

## Mathematical Tools.

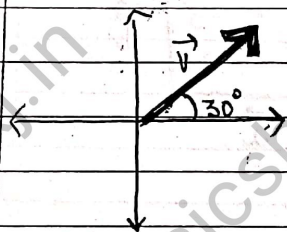
## MATHEMATICS:-

- 1) Trigonometry
- 2) Calculus
- 3) Vectors

## VECTORS

Any physical quantity which has magnitude as well as direction.

## # Representation of a Vector



$\vec{V} = 30^\circ$  North of East  
 $V = 5 \text{ m/sec, NE}$

- A vector can be represented by an arrow, the direction of an arrow gives us direction of the physical quantity and length gives us the magnitude of that physical quantity.
- Any physical quantity represented as a vector by applying an arrow " $\vec{\phantom{A}}$ " over that physical quantity.

## # Unit Vector

A vector which has its magnitude equal to "unit". It is represented by " $\hat{\phantom{A}}$ " over the vector.

$$\hat{A} = \frac{\vec{A}}{|A|} \quad \text{or} \quad \frac{\vec{A}}{A} = 1$$

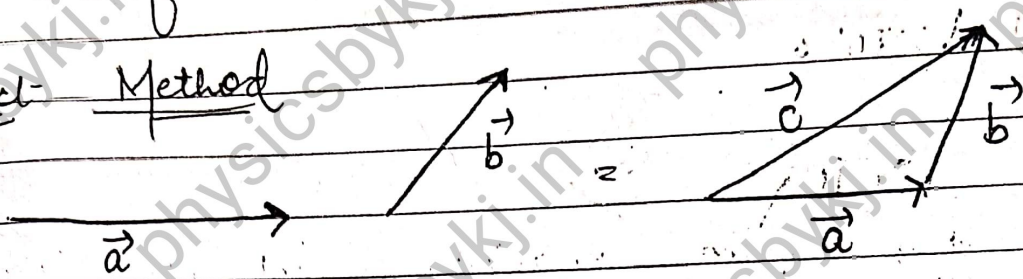


Vector A  
magnitude of vector =  $|\vec{A}|$  or A

## # Addition of Vectors.

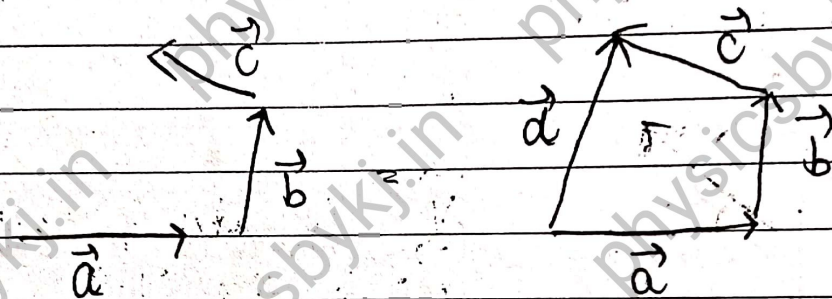
### 1) Direct Method

(1)

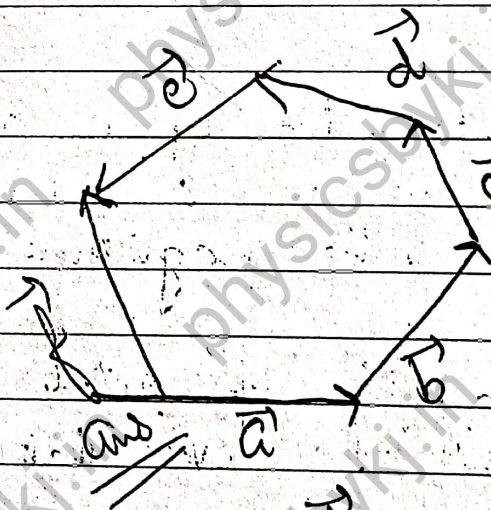


$$\vec{c} = \vec{a} + \vec{b}$$

(2)

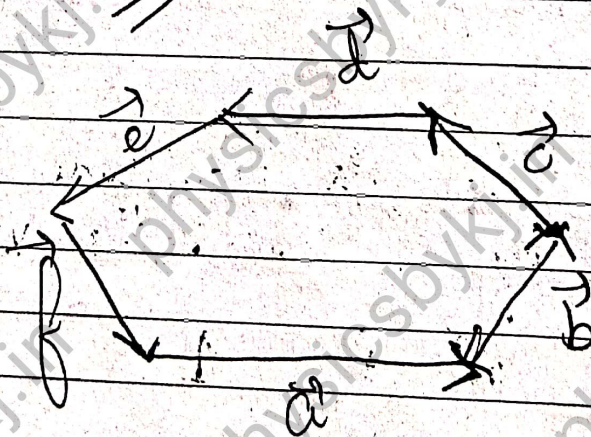


(3)



$$\vec{f} = \vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e}$$

(4)



$$\vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e} + \vec{f} = \vec{0}$$

Null Vector



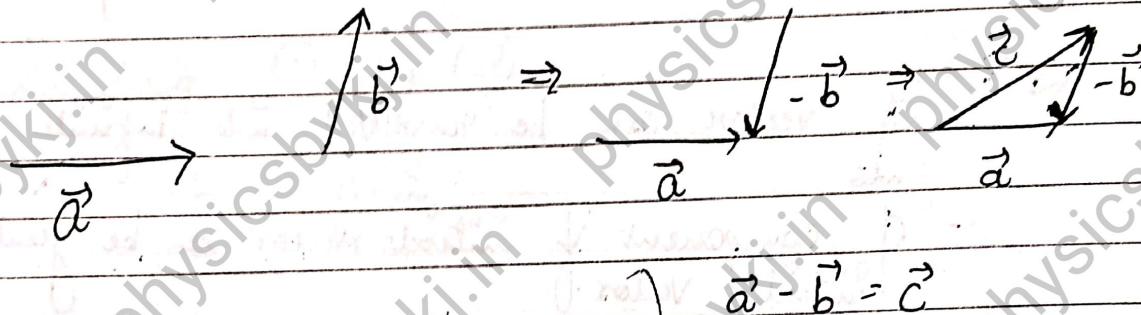


Null Vector  
Vector having its magnitude Zero.

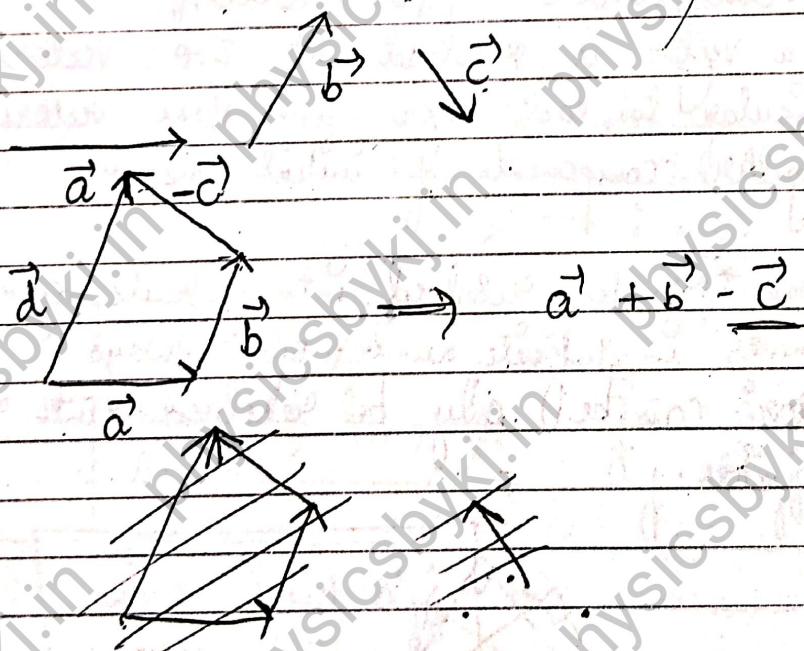
→ If When vectors make a closed polygon, then always, sum of these vectors is a null vector.

## # Subtraction of Vectors

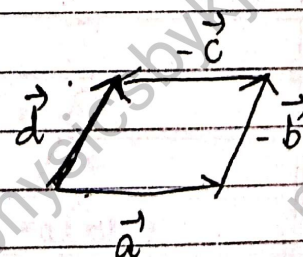
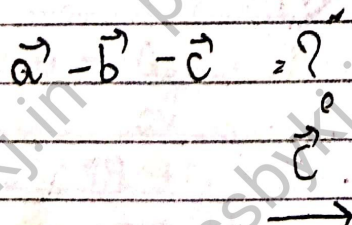
1) Direct Method



2)



3)



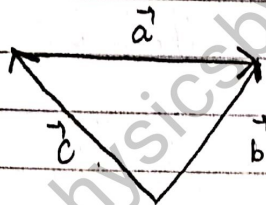




## Resolution of Vectors

### COMPONENTS OF VECTORS

When a vector is resolved into two or more vectors then; all these vectors are called components of initial vector.



$$\vec{a} = \vec{b} + \vec{c}$$

components of  $\vec{a}$

NOTES

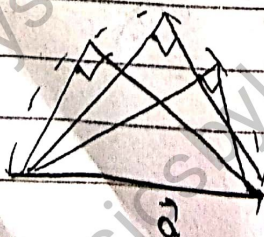
- A vector can be resolved into infinite no. of components.
- A component of initial vector can be greater than the initial vector.

## Rectangular Resolution

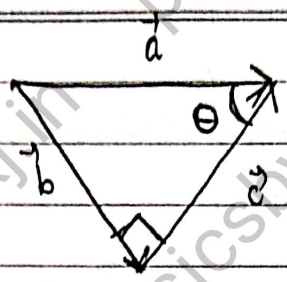
When a vector is resolved into two vectors which are perpendicular to each other, then these vectors are called rectangular components of initial one.

NOTES

- A vector can be resolved into infinite no. of rectangular components in infinite number of ways.
- A vector can be only be resolved into 2 rectangular components.







$$\sin \theta = \frac{b}{a} \Rightarrow b = a \sin \theta \quad (1)$$

$$\cos \theta = \frac{c}{a} \Rightarrow c = a \cos \theta \quad (2)$$

Adding (1)<sup>2</sup> and (2)<sup>2</sup>

$$b^2 + c^2 = a^2 (\sin^2 \theta + \cos^2 \theta)$$

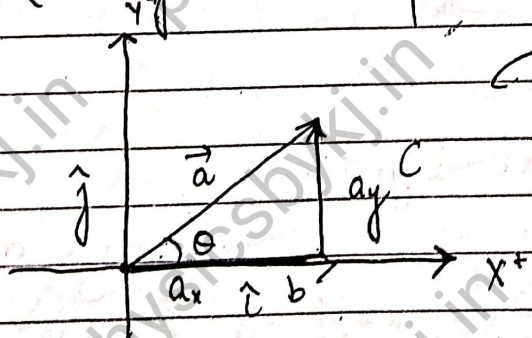
$$a = \sqrt{b^2 + c^2}$$

Dividing (1) by (2)

$$\frac{b}{c} = \frac{a \sin \theta}{a \cos \theta}$$

$$\tan \theta = \frac{b}{c}$$

# Rectangular components along coordinate axis.



$$\hat{i} = \frac{\vec{a}_x}{|\vec{a}_x|} \quad \text{and} \quad \hat{j} = \frac{\vec{a}_y}{|\vec{a}_y|}$$

$$\vec{a}_x = a_x \hat{i}$$

$$\vec{a}_y = a_y \hat{j}$$

$$\vec{a} = \vec{a}_x \hat{i} + \vec{a}_y \hat{j}$$

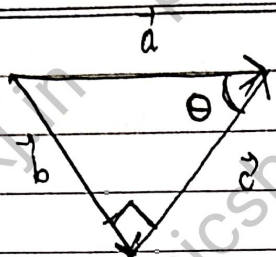
$$\sin \theta = \frac{a_y}{a}$$

$$a_y = a \sin \theta \quad (1)$$

$$\cos \theta = \frac{a_x}{a}$$

$$a_x = a \cos \theta \quad (2)$$





$$\sin \theta = \frac{b}{a} \Rightarrow b = a \sin \theta \quad (1)$$

$$\cos \theta = \frac{c}{a} \Rightarrow c = a \cos \theta \quad (2)$$

Adding (1)<sup>2</sup> and (2)<sup>2</sup>

$$b^2 + c^2 = a^2 (\sin^2 \theta + \cos^2 \theta)$$

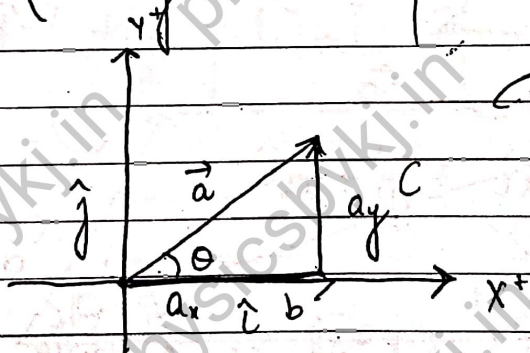
$$a = \sqrt{b^2 + c^2}$$

Dividing (1) by (2)

$$\frac{b}{c} = \frac{a \sin \theta}{a \cos \theta}$$

$$\tan \theta = \frac{b}{c}$$

# Rectangular components along coordinate axis



$$\hat{i} = \frac{\vec{a}_x}{|\vec{a}_x|} \quad \text{and} \quad \hat{j} = \frac{\vec{a}_y}{|\vec{a}_y|}$$

$$\vec{a}_x = a_x \hat{i}$$

$$\vec{a}_y = a_y \hat{j}$$

$$\vec{a} = \vec{a}_x \hat{i} + \vec{a}_y \hat{j}$$

$$\sin \theta = \frac{a_y}{a}$$

$$\cos \theta = \frac{a_x}{a}$$

$$a_y = a \sin \theta \quad (1)$$

$$a_x = a \cos \theta \quad (2)$$



Adding (1)<sup>2</sup> and (2)<sup>2</sup>

$$|a| = \sqrt{a_x^2 + a_y^2}$$

$$\vec{a} = 2\hat{i} + 3\hat{j}$$

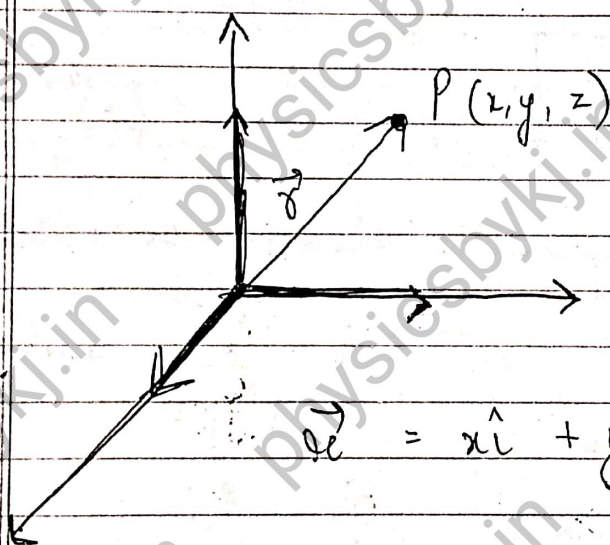
$$|a| = \sqrt{2^2 + 3^2}$$

$$= \sqrt{13}$$

Homework → Race 1

8/Apr/14  
#

## Position Vector

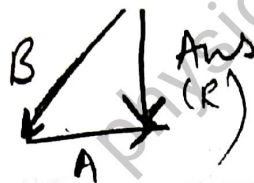
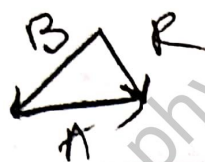


$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

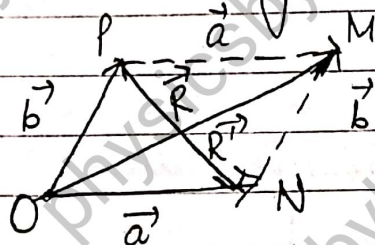
Position Vector is defined as a vector which describe its position with respect to origin of coordinate system.

$$|r| = \sqrt{x^2 + y^2 + z^2}$$





## \* Parallelogram Law of Vector Addition

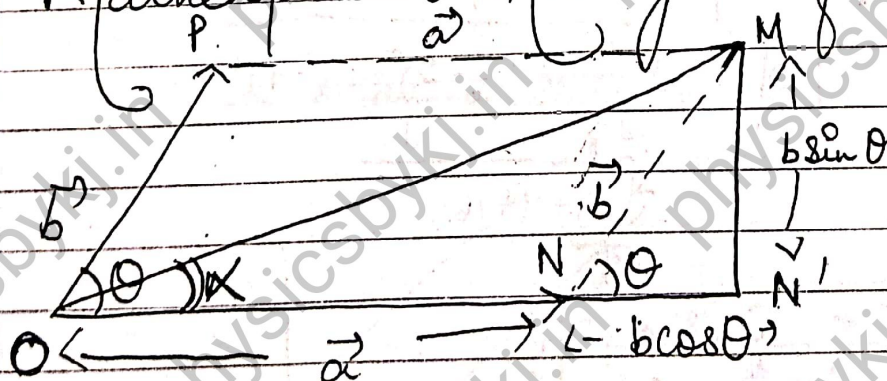


If we have two vectors,  $\vec{a}$  and  $\vec{b}$  then if we place these two vectors as two adjacent sides of a  $\square$  and about a common point O & then complete the  $\square$ , then the diagonal which is passing through the common point O gives us the sum of the 2 vectors.

While the other diagonal will give us the difference of the two vectors.

So,  $\vec{R} = \vec{a} + \vec{b}$   
 $\vec{R}' = \vec{a} - \vec{b}$

## \* Mathematical Analysis of $\square$ Law



In  $\triangle OMN$

$$OM^2 = (ON')^2 + (MN)^2$$

$$\Rightarrow R^2 = (a + b \cos \theta)^2 + (b \sin \theta)^2$$

$$R^2 = a^2 + b^2 \cos^2 \theta + 2ab \cos \theta + b^2 \sin^2 \theta$$

$$R^2 = a^2 + b^2 (\cos^2 \theta + \sin^2 \theta) + 2ab \cos \theta$$

$$R = \sqrt{a^2 + b^2 + 2ab \cos \theta}$$



In  $\triangle OMN'$ ,

$$\tan X = \frac{MN'}{ON'}$$

$$\tan X = \frac{b \sin \theta}{a + b \cos \theta}$$

Ex: Given that  $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ , if  $\vec{B}$  is added to  $\vec{A}$ , we get a unit vector along y axis, then find  $\vec{B}$  and  $|\vec{B}|$ .

$$\begin{aligned} \vec{A} + \vec{B} &= \hat{j} \\ \Rightarrow 2\hat{i} + 3\hat{j} + 4\hat{k} + \vec{B} &= \hat{j} \\ \vec{B} &= -2\hat{i} - 2\hat{j} - 4\hat{k} \\ \vec{B} &= -(2\hat{i} + 2\hat{j} + 4\hat{k}) \quad \text{Ans.} \end{aligned}$$

$$|\vec{B}| = \sqrt{(-2)^2 + (-2)^2 + (-4)^2}$$

$$= \sqrt{4 + 4 + 16}$$

$$= \sqrt{24}$$

$$\begin{aligned} \hat{B} &= \frac{\vec{B}}{|\vec{B}|} \\ &= \frac{-2\hat{i} - 2\hat{j} - 4\hat{k}}{\sqrt{24}} \end{aligned}$$





Ex: Given that  $\vec{A} = -2\hat{i} - 3\hat{j} + 4\hat{k}$ , if vector  $\vec{B}$  is subtracted from  $\vec{A}$ , we get  $2\hat{k}$ .  
Find  $\vec{B}$

$$\vec{A} - \vec{B} = 2\hat{k}$$

$$-2\hat{i} - 3\hat{j} - 2\hat{k} = -\vec{B}$$

$$-2\hat{i} - 3\hat{j} + 4\hat{k} - \vec{B} = 2\hat{k}$$

$$\vec{B} = -2\hat{i} - 3\hat{j} + 2\hat{k} \quad \text{Ans.}$$

$$|\vec{B}| = \sqrt{4+9+4}$$

$$= \sqrt{17}$$

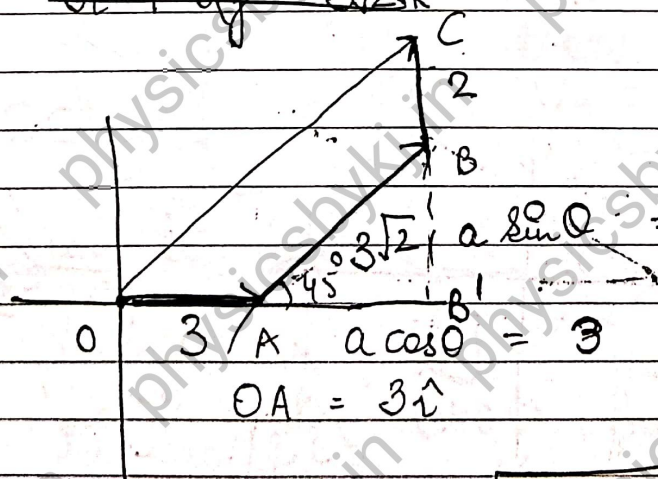
$$\hat{B} = \frac{-2\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{17}}$$

Ex: A ball moves 3 m along east from starting point O, then  $3\sqrt{2}$  N.E. & finally 2 m N. Find final displacement from O.

$$3\hat{i} \neq 2\hat{j} = 3\sqrt{2}\hat{k}$$

$$9+4+18$$

$$\frac{31}{\sqrt{8}}$$



Breaking  
 $AB = 3\hat{i} + 3\hat{j}$

$$\text{Displacement} = \sqrt{6^2 + 5^2}$$

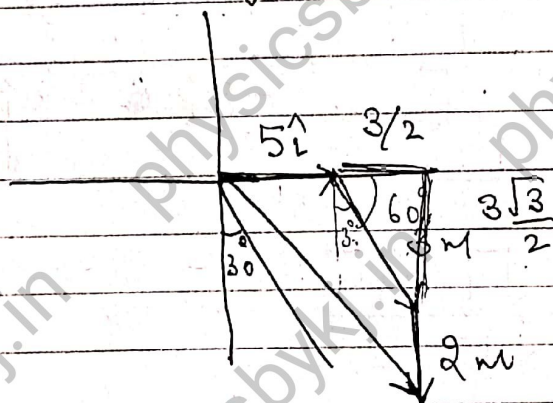
$$= \sqrt{36+25}$$

$$= \sqrt{61} \quad \text{Ans.}$$



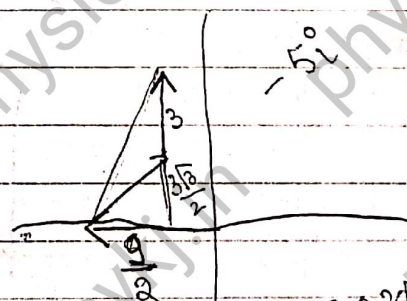
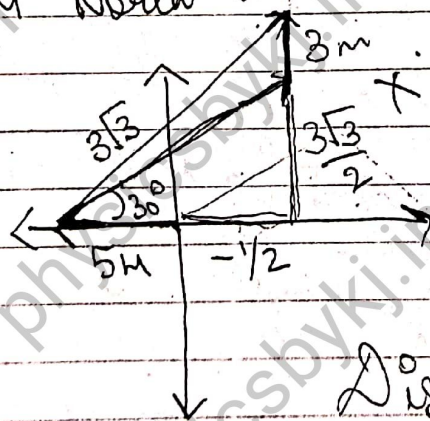
$$\vec{OC} = 6\hat{i} + 5\hat{j}$$

Q. A boy moves 5m along east from starting point O, then 3m,  $30^\circ$  East of South & finally 2m South. Find final displacement of boy from O.



$$\begin{aligned} \text{Displacement} &= \left(5\hat{i} + \frac{3}{2}\hat{i}\right) - \left(\frac{3\sqrt{3}}{2} + 2\right)\hat{j} \\ &= \frac{13}{2}\hat{i} - \frac{3\sqrt{3} + 4}{2}\hat{j} \end{aligned}$$

- Q. 1) 5m along West  
2)  $3\sqrt{3}$  m,  $30^\circ$  North of East  
3) 3m North



$$\begin{aligned} \text{Displacement} &= -5\hat{i} + 4.5\hat{i} + \left(3 + \frac{3\sqrt{3}}{2}\right)\hat{j} \\ &= -0.5\hat{i} + \left(3 + \frac{3\sqrt{3}}{2}\right)\hat{j} \end{aligned}$$



cos 60°



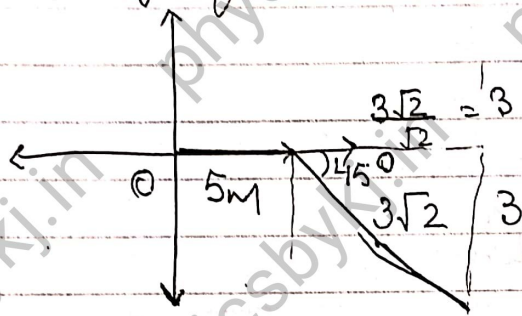
opposite  
sin  $\theta$



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$$= -\frac{1}{2} \hat{i} + \left(3 + \frac{3\sqrt{2}}{2}\right) \hat{j}$$

Q. A boy moves 5m along East from point O, then ~~3m~~  $3\sqrt{2}$  m, East-South, then it moves some displacement BC, but final displacement of boy from O is 2m N. Find BC.



$$5\hat{i} + 3\hat{i} - 3\hat{j} + \vec{BC} = 2\hat{j}$$

$$8\hat{i} - 3\hat{j} + \vec{BC} = 2\hat{j}$$

$$\boxed{\vec{BC} = -8\hat{i} + 5\hat{j}}$$

## # Product of Vectors

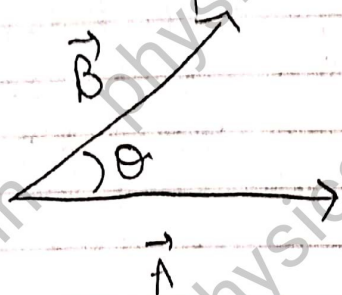
Vectors can be defined in two ways —

- 1) Scalar or Dot Product.
- 2) Vector or Cross Product.



# Scalar Product (Dot Product)

If product of two vectors is a scalar quantity, then this product is called Scalar Product.



$$\vec{A} \cdot \vec{B} = \underbrace{AB}_{\text{Magnitude of A}} \cos \theta \underbrace{\quad}_{\text{of B}}$$

Properties :

1)  $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$   
It is commutative.

2)  $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$   
It is distributive.

3)  $(\vec{A} \cdot \vec{B}) \cdot \vec{C} = \{\vec{A} (\vec{B} \cdot \vec{C})\}$   
It is associative.

4)  $\vec{A} \cdot \vec{A} = AA \cos \theta$   $\vec{A} \rightarrow \vec{A} \quad (\theta = 0^\circ)$   
 $\vec{A} \cdot \vec{A} = AA$   
 $\vec{A} \cdot \vec{A} = A^2$   
 $\sqrt{\vec{A} \cdot \vec{A}} = A$

5)  $\hat{A} \cdot \hat{A} = 1 \times 1 \times \cos \theta$   $\vec{A} \rightarrow \vec{A} \quad (\theta = 0^\circ)$   
 $= 1 \text{ ps.}$

6)  $\hat{i} \cdot \hat{j} = 1 \times 1 \times \cos 90^\circ = \hat{j} \cdot \hat{i} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{j} = \hat{k} \cdot \hat{i} = \hat{i} \cdot \hat{k} = 0$   
 $= 0 \text{ ps.}$



Perpendicular  $\rightarrow$  Dot =

Sol =  
Zero

7) If  $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

and

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

then  $\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$

$\hat{i} \cdot \hat{i} = 1$

$\hat{i} \cdot \hat{j} = 0$

$\hat{i} \cdot \hat{k} = 0$

So it's useless to multiply these and so on (same for other 2 also)

So  $\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$

Ex: If  $\vec{A} = 2\hat{i} + 3\hat{j}$  and  $\vec{B} = 2\hat{i} + 4\hat{j}$   
 $\vec{A} \cdot \vec{B} = 4 + 12$   
 $= 16 \text{ Ans.}$

8) Angle b/w  $\vec{A}$  and  $\vec{B}$  is  
 $\vec{A} \cdot \vec{B} = AB \cos \theta$   
 $\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB}$

$\theta = \cos^{-1} \left( \frac{\vec{A} \cdot \vec{B}}{AB} \right)$

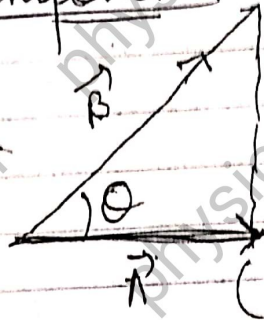
Ex: Given that  $\vec{A} = \hat{i} + 3\hat{j}$  and  $\vec{B} = 2\hat{i} - 3\hat{j}$ , find angle between  $\vec{A}$  and  $\vec{B}$ .  
 $\cos \theta = \frac{(\hat{i} + 3\hat{j}) \cdot (2\hat{i} - 3\hat{j})}{\sqrt{1+9} \sqrt{4+9}}$



$$= -\frac{7}{\sqrt{130}} \text{ Ans.}$$

⑨

Components



∴ Do, component of vector B along  $\vec{A} = B \cos \theta$

$$\vec{A} \cdot \vec{B} = A (B \cos \theta)$$

$$\Rightarrow B \cos \theta = \frac{\vec{A} \cdot \vec{B}}{A}$$

Component of B along A

Component of A along B

$$A \cos \theta = \frac{\vec{A} \cdot \vec{B}}{B}$$

Ex: Given that  $A = 2\hat{i} + 3\hat{j}$  &  $B = \hat{i} - \hat{j}$ , find component of B along A, & A along B.

$$\begin{aligned} \Rightarrow B \cos \theta &= \frac{(2\hat{i} + 3\hat{j})(\hat{i} - \hat{j})}{\sqrt{4+9}} \\ &= \frac{2-3}{\sqrt{13}} \\ &= \frac{-1}{\sqrt{13}} \end{aligned}$$

$$\Rightarrow A \cos \theta = \frac{-1}{\sqrt{1+1}} = \frac{-1}{\sqrt{2}}$$

Homework : Race 1, Race 2, Race 3



Q.9)

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB (\cos (180^\circ - \theta))}$$

$$= \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

$$\frac{|\vec{A} + \vec{B}|}{2} = \frac{|\vec{A} - \vec{B}|}{2}$$

$$A^2 + B^2 + 2AB \cos \theta = \frac{A^2 + B^2 - 2AB \cos \theta}{4}$$

$$4A^2 + 4B^2 + 8AB \cos \theta = A^2 + B^2 - 2AB \cos \theta$$

$$3A^2 + 3B^2 + 10AB \cos \theta = 0$$

$$3(2B)^2 + 3B^2 + 10 \times 2B \times B \times \cos \theta = 0$$

$$3 \times 4B^2 + 3B^2 + 20B^2 \cos \theta = 0$$

$$12B^2 + 3B^2 + 20B^2 \cos \theta = 0$$

$$B^2 (12 + 3 + 20 \cos \theta) = 0$$

$$15 + 20 \cos \theta = 0$$

$$20 \cos \theta = -15$$

$$\cos \theta = -\frac{3}{4}$$

$$\theta = \cos^{-1} \left( -\frac{3}{4} \right) \text{ Ans.}$$

Q.10)

$$\tan \alpha = \frac{B \sin (\alpha + \beta)}{A + B \cos (\alpha + \beta)}$$

$$\Rightarrow \frac{\sin \alpha}{\cos \alpha} = \frac{B \sin (\alpha + \beta)}{A + B \cos (\alpha + \beta)}$$

$$\Rightarrow A \sin \alpha + B \sin \alpha \cos (\alpha + \beta) = B \sin (\alpha + \beta) \cos \alpha$$

$$\Rightarrow A \sin \alpha = B [\sin (\alpha + \beta) \cos \alpha - \sin \alpha \cos (\alpha + \beta)]$$



$$B \sin (\alpha + \beta - \alpha)$$

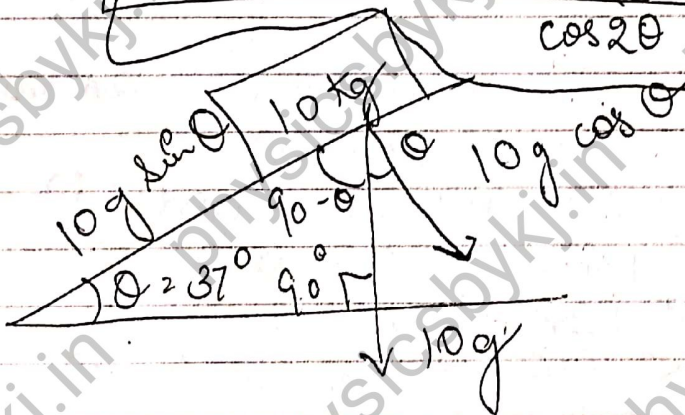
$$A \sin \alpha = B \sin \beta$$

$$\Rightarrow \frac{A}{B} = \frac{\sin \beta}{\sin \alpha}$$

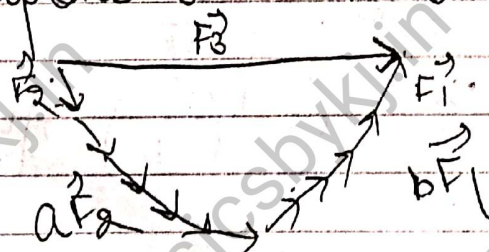
Identities  $\rightarrow$

$$\begin{aligned}\sin (A+B) &= \sin A \cos B + \cos A \sin B \\ \sin (A-B) &= \sin A \cos B - \cos A \sin B \\ \cos (A+B) &= \cos A \cos B - \sin A \sin B \\ \cos (A-B) &= \cos A \cos B + \sin A \sin B \\ 1 - \cos 2\theta &= 2 \sin^2 \theta \\ 1 + \cos 2\theta &= 2 \cos^2 \theta \\ \sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta\end{aligned}$$

Q. 2)



# Component Vectors



$$a\vec{F}_2 + b\vec{F}_1 = \vec{F}_3$$

eg

$$\begin{aligned}\vec{F}_1 &= \hat{i} + 2\hat{j} \\ \vec{F}_2 &= 2\hat{i} - \hat{j} \\ \vec{F}_3 &= 7\hat{i} + 8\hat{j}\end{aligned}$$





Calculate Component Vectors of  $F_1$  &  $F_2$  which will form  $F_3$ ?

Sol:  $a\vec{F}_1 + b\vec{F}_2 = \vec{F}_3$

$$a(\hat{i} + 2\hat{j}) + b(2\hat{i} - \hat{j}) = 7\hat{i} + 8\hat{j}$$

$$a\hat{i} + 2a\hat{j} + b2\hat{i} - b\hat{j} = 7\hat{i} + 8\hat{j}$$

$$\hat{i}(a + 2b) + \hat{j}(2a - b) = 7\hat{i} + 8\hat{j}$$

$$a + 2b = 7$$

$$2a - b = 8$$

$$a = \frac{23}{5}$$

$$b = \frac{6}{5}$$

So, component vectors of  $F_1$  &  $F_2$  are

$$= \frac{23}{5}(\hat{i} + 2\hat{j}) \text{ and } \frac{6}{5}(2\hat{i} - \hat{j}) \text{ Ans.}$$

Q.  $F_1 = \hat{i} - \hat{j}$   
 $F_2 = \hat{i} - 3\hat{j}$   
 $F_3 = 9\hat{i} + 11\hat{j}$

$$a\hat{i} - a\hat{j} + b\hat{i} - 3b\hat{j} = 9\hat{i} + 11\hat{j}$$
$$(a+b)\hat{i} - (a+3b)\hat{j} = 9\hat{i} + 11\hat{j}$$

$$a(\hat{i} - \hat{j}) + b(\hat{i} - 3\hat{j}) = 9\hat{i} + 11\hat{j}$$
$$\hat{i}(a+b) + \hat{j}(-a-3b) = 9\hat{i} + 11\hat{j}$$

$$a+b = 9$$

$$-(a+3b) = 11$$

$$\Rightarrow 3a + 3b = 27$$

$$-a - 3b = -11$$

$$2a = 38$$

$$\frac{27}{11} = \frac{38}{38}$$



$$a = 9$$

$$a+b=9$$

$$b = 19$$

$$b = -10$$

$$F_1 \Rightarrow 9(\hat{i} - \hat{j})$$

$$F_2 \Rightarrow -10(\hat{i} - 3\hat{j})$$

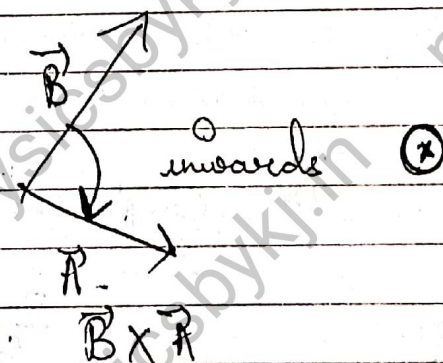
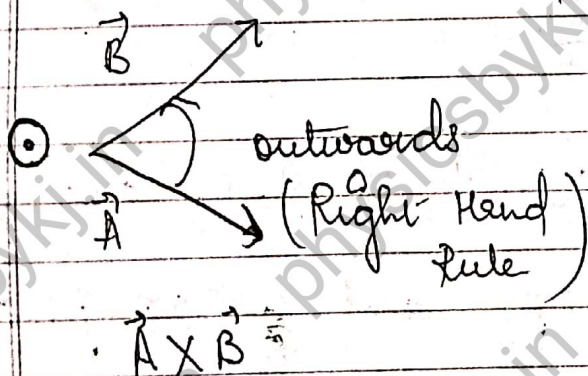
## # Vector Product (Cross Product)

When product of two vector are vector only, that that kind of product is vector product -

$$\vec{A} \times \vec{B} = \vec{C}$$

$$\vec{C} = AB \sin \theta \hat{C}$$

direction of C



NOTE  $\rightarrow \vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

But

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$\rightarrow$  Direction of C is given by RIGHT HAND THUMB RULE





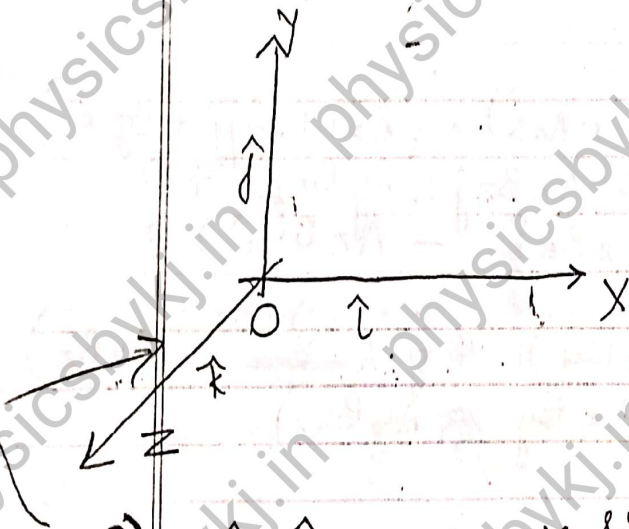
## Right Hand Thumb Rule

Level your fingers along  $\vec{A}$  to  $\vec{B}$  or  $\vec{B}$  to  $\vec{A}$ , then direction of thumb will give us direction of  $\vec{C}$ .

Some Properties / Important Points.

1)  $\vec{A} \times \vec{A} = A \cdot A \sin 0 \hat{C}$   
 $= 0$

2)  $\hat{i} \times \hat{i} = 0$   
 $\hat{j} \times \hat{j} = 0$   
 $\hat{k} \times \hat{k} = 0$



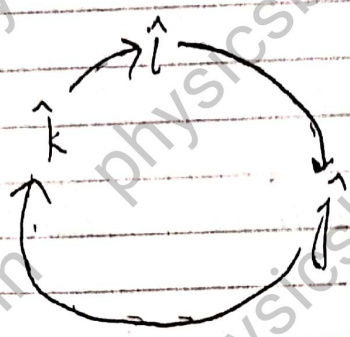
3)  $-\hat{i} \times \hat{j} = 1 \times 1 \times \sin 90 \times \hat{k}$   
 $= \hat{k}$   
 $-\hat{j} \times \hat{k} = \hat{i}$   
 $-\hat{k} \times \hat{i} = \hat{j}$

4)  $\hat{j} \times \hat{i} = -\hat{k}$   
 $\hat{k} \times \hat{j} = -\hat{i}$   
 $\hat{i} \times \hat{k} = -\hat{j}$





How to Remember :



Clockwise  $\rightarrow$  +ve  
Anticlockwise  $\rightarrow$  -ve.

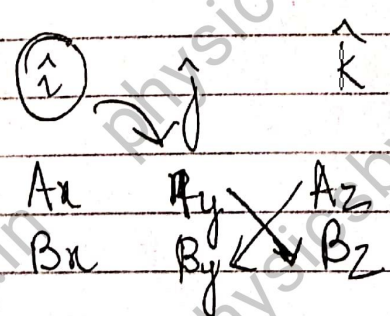
Q. If  $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$   
 $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

Find  $\vec{A} \times \vec{B} = ?$

Solution :-

$$\begin{aligned} \vec{A} \times \vec{B} &= (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\ &= 0 + A_x B_y \hat{k} - A_x B_z \hat{j} - A_y B_x \hat{k} + 0 \\ &\quad + A_y B_z \hat{i} + A_z B_x \hat{j} - A_z B_y \hat{i} + 0 \\ &= (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} \\ &\quad + (A_x B_y - A_y B_x) \hat{k} \end{aligned}$$

How to Remember :



$$\hat{i} (A_y B_z - A_z B_y)$$



$$\hat{i} \quad \hat{j} \quad \hat{k}$$

$$\begin{matrix} A_x & A_y & A_z \\ B_x & B_y & B_z \end{matrix}$$

$$\hat{j} (A_z B_x - A_x B_z)$$

eg:  $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$   
 $\vec{B} = 2\hat{i} - 4\hat{j} - 3\hat{k}$

$$\vec{A} \times \vec{B} = ?$$

$$\hat{i} \quad \hat{j} \quad \hat{k}$$

$$\begin{matrix} 2 & 3 & -4 \\ 2 & -4 & -3 \end{matrix}$$

$$\hat{i} (-9 - 16) + \hat{j} (-8 - (-6)) + \hat{k} (-8 - 6)$$

$$\boxed{-25\hat{i} - 2\hat{j} - 14\hat{k}} \quad \text{Ans}$$

$$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{B} = 2\hat{i} - 3\hat{k}$$

$$\vec{A} \times \vec{B} = ?$$

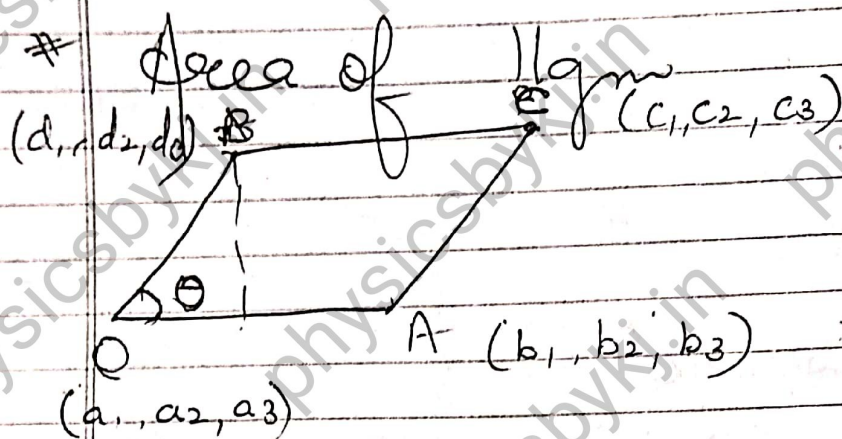
$$\hat{i} \quad \hat{j} \quad \hat{k}$$

$$\begin{matrix} 1 & 2 & 3 \\ 2 & 0 & -3 \end{matrix}$$

$$\hat{i} (-6) + \hat{j} (6+3) + \hat{k} (-4)$$

$$\boxed{-6\hat{i} + 9\hat{j} - 4\hat{k}} \quad \text{Ans}$$





Find the area of  $\triangle OAB$ .

Sol =

$$= \text{Base} \times \text{Height}$$

$$= OA \times BN$$

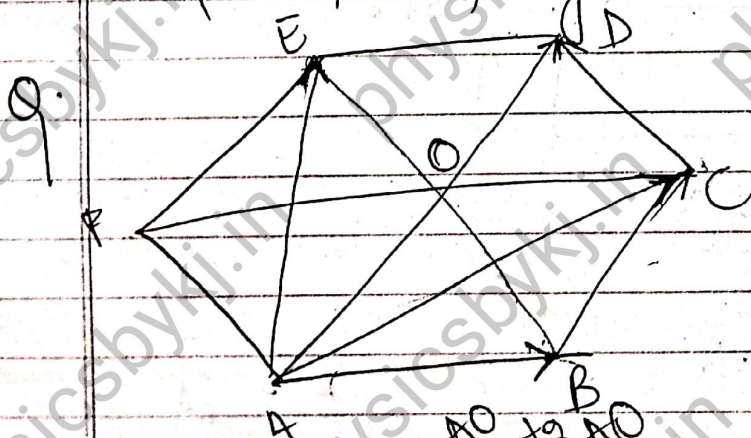
$$= OA \times OB \sin \theta$$

$$= \vec{OA} \times \vec{OB}$$

$$\vec{OA} = (b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$$

$$\vec{OB} = (d_1 - a_1)\hat{i} + (d_2 - a_2)\hat{j} + (d_3 - a_3)\hat{k}$$

Homework: Race 3, 4, Beginner's Box 4

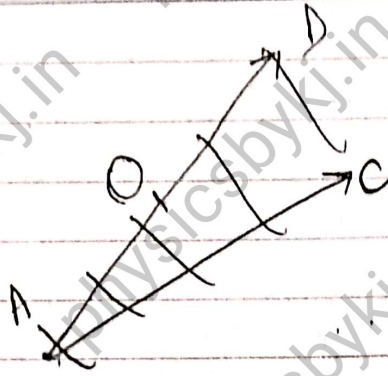


Find  $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$  in terms of  $\vec{AO}$ .

- (a)  $4\vec{AO}$   
(b)  $6\vec{AO}$

- (c)  $3\vec{AO}$   
(d) ?





$$\vec{AB} + \vec{BC} + \vec{CD}$$

$$\vec{AB} = \vec{AB}$$

$$\vec{AO} = \vec{AB} + \vec{BO}$$

$$\vec{AO} = 2\vec{AO}$$

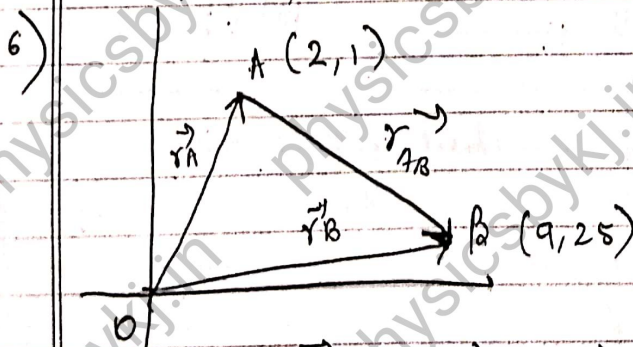
$$\vec{AE} = 2\vec{AO} - \vec{AB}$$

$$\vec{AF} = 2\vec{AO} - \vec{AB} - \vec{AO}$$

$$\vec{AF} = 2\vec{AO} - \vec{AB}$$

$$\begin{aligned} & \vec{AB} + \vec{AB} + \vec{AO} + 2\vec{AO} + 2\vec{AO} - \vec{AB} + 2\vec{AO} - \vec{AB} - \vec{AO} \\ & + \vec{AO} - \vec{AB} \\ & = 6\vec{AO} \end{aligned}$$

Race 3



$$\vec{r}_A = 2\hat{i} + \hat{j}$$

$$\vec{r}_B = 9\hat{i} + 25\hat{j}$$

$$\vec{r}_A + \vec{r}_{AB} = \vec{r}_B$$

$$\begin{aligned} \Rightarrow \vec{r}_{AB} &= \vec{r}_B - \vec{r}_A \\ &= 7\hat{i} + 24\hat{j} \end{aligned}$$





$$V = 50 \text{ m/sec}$$

$$\hat{r}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

$$= \frac{7\hat{i} + 24\hat{j}}{25}$$

$$\vec{V} = 50 \text{ m/sec} \hat{r}_{AB}$$

$$= 50 \times \frac{7\hat{i} + 24\hat{j}}{25}$$

$$= 14\hat{i} + 48\hat{j} \text{ Ans.}$$

\*

$$x \text{ and } x\text{-axis} \rightarrow \cos \theta = \frac{\vec{r} \cdot \hat{i}}{|\vec{r}| |\hat{i}|}$$

$$y \text{ and } y\text{-axis} \rightarrow \frac{\vec{r} \cdot \hat{j}}{|\vec{r}| |\hat{j}|}$$

$$z \text{ and } z\text{-axis} \rightarrow \frac{\vec{r} \cdot \hat{k}}{|\vec{r}| |\hat{k}|}$$

4)

$$|\vec{B}| = 8$$

$$\vec{B} + A\hat{i} = (2A)\hat{j}$$

$$\vec{B} = (2A)\hat{j} - A\hat{i}$$

$$|\vec{B}| = \sqrt{4A^2 + A^2}$$

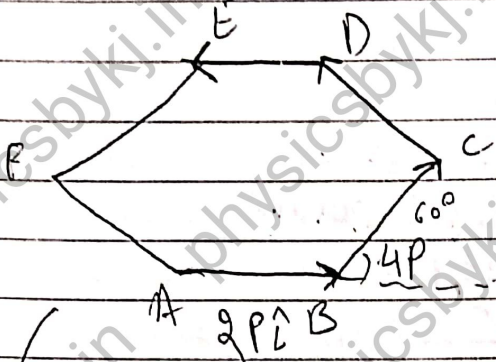
$$64 = \sqrt{5A^2}$$

$$\frac{8}{\sqrt{5}} = A \text{ Ans.}$$

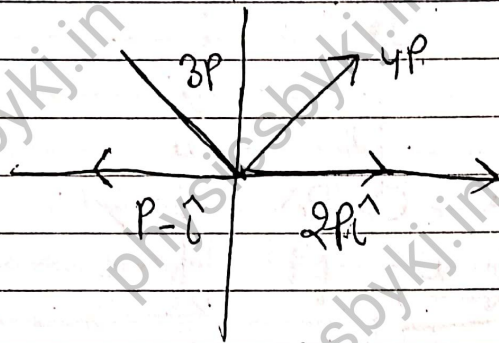




6)



$$BC = 4P \cos 60^\circ + 4P \sin 60^\circ \hat{j}$$



$$\text{Net } = 2P \hat{i} + (4P \cos 60^\circ \hat{i} + 4P \sin 60^\circ \hat{j}) + (3P \sin 60^\circ \hat{j} - 3P \cos 60^\circ \hat{i})$$

$$= \left[ 2P \hat{i} + 2P \hat{i} - 3P \hat{i} - P \hat{i} \right] + \left[ 2P \sqrt{3} \hat{j} + 3\sqrt{3}P \hat{j} \right]$$
$$= \frac{3}{2} \hat{i} + \frac{7\sqrt{3}}{2} \hat{j}$$

$$\text{Net magnitude} = \sqrt{\frac{9P^2}{4} + \frac{49 \times 3P^2}{4}}$$

$$= \sqrt{\frac{9P^2 + 147P^2}{4}}$$

$$= \frac{P}{2} \sqrt{156}$$

$$= \frac{P}{2} \sqrt{39 \times 4}$$

$$= P \sqrt{39}$$

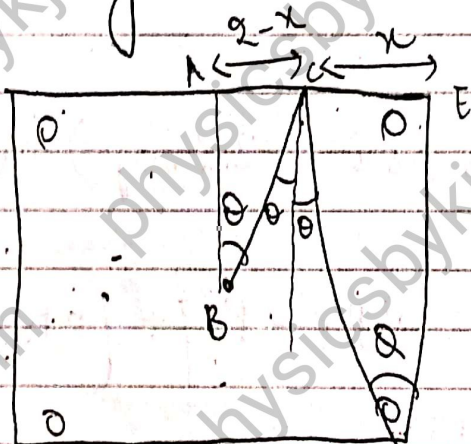


Direction :

$$\begin{aligned} \tan \theta &= \frac{\uparrow \text{component}}{\leftarrow \text{component}} \\ &= \frac{\frac{7\sqrt{3}}{2} p}{\frac{3p}{2}} \\ &= \frac{7}{\sqrt{3}} \text{ Ans.} \end{aligned}$$

Beginner's Box

Q.1)



$$ABC \sim CDE$$

$$\frac{2-x}{2} = \frac{x}{2}$$

$$x = \frac{2}{3}$$

$$BE = \sqrt{2^2 + \left(\frac{2}{3}\right)^2}$$

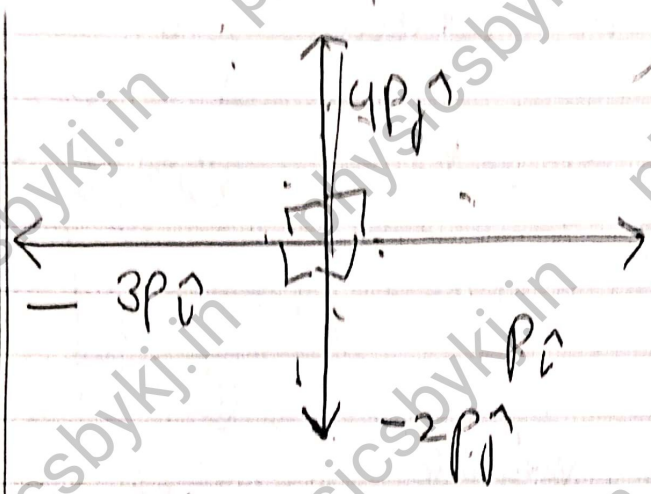
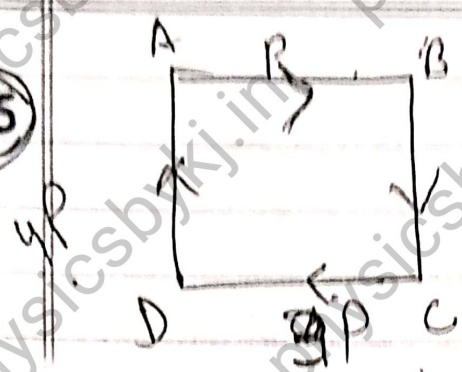
$$= \sqrt{4 + \frac{4}{9}}$$

$$= \sqrt{\frac{40}{9}}$$

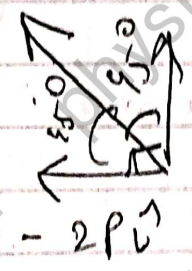
and so on.



(15)



$$4P \uparrow - 2P \downarrow = +2P \uparrow$$



Resultant  $= \sqrt{a^2 + b^2}$   
 $= 2\sqrt{2}$

N-W

Resultant  $\rightarrow 2\sqrt{2}$