



Kinematics :- Study of Motion

Study of motion of point sized objects or objects considered to be point sized with knowing the cause of motion.

Rectilinear Motion or Motion in 1-D

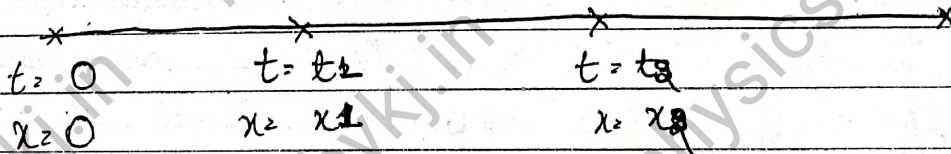
Motion in 2-D

Relative Motion.

- (i) Distance and Displacement
- (ii) Speed and Velocity

Average Velocity :- $\langle \vec{v} \rangle$ or $\bar{v}$ or V_{avg}

$$V = \frac{\text{Total displacement covered by body}}{\text{Total Time taken}}$$



$$V_{avg} = \frac{x_2 - x_1}{t_2 - t_1}$$

Instantaneous Velocity :-

It is defined only for very small instantaneous time dt .

$$\begin{array}{ccc} \leftarrow & x & \rightarrow \\ t=0 & t=t & t=t+dt \\ x=0 & x=x & x=x+dx \end{array}$$

i.e. $V = \frac{dx}{dt}$

NOTE : Velocity of a particle in rectilinear motion can be given in 3 ways :

(i) Velocity can be constant.

Eg : $V = 3 \text{ m/sec}, 5 \text{ m/sec} \dots \text{etc.}$

(ii) Velocity can be function of time.

i.e., $V = f(t)$

Eg : $V = 3t \text{ m/sec}, V = (5t^2 + 2t + 3) \text{ m/sec} \text{ etc.}$

(iii) Velocity can be function of position

i.e., $V = f(x)$

Eg : $V = 3x^2 \text{ m/sec}, V = \sin x \text{ etc.}$

(iv)

Acceleration :

It is defined as rate of change of velocity with respect to time.

i.e., $a = \frac{dv}{dt}$

$$a = \frac{dv}{dt} \times \frac{dx}{dx} = \frac{dv}{dx} \times \frac{dx}{dt}$$

$$a = v \frac{dv}{dx}$$



$$a = \frac{dv}{dt}$$

$$= \frac{d}{dt} \left(\frac{dx}{dt} \right)$$

$$a = \frac{d^2x}{dt^2}$$

can be

$$a = \frac{dv}{dt} = \frac{v dv}{dx} = \frac{d^2x}{dt^2}$$

Note: Acceleration of an object in Rectilinear motion can be given in 4 ways:

(i) $a = \text{constant}$

(ii) $a = f(t)$

(iii) $a = f(x)$

(iv) $a = f(v)$

etc

Problems on Rectilinear Motion having variable acceleration:

Eg: A particle is moving along straight line along acceleration, $a = (2t+3) \text{ m/sec}^2$. It is given that particle starts from $x=0$ at $t=0$ & $v=0$. Calculate

average velocity of particle ^{initial conditions} b/w time $t_1 = 1 \text{ sec}$ to $t_2 = 2 \text{ sec}$.

$$\text{Avg} = \frac{\text{Total Displacement}}{t_2 - t_1}$$

$$= \frac{x_2 - x_1}{1}$$

Given $\Rightarrow a = (2t+3)$

$$\Rightarrow \frac{dv}{dt} = 2t+3$$



$$\Rightarrow \int_0^v dv = \int_0^t (2t+3) dt$$
$$\left[v \right]_0^v = \left[\frac{2t^2}{2} + 3t \right]_0^t$$
$$(v-0) = (t^2 + 3t)$$

$$\boxed{v = (t^2 + 3t)}$$

$$\Rightarrow \frac{dx}{dt} = t^2 + 3t$$

$$\Rightarrow \int_0^{x_1} dx = \int_0^1 (t^2 + 3t) dt$$

$$\left[x \right]_0^{x_1} = \left[\frac{t^3}{3} + \frac{3t^2}{2} \right]_0^1$$

$$x_1 = \frac{1}{3} + \frac{3}{2}$$

$$= \frac{2+9}{6}$$

$$= \frac{11}{6} \text{ m}$$

$$\Rightarrow \int_0^{x_2} dx = \int_2^4 (t^2 + 3t) dt$$

$$\Rightarrow x_2 = \left[\frac{t^3}{3} + \frac{3t^2}{2} \right]_2^4$$

$$= \left[\frac{(4)^3}{3} + \frac{3(4)^2}{2} \right]$$

$$= \frac{64}{3} + 24$$

$$= \frac{112}{3}$$



$$V_{avg} = \frac{\frac{26}{3} - \frac{11}{6}}{\frac{1}{6}}$$

$$= \frac{52 - 11}{6}$$

$$= \frac{41}{6} \text{ m/sec Ans.}$$

Question: A particle is moving along straight line with $V = (3x+2)$ m/sec.

It is given that particle has started from, $x=0$, $t=0$, $V=2$ m/sec. Calculate avg velocity

b/w $t_1 = 1$ sec to $t_2 = 2$ sec

Solution:

$$V = (3x+2)$$

$$\frac{dx}{dt} = (3x+2)$$

$$\frac{dx}{3x+2} = dt$$

$$\int_0^{x_1} \frac{dx}{3x+2} = \int_0^{t_1} dt$$

$$\left[\frac{\ln(3x+2)}{3} \right]_0^{x_1} = 3t_1$$

$$\ln(3x_1+2) - \ln 2 = 3 \times 1$$

$$\ln \frac{3x_1+2}{2} = 3$$

$$\Rightarrow \frac{3x_1+2}{2} = e^3$$

$$\Rightarrow 3x_1+2 = 2e^3$$

$$\Rightarrow x_1 = \frac{2e^3 - 2}{3}$$

$$\Rightarrow x_1 = \frac{2 \times (2.713)^3 - 2}{3}$$



Again,

$$\int_0^{x_2} \frac{dx}{3x+2} = \int_0^{t_2} dt$$

$$\Rightarrow \ln \left(\frac{3x_2+2}{3} \right) = 3 \times t_2$$

$$\Rightarrow \ln \left(\frac{3x_2+2}{3} \right) = 3 \times 2$$

$$\Rightarrow \left(\frac{3x_2+2}{3} \right) = e^6$$

$$\Rightarrow 3x_2 = 2e^6 - 2$$

$$\Rightarrow x_2 = \frac{2(2.718)^6 - 2}{3}$$

$$V_{avg} = \frac{\frac{2(2.718)^6 - 2}{3} - \frac{2(2.718)^3 - 2}{3}}{2-1}$$

Ques: A particle is moving along straight line with acceleration $a = (v^2 + 5) \text{ m/sec}^2$. It starts from $x=0, t=0, v=2 \text{ m/sec}$. Calculate velocity at $x=2 \text{ m}$.

Solution:

$$a = v^2 + 5$$

$$\frac{dv}{dt} = v^2 + 5$$

$$\frac{1}{dt} = \frac{v^2 + 5}{dv}$$

$$\int_{t_1}^{t_2} dt = \int_{v_1}^{v_2} \frac{dv}{v^2 + 5}$$

$$\Rightarrow t = \frac{1}{\sqrt{5}} \ln(v^2 + 5)$$



$$a = v^2 + 5$$

$$v \frac{dv}{dx} = v^2 + 5$$

$$\Rightarrow \int \frac{v dv}{v^2 + 5} = \int_0^2 dx$$

$$\Rightarrow$$

$$\text{Let } v^2 + 5 = t$$

$$\Rightarrow dt = 2v dv + 0$$

$$\Rightarrow \frac{dt}{2} = v dv$$

$$\Rightarrow \frac{1}{2} \int \frac{dt}{t} = \int_0^2 dx$$

$$\Rightarrow \frac{1}{2} \ln(t) = 2 - 0$$

$$\Rightarrow \frac{1}{2} [\ln(v^2 + 5)]_2^v = 2$$

$$\Rightarrow \ln(v^2 + 5) - \ln(2^2 + 5) = 4$$

$$\Rightarrow \ln \frac{v^2 + 5}{9} = 4$$

$$\Rightarrow \frac{v^2 + 5}{9} = e^4$$

JEE - 05

Ques: A particle is moving along straight line with acceleration, $a = -kv$, where k is constant. It is given that at $t=0$, $x=0$, $V=V_0$. Calculate velocity and acceleration of particle as function of time.

$$-kv = \frac{dv}{dt}$$

$$\Rightarrow \int \frac{dv}{v} = -k \int dt$$

$$\Rightarrow \left[\ln v \right]_{V_0}^v = -kt$$

$$a = -kv$$

$$\frac{dv}{dt} = -kv$$

$$\int \frac{dv}{v} = \int -k dt$$

$$\ln v = -kt$$



$$\ln \left(\frac{v}{v_0} \right) = -kt$$

$$\Rightarrow \frac{v}{v_0} = e^{-kt}$$

$$\Rightarrow \frac{v}{v_0} = e^{-kt}$$

$$\Rightarrow \boxed{v = v_0 e^{-kt}}$$

$$\frac{dx}{dt} = v_0 e^{-kt}$$

$$\Rightarrow \int_0^x dx = v_0 \int_0^t e^{-kt} dt$$

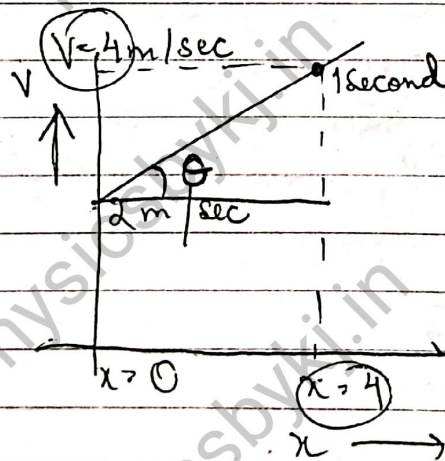
$$\Rightarrow \left[x \right]_0^x = v_0 \left[\frac{e^{-kt}}{-k} \right]_0^t$$

$$\Rightarrow (x-0) = \frac{v_0}{-k} [e^{-kt} - e^{-k \cdot 0}]$$

$$\Rightarrow x = \frac{v_0}{-k} [e^{-kt} - 1]$$

$$\Rightarrow \boxed{x = \frac{v_0}{k} [1 - e^{-kt}]}$$

Ques:



Calculate average displacement at time $t = 2$ sec.

$$x = v \times t$$

$$4 = 4 \times t$$

$$t = 1 \text{ second}$$

Initially, $t = 0$, $x = 0$,

$v = 2 \text{ m/sec}$.

$$\tan \theta = \frac{2}{4}$$

$$y = mx + c$$

$$v = \frac{2}{4}x + 2$$

$$v = \frac{x}{2} + 2$$

$$\frac{dx}{dt} = \left[\frac{1}{2}x + 2 \right]$$

$$\int \frac{dx}{\left(\frac{x}{2} + 2 \right)} = \int_0^2 dt$$

$$\Rightarrow \left[\ln \left(\frac{1}{2}x + 2 \right) \right]_0^x = 2$$

$$2 \left[\ln \left(\frac{1}{2}x + 2 \right) \right]_0^x = 2$$

$$\ln \left(\frac{1}{2}x + 2 \right) - \ln 2 = 1$$

$$\ln \left(\frac{1}{2}x + 2 \right) = 1 + \ln 2$$

$$\left(\frac{1}{2}x + 2 \right) = e^{1 + \ln 2}$$

$$\frac{1}{2}x + 2 = e \cdot 2$$

$$x = 4 \times 1.713$$

Equations of Motion



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Eg: A point object is moving along rectilinear path under uniform acceleration a . It is given that $t=0$, $x=0$, $v=u$, calculate displacement and velocity of particle after time t & covering a displacement x .

Solution $\Rightarrow$

$$v = \frac{dx}{dt} \quad a = \frac{dv}{dt}$$

$$\cancel{dx} = v dt \Rightarrow \int dv = \int a dt$$

$$\Rightarrow \int_u^v dv = a \int_0^t dt$$

$$\Rightarrow [v]_u^v = a[t]_0^t$$

$$\Rightarrow [v-u] = a[t-0]$$

$$\Rightarrow v-u = at$$

$$\boxed{v = u + at}$$

①

Again,

$$a = \frac{v dv}{dx}$$

$$\int_u^v v dv = \int_0^x a dx$$

$$\frac{v^2}{2} - \frac{u^2}{2} = a(x-0)$$

$$\frac{v^2 - u^2}{2} = as$$

$$\boxed{2as = v^2 - u^2}$$

②



path
b.c.
y of
int x.

Substituting the value of v from eq. (1) in (

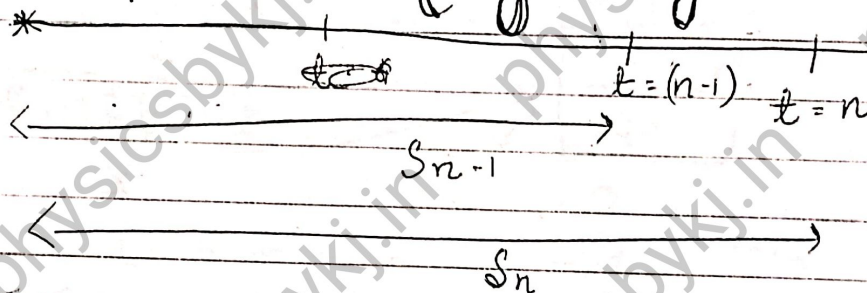
$$(u + at)^2 = u^2 + 2as$$

$$u^2 + a^2t^2 + 2uat = u^2 + 2as$$

$$S = \frac{a^2t^2 + 2uat}{2a}$$

$$S = ut + \frac{1}{2}at^2 \quad \text{--- (3)}$$

Ques * Displacement of body under n^{th} Sec



$$\Delta S_n = S_n - S_{n-1}$$

$$= \left[un + \frac{1}{2}an^2 \right] - \left[u(n-1) + \frac{1}{2}a(n-1)^2 \right]$$

$$S_{nn} = u + \frac{a}{2}(2n-1) \quad \text{--- (4)}$$

Note: In all equations of motion (1), (2), (3), (4) all physical quantities like v , u , a , x & S_n are Vector quantities so we must know in what directions these quantities are applied. So, for applying equations of motion, we must have proper sign conventions.



Short Cut Method ^{can be applied only when initial or final velocity is zero}

Case 1 when $u = 0$

Diagram showing motion from $x=0$ to x with initial velocity $u=0$ and final velocity v at time t .

$t = \frac{v}{a}$
 $t = \sqrt{\frac{2x}{a}}$
 $x = \frac{v^2}{2a}$

$v = 0 + at$
 $x = 0 + \frac{1}{2}at^2$
 $x = \frac{at^2}{2}$
 $x = \frac{v^2}{2a}$

Case 2 when $v = 0$

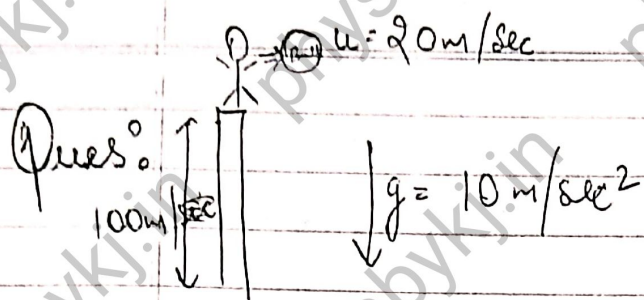
Diagram showing motion from $x=0$ to x with initial velocity $u=v$ and final velocity $v=0$ at time t .

$t = \frac{v}{a}$
 $t = \sqrt{\frac{2x}{a}}$
 $x = \frac{v^2}{2a}$

$0 = v - at$
 $\Rightarrow t = \frac{v}{a}$
 $x = \frac{v^2}{2a}$



only



$$V = u + at$$

$$100 = 20 \times t + \frac{1}{2} \times 10 \times t^2$$

$$200 = 40t + 5t^2$$

$$0 = t^2 + 8t - 40$$

(i) Time taken until the ball hits the ground for the first time.

(ii) Speed with which the ball strikes the ground.

Sol: (i)

$$u = +20 \text{ m/sec}$$

$$a = -10 \text{ m/sec}^2$$

$$S = \text{displacement} = -100 \text{ m}$$

$$S = ut + \frac{1}{2} at^2$$

$$-100 = 20t + \frac{1}{2} \times (-10) \times t^2$$

$$-100 + 20t = -5t^2$$

$$5t^2 - 20t - 100 = 0$$

$$t^2 - 4t - 20 = 0$$

$$t = \frac{-(-4) \pm \sqrt{16 - 4 \times (-20)}}{2}$$

$$= \frac{4 \pm \sqrt{96}}{2}$$

$$= \frac{4 \pm 4\sqrt{6}}{2}$$

$$\frac{4 + 4\sqrt{6}}{2}$$

$$\frac{4 - 4\sqrt{6}}{2}$$

$$\frac{4(1 + \sqrt{6})}{2} \text{ Sec}$$

X

(ii)
$$v^2 = u^2 + 2ax$$

$$= (20)^2 + 2(-10)(-100)$$

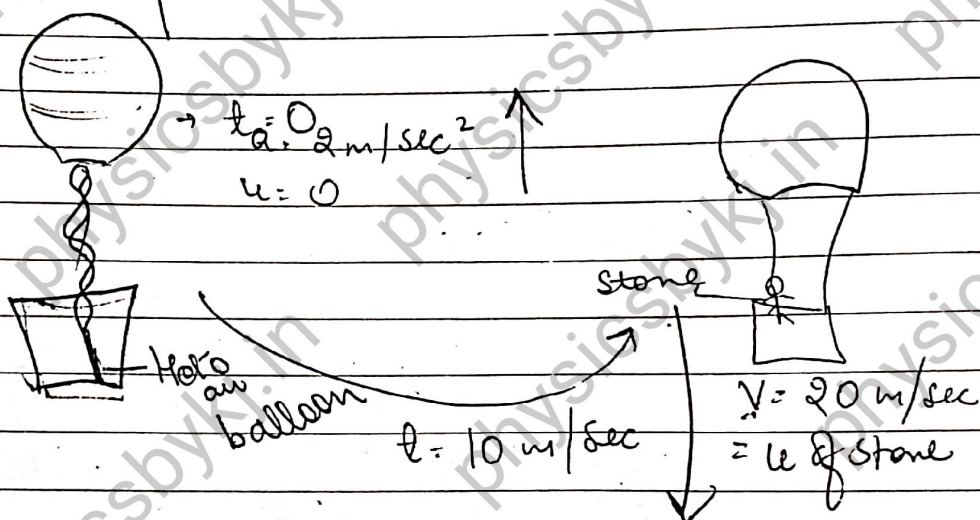
$$= 400 + 2000$$

$$= \sqrt{2400}$$

$$= 10\sqrt{24} \text{ Ans.}$$

Question :

S = 21

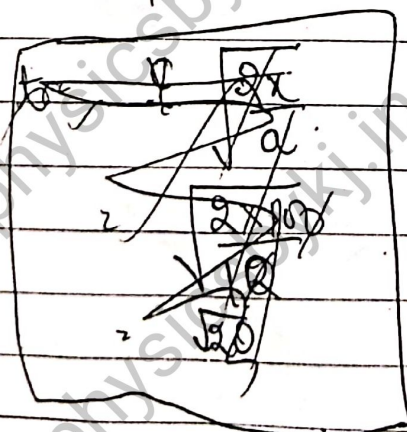


If after $t = 10 \text{ sec}$, boy drops a stone
 Calculate time after which stone will hit
 the ground

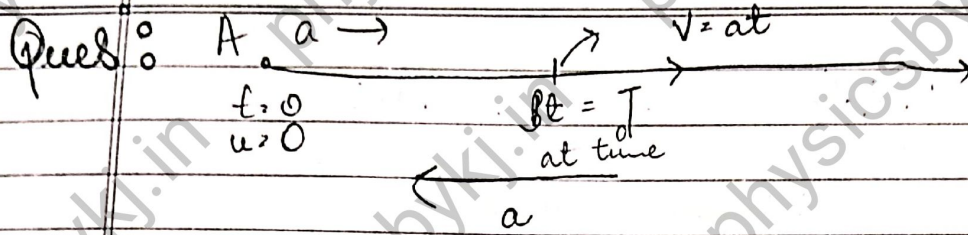
$$S = ut + \frac{1}{2}at^2$$

$$= \frac{1}{2} \times 2 \times 100$$

$$S = 100 \text{ m/sec}$$



Same as last
 problem now.



A particle is moving along a straight line, starts from $t=0$, $u=0$, $x=0$ at origin O, at $t=T$ time; direction of uniform acceleration is reversed. Calculate total time from start when particle again cross point O.

~~$v = u + at$~~ $S = ut + \frac{1}{2}at^2$ Rough

~~$v = at$~~

$S = \frac{1}{2}at^2 \leftarrow AB$

$t_{AB} = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2 \times \frac{1}{2}at^2}{a}}$

$t = \sqrt{\frac{2x}{a}}$

$= \sqrt{\frac{2 \times \frac{1}{2}at^2}{a}}$

$t = t\sqrt{2}$

$u=0$
 $t=0$

$x = \frac{1}{2}at^2$

$t=T, v=at$

$T' = \frac{v}{a} = \frac{aT}{a} = T$

$T = ?$

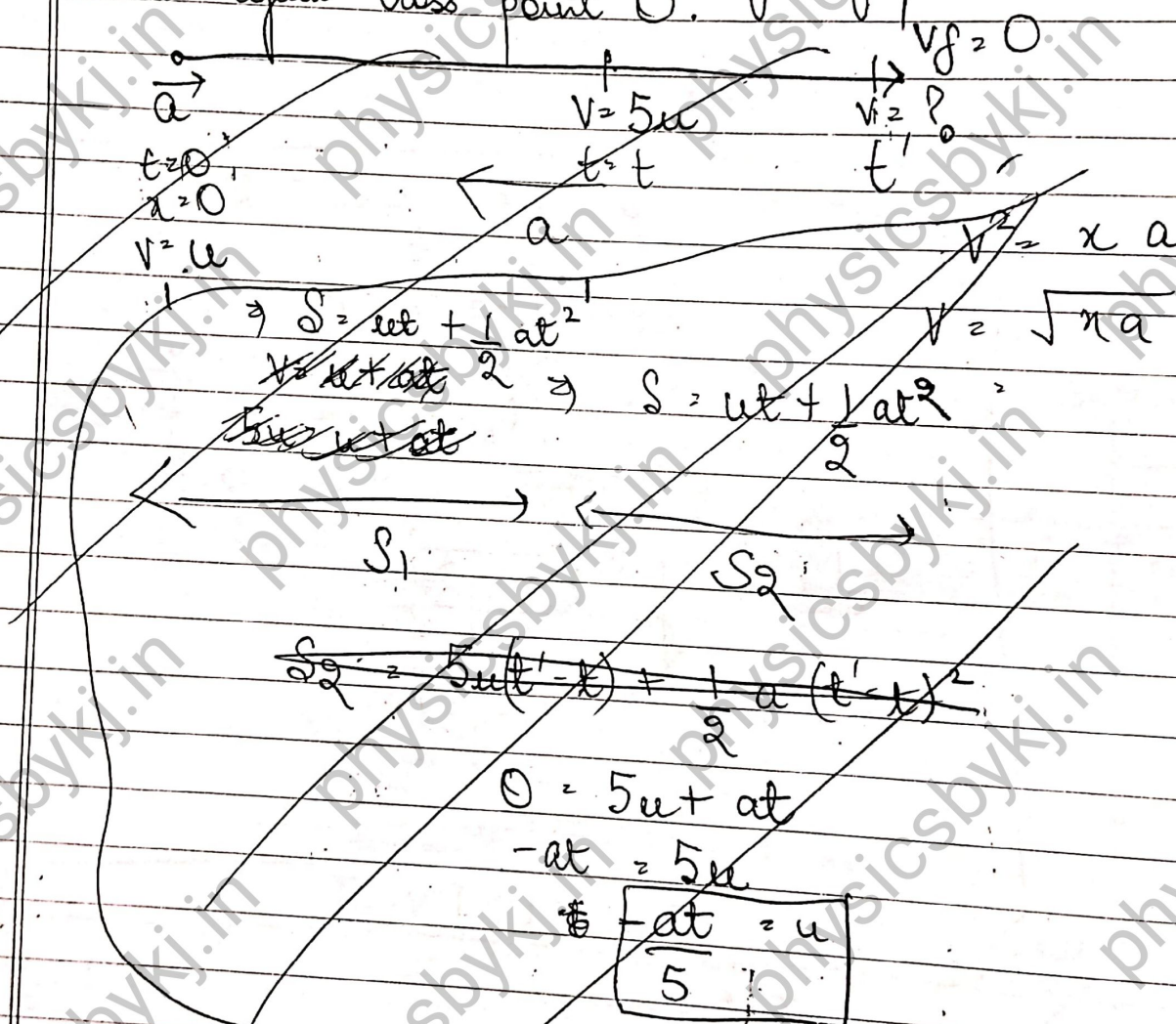
$T = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2 \times \frac{1}{2}aT^2}{a}} = T\sqrt{2}$

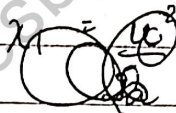
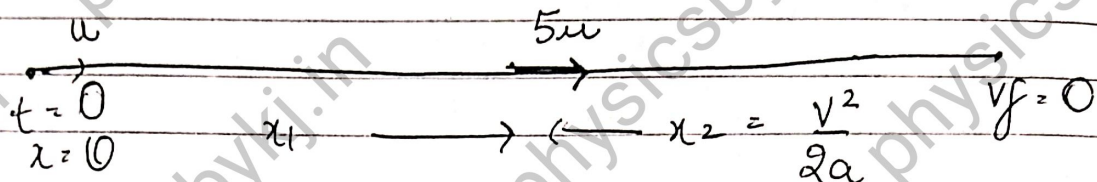


$$\text{Total Time} = 2T + \sqrt{2}T$$

Q:

A particle is moving along straight line under uniform acceleration, particle starts from $t=0$, $x=0$, $v=u$. When velocity of particle becomes 5 times of initial velocity, direction of acceleration is reversed. Calculate velocity of particle when it will again cross point O.





$$\Rightarrow (5u)^2 - u^2 = 2ax_1$$

$$\Rightarrow x_1 = \frac{24u^2}{2a}$$

$$= \frac{(5u)^2}{2a} = \frac{25u^2}{2a}$$

$$\text{Total Displacement} = -x_1 + x_2$$

$$= \frac{25u^2}{2a} - \frac{24u^2}{2a}$$

$$= \frac{u^2}{2a}$$

$$v = \sqrt{2a(x_1 + x_2)}$$

$$= \sqrt{2a \cdot \frac{49u^2}{2a}}$$

$$= 7u$$



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Graphical Analysis of Rectilinear Motion.

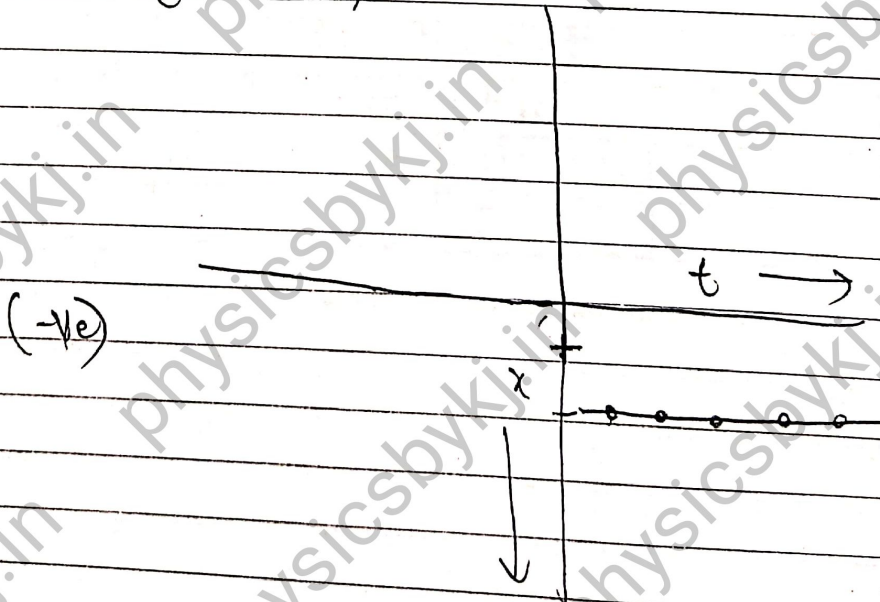
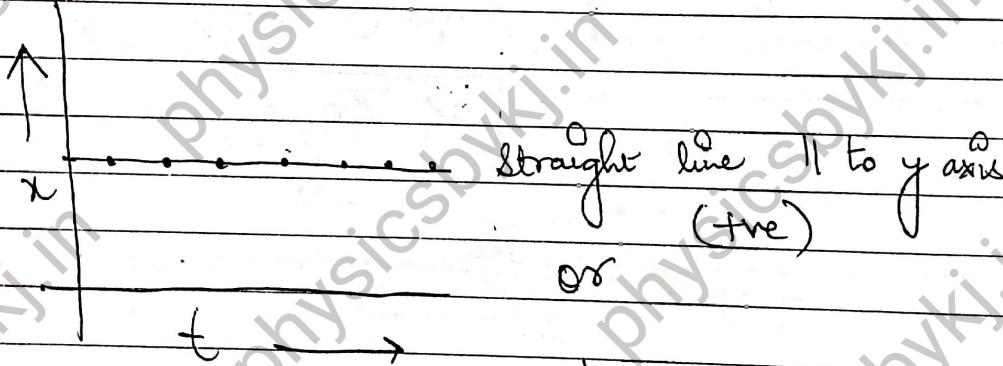
Case

We generally study three types of graph in rectilinear motion :

- 1) Displacement - Time graph
- 2) Velocity - Time Graph
- 3) Acceleration - Time Graph

Displacement Time Graph

Case 1 When particle is at Rest



Case



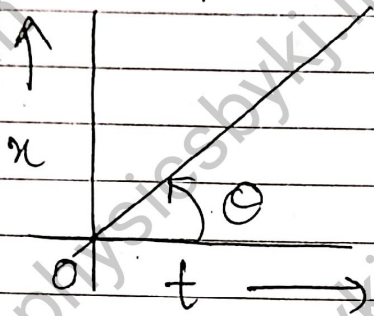
inertion.

Case 2

When particle is in uniform motion

Velocity is constant
 $\Rightarrow$ acceleration = 0

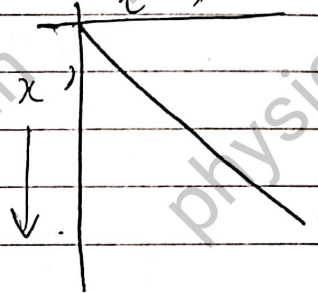
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$$\Rightarrow x = ut + \frac{1}{2}at^2$$

$$x = ut + 0$$

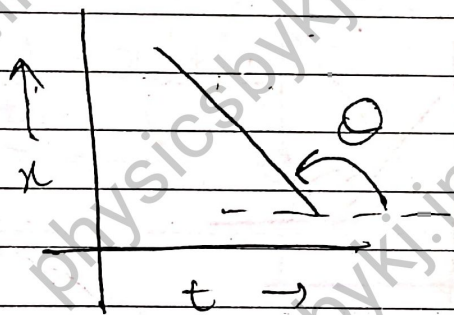
$$x \propto t$$



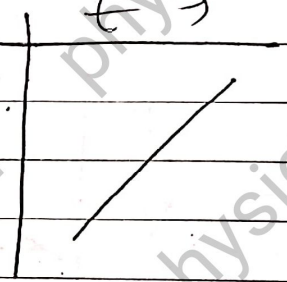
Slope = $\frac{dx}{dt}$
(velocity) = $\tan \theta$

$\leftarrow \theta < \text{acute} \Rightarrow \text{Velocity} = +ve$

②



Velocity
is +ve

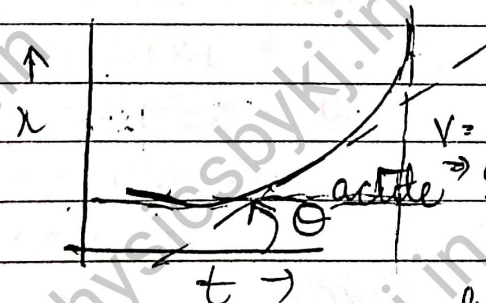


Case 3

When particle is in accelerated motion,
($a \neq 0$) let $u = 0$

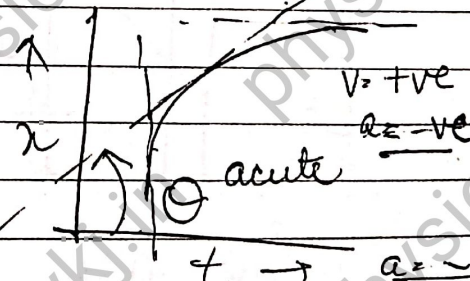
$$x = \frac{1}{2}at^2$$

$$x \propto t^2$$



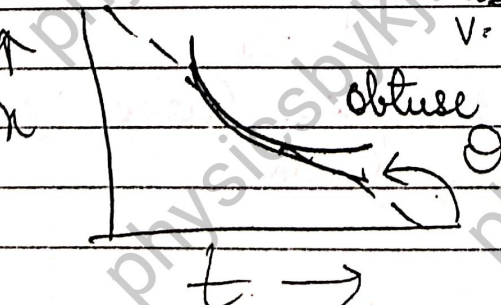
$V = +ve$
 $\Rightarrow a = +ve$

$a = +ve$
 $V = -ve$

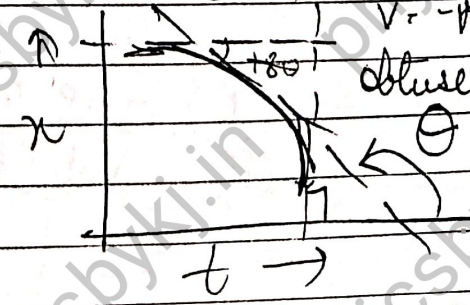


$V = +ve$
 $a = -ve$

$a = -ve$
 $V = +ve$



obtuse



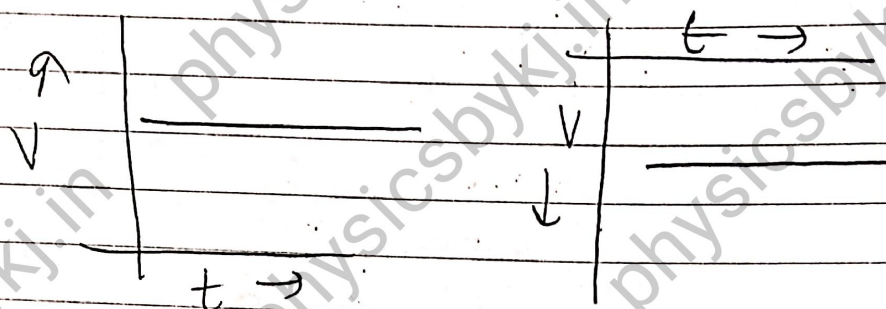
obtuse



Velocity Time Graph

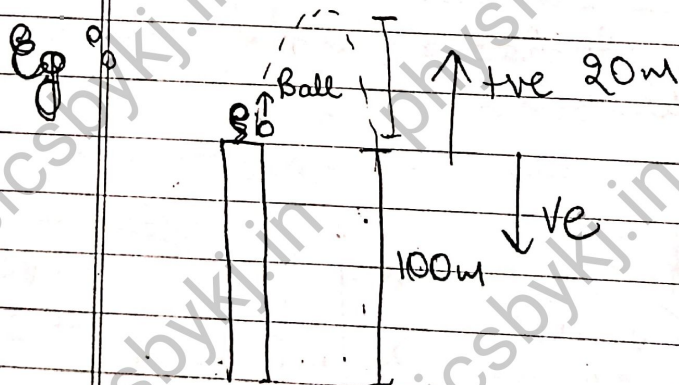
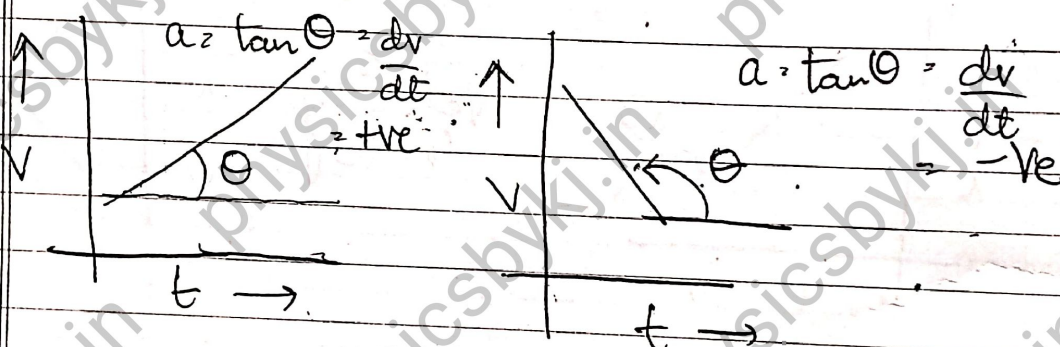
Case 1

When particle is in uniform motion :
 $v = \text{constant}$, $a = 0$



Case 2

When particle is in accelerated motion :
 $a = \text{constant}$



$$t_{\text{time}} = 2 + \sqrt{24} \text{ sec}$$

$$v = \sqrt{2400} \text{ m/sec}$$

$$u = 20 \text{ m/sec}$$

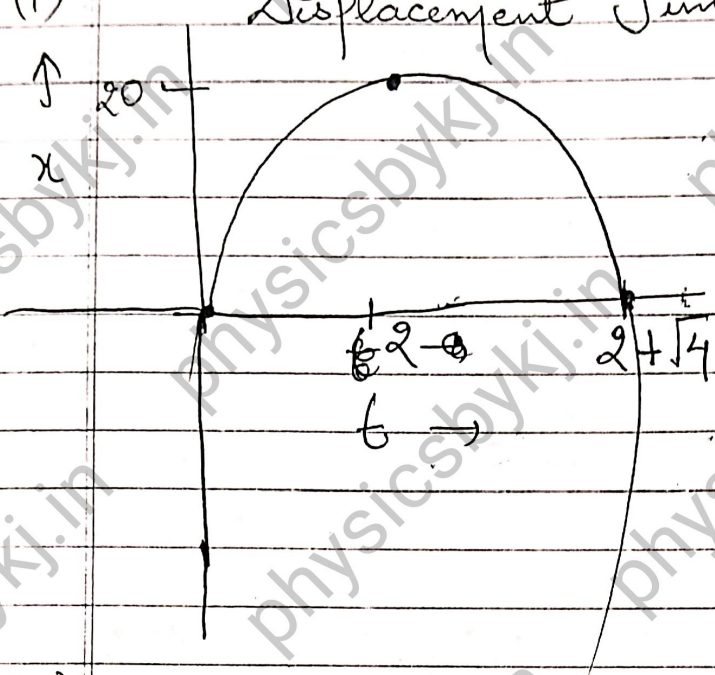
$$0 = 20 + (-10)t$$

$$t = 2 \text{ sec}$$

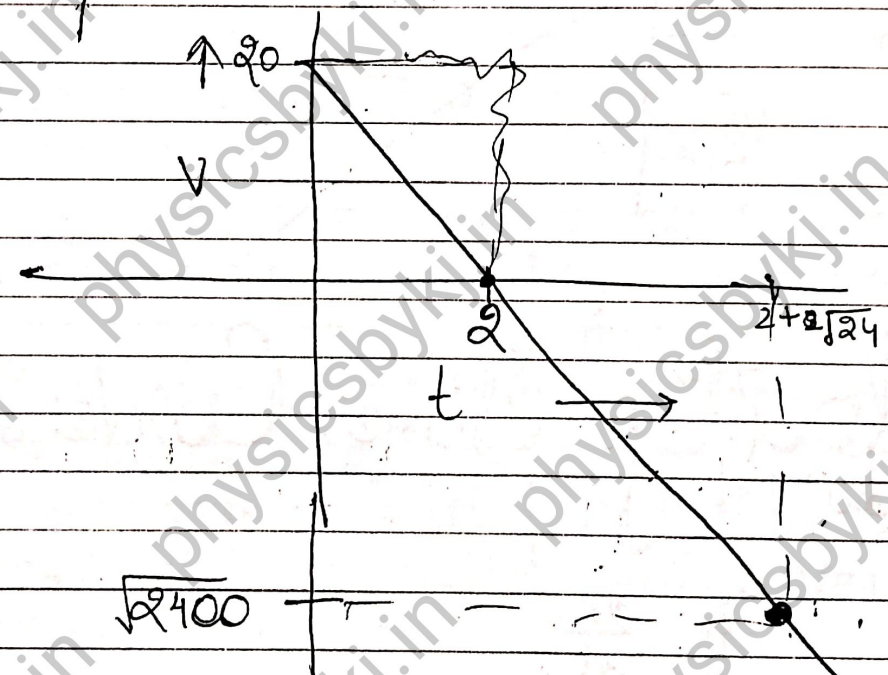
For given problem Sketch

- (i) Displacement Time Curve
- (ii) Velocity Time Curve

(i) Displacement Time curve



(ii)



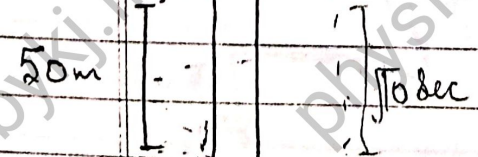
Q: If a ball is dropped from a tower of height 50 m,

Sketch (a) A-T graph

(b) V-T curve.

$$S = ut + \frac{1}{2}at^2$$

10/10 s $\frac{1}{2} \times$



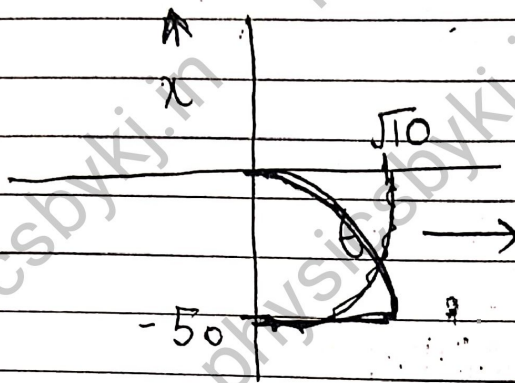
$$V = u + at$$

$$V = 0 + 10/10$$

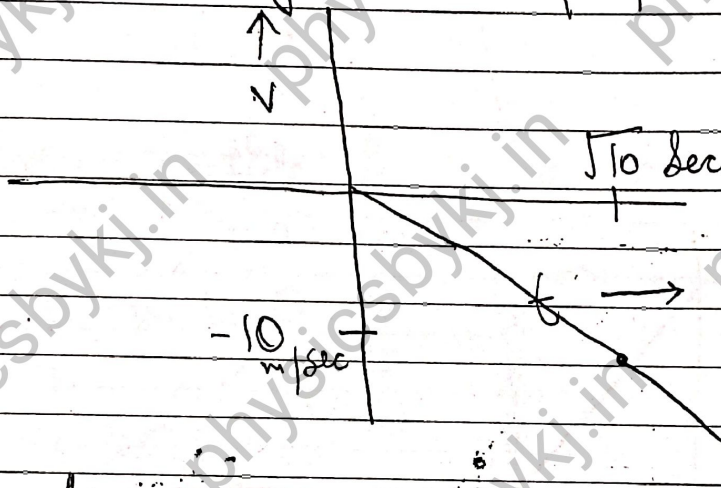
$$50 = \frac{1}{2} \times 10 \times t^2$$

$$V = 10/10 \text{ m/sec}$$

$$10 \text{ sec} = \text{time}$$

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(ii) Velocity Time Graph



Velocity Time Curve:

We know that

$$v = \frac{dx}{dt}$$

$$dx = v dt$$

$$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v dt$$

$$(x_2 - x_1) = \int_{t_1}^{t_2} v dt$$

my companion

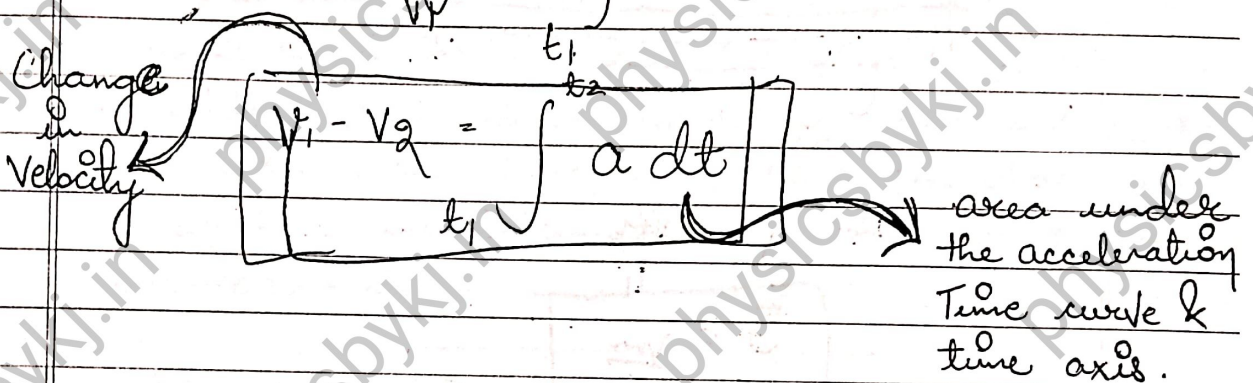
Displacement of

Acceleration Time Curve :

We know that,

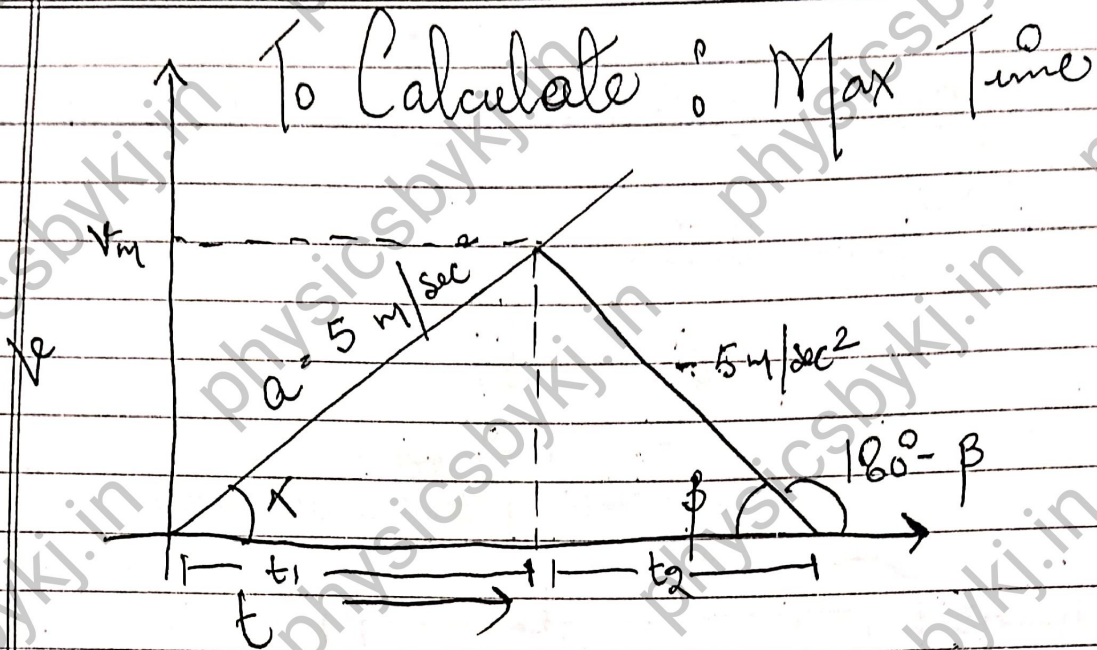
$$a = \frac{dv}{dt}$$

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a dt$$



NOTE: When in a problem, it is given that an object is in accelerating motion for some time, and moving with uniform velocity after some time and then finally retarding motion for some time [if moving from rest to rest] then it is advisable to solve such a problem using **VELOCITY TIME** curve.

Q: A bike accelerate for some time with 5 m/sec^2 acceleration and then achieves maximum velocity, its break can produce maximum retardation of 5 m/sec^2 . Calculate maximum time in which bike can cover 1800 m displacement.



$$\tan \alpha = \frac{V_m}{T_1}$$

$$5 = \frac{V_m}{T_1}$$

$$\tan(\pi - \beta) = \frac{V_m}{T_2}$$

$$\tan \beta = \frac{V_m}{T_2}$$

$$5 = \frac{V_m}{T_2}$$

Area under v-t curve $\rightarrow$ Displacement

$$1800 = \frac{1}{2} \times (T_1 + T_2) \times V_m$$

$$1800 = \frac{1}{2} \times \left(\frac{V_m}{5} + \frac{V_m}{5} \right) \times V_m$$

$$1800 = \frac{1}{2} \times \frac{2V_m}{5} \times V_m$$

$$9000 = V_m^2$$

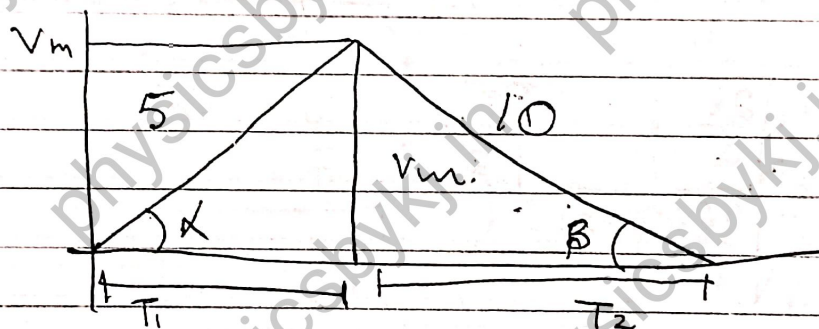
$$30\sqrt{10} = V_m$$

$$T_1 = \frac{V_m}{5} = \frac{30\sqrt{10}}{5} = 6\sqrt{10} \text{ sec}$$

$$T_2 = \frac{V_m}{10} = 6\sqrt{10} \text{ sec}$$

$$\text{Total Time} = 6\sqrt{10} + 6\sqrt{10} = 12\sqrt{10} \text{ Sec}$$

Ques: A bike accelerates for some time with acceleration = 5 m/sec^2 , calculate maximum velocity when its break can produce maximum retardation of 10 m/sec^2 . Also calculate minimum time in which bike can ~~pro~~ cover 1800 m displacement.



$$\tan X = \frac{V_m}{T_1}$$

$$5 = \frac{V_m}{T_1} \quad \text{--- (1)}$$

$$10 = \frac{V_m}{T_2} \quad \text{--- (2)}$$

$$\text{Area under the curve} = \frac{1}{2} \times (T_1 + T_2) \times V_m$$

$$= \frac{1}{2} \times \left(\frac{V_m}{10} + \frac{V_m}{5} \right) V_m$$

$$= \frac{1}{2} \times \frac{3V_m}{10} \times V_m$$



$$1800 = \frac{3v_m^2}{20}$$

$$36000 = 3v_m^2$$

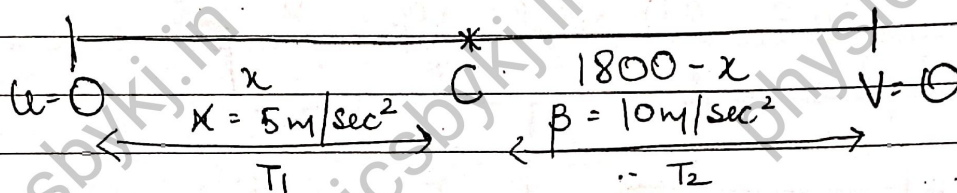
$$12000 = v_m^2$$

$$20\sqrt{30} = v_m$$

$$T_1 = 4\sqrt{30} \text{ Sec}$$

$$T_2 = 2\sqrt{30} \text{ Sec}$$

OR



$$T_1 = \sqrt{\frac{2x}{a}}, \quad \sqrt{\frac{2x}{5}}$$

$$T_2 = \sqrt{\frac{2(1800-x)}{10}}$$

$$v = \sqrt{2 \times 5 \times x} = \sqrt{2 \times 10 \times (1800-x)}$$

$$2 \times 5 \times x = 2 \times 10 \times (1800-x)$$

$$10x = 2 \times 10 \times (1800-x)$$

$$\frac{x}{2} = 1800 - x$$

$$\frac{x}{2}$$

$$x = 3600 - 2x$$

$$3x = 3600$$

$$x = 1200$$

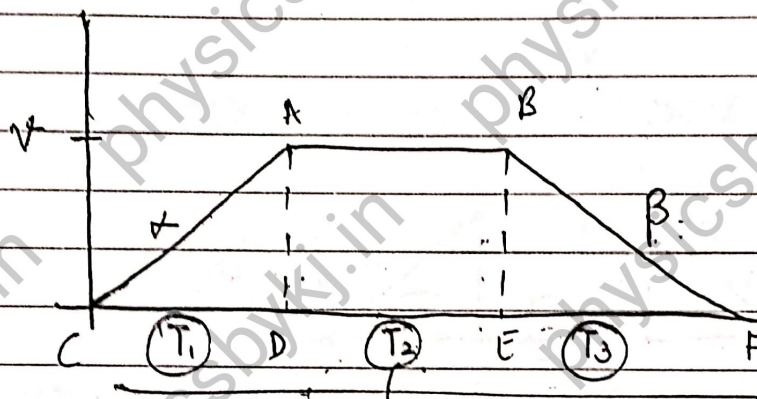
$$T_1 = \sqrt{\frac{1200 \times 2}{5}}$$

$$T_2 = \sqrt{\frac{2 \times 600}{10}}$$

$$= \sqrt{\frac{2400}{5}} = \sqrt{480}$$

$$= \sqrt{120}$$

Ques: A car starts from rest and accelerates with acceleration α for some time and moves with uniform velocity V for some other time and finally breaks of car produce retardation β for some time. It is given that total displacement of body was S in uniform motion. Calculate Total Time.



$$\alpha = \frac{V}{T_1}$$

$$T_1 = \frac{V}{\alpha}$$

$$\beta = \frac{V}{T_3}$$

$$T_3 = \frac{V}{\beta}$$

$T_2 \Rightarrow$ Area of Trapezium

$$\frac{1}{2} \times (AB + DE) \times V = S$$

$$\frac{1}{2} (T_2 + T) \times V = S$$

$$T_2 = \frac{2S}{V} - T$$

$$T = t_1 + t_2 + t_3$$

$$2T = \frac{V}{\alpha} + \frac{V}{\beta} + \frac{2S}{V}$$

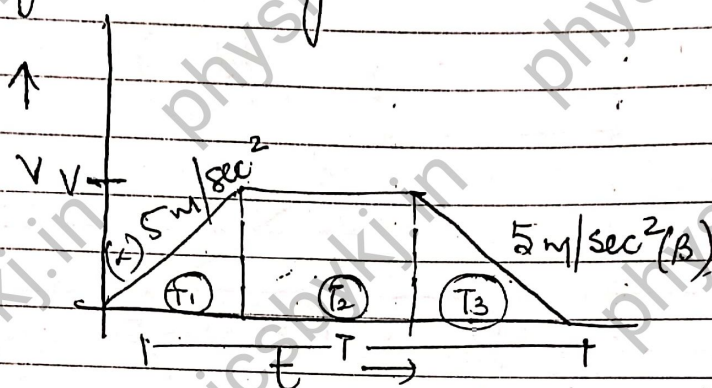
$$T = \frac{V}{2} \left(\frac{1}{\alpha} + \frac{1}{\beta} \right) + \frac{S}{V}$$

Answer



Ques

A car accelerates for some time with acceleration 5 m/sec^2 for some time & moves with uniform velocity for some other time and retards finally with 5 m/sec^2 . If average velocity for whole motion is 20 m/sec and total time for motion is 25 sec . Calculate time for which particle travels with uniform velocity.



$$\text{Average Velocity} = \frac{\text{Total Displacement}}{\text{Total Time}}$$

$$20 = \frac{S}{25}$$

$$500 = S$$

$$a = \frac{V}{T_1}$$

$$5 = \frac{V}{T_1}$$

$$T_1 = \frac{V}{5}$$

$$T_1 = \frac{V}{5}$$

$$T_3 = \frac{V}{5}$$

$$S = \frac{1}{2} \times (T_2 + T_3) \times V$$

$$500 = \frac{1}{2} (T_2 + 25) \times V$$



$$\frac{1000}{V} = T_2 + 25$$

$$\boxed{\frac{1000}{V} - 25 = T_2}$$

$$T_1 + T_2 + T_3 = 25$$

$$\frac{V}{5} + \frac{V}{5} + \frac{1000}{V} - 25 = 25$$

$$V \left(\frac{2}{5} \right) + \frac{1000}{V} = 50$$

$$\frac{1000}{V} + \frac{2V}{5} = 50$$

$$5000 + 2V^2 = 50V$$

$$5V$$

$$500 + 2V^2 = 50V$$

$$V^2 - 50V + 250 = 0$$

$$V = \frac{50 \pm \sqrt{2500 - 4 \times 250}}{2 \times 1}$$

$$= \frac{50 \pm \sqrt{2500 - 1000}}{2}$$

$$= \frac{50 \pm \sqrt{1500}}{2}$$

$$= \frac{50 \pm 10\sqrt{15}}{2}$$

$$\frac{50 + 10\sqrt{15}}{2}$$

$$\frac{50 - 10\sqrt{15}}{2}$$

Find V & substitute its value in

Ans: $T_2 = 15 \text{ sec}$ 15

$$\frac{1000}{50 + 10\sqrt{15}} - 25 = T_2$$

$$\frac{2000}{50 + 10\sqrt{15}} - 25 = T_2$$

$$2000 - 25(50 + 10\sqrt{15}) = T_2(50 + 10\sqrt{15})$$

$$2000 - 1250 - 250\sqrt{15} = T_2(50 + 10\sqrt{15})$$

$$750 - 250\sqrt{15} = T_2(50 + 10\sqrt{15})$$

$$T_2 = \frac{750 - 250\sqrt{15}}{50 + 10\sqrt{15}}$$

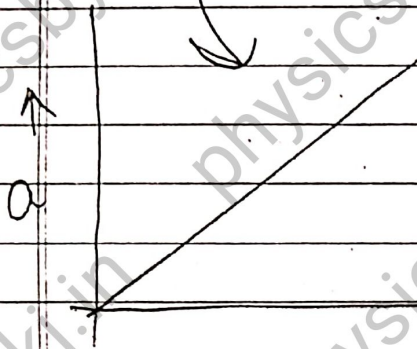
$$T_2 = 15$$



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Paper 1 Paragraph 2 (Q. 17 to 19)



$$y = mx + c$$

$$a = 10t + 0$$

$$10$$

$$a = t$$

$$\frac{dv}{dt} = t$$

$$\int_0^v dv = \int_0^t t dt$$

$$v = \frac{t^2}{2}$$

$$\frac{dx}{dt} = \frac{t^2}{2}$$

$$x = \frac{t^3}{6}$$

Average speed

$$\frac{x_{10} - x_0}{10 - 0}$$

$$= \frac{(10)^3 - 0}{6}$$

$$= \frac{1000}{6}$$

$$= 166.67$$

$$= 166.67 \text{ m/s}$$

$$= 166.67 \text{ m/s}$$

$$= 166.67 \text{ m/s}$$

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$$= 166.67 \text{ m/s}$$

$$= 166.67 \text{ m/s}$$

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$$= 166.67 \text{ m/s}$$

$$= 166.67 \text{ m/s}$$

$$= 166.67 \text{ m/s}$$

$$= 166.67 \text{ m/s}$$

In Paper 2

Q.

$$v_1 + v_2$$

$$v_1 - v_2$$

$$(\vec{v}_1 + \vec{v}_2) \cdot (\vec{v}_1 - \vec{v}_2) = 0$$

$$v_1^2 + v_2^2 + v_1 v_2 - v_1 v_2 = 0$$

$$|v_1| = |v_2|$$



paper 2

Paragraph 1 (Q. 38 to 40)

$$u = 0$$

$$\vec{r}_1 = \hat{i} + \hat{j}$$

F_1 & F_2

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$$

$$m\vec{a}_1 = 3\hat{i} + 5\hat{j} + 2\hat{k}$$

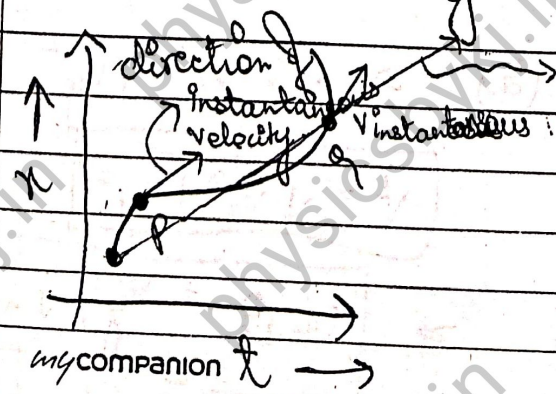
$$\vec{a}_1 = (3\hat{i} + 5\hat{j} + 2\hat{k})$$

$$s = 0 + \frac{1}{2} at^2$$

$$= \frac{1}{2} (3\hat{i} + 5\hat{j} + 2\hat{k})$$

$$= \frac{3}{2}\hat{i} + \frac{5}{2}\hat{j} + \hat{k}$$

Representation of Instantaneous Velocity and Avg. Velocity Graphically

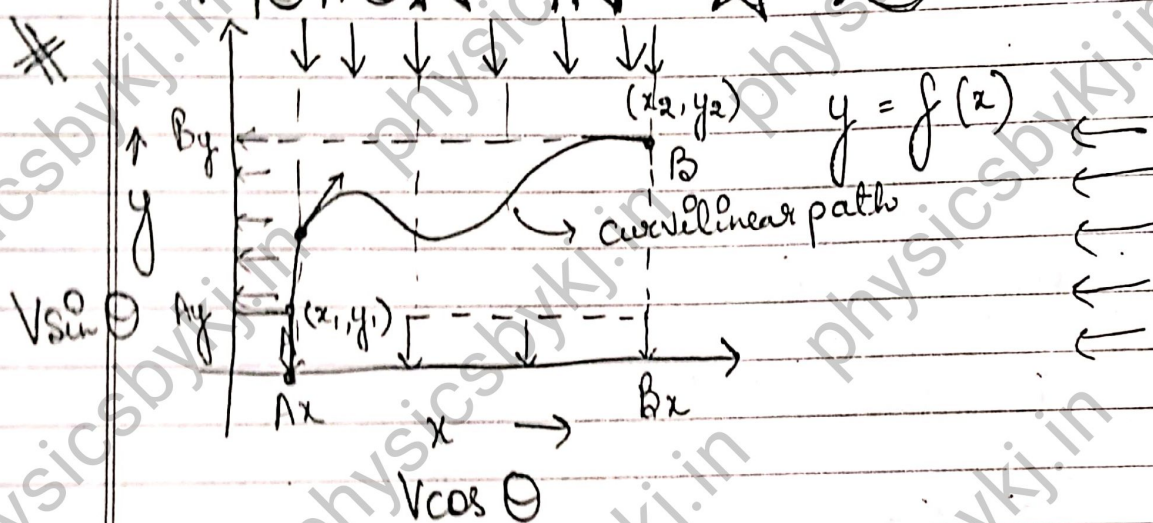




- For direction of instantaneous velocity, just drop a tangent on a point, direction of tangent will give us direction of instantaneous velocity.

- For direction of average velocity at any point, join this point to initial point & in direction joining line, we will get direction of average velocity.

MOTION IN 2 D



- Motion in 2-d can be split into separate rectilinear motions. One is along x-axis and another is along y-axis. Both rectilinear motions are independent motion. Only one parameter is common, between these two vectors motions which is time.

$$v_x = v \cos \theta \quad \text{--- (1)}$$

$$v_y = v \sin \theta \quad \text{--- (2)}$$

$$(1)^2 + (2)^2 \Rightarrow v^2 = v_x^2 + v_y^2$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$(2)/(1)$$

$$\Rightarrow \tan \theta = v_y / v_x$$