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Ans 1 (a) $S = \{(a_1, a_2, a_3, a_4) ; a_1 + a_2 + a_3 + a_4 = 0\}$

$T = \{(a_1, a_2, a_3, a_4) ; a_1 - a_2 + a_3 - a_4 = 0\}$.

129

Calculation Mistakes could be avoided

Let $W = S \cap T = \{(a_1, a_2, a_3, a_4) ; \begin{array}{l} a_1 + a_2 + a_3 + a_4 = 0 \\ a_1 - a_2 + a_3 - a_4 = 0 \end{array}\}$.

Now $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Basis should be set of 4 elements

$R_2 \to R_2 - R_1 \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (5)

↓

Set which generates S & T

Hence dimension of $W = 4 - 2 = 2$

and basis $= \{(1, 0), (1, -2)\}^T$

Ans 1 (c) $f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$.

Let $\varepsilon > 0$, then $|f(x, y) - f(0, 0)| = \left| \dfrac{xy}{\sqrt{x^2+y^2}} - 0 \right|$

Let $x \to r\cos\theta$
$y \to r\sin\theta$

then $|f(x, y) - f(0, 0)| = |r\sin\theta\cos\theta| \leq \left|\dfrac{r}{2}\right| |\sin 2\theta| < \dfrac{r}{2}$

$\therefore$ If $|f(x, y) - f(0, 0)| < \varepsilon$ ie $\dfrac{r}{2} < \varepsilon \Rightarrow \sqrt{x^2+y^2} < 2\varepsilon$

we have $\delta = 2\varepsilon^2$ s.t $|x| < 2\varepsilon^2$ and $|y| < 2\varepsilon^2$

Hence $f(x, y)$ is continuous at $(0, 0)$

Now $f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = 0$

(2)

Similarly $f_y(0,0) = \lim_{k\to 0} \frac{f(0,k) - f(0,0)}{k} = 0$

Hence partial derivatives also exist at $(0,0)$

Differentiability at $(0,0)$

Now if it is differentiable at $(0,0)$ following will hold true

$f(h,k) - f(0,0) = h f_x(0,0) + k f_y(0,0) + h\phi + k\psi$

where ϕ and ψ are functions which tend to 0

if $x, y \to 0$

Now $\frac{hk}{\sqrt{h^2+k^2}} = h\phi + k\psi$

if $h \to r\cos\theta$, $k \to r\sin\theta$, we have

$r\sin\theta\cos\theta = r\cos\theta\,\phi + r\sin\theta\,\psi$

8

$\sin\theta\cos\theta = \cos\theta\,\phi + \sin\theta\,\psi$

Now ϕ and $\psi \to 0$ as $r \to 0$

$\therefore \sin\theta\cos\theta = 0$ which is not true for all θ,

hence $f(x,y)$ is not differentiable at $(0,0)$

Ans 1(d) $\lim_{x\to 0} \frac{x(1+a\cos 2x)+b\sin 2x}{x^3} = 1$ (3)

LHS is $\frac{0}{0}$ hence applying L'hospital rule

$\Rightarrow \lim_{x\to 0} \frac{(1+a\cos 2x) - 2ax\sin 2x + 2b\cos 2x}{3x^2}$

It will be $\frac{0}{0}$ form only if $1+a+2b=0$ — (1)

Again applying L hospital rule we get

$\Rightarrow \lim_{x\to 0} \frac{1 - 2a\sin 2x - 2a\sin 2x - 4ax\cos 2x - 2^2 b\sin 2x}{6x}$

~~It will be $\frac{0}{0}$ only if~~ It is $\frac{0}{0}$ form

hence again applying same rule

$\Rightarrow \lim_{x\to 0} \frac{-8a\cos 2x - 4a\cos 2x + 8ax\sin 2x - 8b\cos 2x}{6}$

7

$= \frac{-8a-4a-8b}{6} = 1$

$\Rightarrow 12a+8b+6=0$ — (2)

from (1) and (2) we have

$12(-2b-1)+8b+6=0$

$-24b-12+8b+6=0 \Rightarrow -18b=6$ $\boxed{\Rightarrow b = \frac{-1}{3}}$

$-16b=6 \quad b = \frac{-6}{16} = \frac{-3}{8}$

and $\boxed{a = \frac{-1}{3}}$ Ans

Ans 1(e) Equation of plane containing $\frac{y}{b}+\frac{z}{c}=1, x=0$ is (4)

$$\frac{y}{b}+\frac{z}{c}-1+\lambda x=0 \quad -(1)$$

Symmetric form of line $\frac{x}{a}-\frac{z}{c}=1, y=0$ is given by

$$\frac{x}{a}=\frac{y}{0}=\frac{z+c}{c} \quad -(2) = r_1 \text{ (say)}$$

If (2) is parallel to (1) then

$$\lambda a+\frac{c}{c}=0 \Rightarrow \lambda=-\frac{1}{a}$$

hence given plane is $\boxed{\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0}$ Ans.

Now 2d is given shortest distance b/w lines

Symmetrical form of line $\frac{y}{b}+\frac{z}{c}=1, x=0$ is

$$\frac{x}{0}=\frac{y}{b}=\frac{z-c}{-c} \quad -(3) = r_2 \text{ (say)}$$

Any point on line (2) is $(ar_1, 0, cr_1-c)$

Any point on line (3) is $(0, br_2, -cr_2+c)$

$\therefore$ Directional ratio of SD is $(ar_1, -br_2, c(r_1+r_2)-2c)$

If it is $\perp$ ~~parallel~~ to (2) then

$$a^2r_1+c^2(r_1+r_2)-2c^2=0 \quad -(4)$$

Similarly if it ~~parallel~~ $\perp$ to (3) then

$$-b^2r_2-c^2(r_1+r_2)+2c^2=0 \quad -(5)$$

Ans. 2(a) $u_1 = (1,-2,0,3)$ $u_2 = (2,3,0,-1)$, $u_3 = (2,-1,2,1)$ (8)

Now $v = (3,9,-4,-2)$

Let $(3,9,-4,-2) = a(1,-2,0,3) + b(2,3,0,-1) + c(3,9,-4,-2)$

$$3 = a+2b+3c$$
$$9 = -2a+3b+9c$$
$$-4 = -4c$$
$$-2 = 3a-b-2c$$

$\Rightarrow$ $c=1$, hence equations become

$$a+2b=0$$
$$-2a+3b=0$$
$$3a-b=0$$

which is not true except for $a=b=0$

Ans 2(a)(i) $u_1 = (1,-2,0,3)$, $u_2 = (2,3,0,-1)$, $u_3 = (2,-1,2,1)$

Now let $v = (3,9,-4,-2) = a(1,-2,0,3) + b(2,3,0,-1) + c(2,-1,2,1)$

$\Rightarrow$ $3 = a+2b+2c$

$$9 = -2a+3b-c$$
$$-4 = 2c$$
$$-2 = 3a-b+c$$

$\Rightarrow$ $\boxed{c=-2}$ hence equations become

$\Rightarrow$ $a+2b=7$

$2a-3b=-7$ $\Rightarrow$ $a=1,\ b=3$

$3a-b=0$

hence v is linear combination of given vectors

Ans 2(a)(ii) Let $(a,b,c) = p(2,1,0) + q(1,-1,2) + r(0,3,-4)$

$$2p + q = a$$
$$p - q + 3r = b$$
$$2q - 4r = c$$

$$\cancel{\Rightarrow \frac{a-q}{2} - q + 3\left(\frac{2q-c}{4}\right) = b \Rightarrow 2a - 2q - 4q + 6q - 3c = 4b}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} b \\ a \\ c \end{bmatrix}$$

$R_2 \to R_2 - 2R_1 \Rightarrow$
$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & 3 & -6 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} b \\ a-2b \\ c \end{bmatrix}$$

$R_3 \to \frac{R_3}{2}$, $R_2 \to \frac{R_2}{3}$, $R_3 \to R_3 - R_2$ we get

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} b \\ \frac{a-2b}{3} \\ \frac{c}{2} - \frac{a}{3} + \frac{2b}{3} \end{bmatrix}$$

Now given system will be consistent only if

$$\frac{c}{2} - \frac{a}{3} + \frac{2b}{3} = 0 \Rightarrow \cancel{\boxed{3a - 2a + 4b = 0}}$$ Ans.

$3c - 2a + 4b = 0$

10

Ans 2(b) Both matrices will have same column space if their transpose have rowspace, hence

No need to transpose Directly do column operation

$$A^T = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 1 \\ 5 & 3 & 9 \end{bmatrix}, \quad B^T = \begin{bmatrix} 1 & -2 & 7 \\ 2 & -3 & 12 \\ 3 & -4 & 17 \end{bmatrix}$$

(b)

A^T.

$R_2 \to R_2 - 3R_1$
$R_3 \to R_3 - 5R_1$ $\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & -2 & 4 \end{bmatrix}$

$R_3 \to R_3 + 2R_2 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ — (1)

B^T

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$ $\Rightarrow \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 2 & -4 \end{bmatrix}$

$R_3 \to R_3 - 2R_2 \Rightarrow \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ — (2)

Using (1), and applying $R_1 \to R_1 - R_2$ we get

$\Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ — (3)

Using (2) and apply $R_1 \to R_1 + 2R_2$ we get

6

$\Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ — (4)

$\boxed{\text{Since (3) and (4) are same, hence given matrices have same column space}}$

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Ans 2(c)

$$f(x,y,z) = x - 2y + 5z \quad -(1)$$

Let $\quad g(x,y,z) = x^2 + y^2 + z^2 - 30 \quad (2)$

Differentiating both equation partially, multiplying (2) then by λ and adding corresponding partial elements coeffs

$\Rightarrow \quad 1 + 2\lambda x = 0 \quad \Rightarrow \quad x = -1/2\lambda$

$-2 + 2\lambda y = 0 \quad \Rightarrow \quad y \Rightarrow 2/2\lambda$

$5 + 2\lambda z = 0 \quad \Rightarrow \quad z = -5/2\lambda$

Using in (2) we get $\dfrac{1+4+25}{4\lambda^2} = 30 \Rightarrow \lambda = \pm\dfrac{1}{2}$

hence two possible points are $(-1, 2, -5)$ and $(1, -2, 5)$

Now At $(-1, 2, -5)$ $\quad \boxed{f = \cancel{11} \cdot -1 - 4 - 25 = -30}$

At $(1, -2, 5)$ $\quad \boxed{f = 1 + 4 + 25 = 30}$

Verification. Take y as free variable s.t z is only dependent on y.

Diff (1) wrt x $\Rightarrow \dfrac{\partial f}{\partial x} = 1 + 5\dfrac{\partial z}{\partial x} \quad -(3)$

Diff (2) wrt x $\Rightarrow \quad 2x + 2z\dfrac{\partial z}{\partial x} = 0 \Rightarrow \dfrac{\partial z}{\partial x} = -\dfrac{x}{z}$

13

hence $\dfrac{\partial f}{\partial x} = 1 - \dfrac{5x}{z}$, Again diff wrt x

$\Rightarrow \dfrac{\partial^2 f}{\partial x^2} = -\dfrac{5}{z} + \dfrac{5x}{z^2}\dfrac{\partial z}{\partial x} = -\dfrac{5}{z} - \dfrac{5x^2}{z^3}$

Now $\dfrac{\partial^2 f}{\partial x^2}$ at $(-1, 2, -5) > 0$, hence -30 is minimum at $(-1, 2, -5)$ and $\dfrac{\partial^2 f}{\partial x^2}$ at $(1, -2, 5) < 0$ hence 30 is max at $(1, -2, 5)$

Ans 3(a)

$T(x,y,z) = (x+y+z,\ x+2y-3z,\ 2x+3y-2z,\ 3x+4y-z)$

Kernal is given by $x+y+z=0$, $2x+3y-2z=0$

$x+2y-3z=0$, $3x+4y-z=0$

Take $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 1 & 2 & -3 \\ 3 & 4 & -1 \end{bmatrix}$

$R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - R_1,\ R_4 \to R_4 - 3R_1 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -4 \\ 0 & 1 & -4 \\ 0 & 1 & -4 \end{bmatrix}$

$R_3 \to R_3 - R_2,\ R_4 \to R_4 - R_2 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Hence Nullity $= \boxed{3-2=1}$ and Basis for kernal is given by

$x+y+z=0$

$y-4z=0$ $\Rightarrow x=-5z,\ y=4z,\ z$

ie $\boxed{\text{Kernal} = \{(-5,4,1)\}}$

Now Rank + Nullity = Dim R^3

ie Rank + 1 = 3 $\Rightarrow$ $\boxed{\text{Rank}=2}$

~~Rank~~ Range

$T(x,y,z) = x(1,1,2,3) + y(1,2,3,4) + z(1,-3,-2,-1)$

12

Take $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & -3 & -2 & -1 \end{bmatrix}$, $\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix} \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & -4 & -4 & -4 \end{bmatrix}$

$R_3 \to R_3 + 4R_2 \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

hence $\boxed{\text{Range} = \{(1,1,2,3),\ (0,1,1,1)\}}$

Ans. 3(b)(i) Let A be non singular, hence A^{-1} exists. s.t (9)

$AA^{-1} = I \Rightarrow |AA^{-1}| = 1 \Rightarrow |A||A^{-1}| = 1$

hence $\boxed{|A^{-1}| = |A|^{-1}}$ hence proved.

Ans 3(b)(ii) Take $C = B^{-1}AB$

$\Rightarrow |C| = |B^{-1}AB| = |B^{-1}||A||B| = |B^{-1}||B||A|$

$= |B^{-1}B||A| = |A|$

5

hence $\boxed{B^{-1}AB \text{ and } A \text{ have same determinant}}$

Ans 3(E) $f(x,y) = \begin{cases} (x^2+y^2)\log(x^2+y^2) & (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

Now $f_{xy}(0,0) = \lim_{k \to 0} \dfrac{f_x(0,k) - f_x(0,0)}{k}$

where $f_x(0,k) = \lim_{h \to 0} \dfrac{f(h,k) - f(0,k)}{h} = \dfrac{(h^2+k^2)\log(h^2+k^2) - k^2\log k^2}{h}$

$= \lim_{h \to 0} h\log(h^2+k^2) + \dfrac{k^2 \log\frac{(h^2+k^2)}{k^2}}{h}$ $\left(\frac{0}{0}\right)$ form

$= \lim_{h \to 0} k^2 \cdot \left[\dfrac{2h}{h^2+k^2}\right] = 0$

Similarly $f_x(0,0) = \lim_{h \to 0} \dfrac{f(h,0) - f(0,0)}{h} = \dfrac{h^2 \log h^2}{h} = 0$

hence $\boxed{f_{xy}(0,0) = 0 \text{ and similarly } f_{yx}(0,0) = 0}$

ie $\boxed{f_{xy}(0,0) = f_{yx}(0,0) = 0}$

Differentiating given fn w.r.t. x we get

$$f_x = 2x \log(x^2+y^2) + \frac{(x^2+y^2)}{x^2+y^2} \cdot 2x$$

$$= 2x\left(1+\log(x^2+y^2)\right) \quad -(1)$$

Now diff (1) w.r.t. y

$$f_{xy} = \frac{2x}{x^2+y^2} \cdot 2y = \frac{4xy}{x^2+y^2}$$

Let $y \to mx$ s.t $y \to 0$, $x \to 0$

hence $f_{xy} = \frac{4 \cdot x \cdot mx}{x^2+m^2x^2} = \frac{4m}{1+m^2}$

which is different for different paths, hence

$f_{xy}(0,0) \neq 0$ for all paths, hence f_{xy} is not continuous at (0,0)

Similarly f_{yx} is not continuous at (0,0)

12

Ans 3(d) Locus of line of intersection of perpendicular tangent planes to $ax^2+by^2+cz^2=0$

Ans 5(a) $16(x+1)^4 y_4 + 96(x+1)^3 y_3 + 104(x+1)^2 y_2 + 8(x+1) y_1 + y = x^2+4x+3$ (11)

Let $x+1 = e^z \Rightarrow \log(x+1) = z \Rightarrow \frac{1}{x+1}\cdot\frac{dx}{dy} = \frac{dz}{dy}$

$\Rightarrow D_1 \equiv \frac{\partial}{\partial z} = (x+1)\frac{\partial}{\partial x}$

hence equation becomes -

$(16 D_1(D_1-1)(D_1-2)(D_1-3) + 96 D_1(D_1-1)(D_1-2) + 104 D_1(D_1-1) + 8D_1 + 1) = e^z(e^z+2) = e^{2z} + 2e^z$

Complementary Solution

$16(D_1^2-D_1)(D_1^2-5D_1+6) + 96D_1(D_1^2-3D_1+2) + 104D_1^2 - 104D_1 + 8D_1 + 1 = 0$

$\Rightarrow (4D_1^2-1)^2 = 0 \Rightarrow D_1 = \pm\frac{1}{2}$ twice

hence Complementary function is $y = (c_1 + zc_2)e^{\frac{1}{2}z} + (c_3 + zc_4)e^{-z/2}$

Particular Integral

$\Rightarrow \frac{e^{2z} + 2e^z}{(4D_1^2-1)^2} = \frac{e^{2z}}{(4\cdot4-1)^2} + \frac{2e^z}{(4-1)^2}$

$= \frac{e^{2z}}{225} + \frac{2e^z}{9}$ 8

hence $\boxed{\text{solution is } y = (c_1 + \log(x+1)c_2)\log\sqrt{x+1} + (c_3 + \log(x+1)c_4)\log\frac{1}{\sqrt{x+1}} + \cancel{\frac{\log(x+1)^2}{225}} + \cancel{2} + \frac{(x+1)^2}{225} + \frac{2(x+1)}{9}}$

Ans

Ans 5(b) $r = \dfrac{2a}{1+\cos\theta} \Rightarrow \log r = \log 2a - \log(1+\cos\theta)$

Diff wrt $\theta \Rightarrow \dfrac{1}{r}\dfrac{dr}{d\theta} = \dfrac{\sin\theta}{1+\cos\theta} \Rightarrow \dfrac{1}{r}\dfrac{dr}{d\theta} = \dfrac{2\sin\theta/2\cos\theta/2}{2\cos^2\theta/2}$

replacing $\dfrac{d\theta}{dr} \rightarrow \dfrac{-1}{r^2}\dfrac{dr}{d\theta}$, hence eqn. become

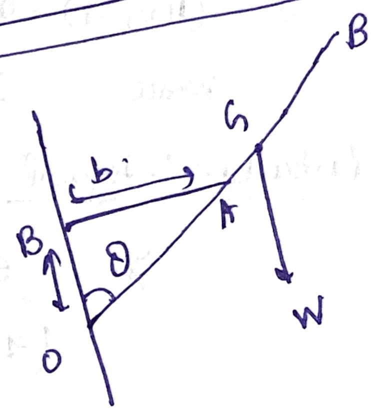
$\dfrac{1}{r}\left(-r^2\dfrac{d\theta}{dr}\right) = \tan\theta/2 \Rightarrow \dfrac{d\theta}{\tan\theta/2} + \dfrac{dr}{r} = 0$

$\Rightarrow 2\log\sin\theta/2 + \log r = \log c$ where c is constant

$\sin^2\theta/2 \cdot r = c \Rightarrow \boxed{r = c\,\mathrm{cosec}^2\theta/2}$ Ans.

8

Ans. 5(c) Let OB be given rod of length $2a$ with centre at G. Let AB be peg s.t $AB = b$



Now z be distance of G from A

Then $z = a\cos\theta - b\cot\theta$

$\Rightarrow \dfrac{\partial z}{\partial\theta} = -a\sin\theta + b\,\mathrm{cosec}^2\theta$

for position $\dfrac{\partial z}{\partial\theta} = 0 \Rightarrow \boxed{\sin^3\theta = b/a}$

8

Now $\dfrac{\partial^2 z}{\partial\theta^2} = -a\cos\theta - 2b\,\mathrm{cosec}^2\theta\cot\theta < 0$

Hence System is in equlibrium stable.

Ans 5(e) $f(x,y) = \oint_C (4x-2y)\,dx + (2x-4y)\,dy$

13

Now According to green Theorem

$$\oint_C (4x-2y)\,dx + (2x-4y)\,dy = \iint_R (2+2)\,dx\,dy$$

$$= \iint_R 4\,dx\,dy = 4 \cdot 4\pi = \boxed{16\pi} \text{ Ans.}$$

8

Ans 6(a) $x^2 + y^2 + 2gx + c = 0$ —(1)

diff wrt x $\quad 2x + 2y\frac{dy}{dx} + 2g = 0 \Rightarrow g = -(x + yy')$

using in (1) we get $\quad x^2+y^2 - 2x(x+yy') + c = 0$

Replacing y' by $-\frac{1}{y'}$ we get

$x^2+y^2 - 2x\left(x - \frac{y}{y'}\right) + c = 0 \Rightarrow y^2 - x^2 + \frac{2xy}{y'} + c = 0$

$\cancel{y^2 - x^2 + y^2 + \frac{2xy}{y'} + c = 0} \Rightarrow dy\,\underbrace{(y^2-x^2+c)}_{N} + \underbrace{2xy}_{M}\,dx = 0$

$\frac{dy}{dx}$ ~~Now $\frac{1}{N}\left(\frac{\partial M}{\partial y}\right.$~~ $\Rightarrow e^{\int \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)}$ is integrating fact

i.e $\quad e^{\int \frac{1}{2xy}(-2x-2x)} = e^{\int -\frac{2}{y}dy} = \frac{1}{y^2}$ is integrating.

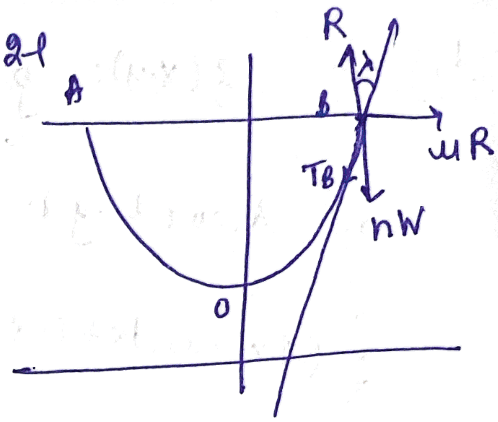
hence exact eq. is $\quad dy\left(1 - \frac{x^2}{y^2} + \frac{c}{y^2}\right) + dx \cdot \frac{2x}{y} = 0$

13

Solution is $\quad \boxed{\frac{x^2}{y} + y - \frac{c}{y} = c_1}$

Ans 6(b)

Let AOB be heavy chain of length $2l$



Then $\tan\lambda = \frac{R}{\mu R} = \frac{1}{\mu}$

Now let Tension at B be T_B

$T_B \cos\lambda + nW = R$

$T_B \sin\lambda = \mu R$

Also $T_B \sin\lambda = WC$ and $T_B \cos\lambda = \frac{W}{2l} \cdot y_B$

hence $\frac{W y_B}{2l} + nW = R$

$WC = \mu R$

Ans 6(c)

Case I

Let actual velocity of wind be $x\hat{i} + y\hat{j}$

when person moving eastwards ; his velocity is $4\hat{i}$

Resultant is blowing from north , hence $x = -4$

$\therefore$ wind velocity $= -4\hat{i} + y\hat{j}$

Case II

when person moving twice velocity ie his velocity is $8\hat{i}$

Resultant is $4\hat{i} + y\hat{j}$ which is north east , hence

$y = -4$

$\therefore$ $\boxed{\text{wind velocity} = -4\hat{i} - 4\hat{j}}$

Ans-6(c)ii

$\phi = xy^2 + yz^3$

$\nabla\phi = y^2\hat{i} + (2xy + z^3)\hat{j} + 3yz^2\hat{k}$ (1)

Normal to surface $x\log z - y^2 = -4$ is

$\left.\log z\,\hat{i} - 2y\hat{j} + \frac{x}{z}\hat{k}\right|_{(-1,2,1)} = 0\hat{i} - 4\hat{j} - \hat{k}$ (2)

Component of (1) in direction of (2) is given by.

$$\left.\frac{-4(2xy + z^3) - 3yz^2}{\sqrt{17}}\right|_{(2,-1,1)} = \frac{-4(2\cdot 2\cdot -1 + 1) - 3\cdot(-1)\cdot 1}{\sqrt{17}}$$

$$= \frac{-4(-3) + 3}{\sqrt{17}} = \boxed{\frac{15}{\sqrt{17}}}$$

6

Ans1 (b) (ii)

(15)

Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$

$\text{adj}A = \begin{bmatrix} 6 & -6 & +8 \\ 0 & 3 & -4 \\ 0 & 0 & 2 \end{bmatrix}$ and $|A| = 8$ 6

Covert to Z_5

hence Inverse of $A = \frac{1}{8}\begin{bmatrix} 6 & -6 & 8 \\ 0 & 3 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

Inverse of A in $Z_5 = \frac{1}{8}\begin{bmatrix} 1 & -1 & 3 \\ 0 & 3 & -4 \\ 0 & 0 & 2 \end{bmatrix}$ Ans.