SYDNEY GRAMMAR SCHOOL

NAME ____________________ MATHS MASTER ____________

2024 **Annual Examination**

Form IV Mathematics

Monday 4th November, 2024

Session A

General Instructions

- Working time — 2 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.

Nine Questions — 108 Marks

- Each question is worth 12 marks.
- Relevant mathematical reasoning and calculations are required.
- Record your answers on the writing paper provided.
- Start each question on a new page.

Collection

- Write your name and master on this page and on each page of writing paper.
- Arrange your solutions in order.
- Staple the sheets of writing paper together.

Classes

4A: KCCT	4B: RCF	4C: MCW	4D: WJM	4E: BR
4F: PC	4G: YH	4H: WJW	4I: LRP/AHSH	4J: PKS

Checklist

- 14 sheets of writing paper per boy
- Candidature: 205 pupils

Writer: YH

QUESTION ONE (12 marks) Start a new page.

(a) Solve:

(i) $(x-3)(x+6)=0$ **1**

(ii) $x^2=16$ **1**

(b) Simplify $3x \times \frac{x}{3}$. **1**

(c) Express $10x^{-3}$ using positive indices. **1**

(d) Express 133 510 in scientific notation correct to 3 significant figures. **2**

(e) Solve $2x+\frac{x}{2}=1$. **1**

(f) Factorise:

(i) a^2-25 **1**

(ii) $6x^2-x-2$ **2**

(g) Consider the polynomial $P(x)=6x^5-5x^3+x^2-11$.

(i) State the degree of the polynomial. **1**

(ii) State the constant term of the polynomial. **1**

The paper continues on the next page

QUESTION TWO (12 marks) Start a new page.

(a) Find the equation of the line parallel to $y = 4x - 11$, passing through the point $(1, 2)$. Give your answer in gradient-intercept form. **2**

(b) If $\log_3 x = 5$, find the value of x. **1**

(c) Consider the polynomials $P(x) = x^3 - x + 5$ and $Q(x) = x + 1$.

(i) Find the value of $P(2)$. **1**

(ii) Simplify the expression $P(x) + Q(x)$. **1**

(d) Use the quadratic formula to solve $2x^2 - x - 2 = 0$. **1**

(e)

Score	1	2	3
Frequency	2	3	6

For the data shown in the table above, find the:

(i) range **1**

(ii) median **1**

(f) A boat is 200 metres away from the base of a vertical cliff. From the top of the cliff the boat can be seen at an angle of depression of 25°.

(i) Sketch a diagram of the scenario, showing all given information. **1**

(ii) Find the height of the cliff. Give your answer correct to the nearest metre. **1**

(g) Three fair coins are tossed and the results recorded. Find the probability of obtaining:

(i) three heads **1**

(ii) at least one tail **1**

The paper continues on the next page

QUESTION THREE (12 marks) Start a new page.

(a) Consider the scores 1, 2, 5, 8.

(i) Find the mean of the scores. **1**

(ii) Find the standard deviation of the scores correct to two decimal places. **1**

(b) Solve $2^x = \dfrac{1}{4}$. **1**

(c) Find the remainder when $P(x) = x^3 - 5x^2 + 3$ is divided by $x - 2$. **1**

(d) (i) Sketch the graph of $y = \sin\theta$ for $0^\circ \le \theta \le 360^\circ$. Clearly label all key features. **2**

(ii) Hence, or otherwise, solve the equation $\sin\theta = 1$ for $0^\circ \le \theta \le 360^\circ$. **1**

(e) Sketch the graph of $y = \log_3(x - 2)$, clearly indicating any intercepts and asymptotes. **2**

(f) Let $\log_2 x = a$ and $\log_2 y = b$, where $b \neq 0$. Express the following in terms of a and b:

(i) $\log_2 xy$ **1**

(ii) $\log_2 \left(\dfrac{x^2}{y}\right)$ **1**

(iii) $\log_y x$ **1**

The paper continues on the next page

QUESTION FOUR (12 marks) Start a new page.

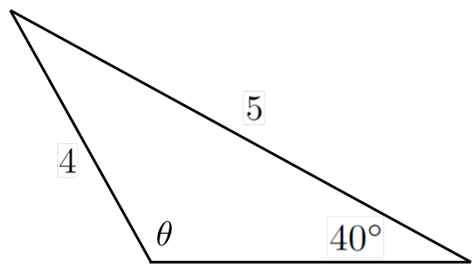
(a) Solve $\cos\theta = \frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$. **2**

(b) Consider the parabola given by the equation $y = x(x-6)$.

(i) State the equation of its axis of symmetry. **1**

(ii) Hence, find the coordinates of its vertex. **1**

(c) **3**



In the diagram above, the angle θ is obtuse. Find the value of θ correct to the nearest degree.

(d) The mass M kg of a decaying radioactive substance is given by the equation **2**

$$M = 800 \times 10^{-t},$$

where t is time measured in hours. How long will it take to reach a mass of 30 kg? Give your answer as an exact value.

(e) Two fair six-sided dice, one red and one blue, are rolled. Find the probability that:

(i) The same number appears on both dice. **1**

(ii) Different numbers appear on the two dice. **1**

(iii) The number on the red die is greater than the number on the blue die. **1**

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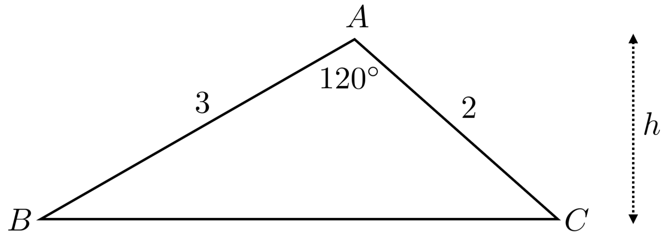
QUESTION FIVE (12 marks) Start a new page.

(a)



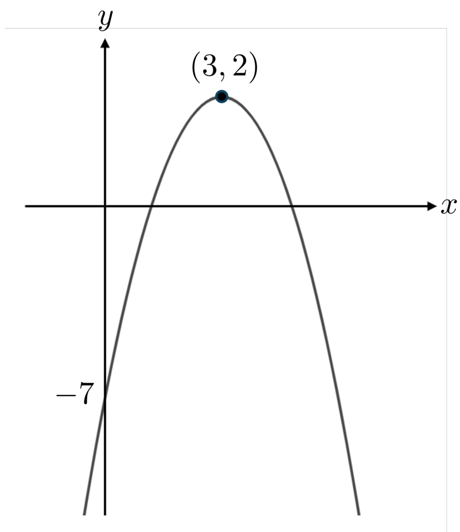
In the triangle above, $AB = 3$, $AC = 2$, $\angle BAC = 120°$ and h is a perpendicular height of the triangle. Find the exact values of the following:

(i) the area of $\triangle ABC$ **1**

(ii) the length BC **2**

(iii) the height h **1**

(b) **2**



Find the equation of the parabola in the diagram above.

(c) Consider the graph of the curve given by the equation $y = \dfrac{1}{x+1} - 2$.

(i) Find its y-intercept. **1**

(ii) Find its x-intercept. **1**

(iii) Sketch the graph of the curve, clearly labeling any intercepts with the axes. Include any asymptotes and their equations on your graph. **2**

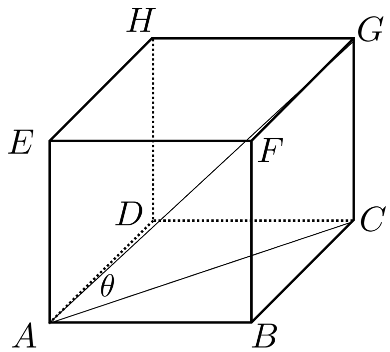
(d) Find the centre and radius of the circle given by the equation $x^2 - 4x + y^2 + 6y + 2 = 0$. **2**

The paper continues on the next page

QUESTION SIX (12 marks) Start a new page.

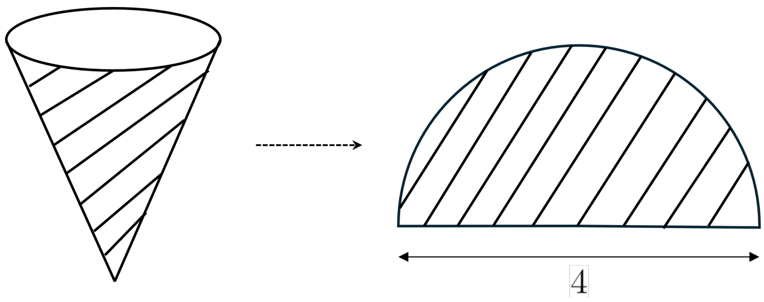
(a) Solve $\tan\theta = -0.3$ for $0° \leq \theta \leq 360°$. Give your answers correct to the nearest degree. **2**

(b) **2**



The cube in the diagram above has side lengths of 1 cm. The space diagonal AG meets the plane $ABCD$ at angle θ. Find θ correct to the nearest degree.

(c)



The net of the curved surface of a cone is a semicircle with diameter 4 cm.

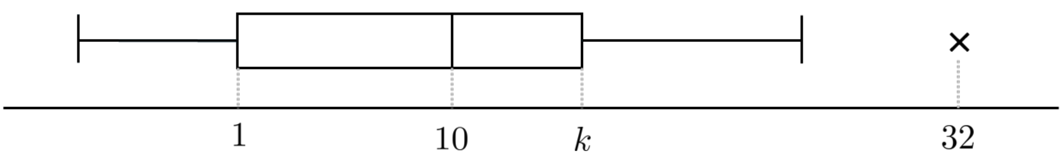
(i) State the slant height of the cone. **1**

(ii) Find the radius of the cone. **1**

(iii) Find the exact volume of the cone in terms of π. **2**

(d) Solve $2\log_3 x + \log_3\left(\frac{6}{x}\right) = 4$. **2**

(e) **2**



The box plot above is not drawn to scale. Given that 32 is an outlier, find the maximum possible integer value of k.

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QUESTION SEVEN (12 marks) Start a new page.

(a) Solve $2\sin 3\theta = \sqrt{3}$ for $0^\circ \leq \theta \leq 180^\circ$. **2**

(b) Consider the polynomial $P(x) = 4x^3 - 21x + 10$.

(i) Find a linear factor of $P(x)$. **1**

(ii) By using long division, or otherwise, fully factorise $P(x)$. **2**

(iii) Hence, solve $P(x) = 0$. **1**

(c) Bag A contains 4 red balls and 3 yellow balls. Bag B contains 25 red balls and 5 yellow balls. A bag is selected at random and a ball is drawn from it.

(i) Draw a tree diagram illustrating the possible outcomes with their probabilities. **1**

(ii) Find the probability that bag A was selected given that a red ball was drawn. **2**

(iii) How many red balls must be added, and to which bag, such that the events of selecting bag A and drawing a red ball are independent? **1**

(d) A single translation can be applied to the graph of $y = 3^x$ so that the new graph passes through the point $(-3, 9)$. Answer the following clearly in words:

(i) Which vertical translation achieves this result? **1**

(ii) Which horizontal translation achieves this result? **1**

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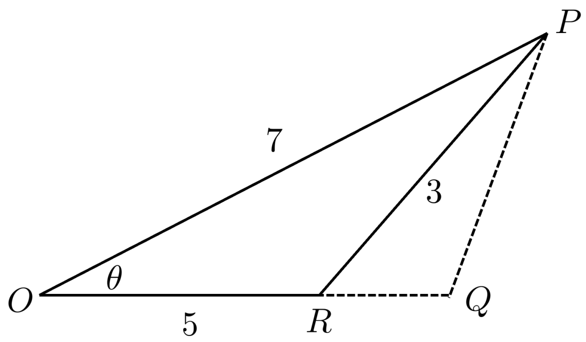
QUESTION EIGHT (12 marks) Start a new page.

(a) Find all possible values of the gradient m so that the graphs of $y = mx + 1$ and $y = \dfrac{1}{x}$ do not intersect. **2**

(b)



In the diagram above, $\angle POR = \theta$ and OR is extended to Q so that $\angle OPQ = 60°$.

(i) Find θ correct to the nearest minute. **2**

(ii) Show that $7PQ - 3RQ = 15$. **2**

(c) If $4^x = 2^{x+2} + 12$, find the value of 8^x. **3**

(d) Consider the polynomial $P(x) = a(x+1)^3 - b(x-1)$, where a and b are constants. Given that $P(x)$ is divisible by x and has a remainder of 16 when divided by $x - 1$, find the remainder when $P(x)$ is divided by $x + 2$. **3**

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QUESTION NINE (12 marks) Start a new page.

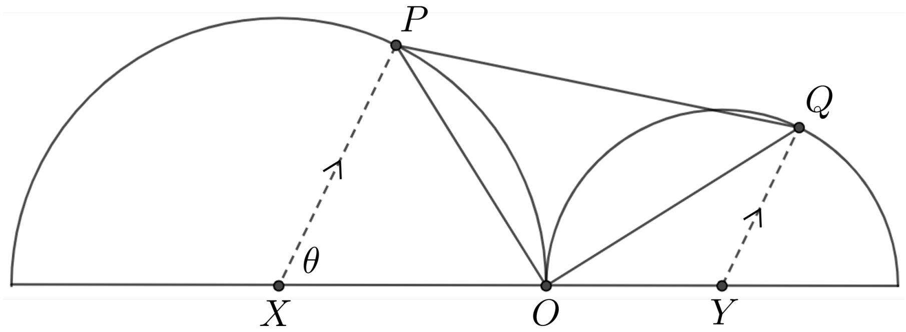
(a) The standard deviation σ of the scores $x_1, x_2, x_3, \cdots, x_n$ is given by: **3**

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}},$$

where $\bar{x}$ is the mean of these scores. If $\bar{x}$ and σ are equal, show that:

$$2n\sigma^2 = x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2.$$

(b)



In the diagram above, the semicircle centered at X has a radius of R, while the semicircle centered at Y has radius of r. Points P and Q are chosen on the semicircles so that PX is parallel to QY. Let $\angle PXO = \theta$, where $0° < \theta < 180°$.

(i) Find the area of the trapezium $PQYX$ in terms of R, r and θ. **2**

(ii) Find the maximum possible area of $\triangle POQ$. **2**

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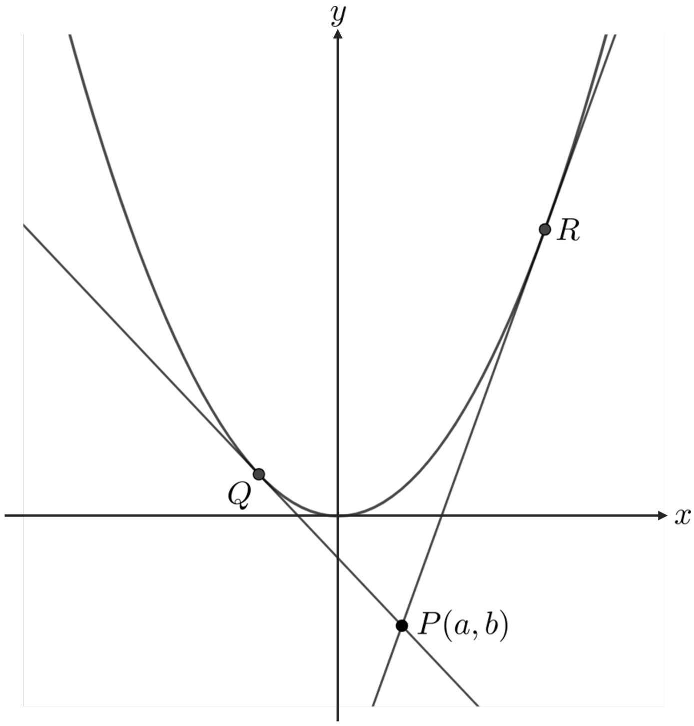
(c)



Two non-vertical lines pass through $P(a,b)$ and intersect the parabola $y = x^2$ exactly once each at points Q and R as shown above.

(i) Write down the equation of the line with gradient m passing through $P(a,b)$. **1**

(ii) Find the required conditions on a and b so that $\angle RPQ = 90°$. **2**

(d) Consider the remainder when $P(x) = x^2 + x + 1$ is divided by $x - \alpha$. Find the value of α so that this remainder cannot be obtained when any polynomial formed from any horizontal translation of $P(x)$ is divided by $x - \alpha$. **2**

END OF PAPER