

JEE RANK COMPANION

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MATHEMATICS

For JEE Main & Advanced
Aligned with Latest JEE Pattern

FUNCTION

1

Function

1. Number System

(i) Natural Numbers

The set of numbers $\{1, 2, 3, 4, \dots\}$ are called natural numbers, and is denoted by N .

i.e., $N = \{1, 2, 3, 4, \dots\}$

(ii) Integers

The set of numbers $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ are called integers and the set is denoted by I or Z .

• Where we represent;

(A) Positive integers by $I^+ = \{1, 2, 3, 4, \dots\}$
= Natural numbers.

(B) Negative integers by $I^- = \{\dots, -4, -3, -2, -1\}$

(C) Non-negative integers I_0^+ (or N_0)
= $\{0, 1, 2, 3, 4, \dots\}$ = Whole numbers

(D) Non-positive integers $I_0^- = \{\dots, -3, -2, -1, 0\}$

(iii) Rational Numbers

A number which can be written as a/b , where a and b are integers, $b \neq 0$ is called a rational number and their set is denoted by Q .

i.e., $Q = \{\frac{a}{b} \mid a, b \in I \text{ and } b \neq 0\}$.

Note:

Every integer is a rational number as it could be written as $Q = \frac{a}{b}$ (where $b = 1$)

All recurring decimals are rational numbers.

e.g., $Q = 0.3333 \dots = \frac{1}{3}$

$Q = 0.9999 \dots = 1$

(iv) Irrational Numbers :

Those values which neither terminate nor could be expressed as recurring decimals are irrational numbers. (i.e., it cannot be expressed as $\frac{a}{b}$ form),

their set is denoted by Q^c (i.e., complement of Q).

e.g., $\sqrt{2}, \pi + \sqrt{2}, \frac{1}{\sqrt{2}}, \frac{-3}{\sqrt{2}}$

$\sqrt{3}, 1 + \sqrt{3}, \pm \frac{1}{\sqrt{3}}, \pi \dots$ etc.

(v) Real Numbers :

The set which contains both rational and irrational numbers is called real number set and is denoted by R . i.e., $R = Q \cup Q^c$

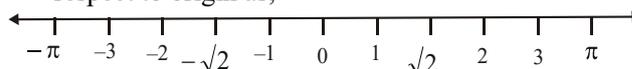
$R = \{\dots -2, -1, 0, 1, 2, 3, \dots\}$

$\frac{5}{6}, \frac{3}{4}, \frac{7}{9}, \frac{1}{3}, \frac{1}{7}, \frac{1}{5}, \dots, \sqrt{2}, \sqrt{3}, \pi, \dots\}$

Note:

As from above definitions;

$N \subset I \subset Q \subset R$, it could be shown that real numbers can be expressed on number line with respect to origin as;



Intervals :

The set of numbers between any two real numbers is called interval. The following are the types of interval.

(i) **Closed Interval:** $[a, b] = \{x : a \leq x \leq b\}$

(ii) **Open Interval:** (a, b) or $]a, b[= \{x : a < x < b\}$

(iii) **Semi open or semi closed interval:**

$[a, b[$ or $]a, b] = \{x : a \leq x < b\}$

$]a, b]$ or $(a, b] = \{x : a < x \leq b\}$

• Inequalities

The following are some very useful points to remember:

- $a \leq b \Rightarrow$ either $a < b$ or $a = b$
- $a < b$ and $b < c \Rightarrow a < c$
- $a < b \Rightarrow -a > -b$ i.e., inequality sign reverses if both sides are multiplied by a negative number
- $a < b$ and $c < d$
 $\Rightarrow a + c < b + d$ and $a - d < b - c, \forall c \in R$
- $a < b \Rightarrow ma < mb$ if $m > 0$ and $ma > mb$ if $m < 0$
- $0 < a < b \Rightarrow a^r < b^r$ if $r > 0$ and $a^r > b^r$ if $r < 0$
- $\left(a + \frac{1}{a}\right) \geq 2$ for $a > 0$ and equality holds for $a = 1$
- $\left(a + \frac{1}{a}\right) \leq -2$ for $a < 0$ and equality holds for $a = -1$

• The Absolute value of a real number

The absolute value (or modulus) of a real number x (written $|x|$) is a non negative real number that satisfies the conditions.

$|x| = x$ if $x \geq 0$

$|x| = -x$ if $x < 0$

Example:

$|2| = 2, |-5| = 5, |0| = 0$

From the definition it follows that the relationship $x \leq |x|$ holds for any x .

The properties of absolute values are

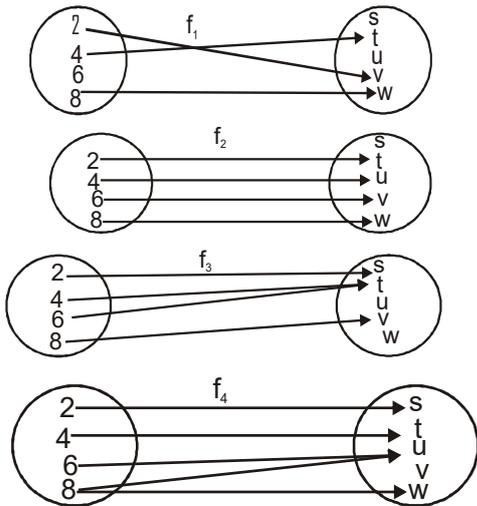
- (i) the inequality $|x| \leq \alpha$ means that $-\alpha \leq x \leq \alpha$; if $\alpha > 0$
- (ii) the inequality $|x| \geq \alpha$ means that $x \geq \alpha$ or $x \leq -\alpha$; if $\alpha > 0$
- (iii) $|x \pm y| \leq |x| + |y|$ (iv) $|x \pm y| \geq ||x| - |y||$
- (v) $|xy| = |x| |y|$ (vi) $\left| \frac{x}{y} \right| = \frac{|x|}{|y|}$ ($y \neq 0$).

2. Definition of Function

Let A and B be two non-empty sets. Then a function 'f' from set A to set B is a rule which associates elements of set A to elements of set B such that an element of set A is associated to a unique element in set B. All elements of set A are associated to element in set B. Terms such as "map" (or mapping), "correspondence" are used as synonyms for function. If f is a function from a set A to set B, then we write $f : A \rightarrow B$ or $A \xrightarrow{f} B$. which is read as f is a function from A to B or f maps A to B.

Example :

Let $A = \{2, 4, 6, 8\}$ and $B = \{s, t, u, v, w\}$ be two sets and let f_1, f_2, f_3 and f_4 be rules associating elements of A to elements of B as shown in the following figures.



Now see that f_1 is not function from set A to set B, since there is an element $6 \in A$ which is not associated to any element of B, but f_2 and f_3 are the function from A to B, because under f_2 and f_3 each elements in A is associated to the element in B. But f_4 is not function from A to B because an elements $8 \in A$ is associated to two elements u and w in B.

- **Domain:** Set A is called domain of f i.e. Set of those elements from which functions is to be defined.
- **Co-Domain :** Here set B is called co-domain of function.
- **Range:** Set of images of each element in A, is called range of f.

Note: Range \subseteq Co-domain

Real Valued Function :

All those functions of which domain and Co-Domain are subsets of R are called real valued functions. In this case for a given function we have to find domain and Range.

Bounded Function :

A function 'f' is said to be bounded if $|f(x)| \leq m$, for some finite 'm' & for every x in Domain of f.

Equality of two functions :

Two function f and g are said to be equal functions, if and only if

- (i) domain of f = domain of g
- (ii) co-domain of f = co-domain of g
- (iii) $f(x) = g(x) \forall x \in$ their domain

Example :

If $A = \{1,2\}$, $B = \{10,13\}$, $f : A \rightarrow B$, $f(x) = x^2 + 9$ and $g : A \rightarrow B$, $g(x) = 3x + 7$, then $f = g$ because domains and co-domains of both f and g are same also $f(A) = 10 = g(A)$; $f(B) = 13 = g(B)$

Examples :- 1

- (i) Determine the values of x satisfying the equality:

$$|(x^2 + 4x + 9) + (2x - 3)| = |x^2 + 4x + 9| + |2x - 3|;$$

- (ii) If $f(x) = \frac{1+x}{1-x}$, show that $\frac{f(x) \cdot f(x^2)}{1+[f(x)]^2} = \frac{1}{2}$.

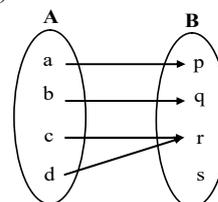
Solution:

- (i) The equality $|a + b| = |a| + |b|$ is valid if and only if both a and b have the same sign. Since $x^2 + 4x + 9 = (x + 2)^2 + 5 > 0$ at any values of x, the equality is satisfied at those values of x at which $2x - 3 \geq 0$, i.e., at $x \geq 3/2$.

$$\begin{aligned} \text{(ii) } f(x^2) &= \frac{1+x^2}{1-x^2} = \frac{1+x^2}{(1+x)(1-x)} \\ \therefore \frac{f(x) \cdot f(x^2)}{1+[f(x)]^2} &= \frac{\left(\frac{1+x}{1-x}\right) \cdot \frac{1+x^2}{(1+x)(1-x)}}{1 + \frac{(1+x)^2}{(1-x)^2}} \\ &= \frac{\frac{1+x^2}{(1-x)^2}}{\frac{(1-x)^2 + (1+x)^2}{(1-x)^2}} = \frac{1+x^2}{2+2x^2} = \frac{1}{2} \end{aligned}$$

3. Domain and Range

Domain = All possible values of x for which f(x) exists. Range = For all values of x, all possible values of f(x).



4. Domain and Range of some important functions

FUNCTION (y = f(x))	DOMAIN (i.e. values taken by x)	RANGE (i.e. values taken by f(x))
Algebraic Functions		
(i) $x^n, (n \in \mathbb{N})$	\mathbb{R} = (set of real numbers)	\mathbb{R} , if n is odd $\mathbb{R}^+ \cup \{0\}$, if n is even
(ii) $\frac{1}{x^n}, (n \in \mathbb{N})$	$\mathbb{R} - \{0\}$	$\mathbb{R} - \{0\}$, if n is odd \mathbb{R}^+ , if n is even
(iii) $x^{1/n}, (n \in \mathbb{N})$	\mathbb{R} , if n is odd $\mathbb{R}^+ \cup \{0\}$, if n is even	\mathbb{R} , if n is odd $\mathbb{R}^+ \cup \{0\}$, if n is even
(iv) $\frac{1}{x^{1/n}}, (n \in \mathbb{N})$	$\mathbb{R} - \{0\}$, if n is odd \mathbb{R}^+ , if n is even	$\mathbb{R} - \{0\}$, if n is odd \mathbb{R}^+ , if n is even
Trigonometric Functions		
(i) $\sin x$	\mathbb{R}	$[-1, 1]$
(ii) $\cos x$	\mathbb{R}	$[-1, 1]$
(iii) $\tan x$	$\mathbb{R} - (2k+1)\frac{\pi}{2}, k \in \mathbb{I}$	\mathbb{R}
(iv) $\sec x$	$\mathbb{R} - (2k+1)\frac{\pi}{2}, k \in \mathbb{I}$	$(-\infty, -1] \cup [1, \infty)$
(v) $\operatorname{cosec} x$	$\mathbb{R} - k\pi, k \in \mathbb{I}$	$(-\infty, -1] \cup [1, \infty)$
(vi) $\cot x$	$\mathbb{R} - k\pi, k \in \mathbb{I}$	\mathbb{R}
Inverse Circular Functions		
(i) $\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(ii) $\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
(iii) $\tan^{-1} x$	\mathbb{R}	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(iv) $\operatorname{cosec}^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
(v) $\sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
(vi) $\cot^{-1} x$	\mathbb{R}	$(0, \pi)$
Exponential Functions		
(i) e^x	\mathbb{R}	\mathbb{R}^+
(ii) $e^{1/x}$	$\mathbb{R} - \{0\}$	$\mathbb{R}^+ - \{1\}$
(iii) $a^x, a > 0$	\mathbb{R}	\mathbb{R}^+
(iv) $a^{1/x}, a > 0$	$\mathbb{R} - \{0\}$	$\mathbb{R}^+ - \{1\}$
Logarithmic Functions		
(i) $\log_a x, (a > 0) (a \neq 1)$	\mathbb{R}^+	\mathbb{R}
(ii) $\log_x a = \frac{1}{\log_a x}$ (a > 0) (a ≠ 1)	$\mathbb{R}^+ - \{1\}$	$\mathbb{R} - \{0\}$
Integral part Functions		
(i) $[x]$	\mathbb{R}	\mathbb{I}
(ii) $\frac{1}{[x]}$	$\mathbb{R} - [0, 1)$	$\left\{\frac{1}{n}, n \in \mathbb{I} - \{0\}\right\}$
Fractional part Functions		
(i) $\{x\}$	\mathbb{R}	$[0, 1)$

(ii) $\frac{1}{\{x\}}$	$\mathbb{R} - I$	$(1, \infty)$
Modulus Functions		
(i) $ x $	\mathbb{R}	$\mathbb{R}^+ \cup \{0\}$
(ii) $\frac{1}{ x }$	$\mathbb{R} - \{0\}$	\mathbb{R}^+
Signum Function		
$\text{sgn}(x) = \frac{ x }{x}, x \neq 0$ $= 0, x = 0$	\mathbb{R}	$\{-1, 0, 1\}$
Constant Function		
say $f(x) = c$	\mathbb{R}	$\{c\}$

5. Rules of domain

- Dom $(f + g + h \dots)$
= $\text{Dom } f \cap \text{Dom } g \cap \text{Dom } h \dots$
- Dom $(f - g) = \text{Dom } f \cap \text{Dom } g$
- Dom $(f \times g \times h \dots)$
= $\text{Dom } f \cap \text{Dom } g \cap \text{Dom } h \dots$
- Dom (f / g)
= $\text{Dom } f \cap \text{Dom } g - \{x : g(x) = 0\}$

Examples :- 2

Find the domain of following functions:

(i) $f(x) = 2^{\sin^{-1}x} + \sqrt{x+2} + \frac{1}{\log_{10}(x+1)}$

(ii) $f(x) = \sin^{-1}\sqrt{4-x^2}$

(iii) $f(x) = \ln(-2+3x-x^2)$

Solution :

- (i) For $f(x)$ to be defined $-1 \leq x \leq 1$,
 $x+2 \geq 0$ i.e., $x \geq -2$,
 $x+1 > 0$ i.e., $x > -1$ and
 $x+1 \neq 1$ i.e., $x \neq 0$
 so, domain of $f : (-1, 0) \cup (0, 1]$

- (ii) $f(x) = \sin^{-1}\sqrt{4-x^2}$
 for $f(x)$ to be defined $0 \leq 4-x^2 \leq 1$
 $\Rightarrow x^2 - 4 \leq 0$ and $x^2 - 3 \leq 0$
 $\Rightarrow x \in [-2, 2]$ and
 $x \in (-\infty, -\frac{1}{16}] \cup [\sqrt{3}, \infty)$
 $\Rightarrow x \in [-2-\sqrt{3}] \cup [\sqrt{3}, 2]$
 so domain $(f) : x \in [-2-\sqrt{3}] \cup [\sqrt{3}, 2]$

- (iii) $f(x) = \ln(-2+3x-x^2)$
 for $f(x)$ to be defined $-2+3x-x^2 > 0$
 $\Rightarrow x^2 - 3x + 2 < 0$
 $\Rightarrow (x-1)(x-2) < 0 \Rightarrow x \in (1, 2)$
 so domain $(f) : x \in (1, 2)$

Examples :- 3

Find the range of the following functions:

(i) $f(x) = \frac{1}{8-3\sin x}$ (ii) $f(x) = x^2 - 7x + 5$

(iii) $f(x) = \log_2(\log_{1/2}(x^2 + 4x + 4))$

Solution: (i) $f(x) = \frac{1}{8-3\sin x}$. We know that

$-1 \leq \sin x \leq 1$
 $\Rightarrow -3 \leq 3 \sin x \leq 3 \Rightarrow 5 \leq 8 - 3 \sin x \leq 11$

$\therefore \text{Range } (f) = \left[\frac{1}{11}, \frac{1}{5}\right]$

(ii) $f(x) = x^2 - 7x + 5$

$\Rightarrow f(x) = \left(x - \frac{7}{2}\right)^2 - \frac{29}{4}$ $\text{Range } (f) = \left[-\frac{29}{4}, \infty\right)$

(iii) $f(x) = \log_2(\log_{1/2}(x^2 + 4x + 4))$

since $0 < \log_{1/2}(x^2 + 4x + 4) < \infty \forall x \in \text{Domain } (f)$
 $\Rightarrow -\infty < \log_2(\log_{1/2}(x^2 + 4x + 4)) < \infty$
 $\text{Range } (f) = (-\infty, \infty)$

Examples :- 4

If $f(x) = x^2 + 1$, $g(x) = \frac{1}{x-1}$, then find $(f \circ g)(x)$ and

$(g \circ f)(x)$.

Solution :

Given, $f(x) = x^2 + 1$ (1)

$g(x) = \frac{1}{x-1}$ (2)

Now $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x-1}\right) = f(z)$,

where $z = \frac{1}{x-1}$

$= z^2 + 1$ [$\because f(x) = x^2 + 1$] $= \left(\frac{1}{x-1}\right)^2 + 1 = \frac{1}{(x-1)^2} + 1$

Note:

Domain of $f \circ g(x)$ is $x \in \mathbb{R} - \{1\}$

$(g \circ f)(x) = g(f(x)) = g(x^2 + 1) = g(u)$,

where $u = x^2 + 1 = \frac{1}{u-1} = \frac{1}{x^2 + 1 - 1} = \frac{1}{x^2}$

Note: Domain of $g \circ f(x)$ is $x \in \mathbb{R} - \{0\}$

Examples :- 5

If $f(x) = \begin{cases} 2+x, & \text{if } x > 0 \\ 2-x, & \text{if } x < 0 \end{cases}$, then find $(f \circ f)(x)$.

Solution :

$$f(x) = 2 + |x|$$

$$f(f(x)) = 2 + |f(x)| = 2 + 2 + |x| = 4 + |x|$$

Examples :- 6

Let $f(x) = \sin^{-1}x$, $g(x) = \sqrt{(3-x)(x+1)}$. Find $f+g$,

$f-g$, $f \cdot g$ and f/g

Solution :

$$(f+g)x = \sin^{-1}x + \sqrt{(3-x)(1+x)}, \text{ defined on } -1 \leq x \leq 1$$

$$(f-g)x = \sin^{-1}x - \sqrt{(3-x)(1+x)}, \text{ defined on } -1 \leq x \leq 1$$

$$(f \cdot g)x = \sin^{-1}x \times \sqrt{(3-x)(1+x)}, \text{ defined on } -1 \leq x \leq 1$$

$$(f/g)x = \frac{\sin^{-1}x}{\sqrt{(3-x)(x+1)}}, \text{ defined on } -1 < x < 1$$

Examples :- 7

Solve for x , $\log_{\frac{1}{4}}x \geq 1$

Key concept:

First make the base of log same both the sides. Then solve.

- **Common Mistake:**

$$\text{Given. } \log_{\frac{1}{4}}x \geq 1 \Rightarrow \log_{\frac{1}{4}}x \geq \log_{\frac{1}{4}}\frac{1}{4}$$

- After this step students used to solve in this manner

$$\log_{\frac{1}{4}}x \geq \log_{\frac{1}{4}}\frac{1}{4} \Rightarrow x \geq \frac{1}{4} \text{ They forget to check the}$$

base of log. Hence the answer is wrong.

Solution :

$$\text{Given. } \log_{\frac{1}{4}}x \geq 1 \Rightarrow \log_{\frac{1}{4}}x \geq \log_{\frac{1}{4}}\frac{1}{4}$$

We know

$$\log_b a_1 \geq \log_b a_2 \Rightarrow \begin{cases} a_1 \geq a_2 & \text{if } b > 1 \\ a_1 \leq a_2 & \text{if } 0 < b < 1 \end{cases}$$

The base of log lying between 0 and 1 hence inequality sign will change, hence $x \leq \frac{1}{4}$. Also log x is defined

only when $x > 0$. Hence the answer is $x \in \left(0, \frac{1}{4}\right]$

6. Classification of Function

The following are the kinds of function :

- (i) **One-One Function (Injective) :**

If each element in the domain of a function has a distinct image in the co-domain the function is said to be one-one function and is also known as Injective Function.

e.g. $f : \mathbb{R} \rightarrow \mathbb{R}^+$ given by $y = e^x$

$g : \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = 3x - 7$

are one-one functions.

or, $f : A \rightarrow B$ is one-one

$\Leftrightarrow a \neq b \Rightarrow f(a) \neq f(b)$ for all $a, b \in A$

$\Leftrightarrow f(A) = f(B) \Rightarrow a = b$ for all $a, b \in A$

- (ii) **Many-One Function :**

If there are two or more than two elements of domain having the same image then $f(x)$ is called Many-One function.

e.g. $f : \mathbb{R} \rightarrow \mathbb{R}^+$ $f(x) = x^2 + 4$

$g : \mathbb{R} \rightarrow \mathbb{R}^+$ $g(x) = x^8 + x^4 + x^2 + 4$

Both functions are many one

If the graph of $y = f(x)$ is given and a line parallel to x -axis cuts the curve at more than one point then function is many one.

or, $f : A \rightarrow B$ is a many-one function if there exist $x, y \in A$ such that $x \neq y$ but $f(x) = f(y)$.

e.g. $y = \sin x$, $y = \cos x$, $y = \tan x$, $y = x^2$, $y = x^4$, etc are many one functions.

- (iii) **Onto Function (Subjective) :**

Let $f : X \rightarrow Y$ be a function. If each element in the co-domain Y has at least one pre-image in the domain X i.e. $\text{Range } f = \text{Co-domain}$, then f is called onto.

Onto function is also called subjective and if function be both one-one and onto then function is called bijective.

or, $f : A \rightarrow B$ is a surjection if for each $b \in B$ [$x \neq -1, 1$ & $f(x) \neq 0$] $a \in A$ such that $f(a) = b$.

e.g. If $f : \mathbb{R}^+ \rightarrow \mathbb{R}$ is defined by $y = \log_2 x$, then $f(x)$ is Onto function.

- (iv) **Into Function :**

If there exist one or more than one element in the Co-domain Y which is not an image of any element in the domain X . Then f is into.

In other words $f : A \rightarrow B$ is an into function if it is not an onto function.

e.g. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $y = x^2 + 1$, then $f(x)$ is an into function. But when $f : \mathbb{R} \rightarrow \mathbb{R}^+$ is defined by $y = x^2 + 1$, then $f(x)$ is not into function.

- (v) **One-one onto Function or bijection :**

A function f is said to be one-one onto if f is one-one and onto both.

- (vi) **One-one into Function:**

A function is said to be one-one into if f is one-one but not onto.

- (vii) **Many one-onto Function:**

A function $f : A \rightarrow B$ is many one-onto if f is onto but not one-one.

(i) $f : \mathbb{R} \rightarrow \mathbb{R}^+ \cup \{0\}$, $f(x) = x^2$.

(ii) $f : \mathbb{R} \rightarrow [0, \infty)$, $f(x) = |x|$

- (viii) **Many one-into Function:**

A function f is said to be many one-into if it is neither one-one nor onto.

(i) $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \sin x$

(ii) $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = |x|$

(ix) Identity Function:

Let A be any set and the function $f : A \rightarrow A$ be defined as $f(x) = x, \forall x \in A$ i.e. if each element of A is mapped by itself then f is called the identity function. It is represented by I_A .

If $A = \{x, y, z\}$ then $I_A = \{(x, x), (y, y), (z, z)\}$

Note :

1. If domain of $f(x)$ is continuous and $\frac{dy}{dx} > 0, \forall x$ in domain then f is One – One, where equality exist at discrete point.
2. If domain of $f(x)$ is continuous and $\frac{dy}{dx} < 0, \forall x$ in domain then f is One – One, where equality exist at discrete point.
3. If a continuous function $f(x)$ which has either local minima or local maxima or both then $f(x)$ will be Many – One
4. Every even function is Many – One
5. Every periodic function is Many – One

Examples :- 8

Let $f : (-\infty, \infty) \rightarrow [2, \infty)$ be a function defined by

$f(x) = \sqrt{x^2 - 2a + a^2}, a \in \mathbf{R}$. Find a for which f is onto.

Solution :

For f to be onto range of the function should be $[2, \infty)$.

So, $a^2 - 2a = 4$

$\Rightarrow a = 1 \pm \sqrt{5}$.

Examples :- 9

If $f: \mathbf{R} \rightarrow \mathbf{R}$ where $f(x) = \frac{x+1}{x^2+x+1}$, find whether

$f(x)$ is one – one or many one.

Key concept :

If $f(x)$ is a rational function then $f(x_1) = f(x_2)$ will always be satisfied when $x_1 = x_2$ in the domain.

Hence we can write

$f(x_1) - f(x_2) = (x_1 - x_2)g(x_1, x_2) = 0$ where

$g(x_1, x_2)$ is some function in x_1 and x_2 . Now if

$g(x_1, x_2) = 0$ gives some solution which is different from $x_1 = x_2$ and which lies in the domain, then f is many – one else one – one.

Solution :

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1 + 1}{x_1^2 + x_1 + 1} = \frac{x_2 + 1}{x_2^2 + x_2 + 1}$$

One solution of this is obviously $x_1 = x_2$. Also we have got a relation in x_1 and x_2 and for each value of x_1 in the domain we get a corresponding value of x_2 which may or may not be same as x_1 .

If $x_1 = 1$ we get $x_2 = \frac{-1}{2} \neq x_1$, and both lies in the domain of f.

Hence we have two different values x_1 and x_2 for which $f(x)$ has the same value.

Hence 'f' is many one.

Examples :- 10

Let $f: \mathbf{R} \rightarrow [2, \infty)$ defined by

$f(x) = x^2 + 2x + b$ is an onto function, then find the value of b.

Key concept:

Given function is an onto function. Hence range should be equal to co-domain.

Solution :

Given $f(x) = x^2 + 2x + b = (x + 1)^2 + b - 1$.

Hence range of $f(x)$ is $[b-1, \infty)$.

Since $f(x)$ is an onto function $b-1 = 2 \Rightarrow b = 3$

7. Even and Odd function

Even function

If we put $(-x)$ in place of x in the given function and if $f(-x) = f(x), \forall x \in \text{domain}$ then function $f(x)$ is called even function.

Odd function

If we put $(-x)$ in place of x in the given function and if $f(-x) = -f(x), \forall x \in \text{domain}$ then $f(x)$ is called odd function.

Properties of even and odd Function

- (i) The product of two even functions is even function.
- (ii) The sum and difference of two even functions is even function.
- (iii) The sum and difference of two odd functions is odd function.
- (iv) The product of two odd functions is even function.
- (v) The product of an even and an odd function is odd function.
- (vi) The sum of even and odd function is neither even nor odd function.

Examples :- 11

Determine the nature of the following function for even and odd:

(i) $f(x) = \log(x + \sqrt{x^2 + 1})$

(ii) $f(x) = x \left(\frac{a^x - 1}{a^x + 1} \right)$

Solution :

(i) $f(x) = \log(x + \sqrt{x^2 + 1})$

$\Rightarrow f(-x) = \log(-x + \sqrt{x^2 + 1})$

$= \log\left(\frac{1}{x + \sqrt{x^2 + 1}}\right) = \log(x + \sqrt{x^2 + 1})^{-1}$

$= -\log(x + \sqrt{x^2 + 1}) = -f(x)$

So, $f(x)$ is an odd function

(ii) We have $f(x) = x \left(\frac{a^x - 1}{a^x + 1} \right)$

$f(-x) = -x = f(x)$

So, $f(x)$ is an even function.

8. Explicit and Implicit Function

Explicit Function

A function is said to be explicit if it can be expressed directly in terms of the independent variable. $y = f(x)$ or $x = \phi(y)$

Implicit Function

A function is said to be implicit if it cannot be expressed directly in terms of the independent variable.

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

9. Increasing and Decreasing Function

• **Increasing Function**

A function $f(x)$ is called increasing function in the domain D if the value of the function does not decrease by increasing the value of x .

If $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$

or $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ or $f'(x) > 0$ for increasing and $f'(x) \geq 0$ for not decreasing.

• **Decreasing Function**

A function $f(x)$ is said to be decreasing function in the domain D if the value of the function does not increase by increasing the value of x (variable).

If $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

or $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

or $f'(x) < 0$ for decreasing and $f'(x) \leq 0$ for not Increasing.

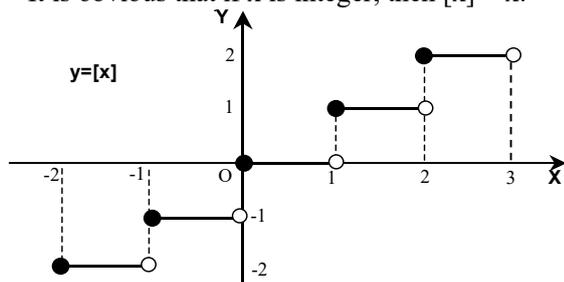
10. Greatest Integer and Fractional Part

• **Greatest Integer:**

Any real number x can always think of lying between two consecutive integers say P and $P+1$. i.e. $P \leq x < (P + 1)$. That means, there always exist an integer, say 'P' which is just less than or equal to x . This unique 'P' is called the greatest integral value of x and is symbolically denoted as $[x]$ i.e. $[x]$ stands for the greatest integer that is less than or equal to x .

e.g. $x = 3.54 \Rightarrow 3 < x < 4 \Rightarrow [x] = 3, x = -2.95 \Rightarrow -3 < x < -2 \Rightarrow [x] = -3$

It is obvious that if x is integer, then $[x] = x$.



Domain $\rightarrow \mathbb{R}$;

Range $\rightarrow \mathbb{I}$;

Period \rightarrow non periodic;

Nature \rightarrow neither even nor odd

• **Properties of G.I.F :**

(i) $[x] = x$ if x is integer

(ii) $[x + I] = [x] + I$, if I is an integer

(iii) $[x + y] \geq [x] + [y]$

(iv) If $[\phi(x)] \geq I$ then $\phi(x) \geq I$

(v) If $[\phi(x)] \leq I$ then $\phi(x) < I + 1$

(vi) If $[x] > n \Rightarrow x \geq n + 1$

(vii) If $[x] < n \Rightarrow x < n, n \in \mathbb{I}$

(viii) $[-x] = -[x]$ if $\forall x \in \mathbb{I}$

(ix) $[-x] = -[x] - 1$ if $x \notin \text{Integer}$

(x) $[x + y] = [x] + [y + x - [x]] \forall x, y \in \mathbb{R}$

(xi) $[x] + \left[x + \frac{1}{n} \right] + \left[x + \frac{2}{n} \right] + \dots + \left[x + \frac{n-1}{n} \right]$
 $= [nx]; n \in \mathbb{N}$

• **Fractional Part:**

Fractional Part of any real number is defined as the difference between the number 'x' and its integral value ' $[x]$ ' and is symbolically denoted as $\{x\}$.

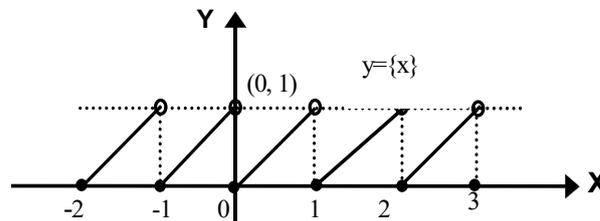
Thus, $\{x\} = x - [x]$, e.g. if $x = 5.68$, then $[x] = 5$ and $\{x\} = 0.68$.

If x is an integer $\Rightarrow x = [x] \Rightarrow \{x\} = 0$

$\Rightarrow \{[x]\} = 0$

If $x \in [0, 1)$, then $[x] = 0 \Rightarrow \{x\} = x$

$x \in [1, 2)$, then $[x] = 1 \Rightarrow \{x\} = x - 1$



Domain $\rightarrow \mathbb{R}$;

Range $\rightarrow [0, 1)$;

Period $\rightarrow 1$;

Nature \rightarrow neither even nor odd

IMPORTANT POINTS:

- $0 \leq \{x\} < 1$
- $\{[x]\} = 0, \{[x]\} = 0$
- $x - 1 < [x] \leq x, 0 \leq \{x\} < 1$
- $\{x\} + \{-x\} = \begin{cases} 0, & \text{if } x \in \text{integer} \\ 1, & \text{if } x \notin \text{integer} \end{cases}$

Examples :- 14

Solve for $x, [x] + \frac{1}{x - 2.7} = 2 + \frac{1}{x - 2.7}$

- **Common Mistake:**
Student used to solve in this manner
- First they cancel the common term both the sides. $\Rightarrow [x] = 2 \Rightarrow x \in [2, 3)$.

Solution :

First step is correct. Cancel common term. But

The important point is that $\frac{1}{x-2.7}$ is not defined when $x=2.7$, hence exclude this from the solution set.

11. Signum Function

The signum function f is defined as

$$\text{Sgn } x = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases} = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

12. Modulus Function

It is given $n \in \mathbb{N}$ by $y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

• **Properties of Modulus function:**

- (i) $|x| \leq a \Rightarrow -a \leq x \leq a$
- (ii) $|x| \geq a \Rightarrow x \leq -a$ or $x \geq a$
- (iii) $|x + y| = |x| + |y| \Rightarrow x, y \geq 0$ or $x \leq 0, y \leq 0$
- (iv) $|x - y| = |x| - |y| \Rightarrow x \geq 0$ & $|x| \geq |y|$ or $x \leq 0$ and $y \leq 0$ and $|x| \geq |y|$
- (v) $|x \pm y| \leq |x| + |y|$
- (vi) $|x \pm y| \geq |x| - |y|$

13. Periodic Function

• **Definition :**

A function $f(x)$ is said to be periodic function if, there exists a fixed positive real number T independent of x , such that, $f(x + T) = f(x) \forall x \in \text{Domain}$ & $x + T \in \text{domain}$.

T is called one of the period of the function

In other words, a function is said to be periodic function if its each value is repeated after a definite interval.

Here the least positive value of T (independent of x) is called the fundamental period of the function.

Clearly

$$f(x) = f(x + T) = f(x + 2T) = f(x + 3T) = \dots$$

• **For example :**

- (A) $\sin x, \cos x, \sec x$ and $\text{cosec } x$ are periodic functions with period 2π .
- (B) $\tan x$ and $\cot x$ are periodic functions with period π .
- (C) $|\sin x|, |\cos x|, |\tan x|, |\cot x|, |\sec x|, |\text{cosec } x|$ are periodic functions with period π .
- (D) $\sin^n x, \cos^n x, \sec^n x, \text{cosec}^n x$ are periodic functions with period 2π or π according as n is odd or even.
- (E) $\tan^n x$ and $\cot^n x$ are periodic function with period π whether n is odd or even.

• **Properties of Periodic Function :**

If $f(x)$ is periodic with period T , then

$f(x)$ is periodic with period T .

$f(x + c)$ is periodic with period T .

$f(x) \pm c$ is periodic with period T .

$f(ax + b)$ has period $\frac{T}{|a|}$, i.e., period is affected only by coefficient of x where; a, b, c , are constants with $a, b \neq 0$.

If $f(x)$ and $g(x)$ be two periodic function with period p & q respectively, then their any combination will be periodic function with one period equal to L.C.M of p & q provided L.C.M of p & q exists.

Note : All periodic functions can be analyzed over an interval of one period within the domain as the same pattern shall be repetitive over the entire domain.

Examples :- 15

Find the period of $f(x) = |\sin x| + |\cos x|$

Solution :

$|\sin x|$ has period π , $|\cos x|$ has period π

Hence, according to the rule of LCM, period of $f(x)$ must be π .

But $|\sin(\frac{\pi}{2} + x)| = |\cos x|$ and

$|\cos(\frac{\pi}{2} + x)| = |\sin x|$ Since, $\frac{\pi}{2} < \pi$, period of $f(x)$ is $\frac{\pi}{2}$

Examples :- 16

If $f(x) = \sin x + \cos ax$ is a periodic function, show that a is a rational number.

Solution :

Period of $\sin x = 2\pi = \frac{2\pi}{1}$ and period of $\cos ax = \frac{2\pi}{a}$

\therefore Period of $\sin x + \cos ax = \text{L.C.M of } \frac{2\pi}{1} \text{ and } \frac{2\pi}{a}$

$\frac{\text{LCM of } 2\pi \text{ and } 2\pi}{\text{H.C.F of } 1 \text{ and } a} = \frac{2\pi}{\lambda}$ where λ is the H.C.F. of 1 and a .

Since λ is the H.C.F of 1 and a , $\frac{1}{\lambda}$ and $\frac{a}{\lambda}$ should be both integers.

Suppose $\frac{1}{\lambda} = m$ and $\frac{a}{\lambda} = n$, then $\frac{1}{\frac{a}{n}} = \frac{n}{m}$, where $n, m \in \mathbb{I}$

i.e., $a = \frac{n}{m}$. Hence, a is rational number with period = 2

14. Inverse Function

If $f : X \rightarrow Y$ be a function defined by $y = f(x)$ such that f is both one-one and onto, then there exists a unique function $g : Y \rightarrow X$ such that for each $y \in Y$, $g(y) = x$. The function g so defined is called the inverse of f and denoted by f^{-1} . Also f is the inverse of g and the two functions f and g are said to be inverse of each other.

$f(f^{-1}(x)) = x, \forall x \in Y$ and $f^{-1}(f(x)) = x, \forall x \in X$

Note :- That f and f^{-1} are symmetric about the line $y = x$.

• **Method of finding inverse of a function :**

1. If you are asked to check whether the given function $y = f(x)$ is invertible, you need to check that $y = f(x)$ is one-one and onto.

2. If you are asked to find the inverse of a bijective function $f(x)$, you do the following : if f^{-1} be the inverse of 'f', then $f(f^{-1}(x)) = x$. Apply the formula of f on $f^{-1}(x)$ and use of the above identity to solve for $f^{-1}(x)$.

Some standard functions given below along with their inverse functions

	FUNCTION	INVERSE FUNCTION
(i)	$f : [0, \infty) \rightarrow [0, \infty)$ defined by $f(x) = x^2$	$f^{-1} : [0, \infty) \rightarrow [0, \infty)$ defined by $f^{-1}(x) = \sqrt{x}$
(ii)	$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow [-1, 1]$ defined by $f(x) = \sin x$	$f^{-1} : [-1, 1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ defined by $f^{-1}(x) = \sin^{-1}x$
(iii)	$f : [0, \pi] \rightarrow [-1, 1]$ defined by $f(x) = \cos x$	$f^{-1} : [-1, 1] \rightarrow [0, \pi]$ defined by $f^{-1}(x) = \cos^{-1}x$

Examples :- 17

Find the inverse of the function

$f(x) = \ln(x^2 + 3x + 1)$; $x \in [1, 3]$ assuming it to be an onto function.

Solution :

Given $f(x) = \ln(x^2 + 3x + 1)$

$\therefore f'(x) = \frac{2x+3}{(x^2+3x+1)} > 0 \forall x \in [1, 3]$

Which is a strictly increasing function. Thus $f(x)$ is injective, given that $f(x)$ is onto. Hence the given function $f(x)$ is invertible.

Now $f(f^{-1}(x)) = x$

$\Rightarrow \ln((f^{-1}(x))^2 + 3(f^{-1}(x)) + 1) = x$

$\Rightarrow (f^{-1}(x))^2 + 3(f^{-1}(x)) + 1 - e^x = 0$

$\therefore f^{-1}(x) = \frac{-3 \pm \sqrt{9 - 4 \cdot 1(1 - e^x)}}{2} = \frac{-3 \pm \sqrt{5 + 4e^x}}{2}$

$\Rightarrow f^{-1}(x) = \frac{-3 + \sqrt{5 + 4e^x}}{2}$ (as $f^{-1}(x) \in [1, 3]$)

Hence $f^{-1}(x) = \frac{-3 + \sqrt{5 + 4e^x}}{2}$

Examples :- 18

Find the inverse of the function

$$f(x) = \begin{cases} x, & x < 1 \\ x^2, & 1 \leq x \leq 4 \\ 8\sqrt{x}, & x > 4 \end{cases}$$

Solution :

Given $f(x) = \begin{cases} x, & x < 1 \\ x^2, & 1 \leq x \leq 4 \\ 8\sqrt{x}, & x > 4 \end{cases}$

Let $f(x) = y \Rightarrow x = f^{-1}(y)$ (i)

$$\therefore x = \begin{cases} y, & y < 1 \\ \sqrt{y}, & 1 \leq \sqrt{y} \leq 4 \\ \frac{y^2}{64}, & \frac{y^2}{64} > 4 \end{cases}$$

$$\Rightarrow f^{-1}(y) = \begin{cases} y, & y < 1 \\ \sqrt{y}, & 1 \leq y \leq 16 \\ \frac{y^2}{64}, & y > 16 \end{cases} \quad [\text{From (i)}]$$

$$\text{Hence } f^{-1}(x) = \begin{cases} x, & x < 1 \\ \sqrt{x}, & 1 \leq x \leq 16 \\ \frac{x^2}{64}, & x > 16 \end{cases}$$

Examples :- 19

Find the period of function $\sin 4x + \tan 2x$.

Solution :

Period of $\sin 4x$ is $\frac{\pi}{2}$, also period of $\tan 2x$ is $\frac{\pi}{2}$.

Hence period of $f(x)$ is $\frac{\pi}{2}$

• **Properties of Inverse Function**

- (i) The inverse of a bisection is unique.
- (ii) If $f: A \rightarrow B$ is a bisection & $g: B \rightarrow A$ is the inverse of f , then $f \circ g = I_B$ and $g \circ f = I_A$, where I_A and I_B are identity functions on the sets A and B respectively. Note that the graphs of f and g are the mirror images of each other in the line $y = x$. As shown in the figure given below a point (x', y') corresponding to $y = x^2 (x \geq 0)$ changes to (y', x') corresponding to $y = +\sqrt{x}$, the changed form of $x = \sqrt{y}$.

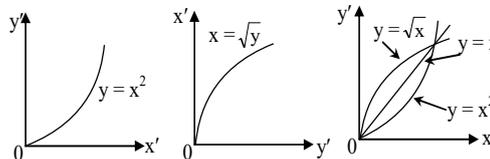


fig. 1 fig. 2 fig. 3

- (iii) The inverse of a bisection is also a bisection.
- (iv) If f & g are two bisections $f: A \rightarrow B$, $g: B \rightarrow C$ then the inverse of $g \circ f$ exists & $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Examples :- 20

The function $f: [1, \infty) \rightarrow [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, find $f^{-1}(x)$.

Key concept:

First check the function for one – one and onto. And if function is one – one and onto then find inverse using the identity $f(f^{-1}(x)) = x$

Solution :

Given, $f(x) = 2^{x(x-1)}$

$\Rightarrow \log f(x) = x(x-1) \log_e 2$

$\Rightarrow \frac{1}{f(x)} f'(x) = (2x-1) \log_e 2$

$\Rightarrow f'(x) = 2x^{(x-1)} \log_e 2 (2x-1)$

Thus $f(x)$ is an increasing function in $[1, \infty)$, therefore, $f(x)$ is a one – one function.

Also range of $f(x)$ is $[1, \infty)$ which is equal to co – domain.

Hence the function is also onto.

• TO FIND $f^{-1}(x)$:

Let f^{-1} be the inverse function of f , then by rule of identity $f(f^{-1}(x)) = x$

$f(f^{-1}(x)) = 2^{f^{-1}(x)(f^{-1}(x)-1)} = x$

$\Rightarrow \log_2 x = (f^{-1}(x))^2 - f^{-1}(x)$

$f^{-1}(x) = \frac{1 \pm \sqrt{1+4\log_2 x}}{2}$

but $f^{-1}(x) = \frac{1 + \sqrt{1+4\log_2 x}}{2}$

$(\because 1 \leq f^{-1}(x) < \infty : 1 + 4\log_2 x \geq 1 \text{ and } x \geq 1)$
 (Therefore only positive sign is acceptable.)

15. Composite Function

If $f : A \rightarrow B$ and $g : B \rightarrow C$ are two function then the composite function of f and g , $g \circ f : A \rightarrow C$ will be defined as $g \circ f(x) = g[f(x)]$, $\forall x \in A$.

• Properties of Composite Function

- (i) If f and g are two functions then for composite of two functions $f \circ g \neq g \circ f$.
- (ii) Composite functions obeys the property of associatively i.e. $f \circ (g \circ h) = (f \circ g) \circ h$.
- (iii) Composite function of two one-one onto functions if exist, will also be a one-one onto function.

Algebra of function

- (i) $(f \circ g)(x) = f[g(x)]$ (ii) $(f \circ f)(x) = f[f(x)]$
- (iii) $(g \circ g)(x) = g[g(x)]$ (iv) $(fg)(x) = f(x) \cdot g(x)$
- (v) $(f \pm g)(x) = f(x) \pm g(x)$
- (vi) $(f/g)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

- Composite functions in not commutative
- Let f and g are two functions then if f & g are injective or subjective or bijective then " $g \circ f$ " also injective or subjective or bijective.

16. Even and Odd function

• Even Extension:

If a function $f(x)$ is defined on the interval $[0, a]$, $0 \leq x \leq a \Rightarrow -a \leq -x \leq 0$ we define $f(x)$ in the $[-a, 0]$ such that $f(x) = f(-x)$.

Let $I(x) = \begin{cases} f(x) & : x \in [0, a] \\ f(-x) & : x \in [-a, 0] \end{cases}$

• Odd Extension:

If a function $f(x)$ is defined on the interval $[0, a]$, $0 \leq x \leq a \Rightarrow -a \leq -x \leq 0$

$\therefore x \in [-a, 0]$, we define $f(x) = -f(-x)$ Let I be the

odd extension then $I(x) = \begin{cases} f(x), & x \in [0, a] \\ -f(-x), & x \in [-a, 0] \end{cases}$

17. Some very important point

(A) If x, y are independent variables then:

- (i) If $f(x, y) = f(x) + f(y)$ then $\Rightarrow f(x) = k \log x$
- (ii) If $f(x, y) = f(x) \cdot f(y)$ then $\Rightarrow f(x) = x^n$, $n \in \mathbb{R}$
- (iii) If $f(x + y) = f(x) \cdot f(y) \Rightarrow f(x) = a^{kx}$
- (iv) If $f(x + y) = f(x) + f(y) \Rightarrow f(x) = x$
- (v) If $f(x + y) = f(x) = f(y) \Rightarrow f(x) = k$, here k is constant
- (vi) By considering a general n th degree polynomial and writing the expression

$f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) \Rightarrow f(x) = \pm x^n + 1$

(B) Algebraic Functions

(i) Polynomial function:

A function having the form $y = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n$, where $a_0, a_1, a_2, \dots, a_n$ are real constant, $a_n \neq 0$ and $n \in \mathbb{N}$ called rational integral function or polynomial of degree n .

(ii) Rational Function:

The ratio of two polynomial is called Fraction Rational function or simply rational function.

e.g. $y = \frac{x^{12} + x^2 - 1}{x^6 + x^4 + 1}$

(iii) Irrational Function:

Functions with operations of addition, subtraction, multiplication, division and raising to power with non-integral rational exponent are called irrational functions.

(I) $y = \sqrt{x}$ (II) $y = \frac{\sqrt{x^3+1} - \sqrt{x^{11}}}{\sqrt{x^2+x+1}}$

Such type of function are called Irrational function.

(iv) Transcendental function :

All those function who has infinite terms while expanded are called transcendental function. for example all trigonometrical function.

Inverse trigonometrical function, exponential function, logarithmic function etc.

e.g. $f(x) = \sin x, y = \cos^{-1} x$
 $y = \log_e x, y = \sqrt{\log_e x - \sin^{-1} x}$

(C) Mapping :

One-one or injective mapping or homomorphism.

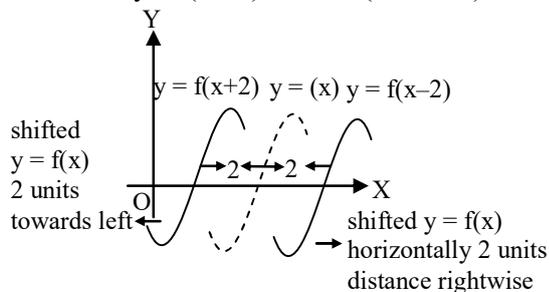
If $f : A \rightarrow B$ is one-one mapping A has m element and B has n element hence the no. of mappings =

$$\begin{cases} n^m, & n \geq m \\ 0, & n < m \end{cases}$$

(D) If graph of $y = f(x)$ be known then to find the graph of

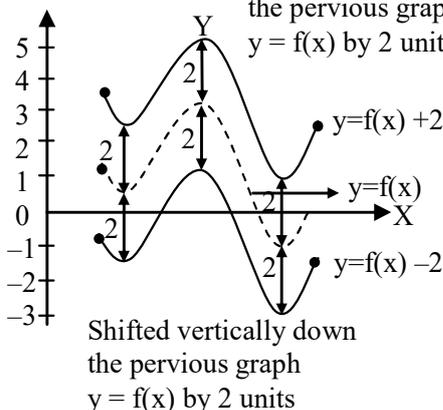
(i) $y = f(x - a)$ or $y = f(x + a)$

To find $y = f(x - a)$ (Let $a = 2$)

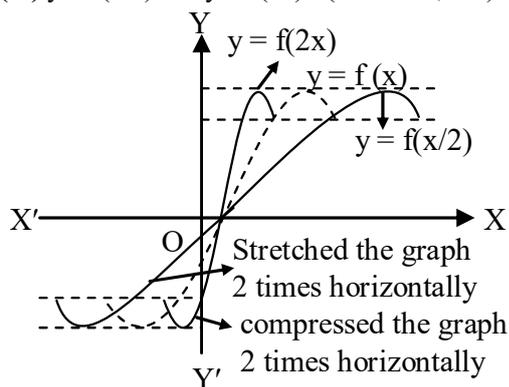


(ii) $y = f(x) + a$: or $y = f(x) - a$ (Let $a = 2$)

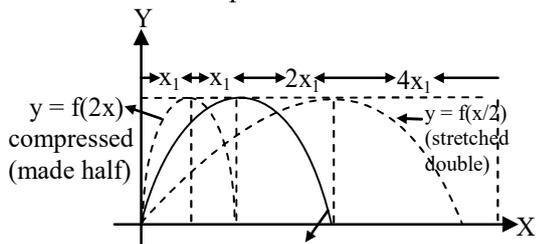
Shifted vertically up the pervious graph $y = f(x)$ by 2 units



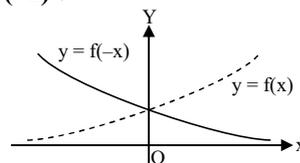
(iii) $y = f(x/a)$ or $y = f(ax)$: (Let $a = 2, 1/2$)



See more examples about the same.



(iv) $y = f(-x)$:

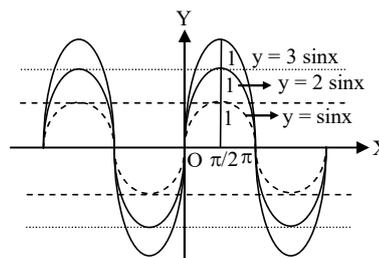


Reflection of $y = f(x)$ w.r.t. axis of y is $y = f(-x)$

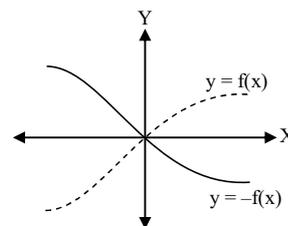
(v) To find $y = k f(x)$:

- **Rule** – Stretch the previous graph k times vertically

e.g. see below $y = 2 \sin x, y = 3 \sin x$

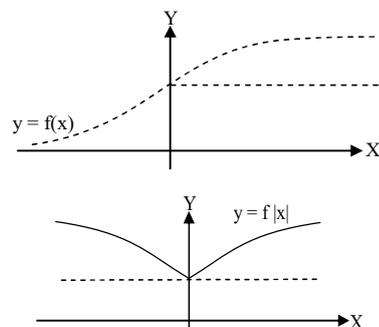


(vi) $y = -f(x)$:



Reflection of $y = f(x)$ w.r.t. axis of x is $y = -f(x)$

(vii) To find $y = f|x|$:

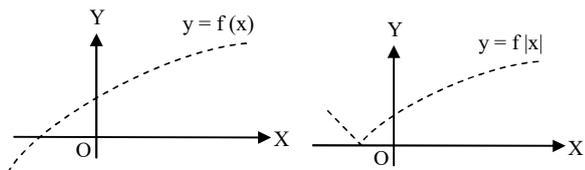


- **RULE:** Neglect the graph lying in IInd and IIIrd quadrant and, Take the image of graph lying in I and IVth quadrant w. r. t. axis of y.

The original graph including its image is called $y = f|x|$.

Here we took the image of the portion lying in first quadrant about axis of y and left the portion which was lying in second quadrant.

(viii) To find $y = |f(x)|$:



❖ **Rule :**

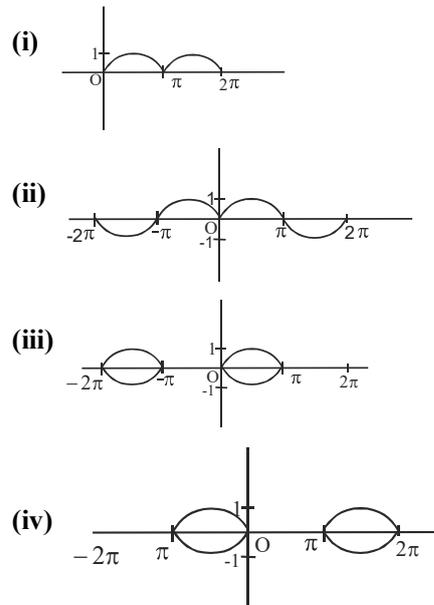
Take the image of the portion line below axis of x about axis of x. Remain as it is the portion above the axis of x.

Examples :- 21

Draw the graphs of the following functions:

- (i) $y = |\sin x|, x \in [0, 2\pi]$
- (ii) $y = \sin |x|, x \in [-2\pi, 2\pi]$
- (iii) $|y| = \sin x, x \in [-2\pi, 2\pi]$
- (iv) $|y| = -\sin x, x \in [-2\pi, 2\pi]$

Solution:



EXERCISE # 1

Based On Domain

- Domain of $y = \log_{10} \left(\frac{5x - x^2}{4} \right)$:
 (A) (0, 5) (B) [1, 4]
 (C) $(-\infty, 0) \cup (5, \infty)$ (D) $(-\infty, 1) \cup (4, \infty)$
- The domain of definition of $f(x) = \frac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{x^2 + 2x + 8}}$ is:
 (A) (1, 4) (B) (-2, 4) (C) (2, 4) (D) [2, ∞)
- The function $f(x) = \cot^{-1} \sqrt{(x+3)x} + \cos^{-1} \sqrt{x^2 + 3x + 1}$ is defined on the set S, where S is equal to:
 (A) {0, 3} (B) (0, 3) (C) {0, -3} (D) [-3, 0]
- The domain of $\sqrt{\sec^{-1} \left(\frac{2-|x|}{4} \right)}$ is
 (A) R (B) $R - (-1, 1)$
 (C) $R - (-3, 3)$ (D) $R - (-6, 6)$
- The domain of the function $f(x) = {}^{24-x}C_{3x-1} + {}^{40-6x}C_{8x-10}$ is -
 (A) {2, 3} (B) {1, 2, 3}
 (C) {1, 2, 3, 4} (D) None of these

Based On Range

- The range of the function $y = \frac{1}{2 - \sin 3x}$ is :
 (A) $\left(\frac{1}{3}, 1 \right)$ (B) $\left[\frac{1}{3}, 1 \right)$ (C) $\left[\frac{1}{3}, 1 \right]$ (D) None
- The value of the function $f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 6}$ lies in the interval -
 (A) $(-\infty, \infty) - \left\{ \frac{1}{5}, 1 \right\}$ (B) $(-\infty, \infty)$
 (C) $(-\infty, \infty) - \{1\}$ (D) None of these
- Find the range of the following function,
 $y = \log_{\sqrt{7}} (\sqrt{2}(\sin x - \cos x) + 5)$
 (A) R (B) Z
 (C) $[\log_7 4, \log_7 5]$ (D) $[2 \log_7 3, 2]$
- Which of the following function (s) has the range [-1, 1]
 (A) $f(x) = \cos(2 \sin x)$
 (B) $g(x) = \cos \left(1 - \frac{1}{1+x^2} \right)$
 (C) $h(x) = \sin(\log_2 x)$
 (D) $k(x) = \cos(e^x)$

Based On Kinds of functions

- Let $f : R \rightarrow R$ be a function defined by $f(x) = \frac{x^2 + 2x + 5}{x^2 + x + 1}$ is :
 (A) One-one and into (B) One-one and onto
 (C) Many-one and onto (D) Many-one and into
- The function $f: [2, \infty) \rightarrow Y$ defined by $f(x) = x^2 - 4x + 5$ is both one-one & onto if:
 (A) $Y = R$ (B) $Y = [1, \infty)$
 (C) $Y = [4, \infty)$ (D) $Y = [5, \infty)$
- Let $f : R \rightarrow R$ be a function defined by $f(x) = x^3 + x^2 + 3x + \sin x$. Then f is:
 (A) One-one & onto (B) One-one & into
 (C) Many one & onto (D) Many one & into
- Which of the following function from $A = \{x: -1 \leq x \leq 1\}$ to itself are bisections-
 (A) $f(x) = x/2$ (B) $g(x) = \sin(\pi x/2)$
 (C) $h(x) = |x|$ (D) $k(x) = x^2$

Based On Inverse function

- If $f(x) = x^3 - 1$ and domain of $f = \{0, 1, 2, 3\}$, then domain of f^{-1} is -
 (A) {0, 1, 2, 3} (B) {1, 0, -7, -26}
 (C) {-1, 0, 7, 26} (D) {0, -1, -2, -3}
- The inverse of the function $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is
 (A) $\frac{1}{2} \log \frac{1+x}{1-x}$ (B) $\frac{1}{2} \log \frac{2+x}{2-x}$
 (C) $\frac{1}{2} \log \frac{1-x}{1+x}$ (D) $2 \log(1+x)$

Based On Composite function

- The function $f(x)$ is defined in $[0, 1]$ then the domain of definition of the function $f[\ln(1-x^2)]$ is given by :
 (A) $x \in \{0\}$
 (B) $x \in [-\sqrt{1+e} - 1] \cup [1 + \sqrt{1+e}]$
 (C) $x \in (-\infty, \infty)$ (D) None

Based On Periodic function

- If $f : R \rightarrow R$ is a function satisfying the property $f(x+1) + f(x+3) = 2 \forall x \in R$ then the period (may not be fundamental period) of $f(x)$ is
 (A) 3 (B) 4 (C) 7 (D) 6
- The fundamental period of the function:
 $f(x) = x + a - [x+b] + \sin \pi x + \cos 2\pi x + \sin 3\pi x + \cos 4\pi x + \dots + \sin(2n-1)\pi x + \cos 2n\pi x$ for every $a, b \in R$ is:
 (where $[.]$ denotes the greatest integer function)
 (A) 2 (B) 4 (C) 1 (D) 0

19. Let $f(x) = \sin \sqrt{[a]} x$ (where $[\]$ denotes the greatest integers function). If f is periodic with fundamental period π , then a belongs to -
 (A) $[2, 3)$ (B) $\{4, 5\}$ (C) $[4, 5]$ (D) $[4, 5)$

Based On Even and odd function

20. Which of the following is an even function?
 (A) $x \frac{a^x - 1}{a^x + 1}$ (B) $\tan x$
 (C) $\frac{a^x - a^{-x}}{2}$ (D) $\frac{a^x + 1}{a^x - 1}$
21. Which of the following function is an odd function
 (A) $f(x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2}$
 (B) $f(x) = x \left(\frac{a^x + 1}{a^x - 1} \right)$
 (C) $f(x) = \log \left(\frac{1-x}{1+x^2} \right)$ (D) $f(x) = k$ (constant)

Based On Miscellaneous

22. The set of points for which $f(x) = \cos(\sin x) > 0$ contains -
 (A) $(-\infty, 0]$ (B) $[-1, 1]$
 (C) $(-\infty, \infty)$ (D) All are correct
23. If $[x]$ stands for the greatest integer function, then the value of $\left[\frac{1}{2} + \frac{1}{1000} \right] + \left[\frac{1}{2} + \frac{2}{1000} \right] + \dots + \left[\frac{1}{2} + \frac{999}{1000} \right]$
 (A) 498 (B) 499 (C) 500 (D) 501
24. Let the function $f(x) = 3x^2 - 4x + 8 \log(1 + |x|)$ be defined on the interval $[0, 1]$. The even extension of $f(x)$ to the interval $[-1, 0]$ is -
 (A) $3x^2 + 4x + 8 \log(1 + |x|)$
 (B) $3x^2 - 4x + 8 \log(1 + |x|)$
 (C) $3x^2 + 4x - 8 \log(1 + |x|)$
 (D) $3x^2 - 4x - 8 \log(1 + |x|)$
25. Let $f: \mathbb{N} \rightarrow \mathbb{N}$ where $f(x) = x + (-1)^{x-1}$ then f is -
 (A) Inverse of itself (B) Even function
 (C) Periodic (D) Identity
26. The function $f(x) = \sqrt{\log_{10} \cos(2\pi x)}$ exists -
 (A) For any rational x
 (B) Only when x is a positive integer
 (C) Only when x is fractional
 (D) For any integer value of x including zero
27. The domain of the function $\sec^{-1}[x^2 - x + 1]$, is given by -
 where $[\cdot]$ is greatest integer function -
 (A) $[0, 1]$ (B) $(-\infty, 0] \cup [1, \infty)$
 (C) $\left[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2} \right]$ (D) None of these

28. The domain of definition of the function $f(x) = \frac{\cot^{-1} x}{\sqrt{\{x^2 - [x^2]\}}}$, where $[x]$ denotes the greatest integer less than or equal to x is -
 (A) \mathbb{R}
 (B) $\mathbb{R} - \{\pm \sqrt{n} : n \in \mathbb{I}^+ \cup \{0\}\}$
 (C) $\mathbb{R} - \{0\}$
 (D) $\mathbb{R} - \{n : n \in \mathbb{I}\}$

29. The domain of the definition of $f(x) = \log\{(\log x)^2 - 5 \log x + 6\}$ is equal to -
 (A) $(0, 10^2)$ (B) $(10^3, \infty)$
 (C) $(10^2, 10^3)$ (D) $(0, 10^2) \cup (10^3, \infty)$

30. If $A = \left\{ x : \frac{\pi}{6} \leq x \leq \frac{\pi}{3} \right\}$ and $f(x) = \cos x - x(1+x)$ then $f(A)$ is equal to -
 (A) $\left[\frac{\pi}{6}, \frac{\pi}{3} \right]$
 (B) $\left[-\frac{\pi}{3}, -\frac{\pi}{6} \right]$

- (C) $\left[\frac{1}{2} - \frac{\pi}{3} \left(1 + \frac{\pi}{3} \right), \frac{\sqrt{3}}{2} - \frac{\pi}{6} \left(1 + \frac{\pi}{6} \right) \right]$
 (D) $\left[\frac{1}{2} + \frac{\pi}{3} \left(1 - \frac{\pi}{3} \right), \frac{\sqrt{3}}{2} + \frac{\pi}{6} \left(1 - \frac{\pi}{6} \right) \right]$

31. If A be the set of all triangles and B that of positive real numbers, then the mapping $f: A \rightarrow B$ given by $f(\Delta) = \text{area of } \Delta, (\Delta \in A)$ is
 (A) One-one into mapping
 (B) One-one onto mapping
 (C) Many-one into mapping
 (D) Many-one onto mapping

32. Let $f: \mathbb{R} \rightarrow A = \left\{ y \mid 0 \leq y < \frac{\pi}{2} \right\}$ be a function such that $f(x) = \tan^{-1}(x^2 + x + k)$, where k is a constant. The value of k for which f is an onto function, is -
 (A) 1 (B) 0
 (C) $1/4$ (D) None of these

33. Which of the following functions are not injective mapping -
 (A) $f(x) = |x + 1|, x \in [-1, \infty)$
 (B) $g(x) = x + \frac{1}{x}; x \in (0, \infty)$
 (C) $h(x) = x^2 + 4x - 5; x \in (0, \infty)$
 (D) $k(x) = e^{-x}; x \in [0, \infty)$

34. Let f be an injective map. with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$, such that exactly one of the following statements is correct and the remaining are false : $f(x) = 1, f(y) \neq 1, f(z) \neq 2$
 The value of $f^{-1}(1)$ is -
 (A) x (B) y (C) z (D) None

35. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be two one-one onto functions such that they are mirror image of each other about the line $y = 0$, then $h(x) = f(x) + g(x)$ is -
 (A) One-one and onto (B) One-one but not onto
 (C) Not one-one but onto (D) Neither one-one nor onto

36. Which of the following functions is inverse of itself -

(A) $f(x) = \frac{1-x}{1+x}$ (B) $g(x) = 5^{\log x}$
 (C) $h(x) = 2^{x(x-1)}$ (D) None of these

37. Period of $f(x) = e^{\cos(x)} + \sin \pi[x]$ is (where, $[.]$ and $\{ \}$ denote the greatest integer function and fractional part of function respectively).

(A) 1 (B) 2 (C) π (D) 2π

38. If $f(x) = \cos(ax) + \sin(bx)$ is periodic, then which of the followings is false -

- (A) a and b both are rational
 (B) Non-periodic if a is rational but b is irrational
 (C) Non-periodic if a is irrational but b is rational
 (D) None of these

39. The function $f(x) = 2(x - [x]) + \sin^2 \pi(x - [x])$ is -
 (Where $[.]$ denotes greatest integer function)

- (A) Non periodic
 (B) Periodic with period 1
 (C) Periodic with period 2
 (D) None of these

40. If $f : [-20, 20] \rightarrow \mathbb{R}$ is defined by $f(x) = \left[\frac{x^2}{a} \right]$

$\sin x + \cos x$, is an even function, then the set of values of a is -

- (A) $(-\infty, 100)$ (B) $(400, \infty)$
 (C) $(-400, 400)$ (D) None of these

41. Let f be a function satisfying $f(x+y) = f(x).f(y)$ for

all $x, y \in \mathbb{R}$. If $f(1) = 3$ then $\sum_{r=1}^n f(r)$ is equal to -

- (A) $\frac{3}{2}(3^n - 1)$ (B) $\frac{3}{2}n(n+1)$
 (C) $3^{n+1} - 3$ (D) None of these

42. If $f(\theta) = \frac{(2 \cos \theta - 1)(2 \cos 2\theta - 1)(2 \cos 4\theta - 1) \dots (2 \cos 2^{n-1}\theta - 1)}{2 \cos 2^n \theta + 1}$

for $n \in \mathbb{N}$ and $\theta \neq 2m\pi \pm \frac{2\pi}{3}$, $m \in \mathbb{I}$,

then $f(\pi/4) =$

- (A) $1 - \sqrt{2}$ (B) $\sqrt{2} - 1$
 (C) $\sqrt{2} + 1$ (D) None of these

43. If $f(x) = [x^2] - [x]^2$ where $[.]$ denotes the greatest integer function and $x \in [0, 2]$, the set of values of $f(x)$ is -

- (A) $\{-1, 0\}$ (B) $\{-1, 0, 1\}$ (C) $\{0\}$ (D) $\{0, 1, 2\}$

44. Domain of $y = \sqrt{-\log_{x+4} \left(\log_2 \frac{2x-1}{3+x} \right)}$

- (A) $(-4, -3) \cup (4, \infty)$ (B) $(-\infty, -3) \cup (4, \infty)$
 (C) $(-\infty, -4) \cup (3, \infty)$ (D) $(-4, -3) \cup (3, 4)$

Based On Domain

45. The domain of definition of

$f(x) = \frac{\log_2(x-3)}{x^2 + 3x + 2}$ is

- (A) $\mathbb{R} - \{-1, -2\}$ (B) $(-2, \infty)$
 (C) $\mathbb{R} - \{-1, -2, -3\}$ (D) $(3, \infty)$

Based On Even and Odd

46. Among the following which is odd function

- (A) $\frac{a^x + a^{-x}}{2}$ (B) $\frac{x}{e^x - 1} + \frac{x}{2}$
 (C) $\sqrt{1+x+x^2} + \sqrt{1-x+x^2}$ (D) $x^3 + \sin x$

47. Let f be a function satisfying $2f(x) - 3f(1/x) = x^2$ for any $x \neq 0$, then the value of $f(2)$ is

- (A) -2 (B) -7/4
 (C) -7/8 (D) 4

48. If $f(x+2y, x-2y) = xy$, then $f(x, y)$ equals

- (A) $\frac{x^2 - y^2}{8}$ (B) $\frac{x^2 - y^2}{4}$
 (C) $\frac{x^2 + y^2}{4}$ (D) $\frac{x^2 - y^2}{2}$

Based On Periodic Function

49. The period of the function

$f(x) = \frac{1}{2} \left(\frac{|\sin x|}{\cos x} + \frac{\sin x}{|\cos x|} \right)$ is

- (A) π (B) 2π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{3}$

50. The function $f(x) = \cos \sqrt{x}$ is

- (A) Periodic with period $2\sqrt{\pi}$
 (B) Periodic with period $\sqrt{2\pi}$
 (C) Periodic with period $4\pi^2$
 (D) Not a periodic function

51. If $f(x) + f(x+a) + f(x+2a) + \dots + f(x+na) = \text{constant}$; $\forall x \in \mathbb{R}$ and $a > 0$ and $f(x)$ is periodic, then period of $f(x)$, is

- (A) $(n+1)a$ (B) $e^{(n+1)a}$
 (C) na (D) e^{na}

52. If $f(x) + f(x+4) = f(x+2) + f(x+6) \forall x \in \mathbb{R}$,

and $f(5) = 10$, then $\sum_{r=1}^{100} f(5+8r)$ equal to

- (A) 1000 (B) 100
 (C) 10000 (D) 10

Based On Composite

53. $f(x) = \log\left(\frac{1+x}{1-x}\right)$, $g(x) = \frac{2x}{1+x^2}$,
 $h(x) = \frac{3x+x^3}{1+3x^2}$, then $f(g(x)) + f(h(x))$
 (A) 0 (B) x (C) $5f(x)$ (D) $3f(x)$
54. If $f(x) = 3x$, $g(x) = \frac{x}{3}$, $h(x) = f(g(x))$ then
 $h(h(\dots n \text{ times}))$ equal to
 (A) x^n (B) x (C) $3^n x$ (D) $2x$

Based On Inverse

55. $f: [2, \infty) \rightarrow (-\infty, 4]$, where $f(x) = x(4-x)$ then
 $f^{-1}(x)$ is
 (A) $2 - \sqrt{4-x}$ (B) $2 + \sqrt{4-x}$
 (C) $-2 + \sqrt{4-x}$ (D) $-2 - \sqrt{4-x}$
56. The value of the parameter α , for which the
 function $f(x) = 1 + \alpha x$, $\alpha \neq 0$ is the inverse of
 itself, is
 (A) -2 (B) -1 (C) 1 (D) 2

Based On General

57. Complete solution set of the inequality
 $x(e^x - 1)(x + 2)(x - 3)^2 \leq 0$ is
 (A) $[-2, 3]$ (B) $(-2, 0]$
 (C) $(-\infty, -2] \cup \{0, 3\}$ (D) $(-\infty, 2) \cup [0, 3]$
58. The entire graphs of the equation
 $y = x^2 + kx - x + 9$ is strictly above the x-axis if
 and only if
 (A) $k < 7$ (B) $-5 < k < 7$ (C) $k > -5$ (D) $-7 < k < 5$

Based On Domain

59. The domain of the function
 $f(x) = \sin^{-1}\left(\log_2 \frac{x^2}{2}\right)$ is given by
 (A) $[-2, -1] \cup [1, 2]$ (B) $[-3, -1] \cup [1, 3]$
 (C) $(-2, -1) \cup (1, 2)$ (D) $[-2, -1) \cup (1, 2]$

Based On Range

60. Range of the function $f(x) = x^2 + \frac{1}{x^2 + 1}$, is
 (A) $[1, \infty)$ (B) $[2, \infty)$ (C) $[3/2, \infty)$ (D) $(-\infty, \infty)$
61. If the function $f: \mathbb{R} \rightarrow A$ given by $f(x) = \frac{x^2}{x^2 + 1}$ is a surjection, then A is
 (A) \mathbb{R} (B) $[0, 1]$ (C) $(0, 1]$ (D) $[0, 1)$
62. If $\theta = \sin^{-1}\left(\ln\left(\frac{x^2 + e}{x^2 + 1}\right)\right)$, then range of θ is
 (A) $(0, \pi/2)$ (B) $[0, \pi/2)$
 (C) $(0, \pi/2]$ (D) $(-\pi/2, \pi/2)$

Based On Periodic

63. Which of the following functions is periodic?
 (A) $f(x) = x - [x]$ where $[x]$ denotes the greatest
 integer less than or equal to the real number x
 (B) $f(x) = \sin \frac{1}{x}$ for $x \neq 0$, $f(0) = 0$
 (C) $f(x) = x \cos x$
 (D) $f(x) = \sin \sqrt{x}$

Based On Inverse

64. Let $f: \mathbb{N} \rightarrow Y$ be a function defined as
 $f(x) = 4x + 3$ where $Y = \{y \in \mathbb{N} : y = 4x + 3 \text{ for}$
 some $x \in \mathbb{N}\}$. its inverse is
 (A) $g(y) = \frac{y+3}{4}$ (B) $g(y) = \frac{y-3}{4}$
 (C) $g(y) = \frac{3y+4}{4}$ (D) $g(y) = \frac{1}{4y+3}$
65. If the function $f: [2, \infty) \rightarrow [-1, \infty)$ is defined
 by $f(x) = x^2 - 4x + 3$. Then its inverse is
 (A) $2 - \sqrt{x+1}$ (B) $2 + \sqrt{x+1}$
 (C) $\frac{2 - \sqrt{x+1}}{2}$ (D) not defined

66. Let $f: [-\pi/3, 2\pi/3] \rightarrow [0, 4]$ be a function defined
 as $f(x) = \sqrt{3} \sin x - \cos x + 2$. Then
 $f^{-1}(x)$ is given by
 (A) $\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{\pi}{6}$ (B) $\sin^{-1}\left(\frac{x-2}{2}\right) + \frac{\pi}{6}$
 (C) $\frac{2\pi}{3} \cos^{-1}\left(\frac{x-2}{2}\right)$ (D) None of these.
67. Let $f(x) = (x+1)^2 - 1, (x \geq -1)$. Then the set
 $S = \{x : f(x) = f^{-1}(x)\}$ is, if f is onto
 (A) $\left\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$
 (B) $\{0, 1, -1\}$ (C) $\{0, -1\}$ (D) Empty

Based On Composite and General

68. If the function $f: [2, \infty) \rightarrow [1, \infty)$ is defined by
 $f(x) = 3^{x(x-2)}$, then $f^{-1}(x)$ is
 (A) $1 + \sqrt{1 + \log_3 x}$ (B) $1 - \sqrt{1 + \log_3 x}$
 (C) $1 + \sqrt{1 - \log_3 x}$ (D) does not exist
69. If $f(x) = \cos(\ell n x)$, then
 $f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$ has the value
 (A) -1 (B) $\frac{1}{2}$ (C) -2 (D) 0

70. Solution set of $\log_{(x^2)} \left(\frac{x}{|x|} - x \right) \geq 0$ is
 (A) $(-\infty, 0) \cup (1, 2)$ (B) $(-\infty, 1) \cup (2, \infty)$
 (C) $(-\infty, -1) \cup (0, 1)$ (D) $(-\infty, -2] \cup (0, 1)$
71. If $f(x) = x^3 + 3x^2 + 12x - 2 \sin x$, where $f : \mathbb{R} \rightarrow \mathbb{R}$, then
 (A) $f(x)$ is many-one and onto
 (B) $f(x)$ is one-one and onto
 (C) $f(x)$ is one-one and into
 (D) $f(x)$ is many-one and into
72. If $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) \forall x \in \mathbb{R} - \{0\}$, where $f(x)$ be a polynomial function and $f(5) = 126$ then $f(3) =$
 (A) 28 (B) 26 (C) 27 (D) 25
73. If $f : [0, \infty) \rightarrow [0, \infty)$ and $f(x) = \frac{x}{1+x}$ then f is
 (A) One - one and into
 (B) Onto but not one - one
 (C) One - one and onto
 (D) Neither one - one nor onto
74. If $f \circ g = |\sin x|$ and $g \circ f = \sin^2 \sqrt{x}$ then $f(x)$ and $g(x)$ are:
 (A) $f(x) = \sqrt{\sin x}$, $g(x) = x^2$
 (B) $f(x) = |x|$, $g(x) = \sin x$
 (C) $f(x) = \sqrt{x}$, $g(x) = \sin^2 x$
 (D) $f(x) = \sin \sqrt{x}$, $g(x) = x^2$
75. If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ in invertible in the Domain
 (A) $\left[0, \frac{\pi}{2}\right]$ (B) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$
 (C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) $[0, \pi]$
76. Let \mathbb{R} be the set of real numbers. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by $f(x) = x^2$ then f is
 (A) Injective but not surjective
 (B) Surjective but not injective
 (C) Injective
 (D) Neither injective nor surjective
77. If $f''(x) = -f(x)$ and $g(x) = f'(x)$ and $F(x) = \left[f\left(\frac{x}{2}\right)^2 + \left(g\left(\frac{x}{2}\right) \right)^2 \right]$ and given that $F(5) = 5$, then $F(10)$ is equal to
 (A) 5 (B) 10 (C) 0 (D) 15
78. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin \sqrt{x})^2$, then
 (A) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$
 (B) $f(x) = \sin x$, $g(x) = |x|$
 (C) $f(x) = x^2$, $g(x) = \sin \sqrt{x}$
 (D) f and g cannot be determined
79. If $\log_3(x^2 - 6x + 11) < 1$, then the exhaustive range of values of x is
 (A) $(-\infty, 2) \cup (4, \infty)$
 (B) $(2, 4)$
 (C) $(-\infty, 1) \cup (1, 3) \cup (4, \infty)$
 (D) $(-\infty, 1) \cup (1, 3)$
80. Given $f(x) = \ln \frac{1+x}{1-x}$ and $g(x) = \frac{3x+x^3}{1+3x^2}$. Then $f(g(x))$ is equal to
 (A) $-f(x)$ (B) $3f(x)$
 (C) $[f(x)]^2$ (D) none of these
81. If $f(x) = \cos^{-1} x$ and $g(x) = x^2$, then range of $f(g(x))$ equal to
 (A) $\left[0, \frac{\pi}{2}\right]$ (B) $[0, \pi]$
 (C) $\left[\frac{\pi}{2}, \pi\right]$ (D) none of these
82. If $f(x) = \ln \left(\frac{x^2 + e}{x^2 + 1} \right)$, then range of $f(x)$ is
 (A) $(0, 1)$ (B) $(0, 1]$
 (C) $[0, 1)$ (D) $\{0, 1\}$
83. If $f(x)$ is a function that is odd and even simultaneously, then $f(3) - f(2)$ is equal to
 (A) 1 (B) -1 (C) 0 (D) 2

EXERCISE # 2

Question One or More Than One Correct Answer Type Questions

- If $f(x) = \sqrt{x^2 - |x|}$, $g(x) = \frac{1}{\sqrt{9 - x^2}}$ then D_{f+g} contains –
 (A) $(-3, -1)$ (B) $[1, 3]$ (C) $[-3, 3]$ (D) $\{0\} \cup [1, 3]$
- If $f(x) = \frac{3x - 1}{3x^3 + 2x^2 - x}$ and $S = \{x | f(x) > 0\}$ then S contains –
 (A) $(-\infty, -2)$ (B) $\left(\frac{1}{3}, 5\right)$
 (C) $(-\infty, -1)$ (D) $(0, \infty) - \left\{\frac{1}{3}\right\}$
- If D is the domain of the function $f(x) = \sqrt{1 - 2x} + 3 \sin^{-1}\left(\frac{3x - 1}{2}\right)$ then D contains-
 (A) $\left[-\frac{1}{3}, \frac{1}{2}\right]$ (B) $\left[-\frac{1}{3}, 0\right]$ (C) $\left[-\frac{1}{3}, 1\right]$ (D) $\left[\frac{1}{2}, 1\right]$
- Let $A = \mathbb{R} - \{2\}$ and $B = \mathbb{R} - \{1\}$. Let $f : A \rightarrow B$ be defined by $f(x) = \frac{x - 3}{x - 2}$ then-
 (A) f is one-one (B) f is onto
 (C) f is injective (D) None of these
- If $F(x) = \frac{\sin \pi[x]}{\{x\}}$, then F(x) is:
 (A) Periodic with fundamental period 1
 (B) Even
 (C) Range is singleton
 (D) Identical to $\text{sign}\left(\text{sgn} \frac{\{x\}}{\sqrt{\{x\}}}\right) - 1$, where $\{x\}$ denotes fractional part function and $[.]$ denotes greatest integer function and $\text{sign}(x)$ is a signum function.
- Let $f: [-1, 1] \rightarrow [0, 2]$ be a linear function which is onto then $f(x)$ is/are.
 (A) $1 - x$ (B) $1 + x$ (C) $x - 1$ (D) $x + 2$
- Function $f(x) = \sin x + \tan x + \text{sgn}(x^2 - 6x + 10)$ is
 (A) Periodic with period 2π
 (B) Periodic with period π
 (C) Non-periodic
 (D) Periodic with period 4π
- In the following functions defined from $[-1, 1]$ to $[-1, 1]$ the functions which are not injective are:
 (A) $\sin(\sin^{-1}x)$ (B) $\frac{2}{\pi} \sin^{-1}(\sin x)$
 (C) $(\text{sgn } x) \ln e^x$ (D) $x^3 \text{sgn } x$

- Let $f(x) = \tan(\log_e x)$, then
 (A) $f\left(\frac{x}{y}\right) = \frac{f(x) - f(y)}{1 + f(x)f(y)}$
 (B) $f(xy) = \frac{f(x) + f(y)}{1 - f(x)f(y)}$
 (C) $f(xyz) = \frac{f(x) + f(y) + f(z) - f(x)f(y)f(z)}{1 - f(x)f(y) - f(y)f(z) - f(z)f(x)}$
 (D) $f(e^{\pi/4}) = 1$
- Let $f(x) = \cos x + \sin x$
 (A) Domain of $f(x)$ is \mathbb{R}
 (B) Range of $f(x)$ is $[-2, 2]$
 (C) Range of $f(x)$ is $[-\sqrt{2}, \sqrt{2}]$
 (D) Period of $f(x)$ is 2π
- Among the following which is / are periodic functions
 (A) $\frac{|x|}{x}$
 (B) $\sin \pi x + \{x\}$ where $\{x\}$ to decimal part of x
 (C) $\cos x + \tan \pi x$
 (D) $\max\left\{\sin x, \frac{1}{2}\right\}$
- Among the following which is / are true
 (A) If f, g are only two functions then $f \circ g = g \circ f$ always
 (B) $f(f(x))$ is always defined, where 'f' is any function
 (C) $f(g(x)) = x \Rightarrow g(x) = f^{-1}(x)$
 (D) None
- If $1 + (x^2 - 1) + (x^2 - 1)^2 + (x^2 - 1)^3 + \dots \infty$ up to terms $= \frac{1}{2 - x^2}$ then x belongs to
 (A) $(0, \sqrt{2})$ (B) $(-\sqrt{2}, \sqrt{2})$
 (C) $(-\sqrt{2}, \sqrt{2}) - \{0\}$ (D) None of these
- The value of $f(x) = 3 \sin x - 4 \cos x + 5$ lies in the interval
 (A) $[0, 10]$ (B) $(0, 10)$
 (C) $[-5, 10]$ (D) none of these
- If $f(x) = \cos^{-1}\left(\frac{2 - |x|}{4}\right) + [\log(3 - x)]^{-1}$, then its domain is
 (A) $[-2, 6]$ (B) $[-6, 2) \cup (2, 3)$
 (C) $[-6, 2]$ (D) $[-2, 2) \cup (2, 3]$
- Let $f: [0, 2] \rightarrow [0, \infty)$ defined as $f(x) = \sqrt{-x^2 + 4}$, then the values of 'x' for which $f(x) = f^{-1}(x)$ is
 (A) $\pm\sqrt{2}$ (B) $\sqrt{2}$ (C) $[0, 2]$ (D) none

17. Let $f(x) = \tan x, g(x) = \cot x$, then
 (A) $f(x) + 2f(2x) + 4f(4x) + 8f(8x) + 16f(16x) = g(x)$
 (B) $f(x) - g(x) = -2g(2x)$
 (C) $f(x) + g(x) = \frac{1+f^2(x)}{f(x)} [f(x) \neq 0]$
 (D) The least value of $|4f(x) + 9g(x)|$ is 12
18. Let $g(x)$ be a function on $[-1, 1]$. If the area of the equilateral triangle with two of its vertices at $(0, 0)$ and $[x, g(x)]$ is $\frac{\sqrt{3}}{4}$, then the function $g(x)$ is
 (A) $g(x) = \pm\sqrt{1-x^2}$ (B) $g(x) = \sqrt{1-x^2}$
 (C) $g(x) = -\sqrt{1-x^2}$ (D) $g(x) = \sqrt{1+x^2}$
19. Let $f(x) = \begin{cases} -x-3 & x < 0 \\ 4 & x = 0 \\ x^2+7 & x > 0 \end{cases}$, then
 (A) $f(-5) = 5$ (B) $f(-5) = -5$
 (C) $f(5) = 32$ (D) $f(-1) + f(A) = 6$
20. If $f(x) = \cos^2 x + \cos^2\left(\frac{\pi}{3} + x\right) - \cos x \cdot \cos\left(\frac{\pi}{3} + x\right)$ then
 (A) $f(x)$ is an even function
 (B) $f\left(\frac{\pi}{8}\right) = f\left(\frac{\pi}{4}\right)$
 (C) $f(x)$ is a constant function
 (D) $f(x)$ is not a periodic function
21. Let $f(x) = \sec^{-1}[1 + \cos^2 x]$ where $[\cdot]$ denotes the greatest integer function. Then
 (A) The domain of f is \mathbb{R}
 (B) The domain of f is $[1, 2]$
 (C) The range of f is $[1, 2]$
 (D) The range of f is $\{\sec^{-1}1, \sec^{-1}2\}$
22. Let $f(x) = \sqrt{9-x^2} + \sqrt{x^2-9}$, then
 (A) domain of $f(x)$ is $\{\pm 3\}$
 (B) $f(x)$ is many one function
 (C) Range of $f(x)$ is $\{0\}$
 (D) 'f' is an even function
23. $f(x) = \sin x, g(x) = x(x^2 - 1), h(x) = (x^2 - 1)x$ then which of the following is/are periodic functions
 (A) $g(f(x))$ (B) $f(g(x))$
 (C) $h(f(x))$ (D) $g(h(x))$
24. Let $f(x) = [x]^2 + [x + 1] - 3$ where $[x] =$ the greatest integer $\leq x$. Where $f: \mathbb{R} \rightarrow \mathbb{R}$. Then
 (A) $f(x)$ is a many-one and into function
 (B) $f(x) = 0$ for infinite number of values of x
 (C) $f(x) = 0$ for only two real values
 (D) None of these
25. Let $f(x) = \cos^2 x + \cos^2\left(\frac{\pi}{3} + x\right) + \cos^2\left(\frac{\pi}{3} - x\right)$, then
 (A) $f(0) = \frac{3}{2}$ (B) $f\left(\frac{\pi}{6}\right) = \frac{3}{2}$
 (C) $f\left(\frac{3\pi}{17}\right) = \frac{3}{2}$ (D) $f(-1) = \frac{3}{2}$
26. If $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x-1}$, then
 (A) $f(g(2)) = 2$ (B) $f(g(3)) = \frac{5}{4}$
 (C) $g(f(4)) = \frac{1}{16}$ (D) $g\left(f\left(\frac{1}{3}\right)\right) = 9$
27. Which of the following statements are true for $|x-1| + |x-2| + |x-3| + \dots + |x-100|$
 (A) Range of the function is $[2500, \infty)$
 (B) The function is symmetrical about $x = 50.5$
 (C) The number of solution of $\max(|x|, |y|) = 2500$ with the curve is 1
 (D) None
28. Which of the following statements are true for the function
 $f(x) = \sqrt{\sin \log\left(\frac{x^2+e}{x^2+1}\right)} + \sqrt{\cos \log\left(\frac{x^2+e}{x^2+1}\right)}$
 (A) Range of the function is $(1, 64^{1/8}]$
 (B) Domain of the function $\in \mathbb{R}$
 (C) Function is even
 (D) Function is invertible
29. Which of the following statements is/are correct
 (A) $f_1(x) = |\sin x| + |\cos x|$ has a period of $\frac{\pi}{2}$ if $-4\pi \leq x \leq 4\pi$
 (B) $f_2(x) = \sqrt{\sin x + \cos x}, x \geq 0$ is a periodic function
 (C) The period of $f_3(x) = \frac{(1+\sin x)(1+\sec x)}{(1+\cos x)(1+\cos \text{ex})}$ is π .
 (D) If derivative of a function is periodic, then the function may be periodic.

Question Passage Based Type Questions

- Passage# 1 (Q.30 to Q.32)**
 Let $f(x) = x^2 - 3x + 2, g(x) = f(|x|)$
 $h(x) = |g(x)|$ and $I(x) = |g(x)| - [x]$
 Are four function, where $[x]$ is the integral part of real x .
30. Find the value of 'a' such that equation $g(x) - a = 0$ has exactly 3 real roots-
 (A) 2 (B) 1
 (C) 0 (D) None of these

31. Find the set of values of 'b' such that equation $h(x) - b = 0$ has exactly 8 real solution
 (A) $b \in [0, 1/4]$ (B) $b \in [0, 1/4)$
 (C) $b \in (0, 1/4)$ (D) None of these
32. Which statement is true for $I(x) = 0$ -
 (A) Two values of x is satisfied for $I(x) = 0$
 (B) One value of x is satisfied for $I(x) = 0$ and that x lie between 1 and 2
 (C) One value of x is satisfied for $I(x) = 0$ and that x lie between 3 and 4
 (D) None of these

Passage # 2 (Q.33 to Q.35)

If $f(x) = 0$; if $x \in Q$
 $= 1$; if $x \notin Q$.

then answer the following questions-

33. $f(x)$ is -
 (A) An even function (B) An odd function
 (C) Neither even nor odd function
 (D) One-one function
34. $f(f(x))$ is-
 (A) A constant function (B) An even function
 (C) An odd function (D) Many one function
35. Domain of $g(x) = \ln(\text{sgn } f(x))$ is-
 (A) R
 (B) set of all rational numbers
 (C) set of all irrational number
 (D) R^+

Passage # 3 (Q.36 to Q.38)

Consider the function

$$f(x) = \begin{cases} x - [x] - \frac{1}{2} & ; \text{ if } x \notin I \\ 0 & ; \text{ if } x \in I \end{cases}$$

where $[.]$ denotes greatest integer function.

If $g(x) = \max. \{x^2, f(x), |x|\}$; $-2 \leq x \leq 2$, then.

36. Range of $f(x)$ is-
 (A) $[0, 1)$ (B) $\left[-\frac{1}{2}, \frac{1}{2}\right)$
 (C) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right)$
37. $f(x)$ is-
 (A) Non periodic
 (B) Periodic with period 1
 (C) Periodic with period 2
 (D) Periodic with period $\frac{1}{2}$
38. The set of values of a, if $g(x) = a$ has three real and distinct solutions, is -
 (A) $(0, 1/2)$ (B) $(0, 1/4)$
 (C) $(1/4, 1/2)$ (D) $(0, 1)$

Passage # 4 (Q.39 to Q.41)

Consider the function

$$f(x) = \begin{cases} x^2 - 1, & -1 \leq x \leq 1 \\ \ln x, & 1 < x \leq e \end{cases}$$

Let $f_1(x) = f(|x|)$

$f_2(x) = |f(|x|)|$ $f_3(x) = f(-x)$

Now answer the following questions.

39. Number of positive solutions of the equation $2f_2(x) - 1 = 0$ is-
 (A) 4 (B) 3 (C) $1/2$ (D) 1
40. Number of integral solution of the equation $f_1(x) = f_2(x)$ is
 (A) 1 (B) 2 (C) 3 (D) 4
41. If $f_4(x) = \log_{27}(f_3(x) + 2)$, then range of $f_4(x)$ is
 (A) $[1, 9]$ (B) $\left[\frac{1}{3}, \infty\right)$ (C) $\left[0, \frac{1}{3}\right]$ (D) $[1, 27]$

Passage # 5 (Q.42 to Q.43)

If notation $[x]$ denotes integer less than or equal to x and $(.)$ denotes integer greater than or equal to x, then

42. The solution set of the equation $(x)^2 + [x]^2 = [x - 1]^2 + (x + 1)^2$ is -
 (A) $\{x; x \in R\}$ (B) $\{x; x \in R - Z\}$
 (C) $\{x; x \in Z\}$ (D) $\{x; x \in \phi\}$
43. Let $f(x) = x + (x)$; $x < 0, 3x - 2(x)$; $x \geq 0$
 Range of $\text{sgn } f(x)$ is -
 (A) $\{-1, 0, 1\}$ (B) $\{-1, 1\}$
 (C) $\{1, 0\}$ (D) $\{-1, 0\}$

Passage # 6 (Q.44 to Q.46)

Let $f(x + y) + f(x - y) = 2f(x) f(y) \forall x, y \in R$ and $f(0) \neq 0$. Then

44. The function $f(x)$ is
 (A) Even (B) Odd
 (C) Both even & odd
 (D) Neither even nor odd
45. The value of $f(2)$, if $f(1) = k$ is
 (A) k (B) $2k^2 + 1$ (C) $2k^2 - 1$ (D) $-k$
46. The value of $f(3) - 2kf(2)$ if $f(1) = k$ is
 (A) k (B) $-k$ (C) $2k$ (D) $-2k$

Passage # 7 (Q.47 to Q.49)

A function $f : A \rightarrow B$ is said to be injective if distinct elements in A have distinct images in B. And surjective if $A = B$. Then answer the following

47. If the function $f: A \rightarrow B$ defined by $f(x) = \sqrt{\frac{1 + \cos 2x}{2}}$ is injective then the set A can be
 (A) $[0, \pi]$ (B) $[-\pi, \pi]$
 (C) $[-\pi/2, 0]$ (D) $[-\pi, 0]$

48. If the function $f: \mathbb{R} \rightarrow \mathbb{B}$ defined by $f(x) = \sqrt{x^2}$ is surjective then the set 'B' can be
 (A) $(-\infty, 0]$ (B) $[0, \infty)$ (C) $(0, \infty)$ (D) \mathbb{R}
49. The function $f: \mathbb{R} \rightarrow \mathbb{B}$ defined by $f(x) = [x] + [-x]$ (where $[.]$ is g.i.f) is surjective then $\mathbb{B} =$
 (A) \mathbb{R} (B) $[0, 1]$ (C) $[-1, 0]$ (D) $\{-1, 0\}$

Passage # 8 (Q.50 to Q.52)

If a function $y = f(x); f: A \rightarrow B$ then the set A is called as domain of the function & B is called co-domain of the function. For all $x \in A$, the values of y thus obtained comprise the set 'C' where C is called as range of function

50. The domain of the function $f(x) = \frac{1}{\ln[\cos^{-1} x]}$ is where $[.]$ indicates greatest integer function
 (A) $[0, 1]$ (B) $[-1, \cos 2]$
 (C) $[-1, \cos 3) \cup (\cos 3, \cos 4)$
 (D) $[-1, \cos 3) \cup (\cos 3, \cos 2)$
51. The domain of the function $f(x) = \sqrt{\cos(\sin x)} + \sqrt{\log_x \{x\}}$ where $\{.\}$ indicates fractional part function
 (A) $[1, \pi)$ (B) $(0, 2\pi) - [1, \pi)$
 (C) $\left(0, \frac{\pi}{2}\right) - \{1\}$ (D) $(0, 1)$

52. The range of the function of $f(x) = \sin^{-1} \sqrt{x^2 + x + 1}$ is
 (A) $\left[0, \frac{\pi}{2}\right]$ (B) $\left[0, \frac{\pi}{3}\right]$ (C) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$ (D) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

Passage # 9 (Q.53 to Q.55)

Mr. X is a teacher of mathematics. His students want to know the ages of his son's S_1 and S_2 . He told that their ages are 'a' and 'b' respectively such that $f(x + y) - axy = f(x) + by^2 \forall x, y \in \mathbb{R}$ after some time students said that information is insufficient, please give more information. Teacher says that $f(1) = 8$ and $f(2) = 32$.

53. The age of S_1 & S_2 will be respectively
 (A) 4, 16 (B) 8, 16
 (C) 16, 8 (D) 32, 8
54. The function $f(x)$ is
 (A) Even (B) Odd
 (C) Neither even nor odd
 (D) Periodic as well as odd
55. The function $f: \mathbb{R} \rightarrow \mathbb{R}$, then function will be
 (A) One one onto (B) One one into
 (C) Many one onto (D) Many one into

7. Match the following with their least periods

Column I

Column II

- (A) $\sin 3x$ (P) $\frac{2\pi}{3}$
- (B) $2x - [2x]$ (where $[.]$ gif) (Q) π
- (C) $\tan x + \tan \frac{x}{3}$ (R) 3π
- (D) $\cos(\sin x)$ (S) $1/2$

8. **Column-I** **Column-II**

- (A) The number of values of 'x' satisfying $e^x \ln x = 1$ is/are (P) 0
- (B) The number of real solution of the equation $x^{2 \log_x(x+3)} = 16$ is (Q) 1
- (C) The number of roots of the equation $x + 2 \tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is (R) 2
- (D) The period of the function $\sin 3\pi t + \sin 4\pi t$ is (S) 3

9. **Column-I** **Column-II**

- (A) The period of the function $\sin \log \left\{ \frac{x}{3} \right\}$ (where $\{x\}$ denotes the fractional part of 'x') is (P) 1
- (B) The total number of solutions of the equation $|\log_e |x|| = |\sin x|$ in $[-\pi/2, \pi/2]$ is (Q) 4
- (C) The least value of the function $f(x) = \sec^2 x + \cos \operatorname{cosec}^2 x$ is (R) 5
- (D) Let $f(x) = |\sin x| + |\cos x|$ then Number of elements in range set Of $[f(x)]$ where $[.]$ is G.I.F is (S) 3
(T) Infinite

Question Numerical Type Questions

- 10. If $f(x) = (100^5 - x^{10})^{1/10}$, then find the value of $\frac{1}{2^{10}} f(f(1024))$.
- 11. If $f(x + f(y)) = f(x) + y \forall x, y \in \mathbb{R}$ and $f(0) = 1$, then find the value of $\frac{f(10)}{5}$. $f(x)$ is not a constant function.
- 12. An odd function is symmetric about the vertical line $x = a$ ($a > 0$) and if $\sum_{r=0}^{\infty} [f(1+4r)]^r = 8$, then find the value of $8f(1)$.

13. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x - f(y)) = f(f(y)) + x f(y) + f(x) - 1 \forall x, y \in \mathbb{R}$. Then find the value of $|(f(16))| - 125$.

14. Let f be a function from the set of positive integers to the set of real number such that (i) $f(1) = 1$

(ii) $\sum_{r=1}^n r f(r) = n(n+1) f(n), \forall n \geq 2$ then find the value of $2126 f(1063)$.

15. Let $f(x)$ be a function such that $f(x-1) + f(x+1) = \sqrt{2} f(x) \forall x \in \mathbb{R}$. If $f(2) = 7$ then find the value of $\sum_{r=0}^{17} [f(2+8r) - 7]$.

16. If $f(x) = x^3 + \frac{1}{x^3} - 4 \left(x^2 + \frac{1}{x^2} \right) + 13$, then find the value of $f(2 + \sqrt{3})$.

17. If $f^3(x) - 3f^2(x) + 3f(x) - 1 = x^6$, then find the value of $f(0)$.

18. If $f(x), g(x)$ are any two real valued functions such that $|f(x) + g(x)| \geq |f(x)| + |g(x)|$ and $g(x) \neq 0, f(x) g(x) \leq 0$ then find the value of $\sum_{r=1}^{100} f(r)$.

19. If $f(x) = \frac{x^4 + x^2 + 1}{x^2 - x + 1}$, then find the value of $f(\omega^n)$ where ' ω ' is the non real root of the equation $z^3 = 1$ and ' n ' is a multiple of '3'.

20. If $f^2(x) f\left(\frac{1-x}{1+x}\right) = x^3 [x \neq -1, 1 \& f(x) \neq 0]$ then find $|(f(-2))|$ (where $[.]$ is the g.i.f).

21. If 'f' is polynomial such that $f\left(\frac{1-x}{1+x}\right) f\left(\frac{1+x}{1-x}\right) = f\left(\frac{1-x}{1+x}\right) + f\left(\frac{1+x}{1-x}\right)$ where $(x \neq 0, \pm 1)$ and $f(3) = 28$, then find the value of $\frac{1}{605} \left(\sum_{n=1}^{10} (f(n) - 1) \right)$.

EXERCISE # 4

Question Previous Year (JEE Main)

- The function $f : \mathbb{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is **[JEE Main-2017]**
 (A) Injective but not subjective
 (B) Subjective but not injective
 (C) Neither injective nor subjective
 (D) Invertible
- For $x \in \mathbb{R} - \{0, 1\}$, let $f_1(x) = 1/x$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2 \circ f_1 \circ f_3)(x) = f_3(x)$ then $J(x)$ is equal to :- **[JEE Main-2019]**
 (A) $f_3(x)$ (B) $f_1(x)$ (C) $f_2(x)$ (D) $(1/x) f_3(x)$
- Let $A = \{x \in \mathbb{R} : x \text{ is not a positive integer}\}$. Define a function $f : A \rightarrow \mathbb{R}$ as $f(x) = \frac{2x}{x-1}$ then f is **[JEE Main-2019]**
 (A) injective but not subjective.
 (B) not injective.
 (C) subjective but not injective.
 (D) neither injective nor subjective
- Let \mathbb{N} be the set of natural numbers and two functions f and g be defined as $f, g : \mathbb{N} \rightarrow \mathbb{N}$ such that : $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ and $g(n) = n - (-1)^n$. The $f \circ g$ is : **[JEE Main-2019]**
 (A) Both one-one and onto.
 (B) One-one but not onto.
 (C) Neither one-one nor onto.
 (D) onto but not one-one.
- If $f(x) = \log_e \left(\frac{1-x}{1+x}\right)$, $|x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to : **[JEE Main-2019]**
 (A) $2f(x)$ (B) $2f(x^2)$
 (C) $(f(x))^2$ (D) $-2f(x)$
- Let $f(x) = a^x$ ($a > 0$) be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function of $f_2(x)$ is an odd function. Then $f_1(x+y) + f_1(x-y)$ equals **[JEE Main-2019]**
 (A) $2f_1(x) f_1(y)$
 (B) $2f_1(x) f_2(y)$
 (C) $2f_1(x+y) f_2(x-y)$
 (D) $2f_1(x+y) f_1(x-y)$
- If the function $f : \mathbb{R} - \{1, -1\} \rightarrow A$ defined by $f(x) = \frac{x^2}{1-x^2}$, is subjective, then A is equal to **[JEE Main-2019]**
 (A) $\mathbb{R} - [-1, 0)$ (B) $\mathbb{R} - (-1, 0)$
 (C) $\mathbb{R} - \{-1\}$ (D) $[0, \infty)$
- The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is - **[JEE Main-2019]**
 (A) $(1, 2) \cup (2, \infty)$
 (B) $(-1, 0) \cup (1, 2) \cup (3, \infty)$
 (C) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
 (D) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$
- Let $f(x) = x^2$, $x \in \mathbb{R}$. For any $A \subseteq \mathbb{R}$, define $g(A) = \{x \in \mathbb{R}, f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true ? **[JEE Main-2019]**
 (A) $f(g(S)) \neq f(S)$ (B) $f(g(S)) = S$
 (C) $g(f(S)) = g(S)$ (D) $g(f(S)) \neq S$
- Let $f(x) = \log_e(\sin x)$, ($0 < x < \pi$) and $g(x) = \sin^{-1}(e^{-x})$, ($x \geq 0$). If α is a positive real number such that $a = (f \circ g)'(\alpha)$ and $b = (f \circ g)(\alpha)$, then : **[JEE Main-2019]**
 (A) $a\alpha^2 - b\alpha - a = 0$ (B) $a\alpha^2 + b\alpha - a = -2\alpha^2$
 (C) $a\alpha^2 + b\alpha + a = 0$ (D) $a\alpha^2 - b\alpha - a = 1$
- For $x \in (0, 3/2)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$. If $\phi(x) = ((h \circ f \circ g)(x))$, then $\phi(\pi/3)$ is equal to : **[JEE Main-2019]**
 (A) $\tan(\pi/12)$ (B) $\tan(7\pi/12)$
 (C) $\tan(11\pi/12)$ (D) $\tan(5\pi/12)$
- For $x \in \mathbb{R}$, let $[x]$ denote the greatest integer $\leq x$, then the sum of the series $\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + \left[-\frac{1}{3} - \frac{2}{100}\right] + \dots + \left[-\frac{1}{3} - \frac{99}{100}\right]$ is - **[JEE Main-2019]**
 (A) -153 (B) -133 (C) -131 (D) -135
- The inverse function of $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$, $x \in (-1, 1)$, is **[JEE Main-2020]**

(A) $\frac{1}{4} \log_8 \left(\frac{1+x}{1-x} \right)$ (B) $\frac{1}{2} \log_8 \left(\frac{1-x}{1+x} \right)$
 (C) $\frac{1}{4} \log_8 \left(\frac{1-x}{1+x} \right)$ (D) $\frac{1}{2} \log_8 \left(\frac{1+x}{1-x} \right)$

14. If $f'(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to :

[JEE Main-2020]

(A) $\frac{\pi-1}{4}$ (B) $\frac{\pi+2}{4}$ (C) $\frac{\pi+1}{4}$ (D) $\frac{1}{4}$

15. If $g(x) = x^2 + x - 1$ and $g \circ f(x) = 4x^2 - 10x + 5$, then find $f(5/4)$.

[JEE Main-2020]

(A) 1/2 (B) -1/2 (C) -1/3 (D) 1/3

16. Let $f(x) = \frac{x[x]}{x^2+1} : (1,3) \rightarrow \mathbb{R}$ then range of $f(x)$ is (where $[.]$ denotes greatest integer function)

[JEE Main-2020]

(A) $\left(0, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{7}{5}\right)$ (B) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right)$
 (C) $\left(\frac{2}{5}, 1\right) \cup \left(1, \frac{4}{5}\right)$ (D) $\left(0, \frac{1}{3}\right) \cup \left(\frac{2}{5}, \frac{4}{5}\right)$

17. Let $[t]$ denote the greatest integer $\leq t$. Then the equation in x , $[x]^2 + 2[x+2] - 7 = 0$ has :

[JEE Main-2020]

- (A) no integral solution
- (B) exactly four integral solutions
- (C) exactly two solutions
- (D) infinitely many solutions

18. If the function $f(x) = \begin{cases} k_1(x-\pi)^2 - 1, & x \leq \pi \\ k_2 \cos x, & x > \pi \end{cases}$

is twice differentiable, then the ordered pair (k_1, k_2) is equal to :

[JEE Main-2020]

(A) (1/2, 1) (B) (1, 1) (C) (1/2, -1) (D) (1, 0)

19. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function which satisfies $f(x+y) = f(x) + f(y) \forall x, y \in \mathbb{R}$. If $f(1) = 2$ and

$$g(n) = \sum_{k=1}^{(n-1)} f(k), n \in \mathbb{N}$$

then the value of n , for which $g(n) = 20$, is :

[JEE Main-2020]

(A) 5 (B) 9 (C) 20 (D) 4

20. If $f(x+y) = f(x)f(y)$ and

$$\sum_{x=1}^{\infty} f(x) = 2, x, y \in \mathbb{N}$$

where \mathbb{N} is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is -

[JEE Main-2020]

(A) 1/9 (B) 4/9 (C) 1/3 (D) 2/3

21. For a suitably chosen real constant a , let a function, $f: \mathbb{R} - \{-a\} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{a-x}{a+x}$. Further suppose that for any real

number $x \neq -a$ and $f(x) \neq -a$, $(f \circ f)(x) = x$. Then $f(-1/2)$ is equal to :

[JEE Main-2020]

(A) 1/3 (B) 3 (C) -3 (D) -1/3

22. The domain of the function

$$f(x) = \sin^{-1} \left(\frac{|x|+5}{x^2+1} \right)$$

is $(-\infty, -a] \cup [a, \infty)$. Then a is equal to :

[JEE Main-2020]

(A) $\frac{1+\sqrt{17}}{2}$ (B) $\frac{\sqrt{17}-1}{2}$
 (C) $\frac{\sqrt{17}}{2} + 1$ (D) $\frac{\sqrt{17}}{2}$

23. Suppose that a function $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$ and

$$f(1) = 3. \text{ If } \sum_{i=1}^n f(i) = 363, \text{ then } n \text{ is equal to}$$

[JEE Main-2020]

24. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of elements in the set $C = \{f: A \rightarrow B \mid 2 \in f(A) \text{ and } f \text{ is not one-one}\}$ is _____.

[JEE Main-2020]

25. The domain of the function $\operatorname{cosec}^{-1} \left(\frac{1+x}{x} \right)$

[JEE Main-2021]

(A) $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$ (B) $\left[-\frac{1}{2}, \infty\right) - \{0\}$
 (C) $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$ (D) $\left(-\frac{1}{2}, \infty\right) - \{0\}$

26. The domain of the function

$$f(x) = \sin^{-1} \left(\frac{3x^2 + x - 1}{(x-1)^2} \right) + \cos^{-1} \left(\frac{x-1}{x+1} \right)$$

is - [JEE Main-2021]

(A) $[0, 1/4]$ (B) $[0, 1/2]$
 (C) $[1/4, 1/2] \cup \{0\}$ (D) $[-2, 0] \cup [1/4, 1/2]$

27. Let $f: \mathbb{N} \rightarrow \mathbb{N}$ be a function such that $f(m+n) = f(m) + f(n)$ for every $m, n \in \mathbb{N}$. If $f(6) = 18$, then $f(2), f(3)$ is equal to

[JEE Main-2021]

(A) 54 (B) 18 (C) 6 (D) 36

28. The range of the function

$$f(x) = \log_{\sqrt{5}} \left(3 + \cos \left(\frac{3\pi}{4} + x \right) \right) + \cos \left(\frac{\pi}{4} + x \right) + \cos \left(\frac{\pi}{4} - x \right) - \cos \left(\frac{3\pi}{4} - x \right)$$

is - [JEE Main-2021]

(A) $[0, 2]$ (B) $[-2, 2]$
 (C) $(0, \sqrt{5})$ (D) $[1/\sqrt{5}, \sqrt{5}]$

29. Let $f(x) = 2 + |x| - |x - 1| + |x + 1|$, $x \in \mathbb{R}$.
Consider **[JEE Main-2022]**
(S1) : $f'(-\frac{3}{2}) + f'(-\frac{1}{2}) + f'(\frac{1}{2}) + f'(\frac{3}{2}) = 2$

(S2) : $\int_{-2}^2 f(x) dx = 12$

Then,

- (A) Both (S1) and (S2) are correct
- (B) Both (S1) and (S2) are wrong
- (C) Only (S1) is correct
- (D) Only (S2) is correct

30. Let $f, g : \mathbb{N} - \{1\} \rightarrow \mathbb{N}$ be functions defined by $f(a) = \alpha$, where α is the maximum of the powers of those primes p such that p^α divides a , and $g(a) = a + 1$, for all $a \in \mathbb{N} - \{1\}$. Then, the function $f + g$ is **[JEE Main-2022]**

- (A) one-one but not onto
- (B) onto but not one-one
- (C) both one-one and onto
- (D) neither one-one nor onto

31. Let a function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as :

$$f(x) = \begin{cases} \int_0^x (5 - |t - 3|) dt, & x > 4 \\ x^2 + bx, & x \leq 4 \end{cases} \text{ [JEE Main-2022]}$$

Where $b \in \mathbb{R}$, If f is continuous at $x = 4$ then which of the following statements is NOT true ?

- (A) f is not differentiable at $x = 4$
- (B) $f'(4) + f'(5) = \frac{35}{4}$
- (C) f is increasing $(-\infty, \frac{1}{8}) \cup (8, \infty)$
- (D) f has a local minima at $x = \frac{1}{8}$

32. Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in \mathbb{Z} : |f(n)| \leq 800\}$. Then, the value of $\sum_{n \in S} f(n)$ is equal to _____. **[JEE Main-2022]**

33. Let a function $f : \mathbb{N} \rightarrow \mathbb{N}$ be defined by

$$f(n) = \begin{cases} 2n, & n = 2, 4, 6, 8, \dots \\ n - 1, & n = 3, 7, 11, 15, \dots \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, \dots \end{cases}$$

then, f is **[JEE Main-2022]**

- (A) One-one but not onto
- (B) Onto but not one-one
- (C) Neither one-one nor onto
- (D) One-one and onto

34. Let $f(x) = ax^2 + bx + c$ be such that $f(A) = 3$, $f(-2) = \lambda$ and $f(C) = 4$. **[JEE Main-2022]**
If $f(0) + f(A) + f(-2) + f(C) = 14$, then λ is equal to
(A) -4 (B) 13/2 (C) 23/2 (D) 4

35. Let $f(x)$ and $g(x)$ be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$ and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is _____. **[JEE Main-2022]**

36. The sum of absolute maximum and minimum values of the function $f(x) = |x^2 - 5x + 6| - 3x + 2$ in the interval $[-1, 3]$ is equal to :

[JEE Main-2023]

- (A) 24 (B) 13 (C) 12 (D) 10

37. Let $f : \mathbb{R} - \{0, 1\} \rightarrow \mathbb{R}$ be a function such that $f(x) + f(\frac{1}{1-x}) = 1 + x$. then $f(2)$ is equal to :

[JEE Main-2023]

- (A) $\frac{9}{2}$ (B) $\frac{9}{4}$ (C) $\frac{7}{3}$ (D) $\frac{7}{4}$

38. If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\leq x$, is $[2, 6)$, then its range is : **[JEE Main-2023]**

- (A) $(\frac{5}{37}, \frac{2}{5}]$
- (B) $(\frac{5}{37}, \frac{2}{5}] - \{ \frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53} \}$
- (C) $(\frac{5}{26}, \frac{2}{5}]$
- (D) $(\frac{5}{26}, \frac{2}{5}] - \{ \frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53} \}$

39. If $f(x) = \frac{2^{2x}}{2^{2x} + 2}$, $x \in \mathbb{R}$, then $f(\frac{1}{2023}) + f(\frac{2}{2023}) + \dots + f(\frac{2022}{2023})$ is equal to **[JEE Main-2023]**
(A) 1010 (B) 2011 (C) 1011 (D) 2010

40. Let $f(x)$ be a function such that $f(x+y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{N}$. If $f(1) = 3$ and $\sum_{k=1}^n f(k) = 3279$, then the value of n is **[JEE Main-2023]**
(A) 8 (B) 9 (C) 6 (D) 7

41. The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has **[JEE Main-2023]**
(A) Exactly two solutions in $(-\infty, \infty)$
(B) No solution
(C) A unique solution in $(-\infty, 1)$
(D) A unique solution in $(-\infty, \infty)$

42. Let $S = \{1, 2, 3, 4, 5, 6\}$. Then the number of one-one functions $f : S \rightarrow P(S)$, where $P(S)$ denote the power set of S , such that $f(n) \subset f(m)$ where $n < m$ is _____. **[JEE Main-2023]**

43. Let $x = (8\sqrt{3} + 13)^{13}$ and $y = (7\sqrt{2} + 9)^9$. If $[t]$ denotes the greatest integer $\leq t$, then

[JEE Main-2023]

- (A) $[x]$ is odd but $[y]$ is even
- (B) $[x] + [y]$ is even
- (C) $[x]$ and $[y]$ are both odd
- (D) $[x]$ is even but $[y]$ is odd

44. The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is:

[JEE Main-2023]

- (A) $[\sqrt{5}, \sqrt{10}]$
- (B) $[\sqrt{2}, \sqrt{7}]$
- (C) $[\sqrt{5}, \sqrt{13}]$
- (D) $[2\sqrt{2}, \sqrt{11}]$

45. Let $A = \{1, 2, 3, 5, 8, 9\}$. Then the number of possible functions $f: A \rightarrow A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to _____.

[JEE Main-2023]

46. The absolute minimum value, of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where $[t]$ denotes the greatest integer function, in the interval $[-1, 2]$, is:

[JEE Main-2023]

- (A) $\frac{3}{4}$
- (B) $\frac{3}{2}$
- (C) $\frac{1}{4}$
- (D) $\frac{5}{4}$

47. Let $f: \mathbf{R} \rightarrow \mathbf{R}$ and $g: \mathbf{R} \rightarrow \mathbf{R}$ be defined as $f(x) = \begin{cases} \log_e x, & x > 0 \\ e^{-x}, & x \leq 0 \end{cases}$ and $g(x) = \begin{cases} x, & x \geq 0 \\ e^x, & x < 0 \end{cases}$. Then,

$\text{gof}: \mathbf{R} \rightarrow \mathbf{R}$ is:

[JEE Main-2024]

- (A) Neither one-one nor onto
- (B) Onto but not one-one
- (C) Both one-one and onto
- (D) One-one but not onto

48. Let the range of the function $f(x) = \frac{1}{2 + \sin 3x + \cos 3x}$, $x \in \mathbf{IR}$ be $[a, b]$. If α and β are respectively the A.M. and the G.M. of a and

b , then $\frac{\alpha}{\beta}$ is equal to:

[JEE Main-2024]

- (A) $\sqrt{2}$
- (B) 2
- (C) $\sqrt{\pi}$
- (D) π

49. Let $A = \{(x, y) : 2x + 3y = 23, x, y \in \mathbf{N}\}$ and $B = \{x : (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to _____.

[JEE Main-2024]

50. Let $f: \mathbf{R} - \left\{\frac{-1}{2}\right\} \rightarrow \mathbf{R}$ and $g: \mathbf{R} - \left\{\frac{-5}{2}\right\} \rightarrow \mathbf{R}$

be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$.

Then, the domain of the function fog is:

[JEE Main-2024]

(A) \mathbf{R}

(B) $\mathbf{R} - \left\{\frac{-5}{2}\right\}$

(C) $\mathbf{R} - \left\{\frac{-7}{4}\right\}$

(D) $\mathbf{R} - \left\{\frac{-5}{2}, \frac{-7}{4}\right\}$

51. The function $f(x) = \frac{x^2 + 2x - 15}{x^2 - 4x + 9}$, $x \in \mathbf{R}$ is

[JEE Main-2024]

- (A) both one-one and onto.
- (B) onto but not one-one
- (C) neither one-one nor onto
- (D) one-one but not onto.

52. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 4, 9, 16\}$. Then the number of many-one functions $f: A \rightarrow B$ such that $1 \in f(A)$ is equal to:

[JEE Main-2025]

- (A) 127
- (B) 151
- (C) 163
- (D) 139

53. Let $f(x) = \log_e x$ and $g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$.

Then the domain of fog is-

[JEE Main-2025]

- (A) \mathbf{R}
- (B) $(0, \infty)$
- (C) $[0, \infty)$
- (D) $[1, \infty)$

54. If the domain of the function $f(x) = \log_7(1 - \log_4(x^2 - 9x + 18))$ is $(\alpha, \beta) \cup (\gamma, \delta)$, then $\alpha + \beta + \gamma + \delta$ is equal to

[JEE Main-2025]

- (A) 17
- (B) 18
- (C) 15
- (D) 16

55. If the range of the function $f(x) = \frac{5-x}{x^2-3x+2}$, $x \neq 1, 2$, is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^2$ is equal to:

[JEE Main-2025]

- (A) 194
- (B) 192
- (C) 188
- (D) 190

Question Previous Year (JEE Advanced)

56. Let $f(x) = \sin\left(\frac{\pi}{6} \sin\left(\frac{\pi}{2} \sin x\right)\right)$ for all $x \in \mathbf{R}$

and $g(x) = \frac{\pi}{2} \sin x$ for all $x \in \mathbf{R}$, Let $(\text{fog})(x)$

denote $f(g(x))$ and $(\text{gof})(x)$ denote $g(f(x))$. Then which of the following is (are) true?

[JEE Advanced-2015]

(A) Range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(B) Range of fog is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$

(D) There is an $x \in \mathbf{R}$ such that $(\text{gof})(x) = 1$

57. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto function form Y to X , then the value of $\frac{1}{5!}(\beta - \alpha)$ is _____.

[JEE Advanced-2018]

58. Let $E_1 = \left\{ x \in \mathbb{R} : x \neq 1 \text{ and } \frac{x}{x-1} > 0 \right\}$ and

$E_2 = \left\{ x \in E_1 : \sin^{-1} \left(\log_e \left(\frac{x}{x-1} \right) \right) \text{ is a real number} \right\}$

(Here, the inverse trigonometric function $\sin^{-1}x$ assumes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$.)

Let $f: E_1 \rightarrow \mathbb{R}$ be the function defined by

$f(x) = \log_e \left(\frac{x}{x-1} \right)$ and $g: E_2 \rightarrow \mathbb{R}$ be the

function defined by $g(x) = \sin^{-1} \left(\log_e \left(\frac{x}{x-1} \right) \right)$

COLUMN - I		COLUMN - II	
P.	The range of f is	1.	$\left(-\infty, \frac{1}{1-e} \right] \cup \left[\frac{e}{e-1}, \infty \right)$
Q.	The range of g contains	2.	$(0, 1)$
R.	The domain of f contains	3.	$\left[-\frac{1}{2}, \frac{1}{2} \right]$
S.	The domain of g is	4.	$(-\infty, 0) \cup (1, \infty)$
		5.	$\left(-\infty, \frac{e}{e-1} \right]$
		6.	$(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1} \right]$

The correct options is :

[JEE Advanced-2018]

- (A) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 1$
- (B) $P \rightarrow 3; Q \rightarrow 3; R \rightarrow 6; S \rightarrow 5$
- (C) $P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 6$
- (D) $P \rightarrow 4; Q \rightarrow 3; R \rightarrow 6; S \rightarrow 5$

59. Let $S = (0,1) \cup (1,2) \cup (3,4)$ and $T = \{0,1, 2,3\}$. Then which of the following statements is(are) true?

[JEE Advanced-2023]

- (A) There are infinitely many functions from S to T
- (B) There are infinitely many strictly increasing functions from S to T
- (C) The number of continuous functions from S to T is at most 120
- (D) Every continuous function from S to T is differentiable

60. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be functions defined by

$$f(x) = \begin{cases} x |x| \sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases} \text{ and}$$

$$g(x) = \begin{cases} 1 - 2x, & 0 \leq x \leq \frac{1}{2}, \\ 0, & \text{otherwise,} \end{cases}$$

Let $a, b, c, d \in \mathbb{R}$. Define the function $h: \mathbb{R} \rightarrow \mathbb{R}$ by

$$h(x) = af(x) + b \left(g(x) + g\left(\frac{1}{2} - x\right) \right) + c(x - g(x))$$

+ $d g(x), x \in \mathbb{R}$,

Match each entry in **List-I** to the correct entry in **List-II**. [JEE Advanced-2024]

List-I		List-II	
(P)	If $a = 0, b = 1, c = 0,$ and $d = 0,$ then	(1)	h is one-one
(Q)	If $a = 1, b = 0, c = 0,$ and $d = 0,$ then	(2)	h is onto
(R)	If $a = 0, b = 0, c = 1,$ and $d = 0,$ then	(3)	h is differentiable on \mathbb{R}
(S)	If $a = 0, b = 0, c = 0,$ and $d = 1,$ then	(4)	the range of h is $[0, 1]$
		(5)	the range of h is $\{0, 1\}$

The correct option is

- (A) $(P) \rightarrow (4), (Q) \rightarrow (3), (R) \rightarrow (1), (S) \rightarrow (2)$
- (B) $(P) \rightarrow (5), (Q) \rightarrow (2), (R) \rightarrow (4), (S) \rightarrow (3)$
- (C) $(P) \rightarrow (5), (Q) \rightarrow (3), (R) \rightarrow (2), (S) \rightarrow (4)$
- (D) $(P) \rightarrow (4), (Q) \rightarrow (2), (R) \rightarrow (1), (S) \rightarrow (3)$

61. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0 \end{cases}$$

Then which of the following statement is TRUE?

[JEE Advanced-2024]

- (A) $f(x) = 0$ has infinitely many solutions in the interval $\left[\frac{1}{10^{10}}, \infty \right)$.
- (B) $f(x) = 0$ has no solutions in the interval $\left[\frac{1}{\pi}, \infty \right)$.
- (C) The set of solutions of $f(x) = 0$ in the interval $\left(0, \frac{1}{10^{10}} \right)$ is finite.
- (D) $f(x) = 0$ has more than 25 solutions in the interval $\left(\frac{1}{\pi^2}, \frac{1}{\pi} \right)$.

62. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$, and $g: \mathbb{R} \rightarrow (0, \infty)$ be a function such that $g(x + y) = g(x)g(y)$ for all $x, y \in \mathbb{R}$. If $f\left(\frac{-3}{5}\right) = 12$ and $g\left(\frac{-1}{3}\right) = 2$, then the value of $\left(f\left(\frac{1}{4}\right) + g(-2) - 8 \right) g(0)$ is _____.

[JEE Advanced-2024]

63. Let the function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by
 $f(x) = \frac{\sin x (x^{2023} + 2024x + 2025)}{e^{\pi x} (x^2 - x + 3)} + \frac{2 (x^{2023} + 2024x + 2025)}{e^{\pi x} (x^2 - x + 3)}$
 Then the number of solutions of $f(x) = 0$ in \mathbb{R} is _____.
[JEE Advanced-2024]

64. Let \mathbb{R} denote the set of all real numbers. Let $a_i, b_i \in \mathbb{R}$ for $i \in \{1, 2, 3\}$.
 Define the functions $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$, and $h : \mathbb{R} \rightarrow \mathbb{R}$ by
 $f(x) = a_1 + 10x + a_2x^2 + a_3x^3 + x^4$,
 $g(x) = b_1 + 3x + b_2x^2 + b_3x^3 + x^4$,
 $h(x) = f(x + 1) - g(x + 2)$.
 If $f(x) \neq g(x)$ for every $x \in \mathbb{R}$, then the coefficient of x^3 in $h(x)$ is
[JEE Advanced-2025]
 (A) 8 (B) 2 (C) -4 (D) -6

65. Let \mathbb{N} denote the set of all natural numbers, and \mathbb{Z} denote the set of all integers. Consider the functions
 $f : \mathbb{N} \rightarrow \mathbb{Z}$ and $g : \mathbb{Z} \rightarrow \mathbb{N}$ defined by
 $f(n) = \begin{cases} (n + 1)/2 & \text{if } n \text{ is odd,} \\ (4 - n)/2 & \text{if } n \text{ is even,} \end{cases}$
 and
 $g(n) = \begin{cases} 3 + 2n & \text{if } n \geq 0, \\ -2n & \text{if } n < 0, \end{cases}$
 Define $(g \circ f)(n) = g(f(n))$ for all $n \in \mathbb{N}$, and $(f \circ g)(n) = f(g(n))$ for all $n \in \mathbb{Z}$
 Then which of the following statements is (are) TRUE?
[JEE Advanced-2025]
 (A) $g \circ f$ is **NOT** one-one and $g \circ f$ is **NOT** onto
 (B) $f \circ g$ is **NOT** one-one but $f \circ g$ is onto
 (C) g is one-one and g is onto
 (D) f is **NOT** one-one but f is onto

66. Let \mathbb{R} denote the set of all real numbers. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) > 0$ for all $x \in \mathbb{R}$, and $f(x + y) = f(x) f(y)$ for all $x, y \in \mathbb{R}$.
 Let the real numbers a_1, a_2, \dots, a_{50} be in an arithmetic progression. If $f(a_{31}) = 64f(a_{25})$, and

$$\sum_{i=1}^{50} f(a_i) = 3(2^{25} + 1),$$

then the value of

$$\sum_{i=6}^{30} f(a_i)$$

is _____.

[JEE Advanced-2025]

ANSWER KEY

EXERCISE-1

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	D	C	D	A	C	A	D	C,D	D	B	A	B	C	A	A	B	A	C	A
Qus.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	A	D	C	A	A	D	B	B	D	C	D	C	B	B	D	A	A	D	B	B
Qus.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	A	B	D	A	D	D	B	A	B	D	A	A	C	B	B	B	C	B	A	A
Qus.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	D	C	A	B	B	C	C	A	D	D	B	A	A	C	B	D	A	A	B	B
Qus.	81	82	83																	
Ans.	A	B	C																	

EXERCISE-2

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A,B,D	A,B,C,D	A,B	A,B,C	A,B,C,D	A,B	A	B,C,D	A,B,C,D	A,C,D	B,D	D	C	A,C	B
Qus.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B,C	A,B,C,D	A	C,D	A,B,C	A,D	A,B,C,D	A,C	A,B	A,B,C,D	A,B,C,D	A,B	A,B,C	C,D	A
Qus.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	B	C	A	A,B,D	C	C	B	C	C	D	C	B	A	A	C
Qus.	46	47	48	49	50	51	52	53	54	55					
Ans.	B	C	B	D	B	D	C	C	A	D					

EXERCISE-3

[MATCH THE COLUMN]

- [A] → [R], [B] → [P], [C] → [S], [D] → [Q]
- [A] → [Q], [B] → [S], [C] → [P], [D] → [R]
- [A] → [S], [B] → [S], [C] → [S], [D] → [R]
- [A] → [R], [B] → [P], [C] → [Q], [D] → [S]
- [A] → [S], [B] → [R], [C] → [P], [D] → [Q]
- [A] → [Q], [B] → [R]
- [A] → [P], [B] → [S], [C] → [R], [D] → [Q]
- [A] → [Q], [B] → [P], [C] → [S], [D] → [R]
- [A] → [S], [B] → [Q], [C] → [Q], [D] → [P]

[NUMERICAL]

- 10.[1] 11.[2] 12.[7] 13.[2] 14.[2] 15.[0] 16.[9] 17.[1] 18.[0] 19.[3] 20.[2] 21.[5]

EXERCISE-4

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	A	A	D	A	A	A	C	C	D	C	B	A	C	B	B	D	A	A	B
Qus.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	B	A	5	19	B	C	A	A	D	D	C	10620	D	D	18	A	B	A	C	D
Qus.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	D	3240	B	A	432	A	A	A	24	B	Bonus/ 3(NTA)	B	A	B	A	A,B,C	119	A	A,C,D	C
Qus.	61	62	63	64	65	66														
Ans.	D	51	1	C	A,D	96.00														