bar{x}(x_1 + x_2 + x_3 + \cdots + x_n) + n\bar{x}^2$

$0 = x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2 - 2n\bar{x}^2$ (as $n\bar{x}^2 = n\sigma^2$ and $x_1 + x_2 + x_3 + \cdots + x_n = n\bar{x}$)

$2n\bar{x}^2 = x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2$

$2n\sigma^2 = x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2$ (✓✓)

Q9(a) comments

1 mark for expanding the brackets

1 mark for:

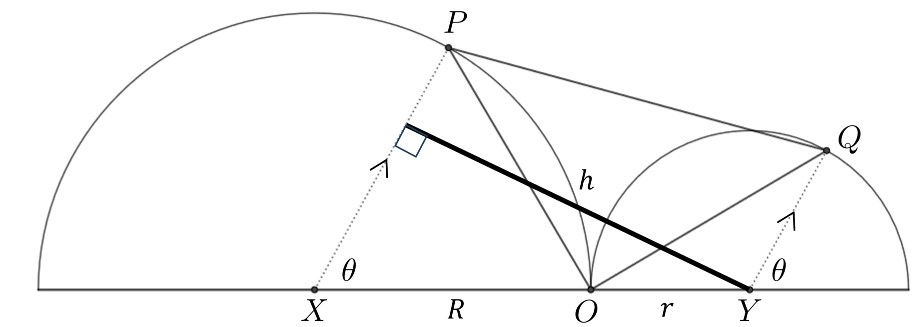
- realising there are n number of $\bar{x}$, gathering these and canceling this with $n\sigma^2$ on the left

or

- using $x_1 + x_2 + x_3 + \cdots + x_n = n\bar{x}$ to get the $2n\bar{x}^2$ term

1 mark for doing both and successfully getting the required result

(b) i. [2]



Let h be the perpendicular height of the trapezium

$$\sin\theta = \frac{h}{R+r}$$

$$h = (R+r)\sin\theta \qquad (\checkmark)$$

The area of the trapezium is given by:

$$A = \frac{1}{2}h(R+r)$$

$$A = \frac{1}{2}(R+r)^2\sin\theta \qquad (\checkmark)$$

ii. [2]

Area of $\triangle POQ$ = area of the trapezium − area of $\triangle POX$− area of $\triangle QOY$

$$\text{Area of } \triangle POQ = \frac{1}{2}(R+r)^2\sin\theta - \frac{1}{2}R^2\sin\theta - \frac{1}{2}r^2\sin(180-\theta) \qquad (\checkmark)$$

$$\text{Area of } \triangle POQ = \frac{1}{2}R^2\sin\theta + Rr\sin\theta + \frac{1}{2}r^2\sin\theta - \frac{1}{2}R^2\sin\theta - \frac{1}{2}r^2\sin\theta$$

$$\text{Area of } \triangle POQ = Rr\sin\theta$$

As $\sin\theta \le 1$, maximum area is Rr $(\checkmark)$

Q9(b)(ii) comments

1 mark for $\frac{1}{2}R^2\sin\theta$ or $\frac{1}{2}r^2\sin(180-\theta)$

(c) i. [1]

$y - b = m(x - a)$

ii. [2]

The intersection between this line and the parabola is given by:

$x^2 - b = m(x - a)$

$x^2 - mx + am - b = 0$

Since each line only cuts the parabola once, we have:

$m^2 - 4(am - b) = 0$

$m^2 - 4am + 4b = 0$

$$m = \frac{4a \pm \sqrt{16a^2 - 4(1)(4b)}}{2(1)}$$

$m = 2a \pm 2\sqrt{a^2 - b}$ (✓)

For $\angle RPQ$ to be $90°$ the product of these two possible gradients has to equal to -1

$(2a + 2\sqrt{a^2 - b})(2a - 2\sqrt{a^2 - b}) = -1$

Using difference of two squares on the left we get: $4a^2 - 4a^2 + 4b = -1$

Condition doesn't involve a, so a can be any value, while b has to equal to $-\frac{1}{4}$.

(✓)

(d) $P(x-k)$ is the translated graph. So $P(\alpha-k)$ is substituting $x=\alpha$ into the translated graph, which gives the remainder by the remainder theorem. We get same remainders when $P(\alpha-k)=P(\alpha)$. This condition cannot be met when the value $P(\alpha)$ is uniquely obtained with $x=\alpha$ and no other x values, giving no solutions for k. This uniqueness only occurs at the vertex of $P(x)$, thus $\alpha=-\frac{b}{2a}=-\frac{1}{2}$. [2]

Alternatively:

The remainders are given by $\alpha^2+\alpha+1$ and $(\alpha-k)^2+(\alpha-k)+1$.

The remainders are equal when:

$$(\alpha-k)^2+(\alpha-k)+1=\alpha^2+\alpha+1$$

$$\alpha^2-2\alpha k+k^2+\alpha-k+1=\alpha^2+\alpha+1$$

$$-2\alpha k+k^2-k=0$$

$$k-(2\alpha+1)=0$$

This condition will not be met for all non zero values of k when $2\alpha+1=0$. Therefore $\alpha=-\frac{1}{2}$.

Q9(d) comments

1 mark for referencing $P(\alpha-k)$

1 mark for correct α value

or

1 mark for $(\alpha-k)^2+(\alpha-k)+1$

1 mark for correct α value