Calculus

The study of rate of change of one physical variable / quantity with respect to another is Calculus.

Function

⇒ $y = f(x)$

dependent variable $y = x + 3$ → free variable

It is the statement of mathematical relation between two variables

Classification of Functions (4 types):

1) Algebraic Function
2) Trigonometric Function
3) Logarithmic Function
4) Exponential Function

- Algebraic Function

Simple functions that we have studied now

⇒ $y = x + 3$ / $y = x^2 + 3x + 8$

- Trigonometric Function

Which includes trigonometric ratios

$y = \sin x$ / $y = \cos^2 x + \tan x$

- Exponential Function

where the power has variable

$$y = 2^x \;/\; y = 3^{x^2}$$

Inverse Function :-

$$\boxed{y = 2x+7} \quad — ①$$

Input x	Output y
0	7
1	9
2	11
3	13

Domain Range

$\Downarrow$

$2x = y - 7$

$x = \dfrac{y-7}{2}$

$$\boxed{y = \frac{x-7}{2}} \rightarrow ②$$

Input x	Output y
7	0
9	1
11	2
13	3

→ Equation 2 is inverse function of equation 1

NOTE : Inverse functions is defined as a function in which value of domain and range are interchanged.

- Logarithmic Function

It is defined as the inverse function of exponential function.

Eg : $y = 2^x$

Input x	Output y
0	1
1	2
2	4
3	8
Domain	Range

~~Inverse function of this~~ $\Rightarrow$

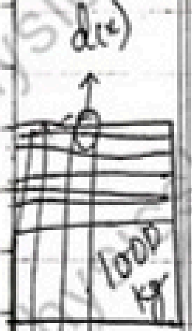
Logarithmic function eg $2^y = x$ $(y = \log_2 x)$

Properties of Logarithmic Function

1) $\log(ab) = \log a + \log b$
2) $\log \frac{a}{b} = \log a - \log b$
3) $\log a^n = n \log a$
4) Base change property

$$y = \log_2 3 \Rightarrow y = \frac{\log_5 3}{\log_5 2}$$

$$y = \log_2 x \Rightarrow y = \frac{\log_{10} x}{\log_{10} 2}$$



They had to measure this weight but they had a machine which couldn't measure more than 10 g. So, they did its differentiation

Differentiation

The process in which a physical quantity can be broken into a large no.'s of very small differential elements (i.e. dx) is called differentiation.

$$dx \rightarrow 0$$

① Eg: $y = x + 5$

If x is increased by differentiation element dx, then y is also increased by differential element dy.

i.e., $(y + dy) = (x + dx) + 5$

$\Rightarrow (x+5) + dy = (x + dx) + 5$

$\Rightarrow dy = dx$

$\Rightarrow \dfrac{dy}{dx} = 1$

↳ differentiation of y with respect to x

② Eg: $y = 5x + 8$, increase

$y + dy = 5(x + dx) + 8$

$\cancel{y} + dy = \cancel{5x} + 5dx + \cancel{8}$

$$\boxed{\frac{dy}{dx} = 5}$$

③ Eg: $y = x^2$

$y + dy = (x + dx)^2$

$y + dy = x^2 + \underbrace{(dx)^2}_{\text{very small}} + 2dx\,x$

(So neglect it)

$$\cancel{y} + dy = \cancel{x^2} + 2dx\, x$$

$$\boxed{\frac{dy}{dx} = 2x}$$

1^{st} ~~differ~~ Principle of differentiation

Direct Method of Differentiation :-

(1) $y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$

Eg : $y = x^4 \Rightarrow \frac{dy}{dx} = 4x^{4-1} = 4x^3$

$y = x^3 + 3x^2 \Rightarrow \frac{dy}{dx} = 3x^2 + 3 \times 2x^{2-1}$

$= 3x^2 + 6x$

$y = 6x^7 + 5x^6 + 3x^4$

$= 6 \times 7x^{7-1} + 5 \times 6x^5 + 4 \times 3x^{4-1}$

$= 42x^6 + 30x^5 + 12x^3$

(2) $y = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

eg : $y = x^{1/2} \Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$ Ans.

(3) $y = \frac{1}{x}$; $\frac{dy}{dx} = -\frac{1}{x^2}$

eg : $y = \frac{1}{x} = x^{-1}$;

$\frac{dy}{dx} = -1x^{-1-1} = -1x^{-2}$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{x^2}$$

(4) $y =$ constant, $\frac{dy}{dx} = 0$

Eg: $y = x^4 + 3\sqrt{x} + \frac{4}{x} + 8$

Solution: $y = 4x^3 + \frac{3}{2\sqrt{x}} - \frac{4}{x^2} + 0$

Eg: $y = 3x^5 + 4\sqrt{x} + 4\sqrt{x} + \frac{3}{x} + 9$

$$\frac{dx}{dy} = 15x^4 + \frac{2}{\sqrt{x}} + \frac{2}{\sqrt{x}} - \frac{3}{x^2} + 0$$

(5) Product Rule

$y = uv$ (where $u = f(x)$ and $v = f(x)$)

$$\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

Eg: $y = \underbrace{(x^2+1)}_{u}\underbrace{(x^3+2x^2)}_{v}$

$$\frac{dy}{dx} = (x^2+1)(3x^2+2x) + (x^3+2x^2)(2x+0)$$

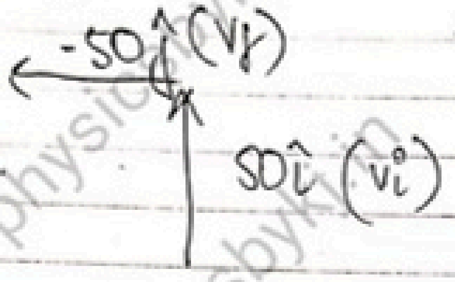
Eg: $y = \left(x + \frac{1}{\sqrt{x}}\right)\left(x^2 + 3x^3 + 4x^4\right)$

$$= \left(x + \frac{1}{\sqrt{x}}\right)\left(2x + 9x^2 + 16x^3\right) +$$

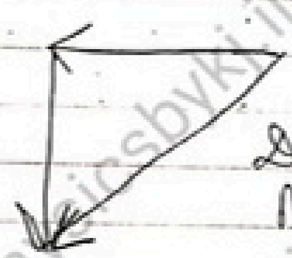
$$\left(x^2 + 3x^3 + 4x^4\right)\left(1 - \frac{1}{2\sqrt{x^3}}\right)$$

HOMEWORK: Ex. 3 Vectors

Exercise 3

10)


Change in velocity $= v_f - v_i$
$= 50\hat{j} - 50\hat{i}$

$\Rightarrow$


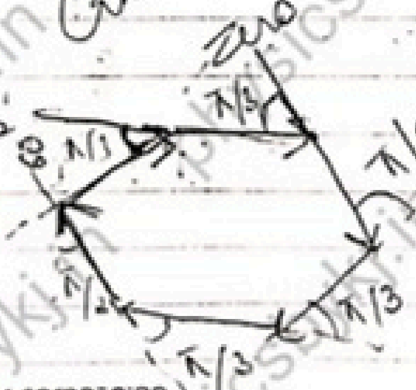
Direction — South West
Mag. — $50\sqrt{2}$ m/s (option C)

(12) (A) (10, 10), 10 → If this sum is ~~greater~~ than or equal to the third one then their sum can be zero

(B) (10, 10), 20

(C) 10, (20, 20)

(D) (10, 20), 40 — Cannot be zero

(14)


= Zero

(6) Quotient Rule

If $y = \frac{u}{v}$ where $u = f(x)$ & $v = g(x)$

then $\frac{dy}{dx} = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2}$

Eg: $y = \frac{x^3+3}{x^2+2x}$, find $\frac{dy}{dx}$ = ?

Sol:

$$\frac{x^2+2x\,(3x^2+0) - x^3+3\,(2x+2)}{(x^2+2x)^2}$$

Eg: $y = \frac{x^7+3x}{x^6+3x^2+9}$, find $\frac{dy}{dx}$

$$= \frac{x^6+3x^2+9\,(7x^6+3) - x^7+3x\,(6x^5+6x)}{(x^6+3x^2+9)^2}$$

Differentiation of Trigonometric Functions

(7) If $y = \sin x$; $\frac{dy}{dx} = \cos x$

(8) If $y = \cos x$; $\frac{dy}{dx} = -\sin x$

(9) If $y = \tan x$; $\frac{dy}{dx} = \sec^2 x$

(10) If $y = \cot x$; $\frac{dy}{dx} = -\operatorname{cosec}^2 x$

(11) If $y = \sec x$; $\frac{dy}{dx} = \sec x \tan x$

(12) If $y = \operatorname{cosec} x$; $\frac{dy}{dx} = -\operatorname{cosec} x \cot x$

Q. $y = x^2 \sin x$, find $\frac{dy}{dx}$.

Ans → ~~$2x \cos x$ Ans~~

$$y = \underbrace{x^2}_{u} \times \underbrace{\sin x}_{v}$$

Applying PRODUCT RULE

Ans : $= x^2 \times \cos x + \sin x \times 2x$

Q. $y = 3x^3 \tan x$, $\frac{dy}{dx}$?

$= 3x^3 \sec^2 x + \tan x \; 9x^2$.

Q. $y = \frac{\sin x}{3x^2+2}$, $\frac{dy}{dx}$?

$$= \frac{3x^2+2(\cos x) - \sin x\,(6x)}{(3x^2+2)^2}$$

Q. $y = \sin 5x$, $\frac{dy}{dx}$?

y : let $5x$ be t

$y = \sin t$

$$\boxed{\frac{dy}{dt} = \cos t} \quad — (1)$$

MyCompanion

$$\boxed{\frac{dt}{dx} = 5} \quad \text{--- (2)}$$

Multiplying (1) and (2)

$$\frac{dy}{dt} \times \frac{dt}{dx} = \cos t \times 5$$

$$= 5\cos 5x$$

Q. $y = \sin^3 x$; $\frac{dy}{dx} = ?$

Let $\sin x = t$ $\Rightarrow \frac{dt}{dx} = \cos x$

So, $y = t^3$ $\Rightarrow \frac{dy}{dt} = 3t^2$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \cancel{3t^2}\ \cos x \times 3t^2$$

$$= 3t^2 \cos x$$

$$\cancel{y = \sqrt{\tan x}}$$
$$\cancel{(\tan^2 x)}$$

Q. $y = \sqrt{\tan x}$

Let $\tan x = t$; $y = \sqrt{t}$

$\frac{dt}{dx} = \sec^2 x$; $\frac{dy}{dt} = \frac{1}{2\sqrt{t}}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan x}} \times \sec^2 x$$

* Assume something as t and do ~~...~~ $\frac{dy}{dt}$ and multiply $\frac{dy}{dt}$ by differentiation of t.

Q: $y = \sin^2 x$

Let $\sin x = t$

$\frac{dy}{dx} = 2\sin x \times \cos x$

Q: $y = \tan x^{1/3}$

$\frac{1}{3} \tan x^{-\frac{2}{3}} \times \sec^2 x$

Q: $y = x^2 \cancel{\cos} \cos 3x$

~~cos 3x~~

~~$-3\sin 3x$~~

~~$= \cos 3x \times 2x + x^2 \cos x \times (-3\sin 3x \cos 3x)$~~

Ans → $x^2 \times \{-\sin(3x)x\} \times 3 + \cos 3x \times 2x$

Q: $y = \cot^4 x$

Ans → $4\cot x^3 \times -\operatorname{cosec}^2 x$

Q: $y = (\operatorname{cosec} x)^{3/2}$

Ans: $\frac{3}{2} \operatorname{cosec} x^{\frac{1}{2}} \times -\operatorname{cosec} x \cot x$

my companion

~~$y = \sqrt{\cot^2 3x}$~~

~~$= (\cot 3x)^{2 \times \frac{1}{2}}$ $\quad \cot t \rightarrow -\operatorname{cosec}^2 t$~~

~~$=$ cot~~

Q. $y = \cot^2 3x$

$= (\cot 3x)^2$

$= 2\cot 3x \times -\operatorname{cosec}^2(3x) \times 3$

Q. $y = (\tan 4x)^{1/2}$

$= \cancel{\frac{1}{2}\tan t} \times \cancel{4\sec^2 x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan 4x}} \times \sec^2(4x) \times 4$

Q. $y = \left(\cot \frac{x}{2}\right)^{1/2}$

$\cancel{\frac{1}{2}\cot}$ $\quad \sqrt{\cot \frac{x}{2}}$

$= \frac{1}{2\sqrt{\cot \frac{x}{2}}} \times -\operatorname{cosec}^2\left(\frac{x}{2}\right) \times \frac{1}{2}$

13 If $y = e^x$ then $\frac{dy}{dx} = e^x$

14 If $y = a^x$ then $\frac{dy}{dx} = a^x \log_e a$
$= a^x \ln a$

15 If $y = \ln x$ then $\log_e x = y$

then $\frac{dy}{dx} = \frac{1}{x}$

Eg: $y = \log_{10} x$, find $\frac{dy}{dx}$

$$y = \frac{\log_e x}{\log_e 10}$$

$$= \frac{1}{\log_e 10} \times \log_e x$$

$$= \frac{1}{\log_e 10} \times \ln x$$

$$= \frac{1}{\log_e 10} \times \frac{1}{x} \text{ Ans.}$$

Eg: If $y = \log_a x$; find $\frac{dy}{dx}$

$$y = \frac{\log_e x}{\log_e a}$$

$$= \frac{1}{\log_e a} \times \log_e x$$

$$= \frac{1}{x} \times a$$

~~If~~ $\cancel{\sqrt{\sin(\ln x)} = y}$

$$\cancel{\sqrt{\sin\left(\frac{1}{x}\right)} = y}$$

$$\cancel{\left(\sin\left(\frac{1}{x}\right)\right)^{1/2}}$$

~~Ans:~~ $\cancel{\frac{\sin\left(\frac{1}{x}\right)^{-1/2}}{2} \times \cos \ln x}$

85

90V

1st

Eg: If $y = \sqrt{\sin(\ln x)}$

find $\frac{dy}{dx} = \frac{1}{2\sqrt{\sin(\ln x)}} \times \cos(\ln x) \times \frac{1}{x}$

Eg: If $y = \tan^2 e^{3x}$; find $\frac{dy}{dx}$

$\tan x \to \sec^2 x$ $\quad$ $2\tan e^{3x} \times e^{3x} \times 3$

$e^x \to e^x$ $\quad$ $\times \sec^2 e^{3x}$

$2\tan e^{3x} \times \sec^2 e^{3x} \times 3 \times e^{3x}$

Eg:

Sol: $y = \sin^4(\ln \sin x)$ $\rightarrow \cancel{(\sin(\ln \sin x))^4}$

$= \cancel{(4\sin(\ln \sin x))^3}$

$\cancel{\sin^{3}\left(\frac{1}{\sin x}\right)}$

$\cancel{\cos^4\left(\frac{1}{\cos x}\right)}$

$\cancel{\left(\sin\left(\frac{1}{\cos x}\right)\right)^4}$

$\cancel{4\sin\left(\frac{1}{\cos x}\right)^3}$

$= 4[\sin(\ln \sin x)]^3 \times \cos(\ln \sin x) \times \frac{1}{\sin x} \times \cos x$

Eg: $a^{4x} = y$

$\frac{dy}{dx} = a^{(4x)} \times \ln a \times 4$

Eg: $y = e^{\sin 2x}$

$e^{\sin 2x} \times \cos 2x \times 2$

companion

→ Eg: $y = e^{\cos^2 x}$

$e^{\cos^2 x} \times 2\cos x \times -\sin x$

→ Eg: $y = e^{\cos^2 4x}$

$e^{\cos^2 4x} \times 2\cos 4x \times 4 \times -\sin 4x$.

→ Eg: $e^{-2\ln x}$ $\quad -2\ln x \Rightarrow \ln x^{-2}$

$-2\ln x$ $\quad \ln x^2$

Ans y $\cancel{e^{-2\ln x} \times \left(-\frac{2}{x}\right) \times -\frac{1}{x^2}}$

$$y = e^{\ln x^{-2}}$$

$$= e^{\log_e x^{-2}}$$

$$= 1 \times x^{-2}$$

$$\boxed{y = \frac{1}{x^2}}$$

$$\frac{dy}{dx} = -2x^{-3}$$

$$= \frac{-2}{x^3} \text{ Ans.}$$

→ Eg: $y = \frac{\sqrt{1-x}}{\sqrt{1+x}}$; $\frac{dy}{dx} = ?$

Sol: $\left(\frac{1-x}{1+x}\right)^{1/2}$

$$t^{1/2} = \frac{t^{1/2}}{2}$$

$$= \frac{\sqrt{\frac{1-x}{1+x}}}{2} = \frac{1}{2\sqrt{\frac{1-x}{1+x}}}$$

$$\frac{1}{2\frac{(1-x)^2}{\sqrt{1-x^2}}} \quad \frac{1}{\frac{2(1-x)}{\sqrt{1-x^2}}} = \frac{1}{2\sqrt{\frac{1-x}{1+x} \times \frac{1-x}{1-x}}}$$

myCOMPANION

$$\frac{dy}{dx} = \frac{\sqrt{1+x} \times \frac{1}{2\sqrt{1-x}}(-1) - \sqrt{1-x} \times \frac{1}{2\sqrt{1+x}}}{(1+x)}$$

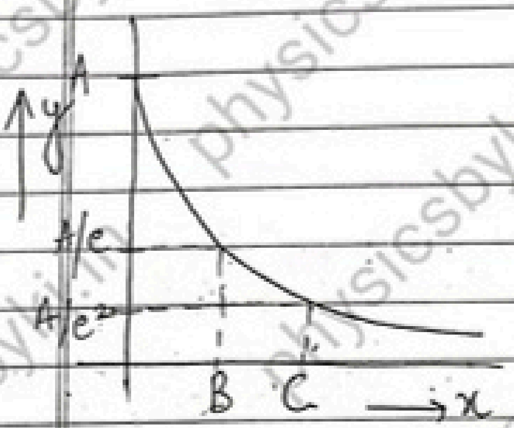
$$= \frac{\frac{1}{2}(-1-x) - \frac{1}{2}(1-x)}{(1+x)\sqrt{(1-x)(1+x)}}$$

$$= \frac{1}{2}\frac{(-1\cancel{-x}-1\cancel{+x})}{(1+x)\sqrt{1-x^2}}$$

$$= \frac{1}{\cancel{2}} \times \frac{-\cancel{2}}{(1+x)\sqrt{1-x^2}}$$

$$= \frac{-1}{(1+x)\sqrt{1-x^2}}$$

Illustrations :



at A ; $y = A$, $x = 0$

$y = 15e^{-x}$

$\Rightarrow A = 15e^{-0}$

$\Rightarrow A = \frac{15}{e^0}$

$\Rightarrow A = \frac{15}{1}$

$\Rightarrow A = 15$

at B , $y = \frac{A}{e}$, $x = B$

$\frac{A}{e} = 15e^{-B}$

$\frac{A}{15} = ee^{-B}$

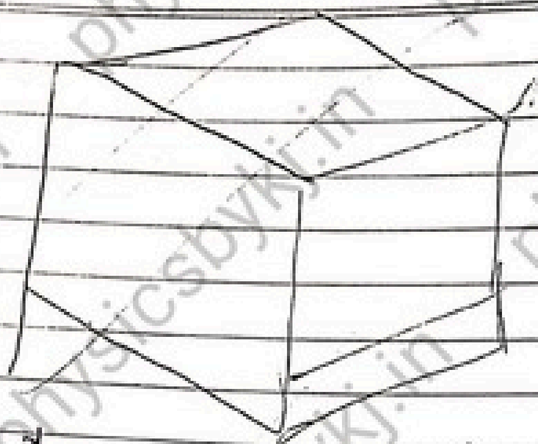
$\frac{15}{15} = e^{-B+1}$

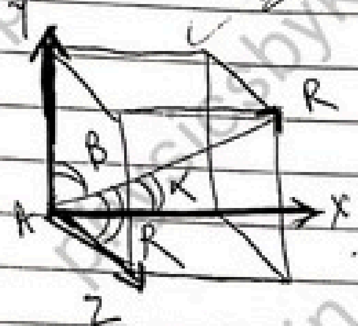
$e^0 = e^{-B+1}$

$1 = B$

$\boxed{C = 2}$ Ans.

⑥





$\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$

$\hat{A} = \hat{i}(\cancel{A}\cos\alpha) + (\cancel{A}\cos\beta)\hat{j} + (\cancel{A}\cos\gamma)\hat{k}$

$1 = \sqrt{\cos^2\alpha + \cos^2\beta + \cos^2\gamma}$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

⑨ For equilibrium, net force = 0

$F_{net} = 0$

Eg: $y = \sqrt{\tan(\sin(\ln x))}$

$= \frac{1}{2\sqrt{\tan(\sin(\ln x))}} \times \sec^2 \sin x \cos x \times \frac{1}{x}$

Differentiation of any function with any 3rd variable.

Eg: $y = x^2$; Calculate $\frac{dy}{dt} = ?$

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Sol: $\frac{dy}{dx} = 2x$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= 2x \cdot \frac{dx}{dt}$ Ans.

Q. $y = \sin t$; Calculate $\frac{dy}{dx}$.

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \cos t \times \frac{dt}{dx}$

Q. $y = \cos 2x$; Calculate $\frac{dy}{dz}$

$\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$= -\sin 2x \times 2 \cdot \frac{dx}{dz}$

$t^2 = 2t = 2\sin t \times \cos t$

Q. $y = \sin t^2$; Calculate $\frac{dy}{dz}$

Ans: ~~$2\cos\ (2\sin t \times \cos t \cdot \frac{dt}{dz}$~~

$2t \times \cos t^2 \cdot \frac{dt}{dz}$

Differentiation of Implicit and Explicit function :

Functions are of two types –

Explicit : In this type of function, y is defined purely in terms of x, then y is explicit function of x

Implicit : In this type of function, y is not defined as a pure term of x, then y is implicit function of x

Explicit $\Rightarrow$ $y = 4x^3 + 3x^2 + 8x + 9$

Implicit $\Rightarrow$ $\underset{u}{x^3}\underset{v}{y} + \underset{u}{y^2}\underset{v}{x} + 3\underset{u}{(\sin x)}\underset{v}{y} + y + 8 = 0$; Calculate $\frac{dy}{dx}$

$\Rightarrow \left(x^3 \times \frac{dy}{dx} + y \times 3x^2\right) + \left(y^2 \times 1 + x \times 2y\frac{dy}{dx}\right) + 3\left(\sin x \times 1 \times \frac{dy}{dx} + y \times \cos x\right)$
$+ 1 \times \frac{dy}{dx} + 0 = 0$

$\Rightarrow \left(x^3 + 2xy + 3\sin x + 1\right)\frac{dy}{dx} + \left(3x^2y + y^2 + 3\cos x \times y\right) = 0$

$\Rightarrow \left(x^3 + 2xy + 3\sin x + 1\right)\frac{dy}{dx} = -\left(3x^2y + y^2 + 3y\cos x\right)$

$$\Rightarrow \frac{dy}{dx} = -\frac{(3x^2y + y^2 + 3y\cos x)}{(x^3 + 2xy + 3\sin x + 1)}$$

$(\cos x)^-$ $\qquad \frac{dy}{dx}\times$

eg: $(\tan x) y^2 + 2(\sin x)y + 3\cos^2 x + \sin y + 3 = 0$

$\Rightarrow \left(\tan x \times 2y\frac{dy}{dx} + y^2\sec^2 x\right) + 2\left(y\cos x + \sin x \frac{dy}{dx}\right) + \left(3\{x\, 2\cos x \times -\sin x\}\right)$

$+ \cos y \frac{dy}{dx} + 0 = 0$

$\Rightarrow \left(2y\tan x + 2\sin x + \cos y\right)\frac{dy}{dx} + \left(y^2\sec^2 x + 2y\cos x - 6\sin x\cos x\right) = 0$

$$\frac{dy}{dx} = -\frac{\left(y^2\sec^2 x + 2y\cos x - 6\sin x\cos x\right)}{\left(2y\tan x + 2\sin x + \cos y\right)}$$

eg: $y^6\sec 7x + x^5y^2 + 3x\cos y + 7 = 0$, find $\frac{dy}{dx}$

$\Rightarrow \left[y^6(\sec 7x\tan 7x \times 7) + \sec 7x \times 6y^5 \times \frac{dy}{dx}\right] + x^5\left(2y\cdot\frac{dy}{dx}\right) +$

$y^2 \times 5x^4\Big) + \left(\cancel{3x(\sin y)\frac{dy}{dx} + 3\cos y}\right)$

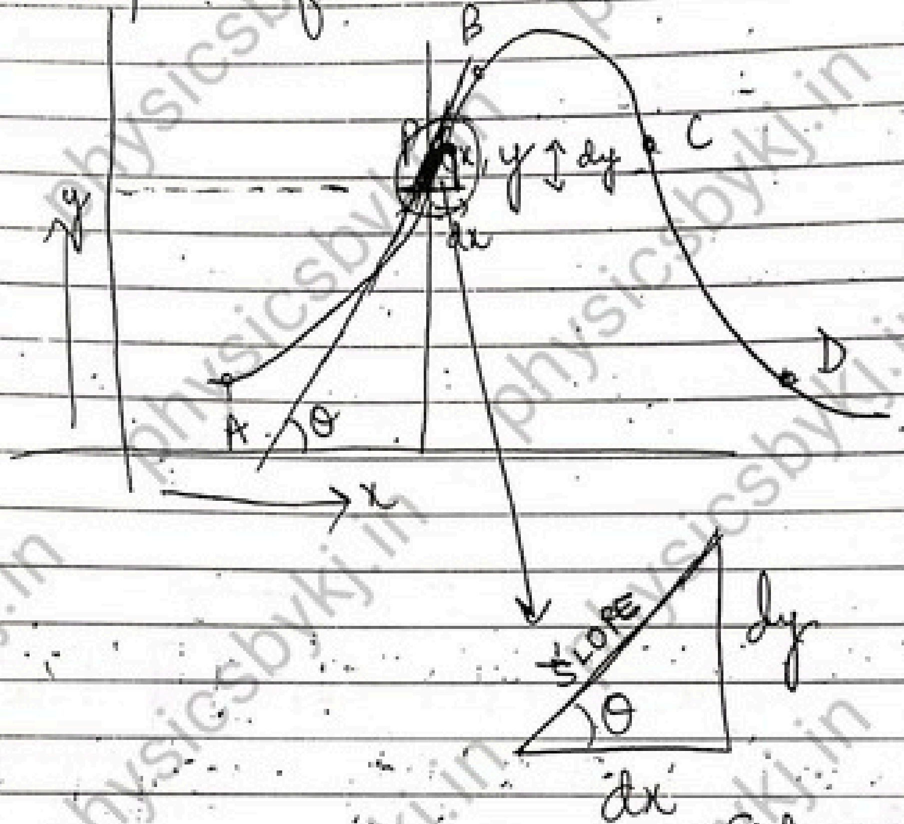
$\left(3x - (\sin y)\frac{dy}{dx} + 3\cos y\right) = 0$

$\Rightarrow \frac{dy}{dx}\left(\sec 7x \times 6y^5 + x^5 2y - 3x(\sin y)\right) +$

$\left(y^6(\sec 7x\tan 7x)7 + y^2 \times 5x^4\right) + 3\cos y = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{\left(y^6(\sec 7x\tan 7x)7 + y^2 \times 5x^4\right)}{\left(\sec 7x \times 6y^5 + x^5 2y - 3x(\sin y)\right)}$$

Slope of a Function



Slope = $\frac{dy}{dx}$

So, here slope at point P can be given by drawing a tangent at point P.

$$\boxed{\text{Slope} = \tan\theta = \frac{dy}{dx}}$$

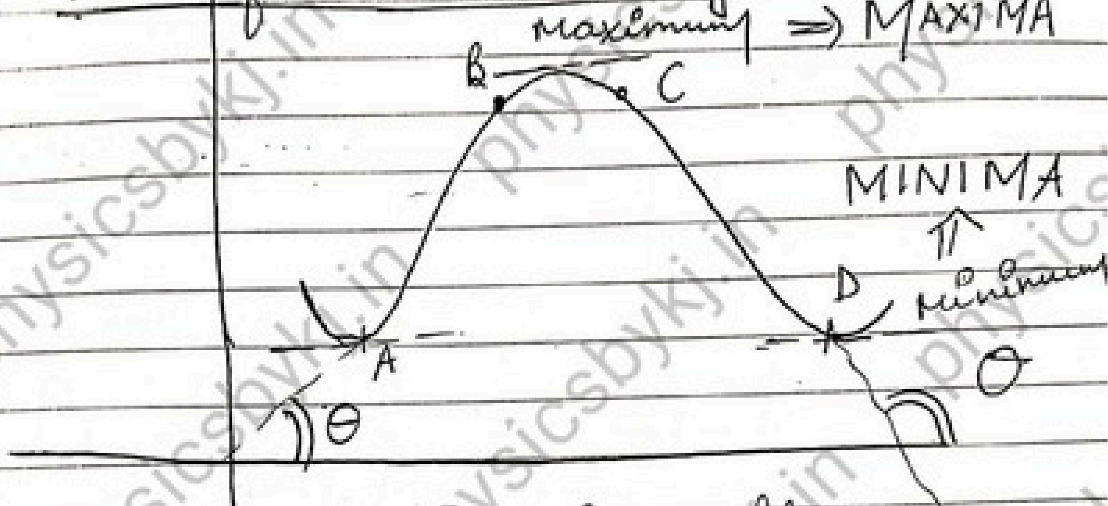
Nature of Curve :

- A to B ⇒ $\tan\theta$ = +ve

$$\boxed{\text{Slope} = \frac{dy}{dx} = \tan\theta = +\text{ve}}$$

sin θ = acute
= So it is +ve

→ Nature of Curve is Increasing.



→ C to D : $\tan\theta$ -ve due θ = obtuse.

→ $\boxed{\text{Slope} = \frac{dy}{dx} = \tan\theta = -ve}$

∴ Nature of Curve is decreasing.

Eg: $y = x^3 + 7x^2 - 8x + 3$

→ Check the nature of function at neighbouring point of $x = 1$

Sol: For nature!

$\frac{dy}{dx} = 3x^2 + 14x - 8$

$= 3\times(1)^2 + 14(1) - 8$

$= 3 + 14 - 8$

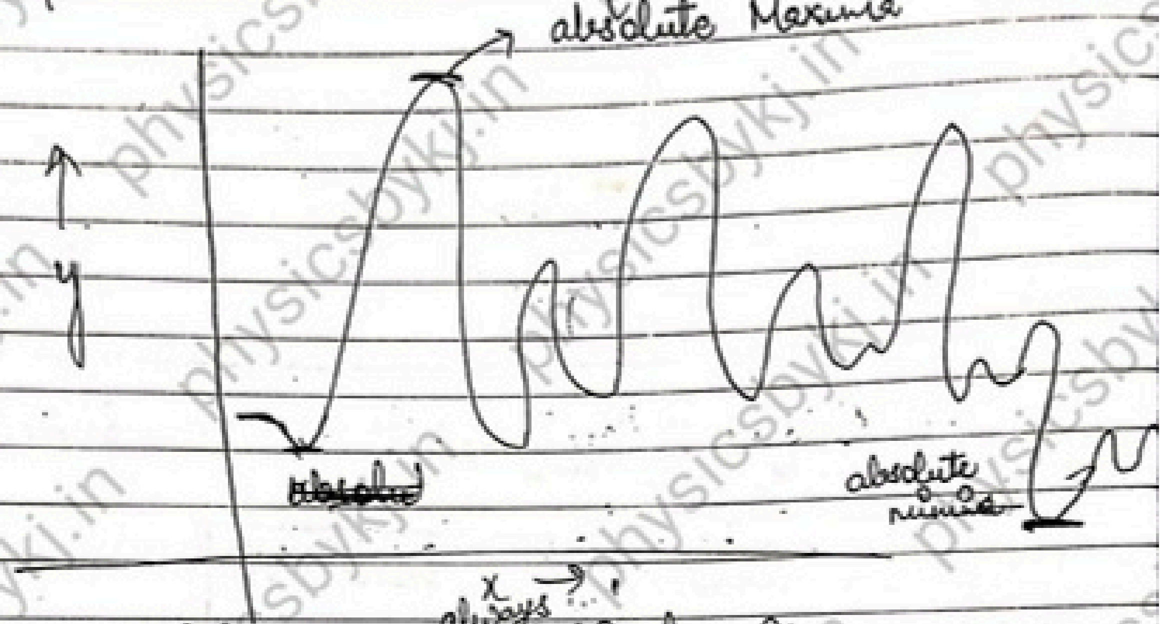
$= 17 - 8$

$= +ve\ (9)$

So, at neighbouring point of $x = 1$, nature of function is increasing.

+ve

Maxima and Minima :



Maxima and Minima are always defined with respect to its neighbouring point.

- A Minima can be greater than a maxima and vice versa.
- If $\frac{dy}{dx}$ or slope $= 0$ at any point then at that point, it will be either Maxima or Minima

~~Question Paper :~~

HOMEWORK : Beginner's Box 2, Exercise 1 (Q.1 to 20)

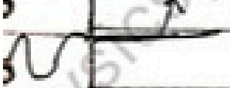
2^{nd} Derivative For Maxima and Minima.

→ As we know that if at any point slope $\frac{dy}{dx} = 0$, then at that point, there will be maxima or minima.

→ If at that point,

$$\frac{d^2y}{dx^2} = +ve$$, then at that point, there will be Minima

No Maxima
no Minima



If, $$\frac{d^2y}{dx^2} = -ve$$, then at that point, there will be Maxima.

& Double Differentiation

$y = \sin x$

$\frac{dy}{dx} = \underbrace{\cos x}$

$\frac{d^2y}{dx^2} = -\sin x$ (doing its differentiation twice)

<u>Polynomial Function</u>

1^{st} Degree polynomial function :- No maxima, No minima

$y = 3x + 7$

equation of straight line

$\boxed{y = mx + c}$ (C → intercept, m → Slope

For position of maxima and minima, $\frac{dy}{dx}$ must be 0

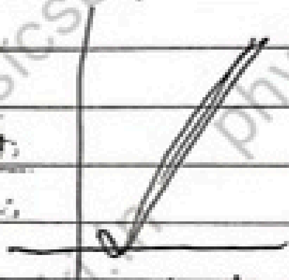
There is no maxima or no minima in 1st degree polynomial as $\frac{dy}{dx} = 3 \neq 0$.

2nd Degree Polynomial - 1 Maxima or 1 Minima

$y = x^2 - 2x + 8$

$\frac{dy}{dx} = 2x - 2$ $\quad 2x - 4$

$\frac{d^2y}{dx^2} = 2 \rightarrow \boxed{+ve}$ minima



→ For Position of Maxima or Minima

$\frac{dy}{dx} = 0$

$2x - 2 = 0$

$\boxed{x = 1}$

$x = 1$ [has Minima $\left(+ve \frac{d^2y}{dx^2}\right)$]

and so, Maxima is at ∞.

3rd Degree Polynomial :

$y = x^3 - 3x^2 + 8x + 3$

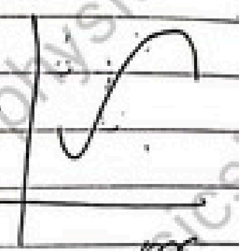
$dy/dx = 3x^2 - 6x + 8$

$d^2y/dx^2 = \underline{6x - 6}$

companion

* For imaginary roots,
There isn't any Maxima or Minima

Graph



or

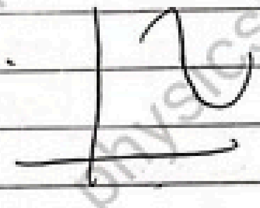
For position of Maxima or Minima

$\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6x + 8 = 0$ Splitting.

~~$3x^2 - 4x - 2x + 8 = 0$~~ $6 \pm \sqrt{36 - 4 \times 8 \times 3}$

x_1 x_2 [must be real roots]



Checking minima and maxima.

At $x_1 \Rightarrow 2$ $\Rightarrow 6x - 6$

$\Rightarrow 6 \times 2 - 6$

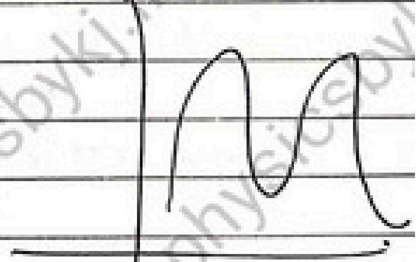
$\Rightarrow 6$ (+ve) $\rightarrow$ Minima.

4th Degree Polynomial

$y = x^4 - 6x^3 + 8x^2 + 7$

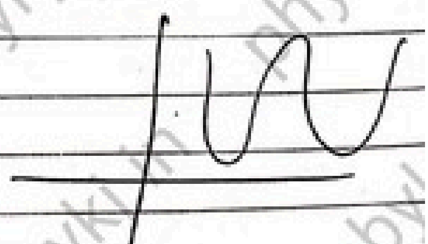
$\frac{dy}{dx} = 4x^3 + \dots$

$\frac{d^2y}{dx^2} = 12x^2 + \dots$



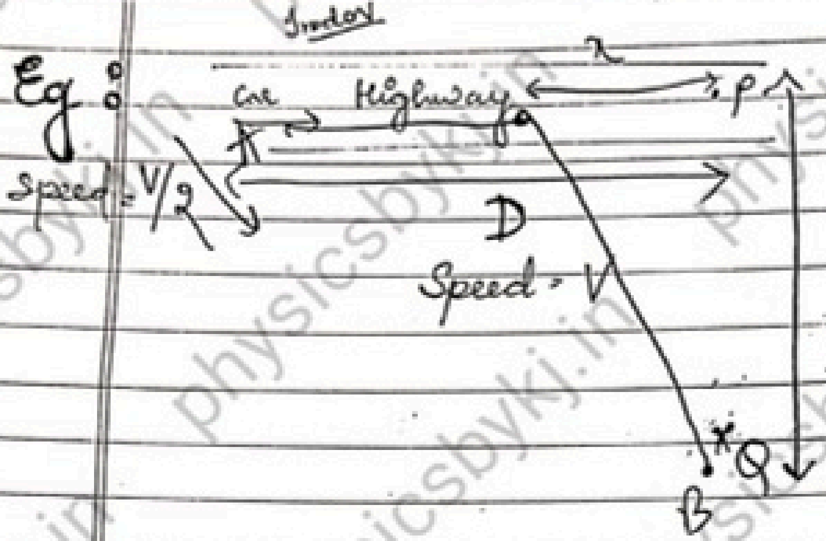
For Maxima or Minima,

find roots (real)



my COMPANION

Eg :



Q: From what distance from point P, should car leave highway so that it can reach in least possible time?

Answer :

Time taken by car to move from C to Q =

$$t = \frac{D-x}{V} + \frac{\sqrt{l^2+x^2}}{V/2}$$

$$= \frac{D-x}{V} + \frac{2}{V}\left(\sqrt{l^2+x^2}\right)$$

Now, for Maxima or minima

$$\frac{dt}{dx} = 0$$

$$0 = \left(\frac{0-1}{V}\right) + \frac{2}{V}\left(\frac{1}{2\sqrt{l^2+x^2}} \times 0+2x\right)$$

$$\cancel{0 = \ominus \frac{1}{V} + \frac{1}{V\sqrt{l^2+x^2}} \times (2x) = \frac{1}{V}}$$

$$\frac{1}{V} \times \frac{1}{\sqrt{l^2+x^2}} (0+2x) = \frac{1}{V}$$

$$\frac{2x}{\sqrt{l^2+x^2}} = 1$$

$$4x^2 = l^2 + x^2$$

$$3x^2 = l^2$$

$$x^2 = \frac{l^2}{3}$$

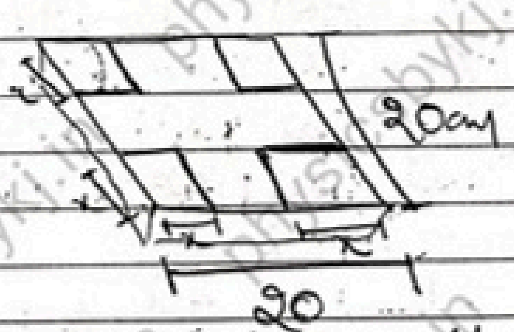
$$\boxed{x = \pm \frac{l}{\sqrt{3}}}$$

* STEPS for Solving Problems on MAXIMA OR MINIMA

We usually follow four steps :

1) Identify that parameter which is to be maximised or minimised.
2) Express that parameter as function of one variable only.
3) Differentiate the parameter which is to be maximised or minimised with respect to taken variable & equate it to zero.
4) Do second Derivative test for Maxima & Minima.

Q:



If we cut squares at corners of cardboard and by folding the sides, we get an open box.

What ~~size of~~ should be the size of square to be cut so that volume of open box formed is maximum.

Volume cut $= \cancel{2(20-2x} \; 4(x^3)$

Volume left $= \cancel{400 - 4x^2}$ $\quad 400x - 4x^2 = 0$

$\frac{dy}{dx} = 400 - 12x^2$ $\quad 400 = 4x^2$

$400 - 12x^2 = 0$ $\quad 100 = x^2$

$\boxed{10 = x}$

$400 = 12x^2$ $\quad 400x^2$

$\frac{\cancel{400}}{\cancel{12}} = x^2$

$\frac{10}{2\sqrt{3}} = x$

$400x - 4x^3$

$400 - 12x^2 = 0$

$400 = 12x^2$

$\frac{100}{3} = x^2$

$400x = 12x^2$

$\frac{400x}{12} = x^2$

$10/\sqrt{3}$

$400x - 12x^2 = 0$

$400x = 12x^2$

$\frac{100}{3} = x$

$-12x^3 + 400x = 0$

Volume of Box $= (20 - 2x)^2 \times x$

$= (400 + 4x^2 - 80x)\, x$

$= 400x + 4x^3 - 80x^2$

$0 = 400 + 12x^2 - 160x$

$0 = 6x^2 - 80x + 200$

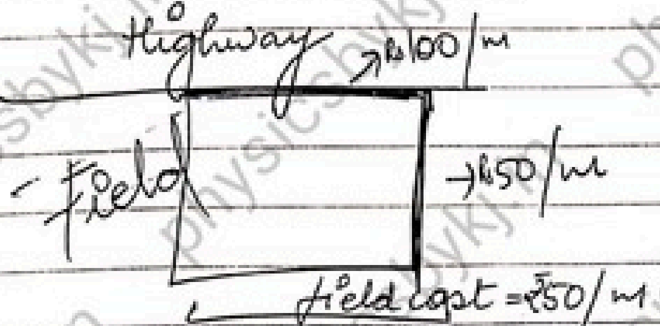
$0 = 3x^2 - \cancel{40x}\, 40x + 100$

$\boxed{x = 10, \frac{10}{3}}$ Ans

$\times$

Area of Square $= \frac{10}{3} \times \frac{10}{3}\ cm^2$

Q:



Calculate the size of plot so that maximum area ~~that~~ can be fenced in Rs. 20,000.

Rectangular plot!

Area.

20000
100 × 250

10 m 20

2000
1

$2(l+b)$ → Area of rectangle

$2(100x + 50y) = 20{,}000$

$200x + 100y = 20{,}000$

$2x + y = 200$

Area $= (100x)(50y)$

$= 5000xy$

$100x + 50y + 100y + 50y = 20000$

$15\cancel{0}x + 10\cancel{0}y = 2000\cancel{0}$

$15x + 10y = 2000$

$$\boxed{y = \frac{2000 - 15x}{10}}$$

area $= xy$

$= x\left(\frac{2000 - 15x}{10}\right)$

Area $= \dfrac{2000x - 15x^2}{10}$

my COMPANION

$$\frac{dy}{dx} = 0$$

$$\frac{2000 - 30x}{10} = 0$$

$$2000 = 30x$$

$$\frac{200\cancel{0}}{3\cancel{0}} = x$$

$$\boxed{\frac{200}{3} = x}$$

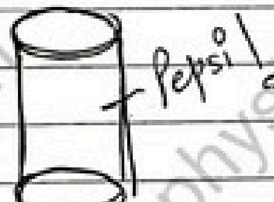
$$\boxed{y = 100\,m}$$

$$Area = xy$$

$$= \frac{200 \times 100}{3}$$

$$= \frac{20000}{3}$$

Q:



$V = 200\,mL$

Height : h

Radius : r

A ~~multip~~ multinational company wants to design a cylinderical can of volume 200 mL by you. Calculate its radius and height.

$\frac{\pi r^2 h}{r}$

$V = Area \times h$

$\pi r^2 h$

$$\cancel{h(2\pi r^2 + 2\pi r h) = 200}$$

$$\pi r^2 h = 200$$

$$h = \frac{200}{\pi r^2}$$

Area $2\pi r^2 + 2\pi r h$

Area $= 2\pi r^2 + 2\pi r \times \frac{200}{\pi r^2}$

Area $= 2\pi r^2 + \frac{400}{r}$

$= \frac{2\pi r^3 + 400}{r}$

$\frac{dy}{dr} = \cancel{(6\pi r^2 + 0)}\left(\frac{2\pi r^3 + 400}{r} \right)$ Quotie... R

$= \frac{r(6\pi r^2) - 2\pi r^3}{r^2} - \frac{400}{r^2}$

$0 = \frac{6\pi r^3 - 2\pi r^3 - 400}{r^2}$

$0 = 4\pi r^3 - 400$

$\frac{400}{4} = \pi r^3$

$100 = \pi r^3$

$\frac{\cancel{100}^{50} \times 7}{\cancel{22}_{11}} = r^3$

$\boxed{r = \sqrt[3]{\frac{350}{11}}}$

$\boxed{h = \frac{200}{\frac{22}{7} \times \left(\frac{350}{11}\right)^{1/3}}}$

Beginner's Box 2

12 (v) $y = \left(a^{2/3} - x^{2/3}\right)^{3/2}$

$= \frac{\cancel{3}}{\cancel{2}}\left(a^{2/3} - x^{2/3}\right)^{1/2}\left(0 - \frac{\cancel{2}}{\cancel{3}}x^{-1/3}\right)$

$= \frac{\left(a^{2/3} - x^{2/3}\right)^{1/2}}{x^{1/3}}$

14 (iii) $y = \frac{x}{2} - \frac{1}{4}\sin 2x$

$= \frac{1}{2} - \frac{1}{\cancel{4}_{2}} \times \cos 2x \times \cancel{2}$

$= \frac{1}{2}(1 - \cos 2x)$

$= \frac{1}{\cancel{2}}\cancel{2}\sin^2\theta$

$= \sin^2\theta$

Identities

$\cos 2\theta = 1 - 2\sin^2\theta$

$1 - \cos 2\theta = 2\sin^2\theta$

$1 + \cos 2\theta = 2\cos^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

14 (vii) $y = x^3 - x^2\cos x + 2x\sin x + 2\cos x$

$\frac{dy}{dx} = 3x^2 - x^2(-\sin x) - \cos x \times 2x + 2[x\cos x + \sin x] - 2\sin x$

$= 3x^2 + x^2\sin x - \cancel{2x\cos x} + \cancel{2x\cos} + \cancel{2\sin x} - \cancel{2\sin x}$

$= 3x^2 + x^2\sin x.$

16 (iii) $y = \cos^3 2x$

$= 3\cos^2 2x \times -\sin 2x \times 2$

$= 6\cos^2 2x \sin 2x$

$\frac{d^2y}{dx^2} = -6\left[\cos^2 2x \times \cos 2x \times 2 + \sin 2x \times 2(\cos 2x) \times -\sin 2x \times 2\right]$

$= -6\left[2\cos^3 2x - 4\cos 2x \sin^2 2x\right]$

$= -12\cos^3 2x + 24\cos 2x(1-\cos^2 2x)$

$= -12\cos^3 2x + 24\cos 2x - 24\cos^3 2x$

$= -36\cos^3 2x + 24\cos 2x$

18) viii) $x^5 e^{-3\ln x}$

$= x^5 e^{\ln x^{-3}}$

$= x^5 \times 1 \times x^{-3}$

$= x^2$

$= 2x$ Ans.

$\log_e n$

$\ln x$

18 (i) $y = \sqrt{\log_{10} x}$

$y = \sqrt{\frac{\log_e x}{\log_e 10}}$

$= \frac{\sqrt{\ln x}}{\sqrt{\log_e 10}}$

$= \frac{1}{\sqrt{\log_e 10}} \times \sqrt{\ln x}$

$= \frac{1}{\sqrt{\ln 10}} \times \frac{1}{2\sqrt{\ln x}} \times \frac{1}{x}$

$= \frac{1}{x} \times \frac{1}{\sqrt{\frac{1}{\log_{10} e}}} \times \frac{1}{2\sqrt{\ln x}}$

$x^{(x)}$

$= 3\ln x$

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Page ______

$= \frac{1}{2x}\sqrt{\frac{\ln_{10} e}{\ln x}}$

Exercise 1

7) $y = \frac{1}{(t+2)(t+1)}$

$= \frac{1}{(t+2)} - \frac{1}{(t+1)}$

$\frac{dy}{dx} = \frac{1}{(t+2)^2} - \frac{1}{(t+1)^2}$

14) Solve $\frac{dy}{dx} = \frac{-1}{(1+x)\sqrt{1-x^2}}$

$= \frac{-1}{\sqrt{1-x}\,(1+x)\sqrt{\frac{1+x}{\sqrt{1-x}}}\times\sqrt{1-x}}$

$= \frac{-1}{(1-x)(1+x)\left(\frac{1}{y}\right)}$

$= \frac{-y}{(1-x^2)}$

$= \frac{y}{x^2-1}$

18) $f'(x) = \sqrt{2x^2-1}$

$y = f(x^2)$ $\quad \frac{dy}{dx} = ?$ at $x=1$

$\frac{dy}{dx} = f'(x^2)\times 2x$

$= \sqrt{2(x^2)^2-1}\times 2x \quad (x=1)$

$= 2$ Ans.

(19) $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = y\sqrt{1+x}$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 + x^2y = y^2 + y^2x$

$\Rightarrow x^2 - y^2 = y^2x - x^2y$

$\Rightarrow \cancel{(x-y)}(x+y) = xy\,\cancel{(x-y)}$

$\Rightarrow (x+y) = xy$

$\Rightarrow x = (xy - y)$

$\Rightarrow x = y(x-1)$

$\frac{dy}{dx} = \cancel{(x-1)x + x(1-}$

20) $ax^2 + 2hxy + by^2 = 0$

$\cancel{a(2x) + 2h\left[(x)\left(\frac{dy}{dx}\right) + y(1)\right] + b\left(2y\left(\frac{dy}{dx}\right)\right)}$

$= 2ax + 2h\left[x\frac{dy}{dx} + y \times 1\right] + 2by\frac{dy}{dx} = 0$

$\Rightarrow (hx + by)\frac{dy}{dx} = -ax - hy$

$\Rightarrow \frac{dy}{dx} = +\frac{(ax + hy)}{hx + by}$

$\Rightarrow \frac{dy}{dx} = \frac{+y}{x}$

H.W : Exercise

Integration : → Definite
→ Indefinite

Integration can be classified into 2 ways :

1) Definite Integration : Result of Integration is either a number or constant.

2) Indefinite Integration : Result of Integration is a function.

Integration can be represented as

$$I = \int_a^b [f(x)]\, dx$$

Symbol of Integration

dx → With respect to which integration will take place.

$f(x)$ → function which is to be Integrated where, a and b are constants

a = lower limit
b = upper limit

Indefinite Integration :-

There are two ways in which Indefinite Integration can take place.

1) Direct Integration

Reverse of Differentiation

2) Integration by Substitution

Eg: $y = x^2$

$\frac{dy}{dx} = 2x$

$\int \frac{dy}{dx} = \int 2x\,dx$

$\Rightarrow y = x^2 + C$

OR

Eg: $y = x^2 + 5$

$\frac{dy}{dx} = 2x$

$\int \frac{dy}{dx} = \int 2x\,dx$

$y = x^2 + C$

Direct Integration
or Reverse of Differention

(1) $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + C$

x^2

$2 \times \dfrac{1}{x^2} = 2 \times x^{-2}$

(2) $\int \dfrac{1}{x^2} \, dx = \dfrac{-1}{x} + c$ $\qquad \dfrac{x^{-2+1}}{-2+1}$

(3) $\int \dfrac{1}{\sqrt{x}} \, dx = 2\sqrt{x} + c$

(4) $\int \cos x \, dx = \sin x + C$

(5) $\int \sin x \, dx = -\cos x + C$

(6) $\int \sec^2 x \, dx = \tan x + C$

(7) $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

(8) $\int \sec x \tan x \, dx = \sec x + C$

(9) $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$ $\qquad x^{\frac{3}{2}+1}$

-3

(10) $\int \dfrac{1}{x} \, dx = \ln x + C$

(11) $\int e^x \, dx = e^x + C$

(12) $\int a^x \, dx = \dfrac{a^x}{\ln a} + c$ $\qquad 2 : \dfrac{1}{2}$

2

$\ln a$

$= \frac{1}{a}$

$\frac{1}{\cos^2 x}$

$(\cos x)^{-2}$

$-2t^{-3}$

$\frac{-2}{\cos^3 x}$

-2

* (13) $\int \tan x\, dx = \underbrace{-\ln(\cos x)}\, + C$

$$\left[\begin{aligned} &= -\frac{1}{\cos x} \times -\sin x \\ &= \frac{\sin x}{\cos x} \\ &= \tan x. \end{aligned}\right]$$

* (14) $\int \cot x\, dx = \ln(\sin x) + C.$

Questions:

1) $I = \int_0^1 x^4 dx = ?$

Sol) $\frac{x^5}{5}$ Ans.

2) $I = \int \left(3x^2 + \frac{2}{x^2} + \frac{1}{x} + 4x^3\right) dx$ $\quad x^{-2+1}$

Solution: $\frac{3x^3}{3} + \frac{2x^{-1}}{-1} + \ln x + \frac{4x^4}{4} + C$ Ans

Ans 2) $I = x^3 - \frac{2}{x} + \ln x + x^4 + C$

3) $I = \int \left(x^7 + 6x^5 + \frac{1}{x^2} + 2\sqrt{x} + \frac{1}{x^{3/2}}\right) dx$

$\frac{x^8}{8} + 6x^6 + \frac{x^{-1}}{-1} + \frac{2x^{3/2}}{(3/2)} + \frac{x^{-1/2}}{(-1/2)} + C$

Ans $= \frac{x^8}{8} + x^6 - \frac{1}{x} + \cancel{\ } - \frac{2}{\sqrt{x}} + C$

Eg: $I = \int (x^2 + 6\sin x + \sec^2 x + 4\sec x\tan x)\,dx$

$= \frac{x^3}{3} - 6\cos x + \tan x + 4\sec x + C$

Exercise 1

(18) $f'(x) = \sqrt{2x^2-1}$

$y = f(x^2)$

$\frac{dy}{dx} = f'(x^2) \times 2x$

$= \sqrt{2(x^2)^2-1} \times 2x$

$\left(\frac{dy}{dx}\right)_{x=1} = \sqrt{2x^4-1} \times 2x$

$= 2\times1\sqrt{2\times1^4-1}$

$= 2\sqrt{2-1}$

$= 2\sqrt{1}$

$= 1$ Ans.

(21) (i) $\rightarrow x^2y + xy^2 = 6$

$x^2 \times \frac{dy}{dx} + y \times 2x + \left(x \times 2y \times \frac{dy}{dx} + y^2\right) = 0$

$\frac{dy}{dx}(x^2 + 2xy) = -(2xy + y^2)$

$$\frac{dy}{dx} = \frac{-(2xy+y^2)}{y^2+2xy}.$$

23) $y = \cos t\sqrt{\cos 2t}$ and $x = \sin t\sqrt{\cos 2t}$

$$\frac{dy}{dx} = \cos t \times \frac{1}{\cancel{2}\sqrt{\cos 2t}} \times -\sin 2t \times \cancel{2} + \sqrt{\cos 2t} \times (-\sin t)$$

$\Rightarrow$

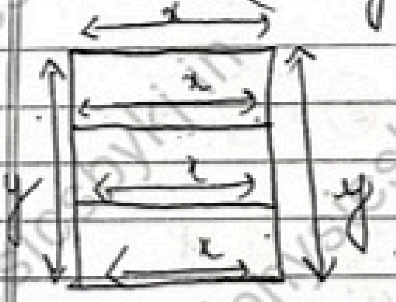
25) (i) $a\,2x + 2h\left(x\frac{dy}{dx} + y\right) + b\,2y\frac{dy}{dx} + 2g +$

$2f \times \frac{dy}{dx} + 0 = 0.$

$\Rightarrow (hx+by+f)\frac{dy}{dx} = -(ax+hy+f)$

$= -\frac{(x+hy+f)}{(hx+by+f)}$

27)



$4x + 2y = 800$

$2x + y = 400 \quad - (1) \Rightarrow y = 400 - 2x$

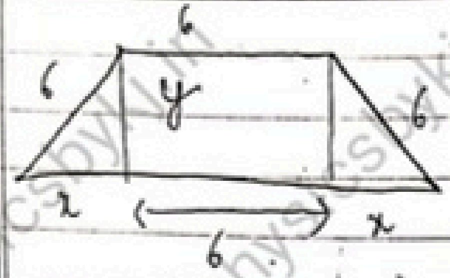
$xy = \text{Area } (A)$

$A = x(400-2x)$

$\frac{dA}{dx} = 0$

$400 - 4x = 0 \Rightarrow x = 100$ feet.

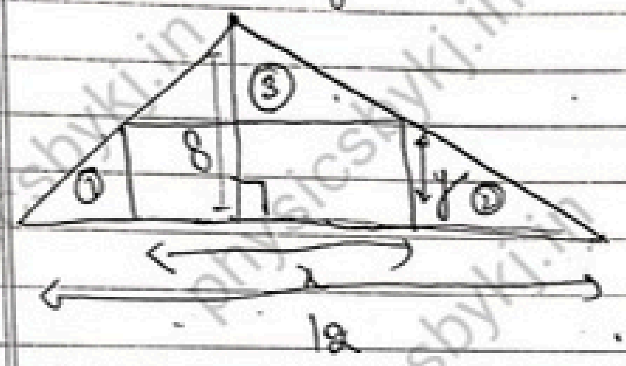
31)



$y = \sqrt{36 - x^2}$

Solve the way you know!

32)



Area of $\Delta = \frac{1}{2} \times 12 \times \cancel{8}4$

$= 48 \text{ feet}^2$

Area of Δ = Area of 1 + 2 + 3 + xy — (i)

$48 = 6y - \frac{\cancel{xy}}{\cancel{2}} + 4x - \frac{\cancel{xy}}{\cancel{2}} + xy$

$\Rightarrow 6y + 4x = 48$

$\Rightarrow 3y + 2x = 24$

$\Rightarrow y = \frac{24 - 2x}{3}$

Area $= xy$

$A = x\left(\frac{24 - 2x}{3}\right)$

$A = \frac{24x - 2x^2}{3}$

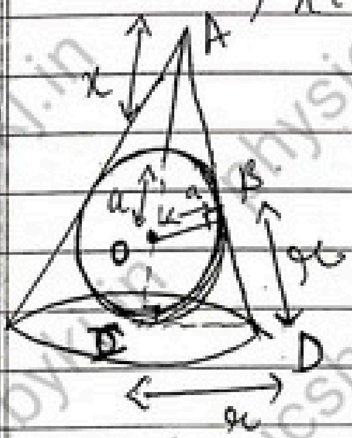
For maximum area,

$$\frac{dA}{dx} = 0$$

$$\Rightarrow 24 - 4x = 0$$

$$\Rightarrow x = 6$$

33)



In $\triangle AOB$,

$$AB = \sqrt{AO^2 - OB^2}$$

$$AB = \sqrt{(x+a)^2 - a^2}$$

$$AB = \sqrt{x^2 + \cancel{a^2} + 2ax - \cancel{a^2}}$$

$$AB = \sqrt{x^2 + 2ax}$$

In $\triangle ACD$

$$AC^2 + CD^2 = AD^2$$

$$(x+2a)^2 + r^2 = \left(\sqrt{x^2+2ax} + r\right)^2$$

$$\Rightarrow x^2 + 4a^2 + 2ax + r^2 = x^2 + 2ax + r^2 + 2r\sqrt{x^2+2ax}$$

$$\Rightarrow 2a^2 + \cancel{2}ax = \cancel{2}r\sqrt{x^2+2ax}$$

$$\Rightarrow (2a^2 + ax)^2 = r^2(x^2 + 2ax)$$

$$\Rightarrow r^2 = \frac{(2a^2+ax)^2}{x^2+2ax}$$

$$r = \sqrt{\frac{(2a^2+ax)^2}{(x^2+2ax)}}$$

VOLUME $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi \frac{(2a^2+ax)^2}{(x^2+2ax)} \times (x+2a)$$

$$= \frac{1}{3}\pi \frac{(2a^2+ax)^2}{x\cancel{(x+2a)}} \times \cancel{(x+2a)}$$

$$= \frac{1}{3}\pi \frac{(2a^2+ax)^2}{x}$$

For Maximum Value

$$\frac{dv}{dx} = 0$$

$$0 = \frac{1}{3}\pi \left[\frac{x \times 2(2a^2+ax)(0+a) - (2a^2+ax)^2 \times 1}{x^2}\right]$$

$$\Rightarrow (2a^2+ax)\left[2ax - (2a^2+ax)\right] = 0$$

$$\Rightarrow ax = 2a^2$$

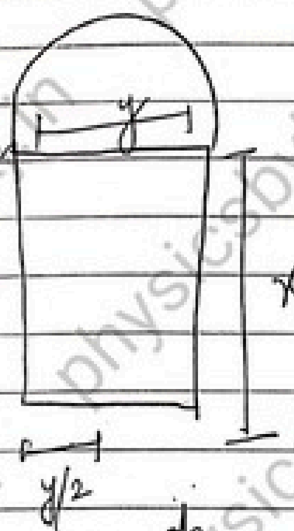
$$\Rightarrow \boxed{x = 2a}$$

Height : $x+2a$

$= 2a+2a$

$= 4a$

36)



$$P = 2x + 2y + \frac{\pi y}{2}$$

$$P = 2x + \left(2 + \frac{\pi}{2}\right) y$$

$$P - 2y - \frac{\pi y}{2} = 2x$$

$$\text{Area} = xy + \frac{\pi y^2}{8}$$

$$= \frac{\left(P - 2y - \frac{\pi y}{2}\right)}{2} \cancel{+\frac{\pi y^2}{x}}\, y + \frac{\pi y^2}{8}$$

$$= Py - y^2 - \frac{\pi y^2}{4} + \frac{\pi y^2}{8}$$

$$\text{Area} = \frac{\left(P - 2y - \frac{\pi y}{2}\right)}{2} y + \frac{\pi y^2}{8}$$

$$= \frac{Py}{2} - y^2 - \frac{\pi y^2}{4} + \frac{\pi y^2}{8}$$

$$= \frac{Py}{2} - y^2 - \frac{\pi y^2}{8}$$

$$\frac{dA}{dy} = 0$$

$$\Rightarrow \frac{P}{2} - 2y - \frac{\pi y}{4} = 0$$

$$\Rightarrow x + y + \cancel{\frac{\pi y}{4}} - 2y - \cancel{\frac{\pi y}{4}} = 0$$

$$\Rightarrow x - y = 0$$

$$\boxed{x = y}$$

Cost per hour

37) $C \propto K v^3$; $\Rightarrow 27 \; K \; 12^3$

$C = K v^3$ $\qquad \frac{27}{(12)^3} = K$

Constant

Cost Per Hour

$C = \frac{27}{12 \times 12 \times 12} \times v^3 + 128 \$$

Dollar

Cost per miles = $\frac{\text{Cost per hour}}{\text{Miles per hour}}$

$\frac{C}{V} = \frac{27}{12 \times 12 \times 12} \times \frac{V^{\cancel{3}2}}{\cancel{V}} + \frac{128}{V}$

$C' \Leftarrow \boxed{\frac{C}{V} = \frac{27}{144 \times 12} V^2 + \frac{128}{V}}$

$\frac{V \times 0 + 128 \times 1}{V^2}$

$\frac{dc'}{dv} = 0$

$\Rightarrow \frac{128}{V^2}$

$\Rightarrow \frac{27}{144 \times \cancel{12}6} \times \cancel{2}V - \frac{128}{V^2} = 0$

$\Rightarrow \frac{27}{144 \times 6} V = \frac{128}{V^2}$

$V^3 = \frac{128 \times \cancel{144}16 \times 6}{\cancel{27} \times 9}$

$V = \sqrt[3]{\cancel{128 \times 144}}$

$V = \sqrt[3]{128 \times 16 \times 6}$

V_2

$$v^3 = \frac{128 \times 144 \times 6^2}{2 \times 9}$$

$$\boxed{v = 16} \text{ Ans.}$$

41) $F = \frac{dp}{dt}$

$$F = \frac{d}{dt}(t \ln t)$$

$$\cancel{t} \times \frac{1}{\cancel{t}} + \ln t \times 1 = 0$$

$$1 + \ln t = 0$$

$$\ln t = -1$$

$$t = e^{-1}$$

$$\boxed{t = \frac{1}{e}} \text{ (B) Ans.}$$

INTEGRATION

Q: $I = \int \left(x^2 + 3x^3 + 4x^7 + \frac{3}{x} + \frac{3}{x^3} + \frac{4}{x^5}\right) dx.$

$$= \frac{x^{2+1}}{2+1} + \frac{3x^{3+1}}{3+1} + \frac{4x^{7+1}}{7+1} + 3\ln x + \frac{3x^{-3+1}}{-3+1} + \frac{4x^{-5+1}}{-5+1} + C$$

Q: $\int \left(x^4 + \frac{4}{x^{1/2}} + \frac{5}{x^{3/2}} - 3x^6 + \sin x + \sec x \tan x\right) dx$

Sol: $\frac{x^5}{5} + \frac{4x^{1/2}}{1/2} + \frac{5x^{-3/2+1}}{-3/2+1} - \frac{3x^{6+1}}{6+1} - \cos x + \sec x + C$

NOTE: If a function which is to be integrated have any coefficient of x, then, after integration, that co-efficient will come to denominator only.

Q: $I = \int \sin 2x \cdot dx$

$= -\frac{\cos 2x}{2} + C$

Q: $I = \int \sec 5x \cdot \tan 5x \, dx$

$= \frac{\sec 5x}{5} + C$

Q: $I = \int \frac{1}{3x+5} \, dx$

$= \frac{\ln(3x+5)}{3} + C$

① $\int \sin(ax+b) \, dx = -\frac{\cos(ax+b)}{a} + C$

② $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{(n+1)a} + C$

③ $\int \frac{1}{ax+b} \, dx = \frac{\ln(ax+b)}{a} + C$

Integration by Substitution

Three Examples:

have

so-

(1) $I = \int \frac{\sin x}{\cos x} dx$

Let $t = \cos x$

$\frac{dt}{dx} = -\sin x$

$dt = -\sin x \, dx$

$-dt = \sin x \, dx$

$= \int \frac{-dt}{t}$

$= -\int \frac{1}{t} dt$.

$= -\ln t + C$

$= -\ln(\cos x) + C$ Ans.

(2) $I = \int \tan x \cdot \sec^2 x \, dx$

Let $t = \tan x$

$\Rightarrow \frac{dt}{dx} = \sec^2 x$

$\Rightarrow dt = \sec^2 x \, dx$

$= \int t \, dt$

$= \frac{t^2}{2} + C$

$= \frac{(\tan x)^2}{2} + C$

(3) $I = \int \frac{1}{x} (\ln x) dx$

Let $t = \ln x$

$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{1}{x} dx$

$= \int t \cdot dt$

$= \frac{t^2}{2} + C$

eg: $I = \int \cot x \cdot \operatorname{cosec}^2 x \cdot dx$

Let $t = \cot x$

$\frac{dt}{dx} = -\operatorname{cosec}^2 x$

$-dt = \operatorname{cosec}^2 x \, dx$

$= -\int t \cdot dt$

$= -\frac{t^2}{2} + C$

my companion

$= \frac{1}{2}(\cot^2 x + C$

Eg: $I = \int x\, e^{x^2}\, dx$

Let $t = x^2$

$\Rightarrow dt = 2x\,dx$

$\Rightarrow \frac{dt}{2} = x\,dx$

$= \int \frac{dt}{2} e^t$

$= \frac{1}{2}\int e^t + C$

$= \frac{1}{2} e^t + C$

$= \frac{1}{2} e^{x^2}$ Ans.

Eg: $I = \int \sec^3 x \tan x\, dx$

~~Let $\tan x = t$~~

~~$\frac{dt}{dx} = \sec^2 x$~~

~~$dt = \sec^2 dx$~~

Let $t = \sec x$

$dt = \sec x \tan x\, dx$

$= \int \sec^2 x (\sec x \tan x)\, dx$

$= \int t^2\, dt$

$= \frac{t^3}{3} + C$

$= \frac{\sec^3 x}{3} + C$ Ans.

Eg: $I = \int \frac{1}{\sqrt{x}} e^{\sqrt{x}}\, dx = \frac{dx}{\sqrt{x}}$

Let $t = \sqrt{x}$

$\frac{dt}{dx} = \frac{1}{2\sqrt{x}}$

$2dt = \frac{dx}{\sqrt{x}}$

$= \int 2dt\, e^{\sqrt{x}}$

$= 2\int e^{t}\, dt$

$= 2e^{\sqrt{x}} + C$ Ans.

Eg: $I = \int x^2 (\ln x^3)\, dx$

$\cancel{\int \frac{x^2 (\ln x^3)}{x}\, dx}$

$= \int \ln t \times \frac{dt}{3}$

$= \frac{1}{3} \int (\ln t)\, dt$ Ans.

Let $t = x^3$

$dt = 3x^2 dx$

$\Rightarrow \frac{dt}{3} = x^2 dx$

Eg: $I = \int \frac{\sec x \tan x \; dx}{\sqrt{e^{\frac{\sec x}{2}}}}$

$\int \frac{dt}{\sqrt{e^{\frac{\sec x}{2}}}}$

$\int dt \times \sqrt{e^{\frac{\sec x \cdot t}{2}}}$

$\int dt \times \frac{1}{2\sqrt{e^t}} \sqrt{e^t}$

$\cancel{\int dt \times}$

$= \frac{1}{2\sqrt{e^{\frac{\sec x}{2}}}} \times \frac{\sec x \tan x}{4}$

Let $t = \sec x$

$\frac{dt}{dx} = \sec x \tan x$

$dt = \sec x \tan x\, dx$

$\frac{\sec x \tan x}{2}$

$\overline{\;\;4\;\;}$

Sol: $I = \int \frac{dt}{\sqrt{e^{t/2}}}$

$= \int \frac{dt}{e^{t/2 \times 1/2}}$

$= \int e^{-t/4}\, dt$

Let $t = \sec x$

$dt = \sec x \tan x\, dx$

$$= \frac{e^{-t/4}}{-1/4} + C$$

$$= -4e^{-t/4} + C$$

$$= -4e^{-\frac{\sec x}{4}} + C$$

Eg: $I = \int \sin^5 x \, dx$

Solution → ~~Let t = x~~ ~~dt/dx~~ ~~dt = dx~~

$$= \int \sin x \cdot \sin^4 x \, dx$$

$$= \int \sin x \, (\sin^2 x)^2 \, dx$$

$$= \int \sin (1 - \cos^2 x)^2 \, dx$$

Let $t = \cos x$

$\Rightarrow \frac{dt}{dx} = -\sin x$

$\Rightarrow dt = -\sin x \cdot dx$

$$= -\int (1 - t^2)^2 \, dt$$

$$= -\int (1 + t^4 - 2t^2) dt = -\left(t + \frac{t^5}{5} - \frac{2t^3}{3}\right) +$$

$$= -\int \cancel{(1+t)} \quad = -\left(\cos x + \frac{\cos 5x}{5} - \frac{2}{3}\cos^3 x\right) +$$

Definite Integration

$I = \int_a^b f(x)\,dx$ $\left(\begin{array}{l} a = \text{lower limit} \\ b = \text{upper limit} \end{array} \right)$

$I = \left[g(x) \right]_a^b = g(b) - g(a)$

Eg $\int_1^2 (x^2 - 2x)\,dx$

$= \left[\frac{x^3}{3} - \frac{2x^2}{2} \right]_1^2$

$= \left[\frac{x^3}{3} - x^2 \right]_1^2$

$= \left[\left\{ \frac{(2)^3}{3} - (2)^2 \right\} - \left\{ \frac{(1)^3}{3} - (1)^2 \right\} \right]$

$= \left(\frac{8}{3} - 4 \right) - \left(\frac{1}{3} - 1 \right)$

$= \frac{-4}{3} + \frac{2}{3}$

$= \frac{-4+2}{3} = \frac{-2}{3}$ Ans.

Date / /
Page

eg $I = \int_{0}^{\pi/2} \sin 2x \cdot dx$

$= -\frac{\cos 2x}{2}$

$= -\frac{1}{2}\left[\frac{\cos \cancel{2}\times 90}{\cancel{2}} - \frac{\cos 0^\circ}{2}\right]$

$= -\frac{1}{2}\left[-1-1\right]$

$= -\frac{1}{\cancel{2}} \times -\cancel{2}$

$= 1$ Ans.

eg $I = \int_{-\pi/2}^{\pi/2} \cos 4x \; dx$

integration

$= \left[\frac{\sin 4x}{4}\right]$

$= \frac{\sin 2\times 180^\circ}{4} - \frac{\sin -360^\circ}{4}$

$= \frac{\sin \cancel{360^\circ}\, 2\pi}{4} + \frac{\sin 2\pi}{4}$

$= \frac{1}{4}\left[0+0\right]$

$= 0$ Ans.

Beginner's Box 2

Q.5) (ii) $\int x\,(x^2-2)^2\, dx$

$= \int t^2 \frac{dt}{2}$

Let $t = x^2-2$

$\frac{dt}{\cancel{dx}} = 2x\, dx$

$\frac{dt}{2} = x\, dx$

$= \frac{1}{2}\times\frac{t^3}{3}+c$

$= \frac{1}{6}(x^2-2)^3 + c$

iv) $\int \frac{\sqrt{x}\, dx}{1+x^{3/2}}$

Let $t = 1+x^{3/2}$

$dt = 0+\frac{3}{2}x^{1/2}dx$

$\frac{2}{3}dt = \sqrt{x}\,dx$

$= \frac{2}{3}\int \frac{dt}{t}$

$= \frac{2}{3}\ln t + c$

$= \frac{2}{3}\ln(1+x\sqrt{x})\, c$

Q.7 (i) $\int \cos^3\theta \sin\theta \, d\theta$

Let $t = \cos\theta$

$dt = -\sin\theta\, d\theta$

$-dt = \sin\theta\, d\theta$

$= -\int t^3 dt$

$= -\frac{t^4}{4} + c$

$= -\frac{(\cos\theta)^4}{4} + c$

(ii) $I = \int \frac{e^t dt}{\sqrt{1-e^t}}$

Let $1-e^t = z$

$dz = 0 - e^t dt$

$-dz = e^t dt$

$= \int -\frac{dz}{\sqrt{z}}$

$= -\frac{z^{+1/2}}{1/2} + c$

$= -2\sqrt{z} + C$

$= -2\sqrt{1-e^{t}} + C$

Q.8 i) $I = \int \frac{1+\cos y}{y+\sin y}\, dy$ Let $y + \sin y = t$

$\frac{dt}{dy} = 1+\cos y$

$= \int \frac{dz}{z}$

$= \ln z + C$

$= \ln(y + \sin y) + C$

iv) $I = \int 3^{2y}\, dy$

$= \frac{3^{2y}}{2 \times (\ln 3)} + C$

Q11) i) $\int \sin^3 x \cos^3 x\, dx$ Let $t = \cos x$

$\frac{dt}{dx} = -\sin x$

$-dt = \sin x\, dx$

$= \int \sin x \cdot \sin^2 x \cdot \cos^3 x\, dx$

$= \int \underbrace{\sin x}(1-\cos^2 x) \cdot \cos^3 x\, \underbrace{dx}$

$= -\int (1-t^2)\, t^3\, dt$

$= -\int (t^3 - t^5)\, dt$

$= -\left(\frac{t^4}{4} - \frac{t^6}{6}\right) + C$

$= -\frac{\cos^4 x}{4} + \frac{\cos^6 x}{6} + C$

(ii) $\int \cos^2\left(\frac{y}{2}\right) dx$

$$\left[2\cos^2\frac{y}{2} = 1 + \cos 2\times\frac{y}{2}\right]$$

$I = \int (1+\cos y)\, dy$

$= y + \sin y + C$

(iii) $\int \sec^n\theta \tan\theta\, d\theta$ 　　Let $t = \sec\theta$
　　　　　　　　　　　　　　　　$dt = \sec\theta\tan\theta\, d\theta$

$= \int \sec^{n-1}\theta \cdot \underline{\sec\theta \cdot \tan\theta}\, d\theta$

$= \int t^{n-1}\, dt$

$= \frac{t^n}{n} + C$

$= \frac{\sec^n\theta}{n} + C$

7 (iii) $I = \int \frac{y+4}{y-4}\, dy$

$= \int \frac{y-4+8}{y-4}\, dy$

$= \int \frac{\cancel{y-4}}{\cancel{y-4}}\, dy + \int \frac{8}{y-4}\, dy$

$= \int dy + \int \frac{8}{y-4}\, dy$

$= y + 8\ln(y-4) + C.$

Q.11) i) $\int \sin 3x \cos 5x \, dx$

$\sin(C+D) + \sin(C-D) = \sin C \cos D + \cancel{\sin D \cos C} + \sin C \cos D - \cancel{\sin D \cos C}$

$\sin(C+D) + \sin(C-D) = 2\sin C \cos D$

$\Rightarrow \sin C \cos D = \frac{1}{2}\left[\sin C+D + \sin C-D\right]$

$= \frac{1}{2}\left[\sin 8x + \sin 2x\right]$

$= \frac{1}{2}\int(\sin 8x + \sin 2x)\,dx$

$= \frac{1}{2}\left[-\frac{\cos 8x}{8} - \frac{\cos 2x}{2}\right] + C$

HOMEWORK : Race 5, 6, 7, 8.
Exercise 2

Integration by Parts:

I. $\int uv \, dx = u\int v \cdot dx - \int\left[\frac{du}{dx} \cdot \int v \, dx\right] dx$

Here u and v are taken as ILATE Rule

Inverse, Logarithmic, Algebraic, Trigonometric, Exponential.

Eg: $I = \int \underbrace{x}_{\text{algebraic}} \underbrace{\sin x}_{\text{trigonometric}} \, dx$

$u \quad v$

$$I = x\int \sin x \, dx - \int \left[\frac{dx}{dx}(x) \times \int \sin x \, dx\right] dx$$

$$= x(-\cos x) - \int [1 \times -\cos x] \, dx + C$$

$$= -x\cos x + \sin x + C$$

Eg: $I = \int \underset{u}{x} \; \underset{v}{\ln x} \, dx$

$$= \ln x \int x \, dx - \int \left\{ \frac{d}{dx}(\ln x) \times \int x \, dx \right\} dx$$

$$= \ln x \times \frac{x^2}{2} - \int \left(\frac{1}{x} \times \frac{x^2}{2}\right) dx + C$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x \, dx + C$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2} \times \frac{x^2}{2} + C$$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

Eg: $I = \int \underset{\text{Algeb}}{x^2} \; \underset{\text{Log}}{e^x}$ $\rightarrow u$

v

$$= e^x \int x^2 \, dx - \int \left\{ \frac{d}{dx}(e^x) \int x^2 \, dx \right\} dx$$

$$= \frac{e^x x^3}{3} - \int e^x$$

Algeb Exponential
$\underset{\underset{u}{\downarrow}}{x^2}\ \underset{\underset{v}{\downarrow}}{e^x}$

$= x^2 e^x - 2\int \underset{u}{x}\, \underset{v}{e^x}\, dx + C$

$= x^2 e^x - 2\left[x e^x - \int 1 \times e^x\, dx\right] + C$

$= x^2 e^2 - 2xe^x + 2e^x + C$

Eg: $I = \int \underset{u}{x} \cdot \underset{v}{\sec^2 x}\, dx$ LATE

~~Alge Logarithmic~~

$= \cancel{\sec^2 x \int \frac{x^2}{2} - \int\left[\sec x \tan x\right]}$

$= x \tan x - \int 1 \times \tan x\, dx + C$

$= x \tan x + \ln(\cos x) + C$

Eg: $I = \int \underset{\underset{u}{\downarrow}}{\underset{A}{x}}\, \underset{\underset{v}{\downarrow}}{\underset{T}{\cos x}}\, dx$

$= x\{+\sin x\} - \int\left[1\{\times \sin x\}\right] dx$

$= x\sin x + \cos x + C$ Ans

study companion

Eg:

$$I = \int dx \times \frac{x}{(x^2+r^2)^{3/2}} \quad (r \text{ is constant})$$

$$= \cancel{\int \frac{x^2 \quad x}{2[x^3+r^3+2rx^2]} dx}$$

$$= \cancel{\frac{x^2}{2\left[\frac{x^4}{4}+\frac{r^4}{4}+\frac{2r^2x^3}{3}\right]}}$$

$$= \cancel{\frac{x^2}{\frac{x^4}{2}+\frac{r^4}{2}+\frac{4r^2x^3}{3}}}$$

Let $t = (x^2+r^2)$

$$dt = 2x\,dx$$

$$\frac{dt}{2} = x\,dx$$

$$= \int \frac{dt}{2(x^2+r^2)^{3/2}}$$

$$= \int \frac{dt}{2t^{3/2}}$$

$$= \frac{1}{2}\int t^{-3/2}\,dt$$

$$= \frac{1}{\cancel{2}} \times \frac{t^{-1/2}}{-1/\cancel{2}} + C$$

$$= \frac{-1}{\sqrt{t}} + C$$

$$= \frac{-1}{\sqrt{x^2+r^2}} + C \quad \text{Ans}$$

*eg: I. $\int \frac{dx}{(x^2+r^2)^{3/2}}$

By TRIGONOMETRIC SUBSTITUTION.

Let $x = r\tan\theta$

$\frac{dx}{d\theta} = r\sec^2\theta$

$dx = r\cdot\sec^2\theta\cdot d\theta$

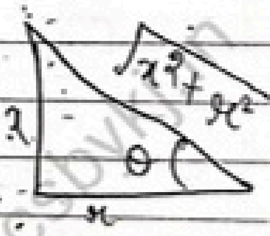
$$= \int \frac{r\sec^2\theta \, d\theta}{(r^2\tan^2\theta + r^2)^{3/2}}$$

$$= \int \frac{r\sec^2\theta \, d\theta}{r^{2\times 3/2}\sec^{2\times 3/2}\theta}$$

$$= \int \frac{\cancel{r}\cancel{\sec^2\theta}\, d\theta}{r^{\cancel{3}2}\sec^{\cancel{3}}\theta}$$

$$= \frac{1}{r^2}\int \frac{1}{\sec\theta} d\theta$$

$$= \frac{1}{r^2}\sin\theta + C$$



$$= \frac{1}{r^2} \times \frac{x}{\sqrt{x^2+r^2}} + C$$

$$= \frac{x}{r^2\sqrt{x^2+r^2}} + C \quad \text{Ans.}$$

*

I. $\int \sqrt{r^2 - x^2}\, dx$. By TRIGONOMETRIC SUBSTITUTION

$x = r\sin\theta$

$\frac{dx}{d\theta} = r\cos\theta$

$\Rightarrow dx = r\cdot\cos\theta\cdot d\theta$

$$I = \int \sqrt{r^2 - r^2\sin^2\theta} \times r\cos\theta \, d\theta$$

$$= \int \sqrt{r^2(1-\sin^2\theta)} \; r\cos\theta \, d\theta$$

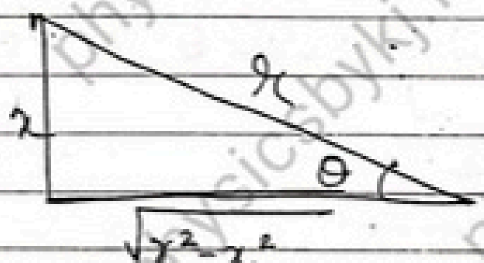
$$= r^2 \int \cos^2\theta \, d\theta$$

$$= r^2 \int \left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= \frac{r^2}{2}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$

$$= \frac{r^2}{2}\left[\theta + \frac{\cancel{2}\sin\theta}{\cancel{2}}\cos\theta\right] + C$$

$$= \frac{r^2}{2}\left[\sin^{-1}\left(\frac{x}{r}\right) + \frac{x}{r} \times \frac{\sqrt{r^2-x^2}}{r}\right] + C \quad \text{Ans}$$



$$\frac{x}{r} = \sin\theta$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{x}{r}\right)$$

Application of Integration

Area of Disc

Area of Elemental ring of radius x and width dx.

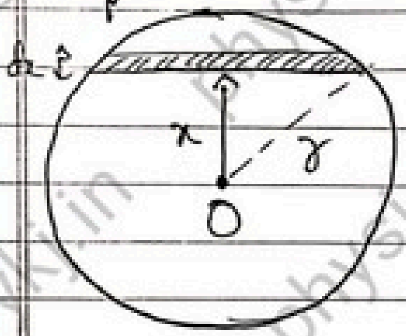
Area of Disc $\Rightarrow \int ds = \int_0^r 2\pi x \, dx$

$$= 2\pi \int_0^r x \, dx$$

$$\Rightarrow 2\pi \left(\frac{x^2}{2}\right)_0^r = \cancel{2}\pi \times \frac{r^2}{\cancel{2}}$$

$= \pi r^2$

Alternate ways :



Area of Elemental Strip $ds = 2\sqrt{r^2 - x^2}\,dx$

So Area of Disc $= \int_{-r}^{r} 2\sqrt{r^2 - x^2}\,dx$

$= 2\int \sqrt{r^2 - r^2\sin^2\theta} \times r\cos\theta\,d\theta$

where $x = r\sin\theta$
$dx = r\cos\theta\,d\theta$

$= 2r^2 \int \cos\theta \times \cos\theta\,d\theta$

$= 2r^2 \int \left(\frac{1+\cos 2\theta}{2}\right) d\theta$

$= \frac{2r^2}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]$

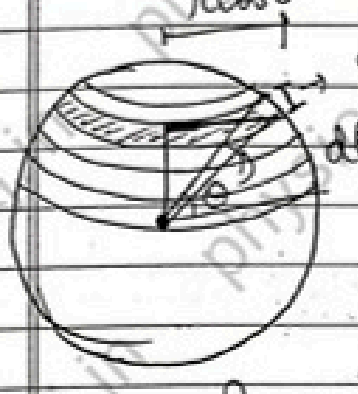
$= r^2\left[\sin^{-1}\frac{x}{r} + \frac{x}{r} \times \frac{\sqrt{r^2 - x^2}}{r}\right]_{-r}^{r}$

$= r^2\left[\left\{\sin^{-1}\frac{r}{r} + 0\right\} - \left\{\sin^{-1}\left(\frac{-r}{r}\right) + 0\right\}\right]$

$= r^2\left[\left\{\frac{\pi}{2} + \frac{\pi}{2}\right\}\right]$

$\boxed{= \pi r^2}$

2) Surface Area of Hollow Sphere



$d\theta = \dfrac{\text{arc}}{\text{radius}}$ (width of ring)

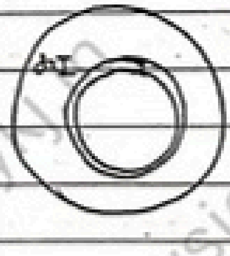
$\boxed{R\,d\theta = \text{arc}}$

Area of Elemental ~~Sphere~~ Ring ds $= 2\pi r\cos\theta \times r\,d\theta$

So Surface Area of Sphere $= \displaystyle\int_{-\pi/2}^{\pi/2} 2\pi r \cos\theta \; r\,d\theta$

$= 4\pi r^2$

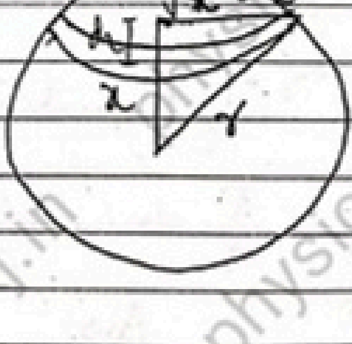
3) Volume of Sphere



$\displaystyle\int dv = \int_0^r 4\pi x^2\,dx$

$= \dfrac{4}{3}\pi r^3$

Other Way :



Race 6

(9) $y = \cos(\sin^{-1} x)$

$= \frac{-\sin(\sin^{-1} x)}{\sqrt{1-x^2}}$

Inverse Functions :

1) $y = \sin^{-1} x$; $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

2) $y = \cos^{-1} x$; $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$

3) $y = \tan^{-1} x$; $\frac{dy}{dx} = \frac{1}{1+x^2}$

4) $y = \cot^{-1} x$; $\frac{dy}{dx} = \frac{-1}{1+x^2}$

Race 7

(9) $I = \int_0^{\pi/4} \tan^2 x \, dx$

$\int_0^{\pi/4} \sec^2 x - 1$

$\sec^2 \frac{\pi}{4} - \frac{\pi}{4}$

$= 1 - \frac{\pi}{4}$ Ans

10) $\cancel{\int_0^{\pi/2} \sin^3\theta \, d\theta}$

Exercise 4

Q.4)

iii) $\int \sqrt{x} \sin^2 (x^{3/2}-1) dx$

$t = x^{3/2} - 1$

$\frac{dt}{dx} = \frac{3}{2} x^{1/2} - 0$

$dt = \frac{3}{2} \sqrt{x} \, dx$

$\frac{2}{3} dt = \sqrt{x} \, dx$

$I = \frac{2}{3} \int \sin^2 t \, dt$

$= \frac{\cancel{2}}{3} \int \left(\frac{1 - \cos 2t}{\cancel{2}} \right) dt$

$= \frac{1}{3} \left(t - \frac{\sin 2t}{2} \right) + C$

$= \frac{t}{3} - \frac{\sin 2t}{6} + C$

$= \frac{x^{3/2} - 1}{3} - \frac{\sin 2(x^{3/2}-1)}{6} + C$

R(i) $\operatorname{cosec} \left(\frac{v-\pi}{2} \right) \cot \left(\frac{v-\pi}{2} \right) dv$

$= -\frac{\operatorname{cosec}\left(\frac{v-\pi}{\cancel{2}}\right)}{1/2} + C$ → coefficient

Q.6) vi) $\int \frac{6\cos t}{(2+\sin t)^3} dt$

$z = 2+\sin t$

$\frac{dz}{dt} = \cos t$

Q.7) iv) $I = \int_0^{\sqrt{\pi}} x \sin x^2 \, dx$

Let $t = x^2$

$\frac{dt}{dx} = 2x$

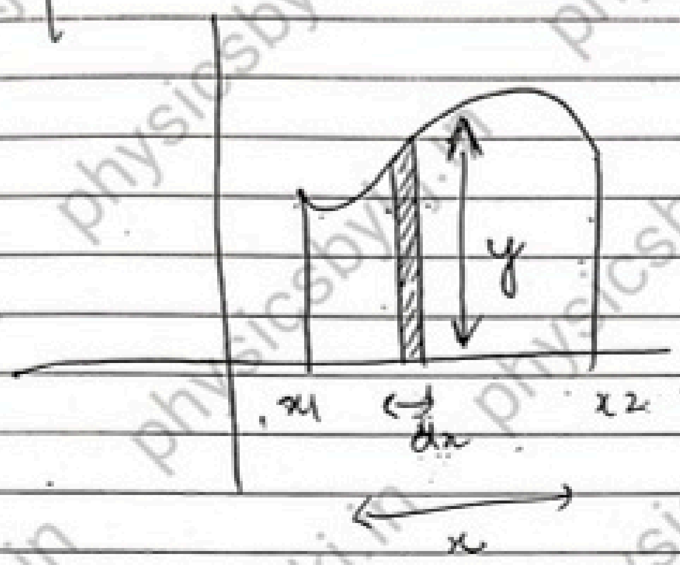
$\Rightarrow \frac{dt}{2} = x\,dx$

$= \frac{1}{2}\int \sin t \, dt$

$= -\frac{1}{2}\cos t$

$= \frac{1}{2}(\cos\pi - \cos 0)$

Area under the curve by Integration

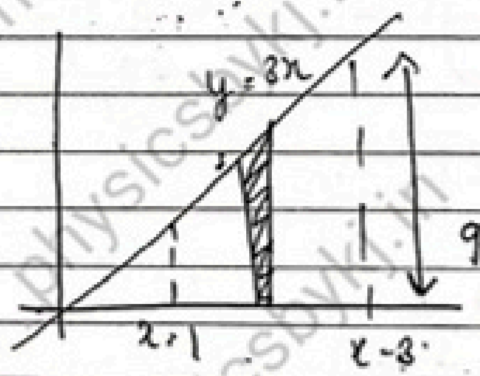


Area of elemental strip =

$$\int dA = \int y\,dx$$

$$\Rightarrow A = \int_{x_1}^{x_2} y\,dx$$

Eg: Calculate area under $y = 3x$ & x-axis between $x = 1$ & $x = 3$



$$A = \int_1^3 y\,dx$$

$$= \int_1^3 3x\,dx$$

$$= \frac{3x^2}{2} = \frac{3(3)^2}{2} - \frac{3(1)^2}{2}$$

$$= \frac{27}{2} - \frac{3}{2} = \frac{24}{2} = 12$$

Exercise 2

Q09) iii) $\int_0^{\pi} \sin^2 x \, dx$

$= \int_0^{\pi} \left(\frac{1-\cos 2x}{2}\right) dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi}$

$= \frac{1}{2}\left[\left(\pi - \frac{\sin 2\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$

$= \frac{\pi}{2}$ Ans

Begginer's Box 3

Q09) iv) $\int_0^1 (e^x + x^e) \, dx$

constant

$= \cancel{x^{e(2.713)}} \left[e^x + \frac{x^{e+1}}{e+1}\right]_0^1$

$= \left[\left(e^1 + \frac{1}{e+1}\right) - \left(1 + \frac{0}{e+1}\right)\right]$

$= e + \frac{1}{e+1} - 1$

$= 2.713 + \frac{1}{3.713} - 1$

$= 1.713 + \frac{1}{3.713}$

Q.10) i) $\int_0^{\pi/3} \frac{\sec\theta \tan\theta}{\sqrt{e^{\sec\theta}}} d\theta$ Let $t = \sec\theta$

$\frac{dt}{d\theta} = \sec\theta \tan\theta$

$dt = \sec\theta \tan\theta \, d\theta$

$= \int \frac{dt}{\sqrt{e^t}}$

$= \int \frac{dt}{e^{t/2}}$

$= \int e^{-t/2} dt$

$= \frac{e^{-t/2}}{-1/2}$

$= \left[-2e^{-\sec\theta/2}\right]_0^{\pi/3}$

$= -2\left[e^{-\sec\pi/6} - e^{-\sec 0}\right]$

$= -2\left[e^{-2/\sqrt{3}} - e^{-1}\right]$

$= -2$

Q.10) iii) $\int_{-1}^{0} \frac{dy}{1+e^y}$

$t = 1+e^y \Rightarrow \boxed{e^y = (t-1)}$

$\frac{dt}{dy} = e^y$

$dt = e^y dy$

$dy = \frac{dt}{e^y}$

$I = \int \frac{dt}{e^y t}$

$= \int \frac{dt}{(t-1)t}$

$= \int \left(\frac{1}{t} - \frac{1}{t-1}\right) dt$

$= \left[\ln(t-1) - \ln(t)\right]_{-1}^{0}$

$= \ln\frac{(t-1)}{t} = \ln\frac{e^y}{1+e^y} = \left[\ln e^y - \ln(1+e^y)\right]$

myCOMPANION

$$= \left[y - \ln(1+e^{y})\right]_{-1}^{0}$$

$$= \left[\{0 - \ln(1+e^{0})\} - \{-1 - \ln(1+e^{-1})\}\right]$$

$$= \left[-\ln 2 + 1 + \ln\left(1+\frac{1}{e}\right)\right]$$

$$= \left[1 - \ln 2 + \ln\left(1+\frac{1}{2.713}\right)\right]$$

Beginner's Box 4

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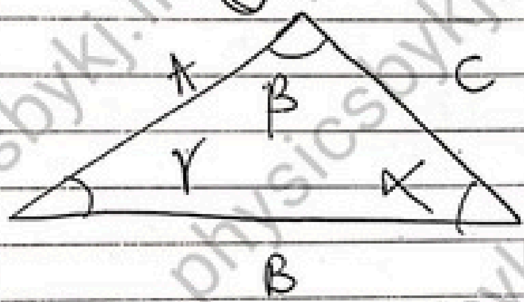
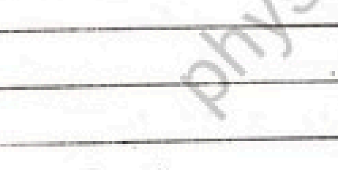


Sine Law:

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Cosine Law:

$$A^2 = B^2 + C^2 - 2BC\cos\alpha$$
$$B^2 = C^2 + A^2 - 2AC\cos\beta$$
$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$



$$\Rightarrow C^2 = (20\sqrt{3})^2 + (20)^2 - 2\times 20\sqrt{3}\times 20 \times \cos 30^\circ$$

$= $ Ans

Exercise - 1

42) $V = a^3$ Volume :

$\frac{dv}{dt} = 3a^2 \frac{da}{dt}$

$= 3 \times (100) \times 3 \ cm^3/sec$

$= 900 \ cm^3/sec$

Area :

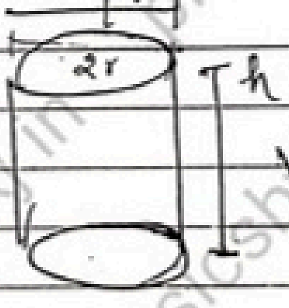
$A = \pi r^2$

$\frac{dA}{dt} = \pi 2r \times \frac{dr}{dt}$

Surface Area :

$4\pi r^2 = S$

$\frac{ds}{dt} = 4\pi \times 3r \frac{dr}{dt}$

80) 

$V = \pi r^2 h$

$\frac{V}{\pi r^2} = h$

Surface Area must be minimum

$2\pi r^2 + 2\pi r h = S$

$2\pi r^2 + 2\cancel{\pi r} \times \frac{V}{\cancel{\pi r^2}} = S$

$2\pi r^2 + 2\frac{V}{r} = S$

x30
30°

$$\frac{dS}{dr} = 0$$

$$2\pi r - \frac{2V}{r^2} = 0$$

$$2\pi r^3 = 2V$$

$$2\pi r^3 = \pi r^2 h$$

$$\boxed{2r = h}$$