

# **Centrifugal Compressor Surge**

## **Modeling and Identification for Control**

This research was financially supported by TNO Science & Industry and Siemens Power Generation and Industrial Applications.

A catalogue record is available from the Eindhoven University of Technology Library

Helvoirt, Jan van

Centrifugal Compressor Surge, Modeling and Identification for Control / by Jan van Helvoirt. – Eindhoven : Technische Universiteit Eindhoven, 2007

Proefschrift. – ISBN-13: 978-90-386-1095-5

NUR 978

Trefwoorden: centrifugaal compressor / surge / modelleren / identificatie / regelen

Subject headings: centrifugal compressor / surge / modeling / identification / control

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# **Centrifugal Compressor Surge Modeling and Identification for Control**

## **PROEFSCHRIFT**

ter verkrijging van de graad van doctor aan de  
Technische Universiteit Eindhoven, op gezag van de  
Rector Magnificus, prof.dr.ir. C.J. van Duijn, voor een  
commissie aangewezen door het College voor  
Promoties in het openbaar te verdedigen  
op woensdag 12 september 2007 om 16.00 uur

door

Jan van Helvoirt

geboren te Utrecht

Dit proefschrift is goedgekeurd door de promotor:

prof.dr.ir. M. Steinbuch

Copromotor:

dr.ir. A.G. de Jager

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## SUMMARY

# Centrifugal Compressor Surge

## Modeling and Identification for Control

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Surge is an unstable operating mode of a compression system that occurs at mass flows below the so-called surge line. The instability is characterized by large limit cycle oscillations in compressor flow and pressure rise that reduce compressor performance. The large thermal and mechanical loads involved can also endanger safe operation of the compression system.

The concept of stabilizing a compression system to the left of the surge line by modifying its dynamics through the use of appropriate feedback has lead to many promising results. However, a real breakthrough in the practical application of this approach has not been made yet. Therefore, the goals of the research presented in this thesis are to determine the critical barriers for the industrial application of active surge control and to investigate how these can be removed. In order to answer these questions, we focused our research on the modeling and identification of the relevant compressor dynamics and subsequently on the design, realization and testing of an active surge control system.

We described the dynamic behavior of two centrifugal compression systems with a low-order model similar to the Greitzer lumped parameter model. To account for the effect of piping acoustics on surge transients in one of the test rigs, we extended the Greitzer model with a transmission line model. Based on experimental validation results we conclude that the developed models describe surge transients of industrial scale centrifugal compression systems with reasonable accuracy under various operating conditions.

The dynamic models were subsequently used to investigate the dynamics of industrial compression systems that are relevant for control design. This analysis confirmed the importance of the so-called stability parameter, motivating the development of two approaches to identify this parameter in the compression systems under study. The first method is based on the geometric approximation of the hydraulic inductance of a centrifugal compressor. The second method uses approximate realization theory to identify

the dominant compressor dynamics from step response data. By combining the results of both methods we conclude that identification can provide more reliable estimates for the stability parameter in comparison with the commonly used parameter tuning approach.

Next, we used the compression system model to design an active surge control system, consisting of a pressure sensor and bleed valve at the compressor discharge pipe and a Linear Quadratic Gaussian controller. Simulations with the nonlinear compression system model showed that stabilization at 95% of surge mass flow is possible at low compressor speeds. Through closed-loop simulations we then determined the required capacity and bandwidth, as well as the allowable time delay for the control valve.

A new high-speed control valve was developed in order to meet the actuator specifications. Experimental tests confirmed the excellent performance of the valve prototype in terms of actuation speed. Given the potential of the electro-mechanical control valve, actuator limitations are no longer considered to be a critical barrier for the application of active surge control.

Finally, the developed surge control system was implemented on one of the compressor test rigs under study. During closed-loop experiments the control system proved to be inadequate for surge suppression in the compression system. The most likely causes for failure are the lack of robustness with respect to model uncertainties, noise and process variations and the small domain of attraction of the implemented controller. Based on the insights gained throughout our research, we propose various approaches to take away these barriers for the successful demonstration of active surge control on an industrial scale centrifugal compression system. Most importantly, future work should focus on resolving the remaining uncertainty in the estimate of the stability parameter and the model-based design of a robust control strategy.

# Nomenclature

---

In the following definitions, a Cartesian coordinate system  $(x, y, z)$  with unit vector base  $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$  applies and following the Einstein summation convention, repeated indices are summed from 1 to 3.

## Roman uppercase

| Symbol | Description                  | Dimensions <sup>1</sup> | Unit         |
|--------|------------------------------|-------------------------|--------------|
| $A$    | area                         | $L^2$                   | $m^2$        |
|        | system matrix                |                         |              |
| $A(s)$ | transfer function            |                         |              |
| $B$    | Greitzer stability parameter | –                       | –            |
|        | general boundary             |                         |              |
|        | input matrix                 |                         |              |
| $B(s)$ | transfer function            |                         |              |
| $C$    | output matrix                |                         |              |
| $D$    | diameter                     | $L$                     | $m$          |
|        | direct feed-through matrix   |                         |              |
| $F$    | lumped model parameter       | –                       | –            |
|        | general field                |                         |              |
| $F(s)$ | frequency dependent friction |                         |              |
| $G$    | lumped model parameter       | –                       | –            |
|        | Markov parameter             |                         |              |
| $G(s)$ | transfer function            |                         |              |
| $H$    | compressor curve semi height |                         |              |
|        | Hankel matrix                |                         |              |
| $H(s)$ | transfer function            |                         |              |
| $I$    | identity matrix              |                         |              |
| $J$    | cost function                |                         |              |
| $K$    | valve flow coefficient       | $L^3 T^{-1}$            | $m^3 h^{-1}$ |
|        | feedback gain                |                         |              |

| Symbol<br>(continued) | Description                     | Dimensions                                     | Unit                               |
|-----------------------|---------------------------------|------------------------------------------------|------------------------------------|
| $K(s)$                | transfer function               |                                                |                                    |
| $L$                   | length                          | L                                              | m                                  |
| $M$                   | Mach number $u/c$               | –                                              | –                                  |
|                       | molar weight                    | $\Psi$                                         | kg mol <sup>-1</sup>               |
| $N$                   | rotor speed                     | T <sup>-1</sup>                                | rpm                                |
| $P_{xx}$              | auto power spectrum             |                                                |                                    |
| $Q$                   | matrix weight                   |                                                |                                    |
| $R$                   | gas constant <sup>2</sup>       | L <sup>2</sup> T <sup>-2</sup> Θ <sup>-1</sup> | J kg <sup>-1</sup> K <sup>-1</sup> |
|                       | radius                          | L                                              | m                                  |
|                       | resistance                      | ML <sup>-3</sup> T <sup>-1</sup>               | kg m <sup>-3</sup> s <sup>-1</sup> |
|                       | scalar weight                   |                                                |                                    |
| $Re$                  | Reynolds number $UL/\nu$        | –                                              | –                                  |
| $S$                   | stress                          | ML <sup>-1</sup> T <sup>-2</sup>               | N m <sup>-2</sup>                  |
|                       | step response                   |                                                |                                    |
| $T$                   | temperature                     | Θ                                              | K                                  |
| $U$                   | velocity                        | LT <sup>-1</sup>                               | m s <sup>-1</sup>                  |
| $V$                   | volume                          | L <sup>3</sup>                                 | m <sup>3</sup>                     |
|                       | noise covariance matrix         |                                                |                                    |
| $W$                   | compressor curve semi width     | –                                              | –                                  |
|                       | process noise covariance matrix |                                                |                                    |
| $Y$                   | expansion factor                | –                                              | –                                  |
| $Z$                   | compressibility factor          | –                                              | –                                  |
| $Z_0$                 | acoustic impedance              | ML <sup>-4</sup> T <sup>-1</sup>               | kg m <sup>-4</sup> s <sup>-1</sup> |

## Roman lowercase

| Symbol | Description                        | Dimensions                                     | Unit                               |
|--------|------------------------------------|------------------------------------------------|------------------------------------|
| $a$    | polynomial coefficient             |                                                |                                    |
| $b$    | polynomial coefficient             |                                                |                                    |
| $c$    | speed of sound                     | LT <sup>-1</sup>                               | m s <sup>-1</sup>                  |
| $c_p$  | specific heat at constant pressure | L <sup>2</sup> T <sup>-2</sup> Θ <sup>-1</sup> | J kg <sup>-1</sup> K <sup>-1</sup> |
| $c_v$  | specific heat at constant volume   | L <sup>2</sup> T <sup>-2</sup> Θ <sup>-1</sup> | J kg <sup>-1</sup> K <sup>-1</sup> |
| $d$    | diameter                           | L                                              | m                                  |
| $f$    | frequency                          | T <sup>-1</sup>                                | Hz                                 |
| $g$    | acceleration due to gravity        | LT <sup>-2</sup>                               | m s <sup>-2</sup>                  |
| $h$    | height                             | L                                              | m                                  |

| Symbol<br>(continued) | Description                        | Dimensions             | Unit                 |
|-----------------------|------------------------------------|------------------------|----------------------|
| $l$                   | path length                        | L                      | m                    |
| $m$                   | mass                               | M                      | kg                   |
| $\dot{m}$             | mass flow                          | $MT^{-1}$              | $kg\ s^{-1}$         |
| $n$                   | unit normal                        |                        |                      |
|                       | number of data points              |                        |                      |
| $p$                   | pressure                           | $ML^{-1}T^{-2}$        | Pa                   |
| $q$                   | volume flow                        | $L^3T^{-1}$            | $m^3\ s^{-1}$        |
| $r$                   | radius                             | L                      | m                    |
|                       | reference variable                 |                        |                      |
| $s$                   | Laplace variable ( $s = j\omega$ ) | $T^{-1}$               | $rad\ s^{-1}$        |
|                       | entropy <sup>3</sup>               | $L^2T^{-2}\Theta^{-1}$ | $J\ kg^{-1}K^{-1}$   |
| $t$                   | time                               | T                      | s                    |
| $u$                   | velocity                           | $LT^{-1}$              | $m\ s^{-1}$          |
|                       | valve opening                      | —                      | —                    |
|                       | input variable                     |                        |                      |
| $v$                   | velocity                           | $LT^{-1}$              | $m\ s^{-1}$          |
|                       | eigenvector                        |                        |                      |
|                       | noise variable                     |                        |                      |
|                       | measurement noise                  |                        |                      |
| $w$                   | width                              | L                      | m                    |
|                       | velocity                           | $LT^{-1}$              | $m\ s^{-1}$          |
|                       | disturbance variable               |                        |                      |
|                       | process noise                      |                        |                      |
| $x$                   | distance from $yz$ -plane          | L                      | m                    |
|                       | state variable                     |                        |                      |
| $x_T$                 | pressure differential ratio factor | —                      | —                    |
| $y$                   | distance from $xz$ -plane          | L                      | m                    |
|                       | output variable                    |                        |                      |
| $z$                   | distance from $xy$ -plane          | L                      | m                    |
| $z_0$                 | specific acoustic impedance        | $ML^{-2}T^{-1}$        | $kg\ m^{-2}\ s^{-1}$ |

## Greek

| Symbol   | Description                       | Dimensions    | Unit     |
|----------|-----------------------------------|---------------|----------|
| $\alpha$ | friction factor                   | $\Theta^{-1}$ | $K^{-1}$ |
| $\gamma$ | ratio of specific heats $c_p/c_v$ | —             | —        |

| Symbol<br>(continued) | Description                        | Dimensions                    | Unit                       |
|-----------------------|------------------------------------|-------------------------------|----------------------------|
| $\eta$                | efficiency                         | —                             | —                          |
| $\theta$              | angle                              | —                             | rad                        |
| $\kappa$              | bulk viscosity                     | $\text{ML}^{-1}\text{T}^{-1}$ | $\text{Pa}\cdot\text{s}$   |
|                       | numerical constant                 | —                             | —                          |
| $\lambda$             | eigenvalue                         | $\text{T}^{-1}$               | $\text{rad s}^{-1}$        |
| $\mu$                 | dynamic viscosity                  | $\text{ML}^{-1}\text{T}^{-1}$ | $\text{Pa}\cdot\text{s}$   |
| $\nu$                 | kinematic viscosity $\mu/\rho$     | $\text{L}^2\text{T}^{-1}$     | $\text{m}^2 \text{s}^{-1}$ |
| $\xi$                 | dimensionless time                 | —                             | —                          |
| $\rho$                | density                            | $\text{ML}^{-3}$              | $\text{kg m}^{-3}$         |
|                       | model order                        |                               |                            |
| $\sigma$              | singular value                     |                               |                            |
| $\varsigma$           | dimensionless time constant        | —                             | —                          |
| $\tau$                | time constant                      | $\text{T}$                    | $\text{s}$                 |
|                       | time delay                         | $\text{T}$                    | $\text{s}$                 |
| $\Upsilon$            | general source term                |                               |                            |
| $\Phi$                | characteristic variable            |                               |                            |
| $\phi$                | dimensionless mass flow            | —                             | —                          |
| $\Psi$                | characteristic variable            |                               |                            |
|                       | dimensionless characteristic curve |                               |                            |
| $\psi$                | dimensionless pressure difference  | —                             | —                          |
| $\omega$              | angular frequency                  | $\text{T}^{-1}$               | $\text{rad s}^{-1}$        |

## Subscripts

| Symbol | Description                                  |
|--------|----------------------------------------------|
| +      | positive, left going                         |
| —      | negative, right going                        |
| 0      | equilibrium value                            |
| 1      | suction side                                 |
| 2      | discharge side                               |
| $a$    | auxiliary                                    |
| amb    | ambient                                      |
| $b$    | bandwidth ( $ H(j\omega)  = -3 \text{ dB}$ ) |
| $bp$   | bandpass                                     |
| $c$    | compressor                                   |
| $d$    | denominator                                  |

## Symbol Description

(continued)

|       |                        |
|-------|------------------------|
| $e$   | impeller tip           |
| $H$   | Helmholtz              |
| $h$   | hub                    |
| $i$   | inducer                |
|       | dummy / boundary index |
| $j$   | dummy / boundary index |
| $k$   | dummy / duct index     |
| $l$   | low-pass               |
| $n$   | numerator              |
| $o$   | stagnation property    |
| $p$   | plenum, pipe           |
| $pp$  | peak-to-peak value     |
| $r$   | control valve          |
| $s$   | shroud                 |
|       | isentropic             |
|       | sample                 |
| $s,x$ | sub-sample             |
| $ss$  | steady-state           |
| $T$   | transmission line      |
| $t$   | throttle valve(s)      |

## Superscripts

### Symbol Description

|   |                                  |
|---|----------------------------------|
| * | surge point, line                |
| ' | dimensionless variable           |
| * | related gas properties $f(p(t))$ |
| ^ | estimate                         |

## Quantities

| Symbol          | Description         |                                    |
|-----------------|---------------------|------------------------------------|
| $a$             | scalar, time series |                                    |
| $\mathbf{a}$    | vector              | $a_i \mathbf{e}_i$                 |
| $\mathbf{A}$    | second order tensor | $A_{ij} \mathbf{e}_i \mathbf{e}_j$ |
| $\underline{a}$ | column              | $[a_i]$                            |
| $\underline{A}$ | matrix              | $[a_{ij}]$                         |

## Symbols and Operations

| Symbol                               | Description                 |                                                         |
|--------------------------------------|-----------------------------|---------------------------------------------------------|
| $\emptyset$                          | empty set                   |                                                         |
| $j$                                  | imaginary number            | $j = \sqrt{-1}$                                         |
| $\text{Re}(a)$                       | real part                   | $\text{Re}(b + cj) = b$                                 |
| $\text{Im}(a)$                       | imaginary part              | $\text{Im}(b + cj) = c$                                 |
| $\underline{a}^T$                    | vector transpose            | $[a_i]^T = [a_j]$                                       |
| $\underline{A}^T$                    | matrix transpose            | $[A_{ij}]^T = [A_{ji}]$                                 |
| $\underline{A}^{-1}$                 | matrix inverse              | $[A_{ij}]^{-1}$                                         |
| $ a $                                | complex modulus (magnitude) | $\sqrt{\text{Re}(a)^2 + \text{Im}(a)^2}$                |
| $\angle a$                           | phase angle                 | $\arg(a) = \arctan \frac{\text{Im}(a)}{\text{Re}(a)}$   |
|                                      | scalar product              | $c = ab$                                                |
|                                      | scalar-vector product       | $\mathbf{c} = a\mathbf{b}$                              |
|                                      | scalar-tensor product       | $\mathbf{C} = a\mathbf{B}$                              |
| $\cdot$                              | vector inner product        | $\mathbf{a} \cdot \mathbf{b} = a_i b_i$                 |
| $\cdot$                              | vector-tensor inner product | $\mathbf{a} \cdot \mathbf{B} = a_j B_{ji} \mathbf{e}_i$ |
| $\nabla$                             | nabla operator              | $\mathbf{e}_j \frac{\partial}{\partial x_j}$            |
| $\frac{\partial a(x,t)}{\partial x}$ | partial derivative          | $\frac{\partial (bx+ct)}{\partial x} = b$               |
| $\dot{a}$                            | time derivative             | $\frac{\partial (bx+ct)}{\partial t} = c$               |

**Symbol**      **Description**
*(continued)*

|                          |                     |                                                                            |
|--------------------------|---------------------|----------------------------------------------------------------------------|
| $\frac{D\mathbf{a}}{Dt}$ | material derivative | $\frac{\partial a_i}{\partial t} + u_j \frac{\partial a_i}{\partial x_j}$  |
| $\Delta a$               | finite difference   | $a_j - a_i$                                                                |
| $da$                     | total differential  | $da = \frac{\partial a}{\partial b} db + \frac{\partial a}{\partial c} dc$ |
| $\tilde{a}$              | perturbed variable  | $a_i - a_0$                                                                |
| $\bar{a}$                | averaged value      | $\frac{1}{N} \sum_1^N a_k$                                                 |

---

<sup>1</sup>M = mass, L = length, T = time,  $\Theta$  = temperature,  $\Psi$  = quantity of substance.

<sup>2</sup>Equal to the universal gas constant (8.314471 J/mol·K), divided by the molar weight  $M$ .

<sup>3</sup>Specific thermodynamic property, i.e. defined per unit mass.



## CHAPTER ONE

# Introduction

---

**Abstract /** In this chapter a general introduction of centrifugal compression systems is presented. The unstable surge phenomenon in compressors is discussed, as well as possible means to avoid or suppress this instability. From an assessment of the current state of the art in control-oriented modeling and identification, the research objectives are formulated.

## 1.1 General introduction

The general topic of this thesis is surge in centrifugal compression systems. In particular it deals with the modeling and identification for control of the surge phenomenon. Surge is an unstable operating mode of a compression system that occurs at low mass flows. Surge not only limits compressor performance and efficiency but it can also damage the compressor and auxiliaries due to the large thermal and mechanical loads involved. Furthermore, the vibrations associated with surge can result in unacceptable noise levels.

A promising technique to cope with surge is the active suppression of aerodynamic flow instabilities. When successful, active surge control enlarges the operational envelope of the compressor towards lower mass flows. Thereby, surge control makes the compression system more versatile and it allows the machine to run at the most efficient operating points, which are usually located near the surge initiation point.

Research efforts over the past decades have led to important advancements in the field of surge control. Progress has been made with the analysis, modeling, and suppression of surge and promising experimental results on laboratory setups were reported. However, a real breakthrough in the practical application of active surge control on industrial scale installations has not been achieved, yet.

The current research originates from work by [Willems \(2000\)](#) and [Meuleman \(2002\)](#) within the *Compressor Surge Control* project at the Department of Mechanical Engineering, Technische Universiteit Eindhoven. The project provided valuable new insights in the

physical phenomena that play a role in centrifugal compressor surge and the suppression thereof. Most importantly, a novel control strategy to suppress surge has been tested with reasonable results on a laboratory scale gas turbine installation. The research presented here is a logical next step towards the implementation of active surge control on industrial scale compression systems.

In this chapter we will first address the basic operating principles and the characteristics of the type of centrifugal compressors that will be considered in this research. Subsequently, we will elaborate on the unstable operating modes of these systems and the means to avoid or suppress them. The main contribution of this chapter is an assessment of the current state of the art in surge suppression in general and of control-oriented modeling and identification of centrifugal compression systems in particular. This assessment will provide insight in the scientific and technological barriers for further advancements in active surge control from which we derive our research objectives. We will end the chapter with an outline of the thesis.

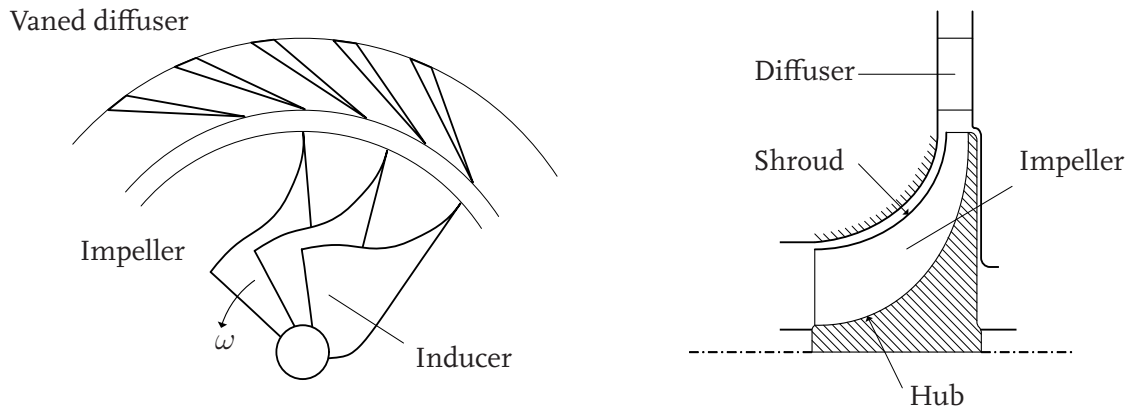
## 1.2 Centrifugal compression systems

Compressors are used, for example, as part of a gas turbine for jet and marine propulsion or power generation, in superchargers and turbochargers for internal combustion engines, and in a wide variety of industrial processes (Cohen et al., 1996; Whitfield and Baines, 1990; Gravdahl and Egeland, 1999b). In this thesis we focus on centrifugal compressors that are used in (petro)chemical process plants and in fluid transportation pipelines. Centrifugal compressors have the same operating principle as axial compressors so whenever appropriate we will refer to the more general class of turbo-compressors that covers all continuous flow compressors.

### 1.2.1 Principle of operation

Centrifugal compressors realize compression by transferring momentum to the fluid and the subsequent diffusion to convert the kinetic energy into pressure. The momentum transfer takes place at the doubly curved blades of the impeller that is mounted on a rotating shaft. Diffusion takes place in the annular channel of increasing radius around the impeller, usually referred to as diffuser. A sketch of a centrifugal compressor is shown in Figure 1.1.

The momentum transfer from the impeller blades to the fluid can be visualized by using so-called velocity triangles. An example is given in Figure 1.2. The velocity triangles illustrate how the absolute fluid velocities  $\mathbf{v}$  and blade velocities  $\mathbf{U}$  with respect to the casing and the resulting relative velocities  $\mathbf{w}$  are influenced by the curvature of the blades.



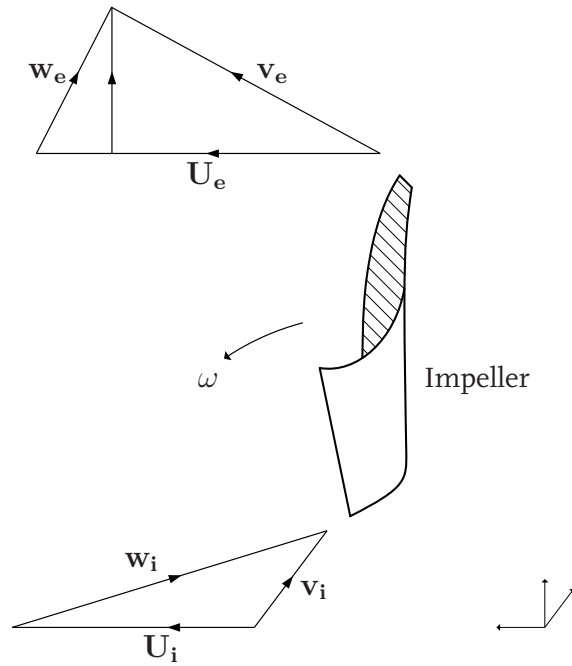
**Figure 1.1** / A centrifugal compressor with a vaned diffuser.

The diffusion process on the other hand can be explained by the Bernoulli<sup>1</sup> equation that states that the sum of kinetic energy  $\frac{1}{2}mu^2$ , potential energy  $mgh$  and pressure head  $p/\rho$  is constant. In the widening diffuser channel, either vaned or vaneless, the flow decelerates and this decrease in kinetic energy implies an increase in potential energy and pressure.

In general, the mechanical and thermodynamic processes in a turbocompressor are described by the continuity equation, the momentum equation, and the first and second law of thermodynamics. However, applying these general principles to the real flow in centrifugal compressors, being three-dimensional, unsteady and viscous, is extremely difficult. Many textbooks on fluid- and thermodynamics are available (e.g. Kundu, 1990; Shavit and Gutfinger, 1995), as well as books dedicated to the modeling and design of turbomachines (e.g. Cumpsty, 1989; Whitfield and Baines, 1990; Cohen et al., 1996), covering the basic concepts, advanced theoretical topics and empirical results acquired over more than a century of research and development.

Ultimately, the performance of any compressor is determined by the complex interactions between the system and the fluid. However, the performance of a turbocompressor can be expressed through a limited number of basic parameters (Cumpsty, 1989; Whitfield and Baines, 1990; Cohen et al., 1996). The number of parameters can be reduced by applying appropriate scaling, yielding a set of six dimensionless parameters ( $\psi, \phi, \eta, M_e, Re, \gamma$ ). Here,  $\psi$  and  $\phi$  are parameters related to the compressor pressure rise and flow rate, respectively,  $\eta$  represents the efficiency,  $M_e$  the scaled impeller speed,  $Re$  describes the amount of turbulence in the flow and  $\gamma$  represents the ratio of specific heats for the compressed fluid. Often the last two parameters are neglected during the preliminary analysis and design phase, but both parameters can have an impact on the overall performance of compression systems in practice (Whitfield and Baines, 1990).

<sup>1</sup>Daniel Bernoulli (1700–1782) was a Swiss mathematician born in Groningen, The Netherlands. In 1738 he published his main work *Hydrodynamica* in which he showed that the sum of all forms of energy in a fluid is constant along a streamline.

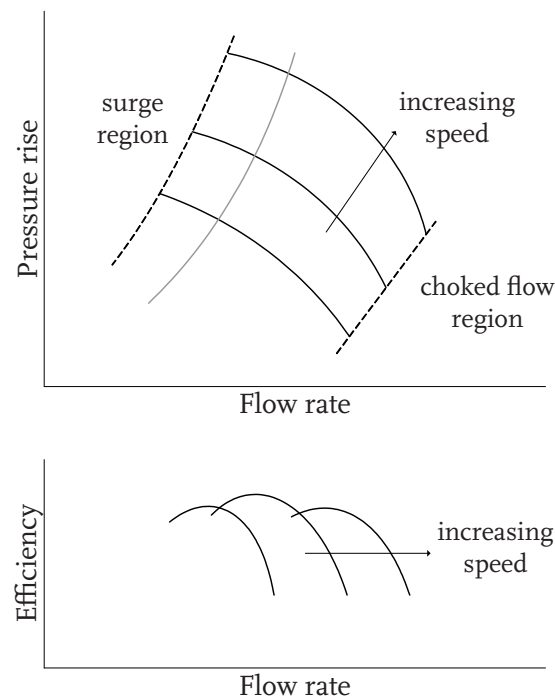


**Figure 1.2 /** Impeller inlet and discharge velocity triangles.

Using the mentioned parameters, the performance of a compressor can be graphically depicted in a so-called compressor map. An example is shown in Figure 1.3. The individual characteristic curves or speed lines are formed by steady-state operating points with the same rotational speed. The achievable flow rates are limited by the occurrence of surge at low flows and the phenomenon known as choking at high flows. Choking occurs when the local velocity, usually in the impeller exit or diffuser, reaches the speed of sound. For completeness also a characteristic curve for the load of the compression system is plotted in the upper part of the figure. This curve represents the total flow resistance of the system that the compressor must supply with pressurized fluid. The intersection of a speed line with the load characteristic determines the operating point of the compressor that, ideally, coincides with the point of maximum efficiency.

### 1.2.2 Industrial centrifugal compressors

In this thesis we focus on industrial scale centrifugal compression systems that are typically used in offshore oil and gas production, (petro-)chemical process plants, and in gas transportation networks. Some typical compressor configurations are depicted in Figures 1.4 and 1.5. Important characteristics of these types of centrifugal compressors are the large pressure rises and flow rates involved, see also Table 1.1. In order to reach these numbers, the compressors have large diameter impellers or operate at high speeds and they consume considerable amounts of power (order of magnitude up to 10 MW), resulting in bulky machines that can withstand the large mechanical loads involved.

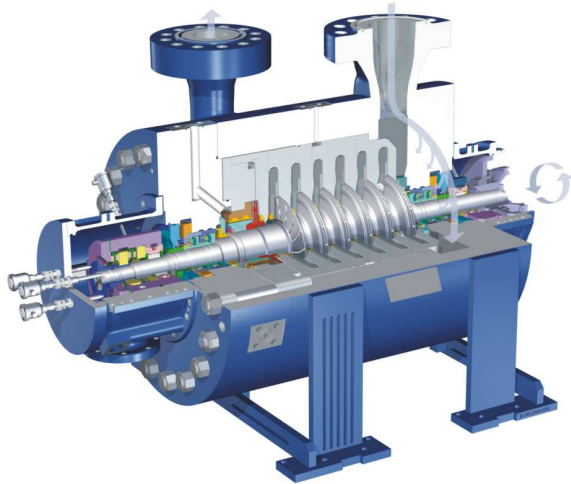


**Figure 1.3** / Illustrative compressor map with constant speed lines (black) and load characteristic (gray).

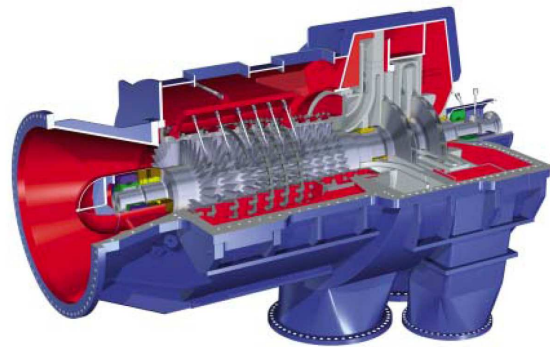
Furthermore, the required pressure rise is seldom realized with a single compressor stage so most industrial compressors contain multiple stages in series, resulting in complicated mechanical and aerodynamic designs, see also Figures 1.4 and 1.5a. In particular we mention the additional ducting that is required to guide the flow from one stage to the other. We point out that in some cases multiple compressors need to operate in series or parallel to realize the required pressure rise or flow.

Another characteristic for industrial compression systems is that they are usually integrated into large and complex systems like, for example, a refinery or an international gas distribution network. Each element in such complex systems, whether it is a reactor vessel, a pipeline, an entire oil field, or a small relief valve, can influence the compressor to which it is connected.

Industrial compression systems are expensive equipment as can be deduced from the above, which in turn makes compressors a critical component of the entire system in which it operates. This imposes restrictions on the equipment used in compression systems and numerous trade-offs need to be made between reliability, maintenance costs and added complexity on the one hand and improvement of functionality and performance on the other hand. Hence, the added value of new technologies needs to be proven through extensive analysis and test programmes, prior to their application in actual compression systems.

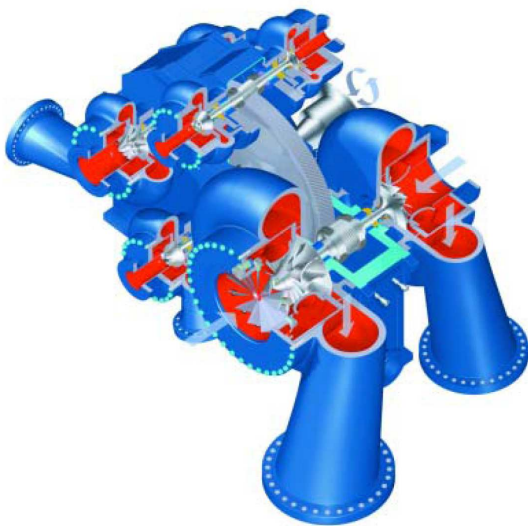


(a) Barrel-type high pressure compressor

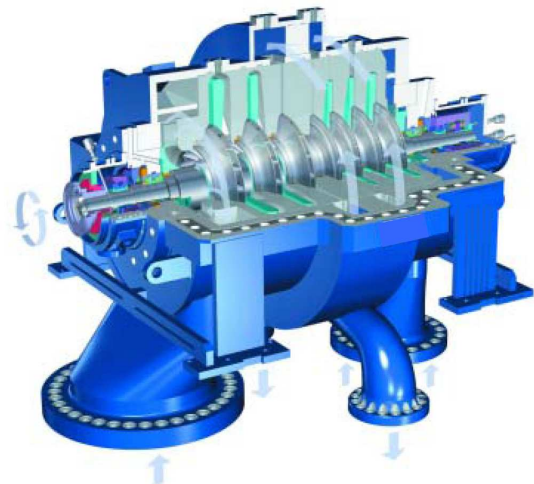


(b) Axial-radial high volume compressor

**Figure 1.4 /** High performance industrial compressors. (Courtesy of Siemens AG.)



(a) Integrally geared compressor



(b) Single shaft compressor

**Figure 1.5 /** Versatile industrial compressors. (Courtesy of Siemens AG.)

**Table 1.1** / Technical performance of industrial compressors. (Courtesy of Siemens AG.)

| Configuration | Capacity<br>(m <sup>3</sup> /h) | Pressure   |           | Speed<br>(rev/min) |
|---------------|---------------------------------|------------|-----------|--------------------|
|               |                                 | exit (bar) | ratio (-) |                    |
| Barrel        | 250–480,000                     | 1–1,000    | n/a       | 3,000–21,000       |
| Axial-radial  | 50,000–1,400,000                | n/a        | 5.8–16    | 2,000–9,000        |
| Geared        | 3,600–120,000                   | 3.5–40     | 3.5–20    | n/a                |
| Overhung      | 200–130,000                     | 1–50       | 1.01–1.45 | 1,500–3,600        |

## 1.3 Review of compressor instabilities

Turbocompressors can exhibit a variety of instabilities under different operating conditions. A review of instabilities found in compression systems is given by [Greitzer \(1981\)](#). Here we will focus on two aerodynamic instabilities in turbocompressors, rotating stall and surge. It is clear that both phenomena can affect each other given the simple fact that they occur in the same operating region of a turbocompressor. More specifically, surge can be regarded as the zeroth order eigenmode of the complete compression system while rotating stall is associated with higher order spatial modes inside the compressor ([Paduano et al., 2001](#)). Therefore, we include only a brief introduction on rotating stall in this thesis that is mainly concerned with surge.

### 1.3.1 Rotating stall

Rotating stall is an aerodynamic instability confined to the compressor internals that is characterized by a distortion of the circumferential flow pattern. One or more regions of stagnant flow, so-called stall cells, travel around the circumference of the compressor at 10–90% of the shaft speed, see also Figure 1.6. The stall cells reduce or completely block the flow, resulting in large vibratory stresses and thermal loads on the compressor blades. While the effects of rotating stall are known, details of stalled flow, for example the number of stall cells, their size and speed relative to the impeller, are far from being completely understood. A recent study of the stall characteristics in a vaneless diffuser was carried out by [Ljevar \(2007\)](#).

Given its distorting effect on the flow, rotating stall can result in a large drop in performance and efficiency ([De Jager, 1995](#)). Rotating stall introduces a gradual or abrupt drop of the pressure rise as is depicted in Figure 1.7. Moreover, rotating stall can introduce hysteresis into the system, implying that the flow rate has to be increased beyond the stall initiation point in order to bring the compression system out of its stalled operating mode ([Greitzer, 1981](#)). A recent review of rotating stall from a system stability perspective is provided by [Paduano et al. \(2001\)](#).

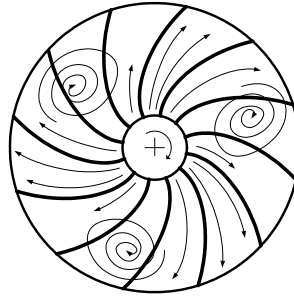


Figure 1.6 / Illustration of three stall cells in an impeller.

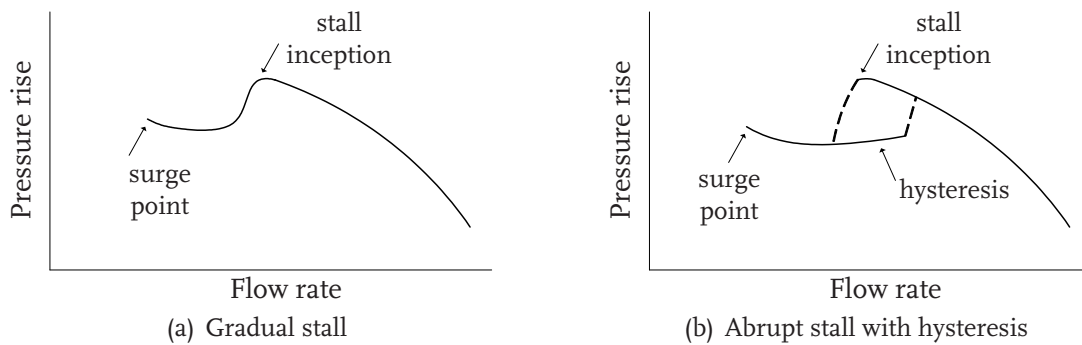


Figure 1.7 / Illustrative compressor maps in the presence of rotating stall.

### 1.3.2 Surge

Surge is an instability that affects the entire compression system. Surge is characterized by a limit cycle oscillation<sup>2</sup> that results in large amplitude fluctuations of the pressure and flow rate. In contrast to rotating stall, during surge the average mass flow is unsteady but circumferentially uniform. In other words, surge is a one-dimensional system instability while rotating stall is a two-dimensional compressor instability. Furthermore, surge only occurs in compressible flow systems whereas rotating stall occurs in both incompressible and compressible flows (Stenning, 1980).

As mentioned before, surge results in considerable loss of performance and efficiency. Furthermore, the power level of the pressure oscillations can approach that of the compressor itself, inducing large mechanical loads on the entire compression system. Moreover, fluctuations in pressure and flow rate can pose a threat to the (chemical) process that is connected to the compression system.

According to De Jager (1995) four types of surge can be distinguished based on the amplitude and frequency of the associated oscillations, namely mild surge, classic surge, deep surge, and modified surge, see also Willems (2000). Deep surge is the most severe form

<sup>2</sup>A limit cycle is an isolated periodic orbit in the phase plane (Khalil, 2000).

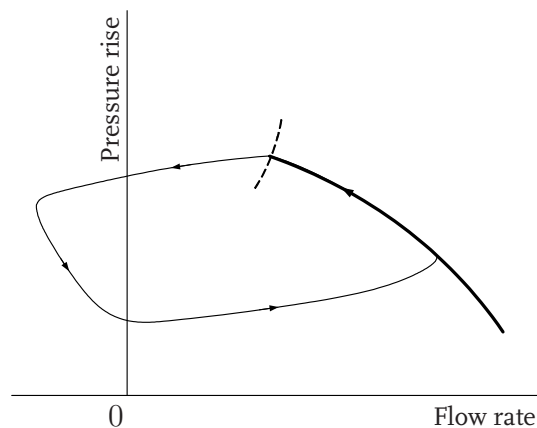


Figure 1.8 / A typical deep surge cycle.

of surge and it is associated with low frequent, large amplitude oscillations and negative flow rates (back flow) during a part of the surge limit cycle. An example of such a limit cycle oscillation is shown in Figure 1.8.

### Surge in practice

Given the potential risk of damage due to violent surge oscillations, turbocompressors are usually operated at a safe distance from the surge line (Cohen et al., 1996). The imposed safety margin limits the achievable pressure rise and ability to operate at off-design conditions. Even for machines operating under non-critical conditions, unexpected events can occur that cause the compressor operating point to cross the surge line.

For example, shutting off downstream processes or changing the production rate imply a load change for the compressor that can cause the system to enter surge. Switching to another compressor with a lower capacity is only a viable option for modest load variations that can be covered with a limited number of compressors.

Furthermore, changing the pressure rise or flow rate demand might require a speed change of the compressor. In particular when a large volume is connected to the compressor, the operating point moves towards the surge line during deceleration because the flow rate will change faster than the pressure in the large volume. An emergency shutdown is an extreme example of compressor deceleration that can trigger surge.

Another phenomenon that can bring the system into surge is a significant temporal change in gas composition, a common situation for compressors used in the production of oil and gas. Significant changes of gas composition cause the compressor characteristic, including the surge line, to change and hence surge can occur when the load for the compression system in terms of pressure rise and flow remain unchanged.

## 1.4 Surge suppression

We argued that surge can endanger the safe operation of a compression system and as a consequence it limits the performance and operating envelope of the compressor. Taking away these limitations requires that surge can be avoided by suppressing the aerodynamic instability of the compression system. Next to the potential performance increase, surge suppression can also have a positive effect on the versatility of compression systems, and on their operating and life cycle costs (Epstein et al., 1989; De Jager, 1995).

There are several approaches possible to remove the limitations that surge impose on turbocompressors. We will give an overview of the known measures to avoid or suppress surge that are known from literature.

Historically, the engineering of turbomachines and their application in the field have been the main driving forces behind the development of turbomachine technology rather than applied science (Cumpsty and Greitzer, 2004). This general observation also holds for the development of measures to suppress surge in turbocompressors.

To acknowledge the importance of the engineering practice and the trends in the centrifugal compressor market we select a rather large scope for our overview of surge suppression technology. Next to the scientific literature on surge suppression, we include patents and non-scientific publications in our assessment of the current state of the art, following a similar approach as Botros (1994a).

Finally, we discuss the available results on modeling and identification of compression system dynamics. Combined with the assessment on the state of the art, this review will form the basis of the problem formulation for our research in the next section.

### 1.4.1 Surge avoidance and suppression techniques

From literature we identified the following approaches to cope with surge in turbocompressors (Botros, 1994a; De Jager, 1995).

- Confine operating point to stable operating envelope through
  - surge avoidance;
  - surge detection and avoidance.
- Extend the stable operating envelope through
  - improved aerodynamic design;
  - reduction of variations in operating conditions;
  - inclusion of components that influence the flow;
  - active suppression of aerodynamic instabilities.

The many available surge avoidance methods all aim to prevent the compressor operating point from crossing the surge line. Usually a safety margin is incorporated by selecting a control line some distance away from the actual surge line (Staroselsky and Ladin, 1979; Botros et al., 1991; Dukle and Narayanan, 2003). In contrast, surge detection and avoidance techniques activate the measures to keep the operating point in the stable regime when the actual (onset of) surge is detected (Dremin et al., 2002). The countermeasures taken in both cases, aim at increasing the flow rate, reducing the downstream pressure or changing the impeller speed to prevent surge from developing.

The aerodynamic design of a compressor affects not only the performance and efficiency of a compression system (Cumpsty, 1989), but also the transition from stable to unstable flow. Elder and Gill (1985) addressed the various design parameters that influence surge in centrifugal compressors. For a discussion of each of these factors and the improvements that are made over the years, consult Whitfield and Baines (1990, Ch. 9) and the references therein.

Another measure to extend the safe operating range of compressors is to reduce the variability in process conditions that can trigger aerodynamic instability or, in other words, the reduction or avoidance of disturbances. Examples are adequate control of the fluid temperature at the inlet and intercoolers, adjusting the safety margins when gas composition and hence the surge line change, and adopting appropriate start-up and shutdown procedures to avoid destabilizing transients (Kurz and White, 2004).

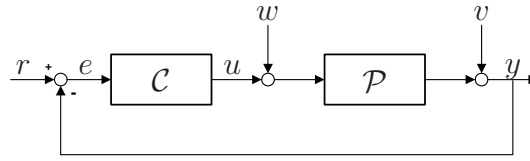
Yet another approach is to equip the compressor with components that can influence the behavior of the flow inside the machine. For example, variable inlet guide vanes or diffuser vanes can be installed to enable adjustment of the relevant flow angles (Simon et al., 1987), passive damping devices like water columns (Arnulfi et al., 2001) or spring-loaded walls (Gysling et al., 1991) in discharge vessels can be used to damp out any destabilizing oscillations, or air can be injected into the flow path to avoid the development of aerodynamic instabilities (Day, 1993; Stein et al., 2000).

A conceptually different approach was brought forward by Epstein et al. (1986) who proposed to use feedback control to suppress surge and rotating stall and thereby increasing the stability and performance of a compressor.<sup>3</sup> In comparison with the other mentioned methods, the potential performance improvements for active suppression of aerodynamic instabilities, also referred to as surge and/or rotating stall control, is large and the generality of the underlying concept makes it applicable to a wide range of machines.

A general feedback control scheme is depicted in Figure 1.9. The feedback controller determines a certain response to an error signal that is fed into the controller. The con-

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<sup>3</sup>The journal paper by Epstein et al. (1989), frequently cited as the first paper mentioning active suppression of aerodynamic instabilities in turbomachines, is based on the cited conference paper that was presented in 1986.



**Figure 1.9** / General feedback control scheme;  $\mathcal{C}$  = controller,  $\mathcal{P}$  = plant or system,  $r$  = reference,  $y$  = output,  $e$  = error,  $u$  = control output,  $w$  = external disturbance,  $v$  = noise.

troller output is sent to the system actuator in order to confine or minimize the error between the actual and desired response.

The proper functioning of a feedback controller depends on many factors. Some factors that are of particular importance in the context of active surge control are the dynamic response of the actuator, its capacity or range, the (inverse) response of the system to actuator changes and the possible time delays between them. Furthermore, a controller must be able to realize the desired behavior of the system despite the presence of external disturbances, measurement noise, and uncertainties in the model on which the controller design is based. This particular aspect of feedback control is usually referred to as robustness.

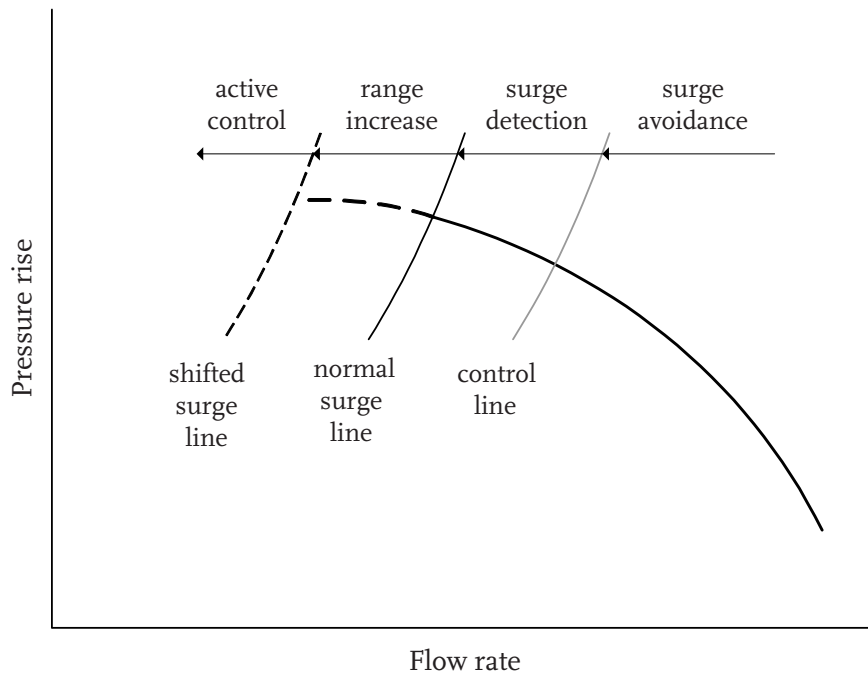
To summarize, the effect of the aforementioned avoidance and suppression techniques on the operating envelope is illustrated in Figure 1.10. Clearly, successful active control offers the largest extension of the stable operating range while surge avoidance is the most restrictive method.

Although we will focus on surge control in this thesis, we will include the other mentioned techniques in our assessment of the current state of the art below. This will enable us to discuss the opportunities and challenges of surge control against the background of the various other ways to cope with surge in centrifugal compression systems.

### 1.4.2 Technology assessment

After the introduction of the concept by Epstein et al. (1986), the research on active surge control focussed on providing a proof of concept through small, laboratory scale demonstrations (Greitzer, 1998). In particular we mention the early work by Ffowcs Williams and Huang (1989) who used a movable wall driven by a loudspeaker to damp out any instabilities in an early stage of development. Pinsley et al. (1991) on the other hand used a variable area throttle valve to achieve surge suppression. Both these studies demonstrated that it is possible to stabilize a small turbocharger system beyond the natural surge line by the use of appropriate feedback.

Following these promising results, Paduano et al. (1993a) and Day (1993) investigated the stabilization of rotating stall, using variable inlet guide vanes (IGV) and pulsed air



**Figure 1.10** / Effect on operating envelope of surge avoidance and suppression methods.

injection, respectively. We remark that all the reported experiments were conducted on axial laboratory scale compression systems, operating at low speeds.

The initial studies on rotating stall control were followed up by experiments on a larger transonic centrifugal compressor as discussed by [Weigl et al. \(1998\)](#). They successfully demonstrated rotating stall and surge suppression by using high-speed (400 Hz) valves to control the air injection rate upstream of the compressor. Around the same time [Freeman et al. \(1998\)](#) conducted extensive experiments on a eight-stage turbojet engine, using different bleed and recirculation schemes for actuation. Bleed and recirculation rates were controlled by fast response (300 Hz) sleeve valves and one conclusion from the experiments was that recirculation schemes are more effective for surge and rotating stall suppression than bleeding of the compressed medium.

More recently, [Willems \(2000\)](#) reported on the implementation of an active surge controller on a laboratory scale turbocharger, using plenum bleed valves and a hybrid one-sided control strategy to minimize bleed losses. [Arnulfi et al. \(2006\)](#) investigated both passive (see also [Gysling et al., 1991](#)) and active suppression of surge in an industrial four-stage centrifugal compressor. Their work showed that both passive and active suppression of surge is feasible with the active approach being the more effective and flexible one. The same installation was used by [Blanchini et al. \(2002\)](#) to investigate high-gain feedback surge controllers with promising experimental results.

The discussed experimental work and the conclusions drawn by various authors, illustrates the importance of actuation methods that improve the flow properties in the com-

pressor. The developed means of actuation, for example the variable area injector nozzles and high-speed control valves, are highly sophisticated components that were specifically designed for the test environment at hand. An overview of different sensor-actuator pairs that were applied in experimental studies is provided by [Willems and De Jager \(1999\)](#).

Despite the promising experimental results, a critical technological barrier is the difficulty of actuation in full-scale applications as stated by [Paduano et al. \(2001\)](#). Actuator bandwidth and the harsh environment need to be addressed in order to realize adequate and economical actuation systems to enable active control of aerodynamic instabilities in compression systems. This conclusion is supported by the absence of any literature on successful applications of active surge control in large, industrial scale turbocompressors.

## Patent literature

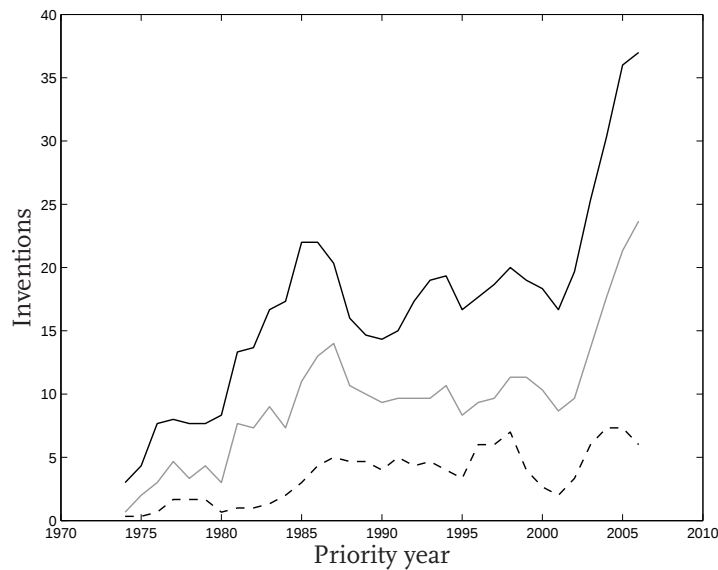
According to [Botros \(1994a\)](#) patent documents offer a unique and rich source of state of the art information, new ideas, and problem-solving technology. Hence, we conducted a patent search to identify disclosures of surge suppression technology over the past 35 years. While patent rights expire after 20 years, we included the period 1972–1986 in our search to provide sufficient overlap with the data from [Botros \(1994a\)](#) and the important scientific results in the 1970s.

A thorough review of the search results yielded a set of 239 relevant patents that we subsequently classified in terms of the disclosed technology, type of compression system, and field of application. The most relevant results are depicted in Figure 1.11 and Table 1.2.

The most prominent trend in Figure 1.11 is the large increase in disclosed inventions after 2001. This increase is, among many other factors, correlated with a worldwide growing demand for energy and the increased oil price ([Mitchell, 2006](#)) that generate a higher demand for new compression systems and a reduction of their power consumption. However, we remark that the number of patents on surge detection technology do not follow this trend, which is in line with the conclusion from [Botros \(1994a\)](#) that the prospects for the commercial use of surge detection techniques are poor.

Moreover, Figure 1.11 shows that the number of filed patents also increased between the mid 70s and mid 80s. This increase can be correlated with the large scientific interest in the surge phenomenon and the high energy (oil) prices during that period.

The aforementioned trend can be attributed for a large part to the significantly increased number of patents that focus on surge suppression in turbocharging and supercharging applications. For other applications the number of patents filed after the year 2001 is increasing at a more modest rate.



**Figure 1.11** / Number of patents on compressor surge in general (solid black), surge avoidance (gray), and surge detection (dashed black); data smoothed using three year moving averages.

From Table 1.2 we see that approximately 40% of the patents deals with centrifugal compression systems while only 20% focusses on axial compressors. We remark that the relative contributions of both compressor types are unchanged when the predominant *axial* jet engine and *centrifugal* turbocharger applications are excluded.

The presented patent data and the above findings support some of the conclusions from Botros (1994a). In particular, technological efforts focus mainly on surge avoidance techniques and much less on surge detection methods and improvements of the compressor geometry. Furthermore, most recent patents focus on specific applications, indicating that surge avoidance technology is in a research or perfection stage rather than in a development or a matured or declining stage. The increase in attention for refrigeration applications as signaled by Botros (1994a) did not persist although some highly cited patents have been filed.

The situation for active surge control technology is completely different. The fact that only one patent on active surge control is filed up to now indicates that this technology is still in an early stage of development.

Finally, the presented results show a steep increase in patent activity after the year 2001 that could not yet be foreseen by Botros (1994a). While the increase in patent applications is mainly attributed to turbocharger applications, the expectation is that in the near future the focus on surge suppression will also increase in various industrial applications (gas turbines, process industry and gas transportation) as we will discuss below.

**Table 1.2** / Number of patents across technical areas.

|                    | Patents (#) | Citations (#) |
|--------------------|-------------|---------------|
| <b>Technology</b>  |             |               |
| surge avoidance    | 148         | 432           |
| surge detection    | 53          | 204           |
| process/machine    | 37          | 130           |
| active control     | 1           | 0             |
| <b>Compressor</b>  |             |               |
| axial              | 44          | 142           |
| centrifugal        | 96          | 305           |
| unspecified        | 99          | 319           |
| <b>Application</b> |             |               |
| turbocharging      | 49          | 108           |
| gas turbines       | 31          | 90            |
| jet engines        | 23          | 99            |
| process plants     | 23          | 94            |
| power generation   | 15          | 21            |
| refrigeration      | 13          | 76            |
| gas pipelines      | 9           | 31            |
| unspecified        | 76          | 247           |

## Market trends

As mentioned earlier, the global increase in energy consumption, the dwindling fossil fuel reserves and the associated high prices for crude oil and natural gas have boosted the demand for new compression systems. In particular we mention the large number of planned gas-to-liquids (GTL), coal-to-liquids (CTL) and integrated gasification combined cycle (IGCC) facilities that require extremely large compression systems (Shelley, 2006). Not only do these large compressors account for the majority of the energy consumption, making efficiency a high priority in the design, reliability and safety of operation is of major importance too for such capital intensive machines (Eisenberg and Voss, 2006). Potentially, the integration of active surge suppression technology can contribute to both these aspects.

The increasing demand for natural gas has led to the expansion of existing and development of new compressor applications like gas storage facilities that enable the compensation for varying demands, gas lift and re-injection applications to increase the production rate of depleting oil fields, gas transportation pipelines and various gas treatment processes (Hunt, 2006). These applications are often associated with conflicting require-

ments of both high performance of the compressor and a wide operating range to cover varying load conditions (e.g. [Chellini, 2002](#); [Anderson, 2005](#); [Farmer, 2005](#)). In combination with the high discharge pressures involved, in particular for gas re-injection where pressures up to 700 bar are not uncommon, these requirements ask for effective surge suppression technology.

Furthermore, we mention the recent developments towards subsea compression systems for the exploration of remote, deep water, or arctic oil and gas fields. Given the remote locations, harsh conditions and deep waters, direct compression of the unprocessed gases at the seabed and transporting it to shore through a pipeline is both a promising economical and technological alternative for offshore platforms or floating production, storage and offloading (FPSO) vessels. However, subsea compressor stations must be highly reliable while operating on dirty gases of which the properties can change in unpredictable ways during production ([Kalyanaraman and Haley, 2006](#)). Active surge control can offer a solution to reduce the conservatism of the protective measures to avoid or suppress surge without compromising reliability of the subsea compression system.

Finally, the increased attention for turbocharger and supercharger applications indicate the need for improved surge suppression technology. Not only can active surge control improve the efficiency and operating envelope of supercharged combustion engines, the developed technology can also play a role in the realization of efficient and reliable fuel cells.

### 1.4.3 Modeling and identification for control

Given the responsive nature of feedback control, knowledge on the dynamic behavior of the system on which the control system acts is required to determine the appropriate control action whenever a change in the system is observed. Often this knowledge is captured by a mathematical model that describes the essential dynamics of the system and that can be used for controller design.

The development of dynamic models and their subsequent use for controller design proved to be particularly successful in the field of surge and rotating stall control ([Greitzer, 1998](#); [Paduano et al., 2001](#)). One of the most relevant results in this context is the introduction of the nonlinear lumped parameter model for axial compressor transients by [Greitzer \(1976a\)](#). The so-called *Greitzer model*, based on earlier work by [Emmons et al. \(1955\)](#), describes the underlying physics of compressor surge, which is an essentially one-dimensional flow phenomenon. Moreover, the lumped parameter model is relatively simple and it fits well into the control framework.

Initiated by the work of [Hansen et al. \(1981\)](#), the aforementioned model has also been successfully applied to describe the dynamics of centrifugal compression systems (e.g. [Meuleman et al., 1999](#); [Arnulfi et al., 1999](#)). A valuable extension of the Greitzer

model for variable speed machines was proposed by Fink et al. (1992) who included rotor dynamics to account for compressor speed variations during surge transients. Gravdahl and Egeland (1999a) included an analytical representation of the compressor performance curve in this variable speed model.

Another interesting compressor modeling approach was followed by Botros et al. (1991) who modeled the individual components of a complete compressor station. The adopted transfer matrix formulation allowed connection of the various dynamic and algebraic equations for the compressor unit, piping elements, flow constrictions and valves, and the station control system. The capability of the resulting model to describe surge transients was discussed by Botros (1994b).

Although more relevant for rotating stall, it is worth mentioning the two-dimensional model introduced by Moore and Greitzer (1986) that describes the dynamics associated with surge, rotating stall, and the coupling between both instabilities. This representation formed the basis for many dynamic compressor models that are being used in the design of control schemes for rotating stall stabilization (Willems and De Jager, 1999; Paduano et al., 2001). More recently, Spakovszky (2000) developed a sophisticated two-dimensional compressible flow model for a centrifugal compressor. This model was used to design a rotating stall controller that proved to be highly effective in suppressing both rotating stall and surge (Spakovszky, 2004).

So far we have discussed some of the key publications on the modeling of turbocompressor dynamics. A first step in describing the essential dynamics of a compression system is to develop a suitable model structure. A second step in developing a usable model is the quantitative identification of the model parameters. Paduano et al. (1993b) proposed a formal closed-loop system identification method, based on the instrumental variable technique. Successful open-loop identification and the value of system identification for designing stabilizing compressor control systems was discussed by Paduano et al. (1994).

Other applications of forced-response system identification were reported by Weigl et al. (1998); Nelson et al. (2000) and others. As stated by Paduano et al. (2001) these developments have required both the development of high bandwidth flow actuation technology for compressors and the application of identification techniques that are familiar in the control field.

At this point we remark that the reported techniques for the identification of turbocompressor dynamics are based on linear system theory. Furthermore, applications are mostly limited to small, laboratory scale compression systems with dedicated actuators or stand-alone turbocompressors without any long pipelines or other process equipment attached to them. True compression *system* identification, using either linear or non-linear identification techniques, has not received significant attention in the literature yet.

## 1.5 Research objectives and scope

From the elaborate discussion above some conclusions can be drawn. After summarizing these conclusions we define the scope for this thesis and formulate some general research questions from which we derive our specific research objectives. We will then present and discuss the methodology for our research and summarize the main contributions of this thesis. Finally, a brief outline of the remainder of this thesis is given.

The first conclusion from the previous sections is that the analysis, modeling and suppression of surge and rotating stall have received significant attention over the past 30 years. The basic mechanisms and the underlying physics of the unstable operating modes are understood reasonably well. Mathematical models that describe the dynamic behavior of turbocompressors during surge and rotating stall are available (see Longley, 1994; Gu et al., 1999; Gravdahl and Egeland, 1999b; Paduano et al., 2001). Exploiting these models and the associated insights resulted in various successful implementation of active surge and rotating stall control schemes. On small, laboratory scale compression systems and, to a lesser extend, on larger test systems, up to 20% reduction of the surge point mass flow has been realized.

A second conclusion is that since the last 10 years research has mainly focussed on active control of rotating stall and analysis of the relevant stall inception processes. Control of rotating stall is considered a more challenging problem since it involves multiple spatial modes in addition to the fundamental surge mode. Control of this instability requiring complex models and multidimensional sensing and actuation capabilities at challenging bandwidths (Paduano et al., 2001). Despite the achieved successes with active stabilization of rotating stall, which proved to have a positive effect on surge suppression as well, challenging problems remain to be solved prior to implementation in aeronautic and industrial applications (De Jager, 1995; Gu et al., 1999; Paduano et al., 2001).

Thirdly, from our technology assessment we conclude that there exists a large gap between the scientific achievements and industrial applications in the field of turbocompressor stabilization, which is in line with the statement above. However, our assessment also indicated that the attention from industry for surge suppression techniques is increasing. Given the current growth of the turbomachine market and the emergence of new technologies and applications it is likely that this trend will continue over the coming years. However, in order to be commercially viable, new technologies should yield an increase of the stable operating range of at least 10% in industrial scale compression systems.

Over the years, several full-scale tests of rotating stall control have been conducted by Freeman et al. (1998); Nelson et al. (2000) and Scheidler and Fottner (2006) on axial compressors in turbojet engines. However, the performance penalty from rotating stall on centrifugal compressors is believed to be low (see e.g. Gravdahl and Egeland, 1999b),

while surge poses a large treat to the capital intensive industrial compressors given the violent, high power level oscillations. Despite the promising results achieved by [Willem's \(2000\)](#); [Blanchini et al. \(2002\)](#); [Arnulfi et al. \(2006\)](#) and others, active surge control is still far from being accepted as proven technology by the compressor industry.

By combining the above conclusions on the scientific successes and the apparent gap towards industrial practice we can formulate the following questions: *Is the implementation of surge and rotating stall control on full-scale turbocompressors hampered by technological (practical) barriers only or do there exist fundamental (theoretical) barriers as well?* and *What are these barriers and how can they be removed?* Given the arguments from the previous paragraph, in this thesis we will limit the scope of these questions to surge in industrial centrifugal compression systems.

Both the transformation of past and recent theoretical results to industrial practice and further advancements in this field require in the first place adequate models that describe the relevant dynamics of industrial compression system. Therefore, the specific objectives of this research are formulated as follows:

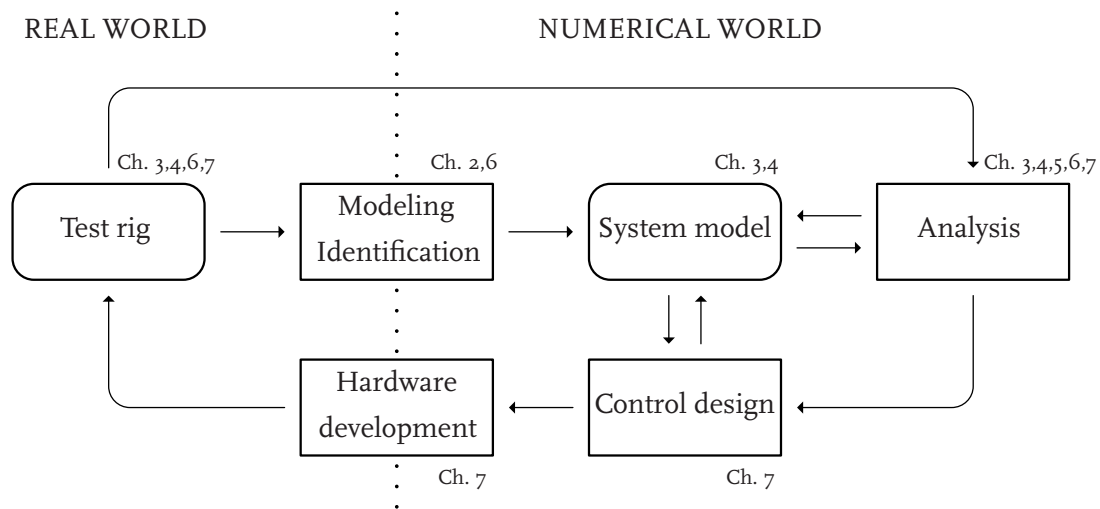
- develop and validate dynamic models for industrial scale centrifugal compressors;
- identify parameters to capture those dynamics that are relevant for control design;
- determine the critical barriers for industrial scale surge control.

In this thesis we aim to present the gained insights in the relevant physics and practical limitations inherent to industrial scale compression systems. Moreover, we aim to pass on the practical experience obtained from compression system modeling, the extensive experimental validation, and the attempts to design and implement stabilizing surge controllers.

The adopted methodology for our research is schematized in [Figure 1.12](#). Central in our approach are the use of modeling and identification techniques to develop dynamic models for the two industrial scale compressor test rigs under study. The resulting models will be used to design and implement an active surge control system. Throughout the process the results are analyzed and compared with experimental data.

The scheme in [Figure 1.12](#) also provides an outline and positioning of the remaining chapters in this thesis. Finally, the main contributions of the research presented in this thesis are that

- lumped parameter models of the behavior of two different industrial compressor test rigs during surge are developed and validated ([Chapters 2, 3, 4](#));
- a model extension to describe the effect of piping acoustics on surge transients is developed and validated ([Chapter 4](#));



**Figure 1.12** / Thesis outline and positioning within the adopted research methodology.

- the dynamics of industrial scale compression systems that are relevant for control design are analyzed (Chapter 5);
- two identification methods to estimate the stability parameter of the compressor model are developed (Chapter 6);
- a high-speed control valve that meets specifications obtained from numerical simulations is developed (Chapter 7);
- valuable information on remaining barriers for full-scale surge control is obtained from unsuccessful control experiments (Chapter 7).

The main conclusions will be summarized in 8 and we will give recommendations for further work.



## CHAPTER TWO

# Theoretical modeling of centrifugal compression systems

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**Abstract** / This chapter deals with the modeling of compression system dynamics. After reviewing the available literature, the nonlinear Greitzer model is introduced as the basis for modeling surge dynamics in centrifugal compression systems. The main assumptions for this model and the essential steps in the derivation are highlighted. After addressing the spatially varying geometry and aerodynamic scaling, the advantages and shortcomings of the model are discussed.

## 2.1 Introduction

A first step when studying the dynamic behavior of a system is to gain insight into the underlying physical principles and relevant phenomena, and translate them into a mathematical model. When the validity of a model is assessed, it can be used to investigate the effect of changes in the system or environment, disturbances and even the effect of a controller on the system behavior. The main requirement for a dynamic model is that it describes the phenomena of interest in the actual system with sufficient accuracy.

Although not necessary, it is desirable when the underlying physics remain recognizable in the model equations. Furthermore, it must be possible to determine the parameters of the model, either directly or through a feasible measurement, identification or estimation step that can provide the parameters with sufficient accuracy. The aim of this approach is to obtain a generic model for the systems of interest that will simplify both the modeling of a specific system and the interpretation of calculation results.

However, when a model is used for control design there are some additional and possibly conflicting requirements on the model itself. When (part of) the model is used in the control algorithm, the model should be of a relatively low order to allow real-time implementation. Furthermore, in practice it is almost never possible to place actuators and

sensors at ideal locations to measure the variables of interest. Therefore, it is desirable to have a model structure that does not depend on the location of actuators and sensors in the system or at least provides flexibility to account for deviations in sensor placement.

In this chapter we discuss the development of a dynamic model for the centrifugal compression systems under study. First we will review the available literature on compression system modeling, including some of the available material that focusses on rotating stall. Despite the differences between surge and rotating stall, the two phenomena are related in such a way that theoretical concepts and modeling techniques for rotating stall can provide information that is useful to understand and model the surge dynamics in centrifugal compression systems. For completeness, we will repeat some of the important results from literature that we already discussed in Chapter 1.

We will then argue that one model in particular meets the above requirements. In the remainder of this chapter we will focus on the so-called Greitzer model for the transient behavior of compression systems. After stating the general model assumptions we will give a step by step derivation. Then we discuss two techniques to make the model more versatile. We will end this chapter with a discussion on the benefits and shortcomings of the resulting dynamic compression system model.

### 2.1.1 Literature on compression system modeling

One of the first models for the dynamic behavior of basic compression systems was derived by [Emmons et al. \(1955\)](#). The authors of this paper exploited the analogy between a self-excited Helmholtz<sup>1</sup> resonator and the small oscillations associated with the onset of surge to develop a linearized compression system model.

A significant step forward in the field was made by [Greitzer \(1976a\)](#) who developed a nonlinear lumped-parameter model for basic compression systems. Although based on the linear analyses by [Emmons et al. \(1955\)](#), the *Greitzer model* was the first model capable of describing the, in essence nonlinear, large amplitude oscillations during a surge cycle. To this day, it is the most widely used dynamic model in the field.

The model by [Greitzer \(1976a\)](#) was developed for axial compressors but [Hansen et al. \(1981\)](#) showed that the model is also applicable to a centrifugal compression system. This publication was followed by more studies that focussed on the analysis and modeling of centrifugal compression system dynamics. A simple but relevant advancement in this evolving field was made by [Fink et al. \(1992\)](#) who included simple rotor dynamics in a Greitzer model to account for the effect of speed variations on system transients.

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<sup>1</sup>Hermann Ludwig Ferdinand von Helmholtz (1821–1894) was a German physician and physicist. In 1863 he published the book *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* in which he discussed the phenomenon of air resonance in a cavity.

The two-part paper by [Moore and Greitzer \(1986\)](#) and [Greitzer and Moore \(1986\)](#) marks another major development in the modeling of compression system dynamics. These papers present the derivation and analysis of a nonlinear dynamic model that describes the growth and possible decay of a rotating stall cell during compressor transients, the development of surge, and the possible coupling between the two instabilities. While surge is considered to be an unsteady axisymmetric oscillation, rotating stall is a steady (in a rotating frame of reference) flow variation in both axial and circumferential direction. To capture both phenomena the so-called Moore-Greitzer model is formed by coupling two-dimensional unsteady flow descriptions to a lumped-parameter system model.

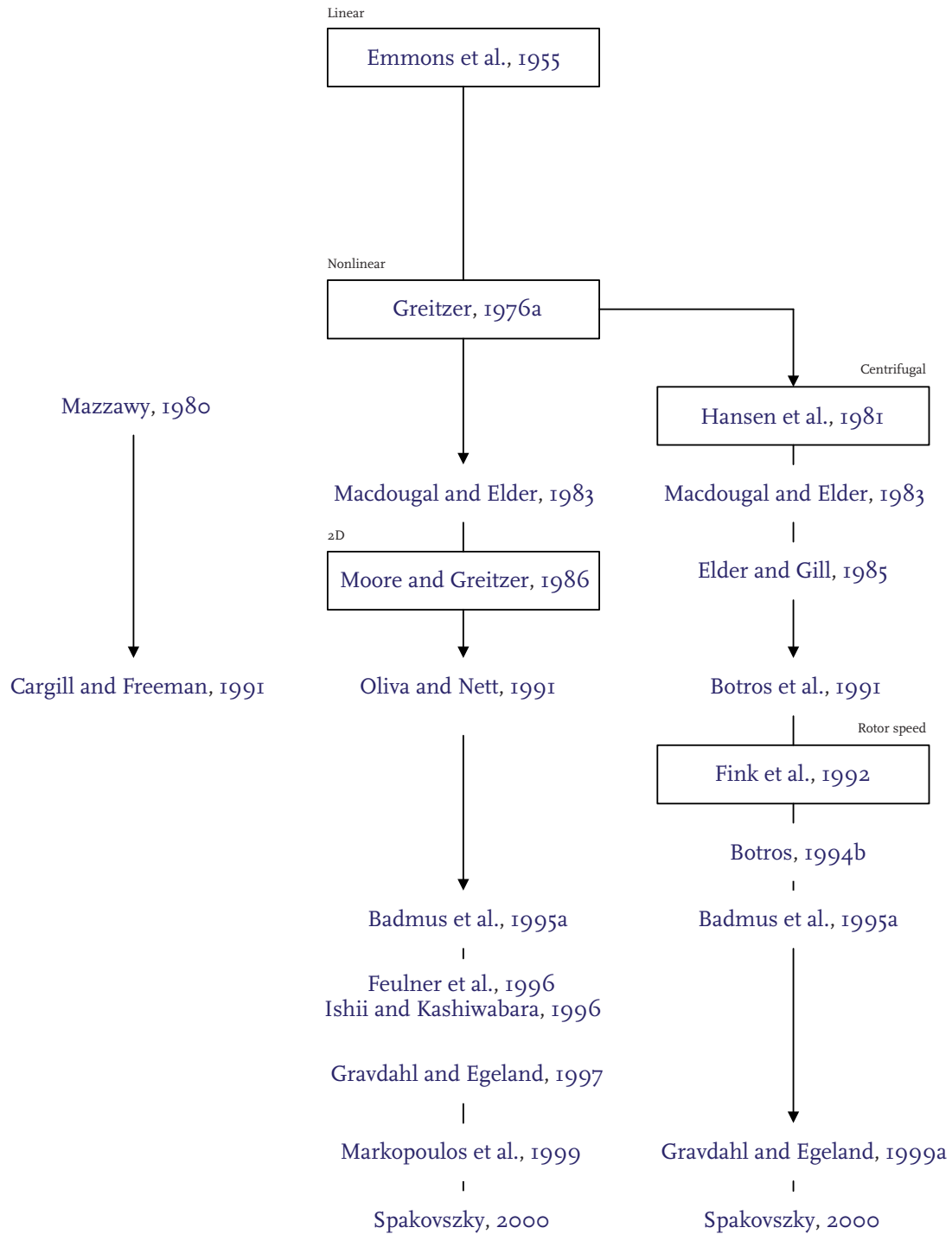
At this point it is worthwhile to categorize the different developments within the field. Figure 2.1 gives a partial overview of the available literature on compression system modeling. The boxed references discriminate chronologically between linearized and nonlinear dynamic models, models for axial and centrifugal compressors, models that describe surge (1D) and rotating stall (2D), and between fixed and variable rotor speed descriptions.

An important aspect of the Greitzer model is that the flow in the ducts is assumed to be incompressible. [Macdougall and Elder \(1983\)](#) derived a model, similar to that of [Greitzer \(1976a\)](#). However, they allow the duct flow to be compressible and their model can deal with non-ideal gases and varying gas compositions. A similar modeling approach was used by [Elder and Gill \(1985\)](#) to model various components of a centrifugal compressor to study the effect of these individual components on overall system stability.

A different modeling approach for centrifugal compressors was followed by [Botros et al. \(1991\)](#) while [Badmus et al. \(1995a\)](#) focussed on axial compressors. They developed component models from the principles of mass, momentum, and energy conservation. The modular structure makes them, without serious modifications, suitable for describing surge dynamics of a variety of both axial and centrifugal compression systems. [Botros \(1994b\)](#) modified his earlier model by including rotor dynamics in order to account for speed variations. Rotor dynamics were included in the original Moore-Greitzer model by [Gravdahl and Egeland \(1997\)](#).

The issue of compressible flow was also addressed by [Feulner et al. \(1996\)](#) who developed a linear compressible flow model for a high speed, multi-stage axial compressor. He used a mixture of 1D and 2D flow descriptions for the blade passages and interblade row gaps, respectively. A complete two-dimensional, compressible flow model for a high speed, multi-stage axial compressor was developed by [Ishii and Kashiwabara \(1996\)](#). Similar to the Moore-Greitzer model, this model describes the development of both rotating stall and surge, and the coupling between the two unstable modes.

[Markopoulos et al. \(1999\)](#) used Fourier series expansions to derive a quantitative model for the unstable dynamics in axial compressors. Although similar to the model of [Moore and Greitzer \(1986\)](#) in this model finite intake and exit duct lengths were taken into account. A sophisticated two-dimensional, compressible flow model for centrifugal



**Figure 2.1 /** Overview of literature on compression system modeling.

and axial compressors was developed by Spakovszky (2000). This low-order, analytical model describes the effect of unsteady radially swirling flows between the impeller and diffuser in centrifugal compressors and via its dedicated modular structure the possible significance of interblade row gap flow is accommodated.

In the Greitzer model, the pressure rise characteristics of the compressor and throttle are described by quasi-static maps that are lumped onto actuator disks. Oliva and Nett (1991) presented a nonlinear dynamic analysis of a reduced Greitzer model in which the pressure rise and throttle characteristics, assumed fixed by Greitzer (1976a), can vary.

Common practice in the literature was and still is to use an approximation of the pressure rise characteristic, for example the cubic polynomial suggested by Greitzer and Moore (1986). Gravdahl and Egeland (1999a) derived an analytical expression for the compressor map, based on compressor geometry and energy considerations.

In Figure 2.1 two more papers are mentioned. Mazzawy (1980) takes an entirely different approach in modeling high speed surge transients by focussing on the shock wave propagation through an axial compressor. These shock waves arise from the flow reversal during a surge cycle and despite the extremely short duration of these 'blast waves' ( $< 0.1$  s) it is argued that they cannot be neglected. This statement is repeated by Cargill and Freeman (1991) who studied how the process of shock wave propagation can be connected to other types of instability models.

### Modeling for control

A review of the early developments on nonlinear, two-dimensional compressor models is given by Longley (1994). In literature good qualitative and quantitative match of these models with experimental observations were reported and significant advancements have been made since the publication by Moore and Greitzer (1986). However, their complexity makes them less suitable for the application in active surge control design.

A similar comment can be made for the rather complex one-dimensional flow models that we discussed (e.g. Macdougall and Elder, 1983; Elder and Gill, 1985; Botros, 1994b; Badmus et al., 1995a). Their modular structure makes them very suitable for describing the dynamic behavior of large systems like power plants and compressor stations, at the cost of increased complexity. However, we point out that the control-oriented model has been successfully applied for controller design by Badmus et al. (1995a).

For the task of designing active surge controllers for centrifugal compression systems, the model of Greitzer (1976a) and the extended model by Fink et al. (1992) seem most appropriate. Although the inclusion of the equations for the rotor dynamics is rather straightforward, identification of the motor parameters and loss terms is far from trivial. Hence, we select the Greitzer model as a starting point for modeling the compression systems under study.

The Greitzer model provides a good qualitative description of the relevant phenomena and its simplicity facilitates the physical interpretation of the model parameters and their influence on the overall dynamics. Furthermore, the set of coupled ordinary differential equations (ODE) is not likely to cause computational problems when implemented in real-time. For further details on compression system models and their application in surge control design we refer to the extensive reviews by [Gravdahl and Egeland \(1999b\)](#) and [Willems and De Jager \(1999\)](#).

## 2.2 The Greitzer lumped parameter model

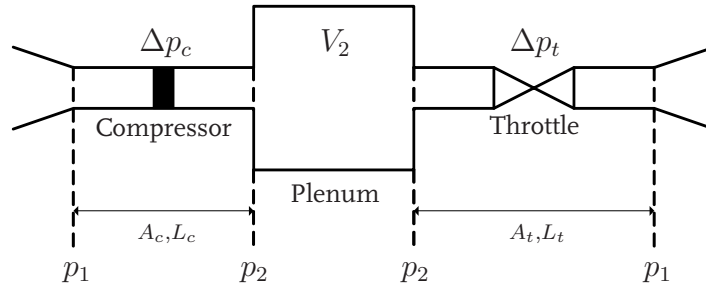
The Greitzer model is particularly suited for control-oriented modeling of compression systems as argued above. However, the benefits of the Greitzer model come at the price of some drastic simplifications that are introduced on pragmatic grounds, rather than strictly physical ones. In order to assess the validity and usability of the Greitzer model, we will therefore present a detailed derivation analogous to the approach followed by [Greitzer \(1976a\)](#). This detailed approach allows us to highlight and discuss the various assumptions, simplifications, and resulting limitations that enables a judicious use of the resulting model.

For this purpose we will first discuss the modeling concept and general assumptions. Then we will present the derivation of the various nonlinear differential equations that constitute the Greitzer model.

### 2.2.1 System boundaries and model assumptions

The model developed by [Greitzer \(1976a\)](#) describes the behavior of the gas in terms of mass flow and pressure rise as it moves through the compression system. The system boundaries are formed by the compressor internals, a throttle valve, the interconnecting plenum volume and other piping that confine the gas as it flows through the system. The model is graphically depicted in Figure 2.2 and we will now state the assumptions on which the mathematical model is based.

For a compression system the compressor and throttle are usually placed in ducts of much smaller diameter than the connecting plenum or discharge volume, indicating that the oscillations in the system can be modeled in a manner analogous to those of a Helmholtz resonator. This idea was first presented by [Emmons et al. \(1955\)](#) as we saw earlier. The above assumption concerning the geometry implies that the kinetic energy of the oscillations is associated with motion of the fluid in the compressor and throttle ducts, while the potential energy is associated with compression of the fluid inside the plenum.



**Figure 2.2** / Schematic representation of the Greitzer model.

We limit our study to those compression systems that have low inlet Mach<sup>2</sup> numbers and generate pressure rises which are small compared to the ambient pressure. However, no restrictions are placed on the amplitude of the oscillations in pressure rise and mass flow, compared to their steady-state values. Therefore, the essentially nonlinear behavior of the system is retained. Furthermore, we assume that the oscillations associated with compressor surge have quite a low frequency. From these two assumptions it follows that it is reasonable to consider the fluid flowing through the ducts to be incompressible, with the density taken equal to the ambient value. Moreover, we assume that the flow in the compression system can be considered to be one-dimensional.

The compressor and throttle ducts are replaced by actuator disks and a constant area pipe of a certain length. The actuator disks<sup>3</sup> account for the pressure rise and drop caused by the compressor and throttle, respectively and the pipes account for the dynamics of the fluid in the ducts. For comments on the use of actuator disks, see for example [Macdougall and Elder \(1983\)](#) and [Chernyshenko and Privalov \(2004\)](#). The length of these ducts should be chosen in such a way that they result in the same unsteady pressure difference for a given rate of change of mass flow as in the actual ducts. As will be shown below, this requirement yields

$$\left(\frac{L}{A}\right)_{model} = \int_{actual\ ducting} \frac{dl}{A(l)} \quad (2.1)$$

We point out that the introduction of a compressor duct as mentioned above is quite a natural simplification for axial compressors. However, for a centrifugal compressor the straight duct implies a much more drastic simplification of the actual compressor geometry, see for example [Figures 1.4 and 1.5](#).

<sup>2</sup>Ernst Mach (1838–1916) was a Czech physicist and philosopher. In 1877 he published the paper 'Photographische Fixierung der durch Projektile in der Luft eingeleiteten Vorgänge' in which he presented the first photographs of a shock wave caused by a supersonic projectile.

<sup>3</sup>In our context an actuator disk represents a discontinuous change in pressure while the mass flow remains continuous.

For the plenum we assume that the fluid velocity inside the plenum is negligible. Furthermore, the dimensions of the plenum are assumed to be smaller than the wavelength of acoustic waves with frequencies in the order of that associated with surge. Therefore, the static pressure is assumed to be uniform throughout the plenum. In addition, the processes in the plenum can be considered as being isentropic according to Greitzer (1976b). Finally, we assume that the overall pressure and temperature ratios for the compression system are near unity, yielding a trivial energy balance. Therefore, we will not consider the energy equation in the remainder of this chapter.

As a starting point for the derivation we apply the principles of mass and momentum conservation. As a result of the last assumption, the principle of energy conservation is not taken into account. The corresponding fundamental equations for a Newtonian<sup>4</sup> fluid are (e.g. Kundu, 1990)

$$\frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \mathbf{u} = 0 \quad (2.2)$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{u} + \left( \kappa + \frac{1}{3} \mu \right) \nabla (\nabla \cdot \mathbf{u}) \quad (2.3)$$

### 2.2.2 Compressor and throttle ducts

For an incompressible flow the term  $\rho^{-1} D\rho/Dt$  can be neglected, so Equation (2.2) reduces to the incompressible form  $\nabla \cdot \mathbf{u} = 0$ . Applying Equation (2.3) to the compressor duct and neglecting gravitational and viscous effects gives

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p \quad (2.4)$$

In the one-dimensional case this equation can be written as

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x}$$

and by using  $\nabla \cdot \mathbf{u} = 0$  (incompressible flow) this reduces to

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} \quad (2.5)$$

We now introduce  $\rho u A_c = \dot{m}_c$  where  $\dot{m}_c$  represents the mass flow through the compressor. Note that, due to the incompressibility assumption,  $\dot{m}_c$  is not depending on the

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<sup>4</sup>Sir Isaac Newton (1643–1727) was a British mathematician, physicist, alchemist, and natural philosopher who is generally regarded as one of the most influential scientists and mathematicians in history. In 1686 he published the *Philosophiae Naturalis Principia Mathematica*. One of the postulates in this monumental work is on the linear relation between shear stress and the velocity gradient perpendicular to the plane of shear, defining what we nowadays call a Newtonian fluid.

position in the duct. Substituting  $u = \dot{m}_c/(\rho A_c)$  in Equation (2.5), separating  $\partial x$  and  $\partial p$ , introducing the effect of the actuator disk, and integrating over the compressor duct yields

$$\frac{L_c}{A_c} \frac{d\dot{m}_c}{dt} = \Delta p_c(\dot{m}_c) - \Delta p \quad (2.6)$$

with  $\Delta p = p_2 - p_1$ . The forcing term  $\Delta p_c(\dot{m}_c)$  describes the pressure rise of the gas as it travels through the compressor. This so-called compressor characteristic will be discussed in some more detail in Sections 2.2.4, 2.4 and Chapter 3. The length  $L_c$  is the length of the duct and the area  $A_c$  is a reference area, usually taken equal to the area at the impeller inlet. Note that the boundaries of the compressor duct should be chosen in such a way that they incorporate all regions of the actual ducting in which the flow has significant kinetic energy.

For the throttle duct the same approach is used, yielding

$$\frac{L_t}{A_t} \frac{d\dot{m}_t}{dt} = \Delta p - \Delta p_t(\dot{m}_t) \quad (2.7)$$

where  $\Delta p_t(\dot{m}_t)$  is the throttle characteristic. This nonlinear term describes the pressure drop over the throttle as a function of the mass flow  $\dot{m}_t$  through the throttle.

### 2.2.3 Plenum

For the plenum we have assumed that fluid velocities are negligible, the pressure is uniformly distributed and gravitational and viscous effects can be neglected. In that case, the momentum equation (2.3) reduces to the trivial equality  $0 = 0$ . We now write the law of mass conservation in integral form

$$\int_V \frac{\partial \rho}{\partial t} dV = - \int_A \rho(\mathbf{n} \cdot \mathbf{u}) dA \quad (2.8)$$

A detailed derivation is provided in Appendix A.1. We point out that for a control volume with  $i$  ports,  $-\int_A \rho(\mathbf{n} \cdot \mathbf{u}) dA$  can be written as  $-\sum \dot{m}_i$  where  $\dot{m}_i$  is positive when it leaves the control volume.

From Appendix A.2 we obtain the expression for the speed of sound

$$c^2 = \left( \frac{\partial p}{\partial \rho} \right)_s \quad (2.9)$$

By using Equation (2.9) and the assumption that the process inside the plenum is isentropic, we can write

$$\frac{\partial \rho}{\partial t} = \frac{1}{c^2} \frac{\partial p}{\partial t} \quad (2.10)$$

which gives the following mass balance for the plenum volume

$$\frac{V_2}{c^2} \frac{dp_2}{dt} = \dot{m}_c - \dot{m}_t \quad (2.11)$$

We point out that both mass flows on the right-hand side of Equation (2.11) are given by separate momentum equations. However, the momentum equation for the throttle duct is often neglected because in practice  $L_t \ll L_c$  (e.g. Fink et al., 1992, Willems, 2000). When inertial effects in the throttle duct are neglected, the mass flow  $\dot{m}_t$  is no longer a state in the model. Instead, it can be described by a static mapping from  $\Delta p_t$  to  $\dot{m}_t$  or in other words, by a static throttle characteristic.

### 2.2.4 Transient compressor response

The momentum equation (2.6) contains the forcing term  $\Delta p_c(\dot{m}_c)$ . This compressor characteristic describes the highly nonlinear relation between pressure rise over and mass flow through the compressor. The assumption in previous linearized analyses was that this relation is the same transiently as it is in steady-state operation. However, Greitzer (1976a) states that the compressor does not respond quasi-steadily to changes in mass flow in practice. More specifically, experiments showed that there is a definite time lag between the onset of compressor instability and the establishment of a fully developed rotating stall pattern.

The mentioned time lag is considered long enough to allow a significant change in compressor mass flow during the development of stall cells. To account for the deviations from quasi-steady behavior, a first order dynamic model is proposed

$$\tau \frac{d\Delta p_c}{dt} = (\Delta p_{c,ss}(\dot{m}_c) - \Delta p_c) \quad (2.12)$$

with  $\Delta p_{c,ss}$  denoting the steady-state compressor characteristic. The time constant  $\tau$  can be obtained by measuring the time needed for the stalled mode to fully develop itself.

## 2.3 Model adjustment and scaling

In the previous section we presented the derivation of the dynamic equations that together form the so-called Greitzer model. We will now discuss how we can make the model better suited for describing different compression systems, including the centrifugal compressor test rigs that are studied in this thesis. Firstly, we will present a modification of the momentum equation for the compressor duct to account for velocity variations in the compressor duct that are the result of a spatially varying cross-sectional area. Secondly, we address the commonly applied aerodynamic scaling of the model equations.

This scaling allows for a better comparison of results from different machines or for different operating conditions.

### 2.3.1 Variable cross-sectional area

The Greitzer model is based on the assumption that the flow in the ducts can be considered incompressible. This assumption implies that all the fluid in these ducts will have the same axial velocity. However, by applying Bernoulli's law to the converging and diverging passages (e.g. diffuser, inlet, and outlet channels) inside a centrifugal compressor, we easily see that variations of fluid velocity can be significant. For a steady flow at constant mass flow, the effect of a varying area on the flow velocity follows directly from  $\dot{m}_c = \rho u A_c$ . Hence, the derivation presented in Section 2.2.2 is not accurate for a typical compressor where area variations of 40% are not uncommon.

Therefore, we propose a slightly different derivation of the momentum equation for the compressor duct. We define the mass flow through the compressor as  $\dot{m}_c = \rho(x, t) u(x, t) A_c(x, t)$ . So, in general mass flow can be a function of both position and time. However, we assume that the fluid in the ducts can be considered to be incompressible as we have done before, implying that the density is constant. Furthermore, we assume that changes in fluid velocity  $u(x, t)$  are only caused by a change in the area  $A_c(x)$  such that the product of  $u(x, t) A_c(x)$  is a function of time only. As a result, the mass flow  $\dot{m}_c(t)$  is also independent of the position  $x$ .

We now reformulate the one-dimensional momentum equation for the compressor duct to get

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = - \frac{\partial p}{\partial x}$$

or

$$\rho \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( \frac{1}{2} \rho u^2 \right) = - \frac{\partial p}{\partial x}$$

Note the subtle difference with the derivation of the original Greitzer model where we used  $\nabla \cdot \mathbf{u} = 0$  to cancel the second term on the left-hand side, see Section 2.2.2. We now use the fact that mass flow however, is not depending on the position  $x$ , so we can write

$$\frac{1}{A_c(x)} \frac{d\dot{m}_c}{dt} = - \frac{\partial \left( p + \frac{1}{2} \rho u^2 \right)}{\partial x} \quad (2.13)$$

where  $p + \frac{1}{2} \rho u^2 = p_o$  is the stagnation pressure. After separation of variables, integration over the flow path  $x$  through the compressor duct gives

$$\int_x \frac{dx}{A_c(x)} \frac{d\dot{m}_c}{dt} = \Delta p_{o,c}(\dot{m}_c) - \Delta p_o \quad (2.14)$$

with  $\Delta p_o = p_{o,2} - p_{o,1}$  the pressure difference between the outlet and inlet of the duct. The source term  $\Delta p_{o,c}(\dot{m}_c)$  again represents the pressure rise over the compressor, only this time the characteristic is expressed in terms of stagnation pressure.

Although the above modification is most useful to account for variations in the cross sectional area of the compressor duct, it can be used to take into account diameter changes of the connected piping system as well. This feature will be used in Chapter 4 where we discuss a specific compression system equipped with stagnation pressure probes and that has inlet and discharge pipes with different diameters.

### 2.3.2 Aerodynamic scaling

In general, aerodynamic scaling is used to allow for a better comparison between different turbo machines. A choice of appropriate scaling factors depends for example on the type of machine and the relative importance of the different variables involved. Various suitable scaling methods are discussed by [Cumpsty \(1989\)](#). Next to enabling the comparison between different machines, scaling has another advantage. By choosing appropriate scaling factors the number of relevant parameters in a mathematical model can be reduced. In other words, it are usually not the individual parameters but (dimensionless) groups of parameters that determine the dynamic behavior.

For the Greitzer model it is common practice to reformulate the differential equations in non-dimensional form. For this purpose mass flow is scaled with the quantity  $\rho_1 U_e A_c$ , pressure difference with  $\frac{1}{2} \rho_1 U_e^2$  and time by using the characteristic time  $1/\omega_H$  as was done by [Greitzer \(1976a\)](#). The Helmholtz frequency  $\omega_H$  is defined as

$$\omega_H = c_2 \sqrt{\frac{A_c}{V_2 L_c}} \quad (2.15)$$

Note that the dimensionless mass flow  $\phi = \dot{m}_c / (\rho U_e A_c)$  is equivalent to the flow coefficient  $u/U_e$ . We also point out that the dimensionless pressure difference  $\psi = \Delta p / (\frac{1}{2} \rho U_e^2)$  provides a measure of the actual input to the potential work available, i.e., to the squared impeller tip speed  $U_e^2$ . With these scaling factors it can be shown that for low Mach numbers the impeller performance is almost independent of its rotational speed. The dimensionless time  $\xi = t \omega_H$  is related to the period time of the Helmholtz resonator.

Applying the mentioned scaling factors gives the following dimensionless equations

$$\frac{d\phi_c}{d\xi} = B (\Psi_c - \psi) \quad (2.16)$$

$$\frac{d\phi_t}{d\xi} = \frac{B}{G} (\psi - \Psi_t) \quad (2.17)$$

$$\frac{d\psi}{d\xi} = \frac{1}{B} (\phi_c - \phi_t) \quad (2.18)$$

$$\frac{d\Psi_c}{d\xi} = \frac{1}{\varsigma} (\Psi_{c,ss} - \Psi_c) \quad (2.19)$$

Next to the dimensionless compressor time constant  $\varsigma$ , the above equations also contain the following two dimensionless parameters

$$B = \frac{U_e}{2\omega_H L_c} = \frac{U_e}{2c_2} \sqrt{\frac{V_2}{A_c L_c}} \quad (2.20)$$

and

$$G = \frac{L_t A_c}{L_c A_t} \quad (2.21)$$

Furthermore, the dimensionless time constant  $\varsigma$  can be related to  $B$  in the following way

$$\varsigma = \frac{\pi R N}{L_c B} \quad (2.22)$$

Finally, we remark that the coupled nonlinear equations Equations (2.16)–(2.19) contain three parameters ( $B$ ,  $G$  and  $\varsigma$ ) while there are four parameters in the full dimensional equations, i.e.  $A_c/L_c$ ,  $A_t/L_t$ ,  $c_2^2/V_2$  and  $\tau$ .

As mentioned before in Section 2.2.3 the parameter  $G$  is usually small enough to be neglected. The relevance of the dimensionless time constant  $\varsigma$  is discussed below. The most important dimensionless parameter is  $B$  and in Chapters 5 and 6 we will discuss this parameter in more detail.

## 2.4 Discussion

We recall that, while Greitzer (1976a) developed his nonlinear dynamic model for axial compressors, Hansen et al. (1981) successfully applied the model to predict deep surge behavior in a small single-stage centrifugal compressor. In the literature many more successful applications of the Greitzer model for analysis and control of the dynamics in centrifugal compression system were reported (e.g Macdougall and Elder, 1983; Ffowcs Williams and Huang, 1989; Pinsley et al., 1991; Badmus et al., 1995a; Arnulfi et al., 1999; Willems, 2000).

The extensive usage of this model illustrates the success of the lumped parameter approach in modeling the dynamic behavior of compression systems. One of the reasons for this success is that the approach results in relatively low order models, a feature that is beneficial both for numerical implementation and model-based controller design. However, the developed model has some inherent limitations that we will discuss here.

### Compressor performance

The pressure rise over and mass flow through the compressor at a given rotational speed are represented in the Greitzer model by a compressor characteristic. This compressor characteristic consists of a collection of working points at steady operating conditions and their extrapolation into the unstable flow regime. The exact nonlinear relation between pressure rise and mass flow is determined by the different energy losses (e.g. incidence, friction, mixing, blade loading losses) that arise between the points at which the pressure difference is measured.

In Section 2.2.4 we presented the dynamic correction of the compressor characteristic during unsteady transients as proposed by Greitzer (1976a). The time constant for this dynamic equation is related to the time lag between the onset of instability and a fully developed stall pattern. Hence, for a compression system where no significant rotating stall occurs, the dynamic correction is not applicable.

A different use of this time constant, namely to match theoretical predictions with experimental data of a centrifugal compressor, was discussed by Hansen et al. (1981) and Mazzawy (1981). The relation between the quasi-steadily and the unsteady compressor characteristic was investigated in detail by Meuleman (2002). However, the relevance of accounting for the unsteadiness of the compressor characteristic in a control-oriented compressor model is still an open issue.

Another problem related to the compressor characteristics is the fact that the unstable parts of these performance curves are unknown as mentioned above. To describe the unstable surge behavior of a compression system, the Greitzer model needs an approximation of the entire compressor curve. Unfortunately, measuring the unstable characteristics is not possible when the system operates in surge. Various approaches have been proposed in literature to circumvent this problem but the accuracy of the suggested polynomial or analytical (Gravdahl and Egeland, 1999a) approximations is unknown. In Chapter 3 we address the approximation of the compressor characteristics in more detail.

### Lumped parameter approach

The adopted lumping approach limits the applicability of the model to those compressor configurations that fit in the general structure depicted in Figure 2.2. Unfortunately, this

structure does not always apply to large industrial compression systems. For example, in many of these systems the compressor outlet is connected to a long discharge pipe rather than to a distinct plenum volume. For systems with long discharge lines phenomena of a distributed nature like traveling pressure waves cannot be neglected anymore (Sparks, 1983; Feulner et al., 1996). We will come back to these effects in long pipes in Chapter 4.

In general, lumped parameter models use a limited set of parameters to describe the dynamics of the actual system under study, see also the discussion above on the compressor characteristic. As a consequence, these parameters must be known accurately because large errors will most likely have a large effect on the overall model accuracy. Hence, careful determination of the parameters in the Greitzer model is needed. The issue of parameter accuracy will be addressed in subsequent chapters.

Another issue related to lumped parameter models is the required level of detail for the model in order to describe all the phenomena of interest. The coarse division of the entire compression system used in the Greitzer model, potentially neglects localized dynamics of interest in either the compressor duct or the plenum. Furthermore, the model structure complicates the inclusion of information from sensors and the effect of actuators that are placed inside the compressor duct.

These limitations reveal the significance of representing the complex internal compressor geometry with a straight duct of constant diameter. As a result of this drastic simplification local speed variations inside the compressor cannot be accounted for by the Greitzer model unless local measurements are available. However, introducing the sensor information into the model in a meaningful way requires a division of the compressor duct into multiple elements.

In literature, various models are proposed that use a more detailed division of the compression system in individual components (e.g. Elder and Gill, 1985; Badmus et al., 1995a; Spakovszky, 2000; Venturini, 2005). In subsequent chapters we come back to the question on the required level of detail when using a lumped compressor model to describe surge dynamics and to design an active surge controller.

## Compressibility

At this point we recall that the Greitzer model assumes an incompressible medium in the compressor and throttle ducts. According to Kundu (1990), compressibility effects are negligible when  $M^2 \ll 1$ , see also Appendix A. Hence, for industrial compression systems where Mach numbers are usually well above 0.3 to increase efficiency, the incompressibility assumption is invalid. The validity of the incompressibility assumption was questioned among others by Hansen et al. (1981) and Ishii and Kashiwabara (1996) whereas Day and Freeman (1994); Day (1994) and Gu et al. (1999) stressed the importance of compressibility effects in high-speed machines.

Unfortunately, analytical solutions of the unsteady compressible flow equations do not exist. Despite the advancements in the field of computational fluid dynamics (CFD), the available CFD approximations are generally of a high order and the computational time lies in the range of hours. This (computational) complexity makes CFD models ill-suited for application in active control design.

Over the years several authors have proposed relatively low order approximations of the general flow equations (e.g. Botros et al., 1991; Badmus et al., 1995a; Feulner et al., 1996; Ishii and Kashiwabara, 1996; Spakovszky, 2000). Despite the promising results obtained with the compressible flow models mentioned here, these models are still complex and/or difficult to solve in comparison with the Greitzer model.

In subsequent chapters we will use the Greitzer model as the starting point for our attempt to determine and describe the relevant dynamics of the centrifugal compression systems under study. Throughout our analyses we will keep in mind the limitations of the Greitzer model that we discussed here.

# Centrifugal compression system model

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**Abstract** / In this chapter a lumped parameter model is discussed that describes the dynamic behavior of a centrifugal compression system. The experimental setup and the model development are treated in detail, as well as the selection of appropriate values for model parameters and component characteristics. Results from extensive validation are presented, illustrating that the model captures the essential dynamics during surge transients of the system. Finally, various modifications and extensions of the model are evaluated.

## 3.1 Introduction

An important aspect of the research presented here is the implementation and evaluation of theoretical results in practice. This chapter deals with the application of the modeling approach from the previous chapter to describe the dynamic behavior of a specific centrifugal compression system.

Greitzer (1976b) evaluated the lumped parameter model that forms the basis of our models, by comparing it with experimental results. The experimental program was carried out on a three-stage axial compressor facility with a specially designed plenum chamber. The experimental and theoretical study of surge on a small single-stage centrifugal compressor by Hansen et al. (1981) showed that the Greitzer model can also be applied to describe surge transients in centrifugal compressors. This conclusion is further supported by more recent successes of applying similar lumped parameter models to describe surge transients in a radial turbocharger (e.g. Willems, 2000; Gravdahl et al., 2004) or an industrial four-stage centrifugal compressor (Arnulfi et al., 2006).

As we will discuss below, the centrifugal compression system under study is operating in a closed circuit. A comparable test rig configuration was used by Nakagawa et al. (1994) to investigate active surge suppression in both open and closed circuit configurations. However, for the development of a lumped parameter model that was needed for controller design, they only considered the open-loop configuration and validation results were not

presented. Hunziker and Gyarmathy (1994); Rose et al. (2003) investigated the effect of subcomponents on overall stability and the flow patterns during surge, respectively in another closed circuit test facility of comparable dimensions. However, no attempt was made to describe the experimental data with a lumped parameter model.

Kurz and White (2004) investigated the performance of a surge avoidance scheme during emergency shutdown of a compressor station operating in the field. The reported work has practical relevance given the dimension and operating conditions that were used in the simple lumped parameter model that was adopted. Unfortunately, a careful validation of the developed model on experimental data was not included. A detailed study of surge transients in a realistic compression station was conducted by Botros (1994b), using a more involved but still relatively simple model.

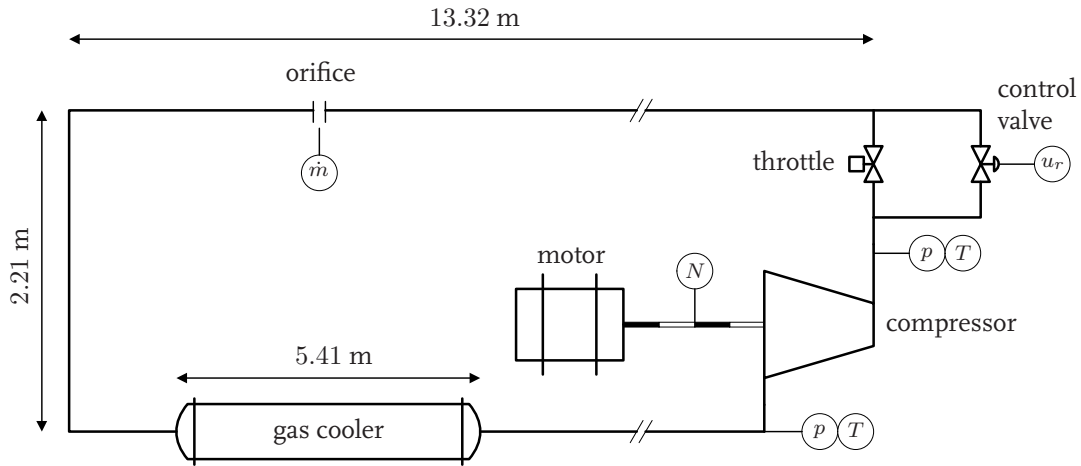
The main contribution of this chapter is the experimental identification and validation of a lumped parameter model for one of the industrial scale compressor test rigs used in our research. We first introduce the experimental setup after which we address the development of a lumped parameter model for this system. We will pay special attention to the way in which values for the various model parameters and component characteristics are obtained. Then, validation results are presented that show a good agreement between simulation and measurement data and that provide justification for the various model assumptions. Finally, various modifications and extensions for the developed model are evaluated and discussed.

## 3.2 Experimental setup

The first of two systems under study is a single stage centrifugal compressor rig that is normally used to test industrial compressors for the oil and gas industry. Throughout this thesis we will denote the system discussed here as *Test rig A*. The whole installation is schematized in Figure 3.1. The compressor is driven by an 1.7 MW electric motor that is connected to the shaft through a gearbox. The rotational speed of the compressor can be varied between 6,000 and 16,000 rpm.

The compressor operates in a closed circuit that makes it possible to use different pure gases or gas mixtures. Furthermore, this configuration allows for varying the average pressure in the system between 1 and 15 bar. All results presented in this chapter were obtained from experiments with  $N_2$  gas ( $28.0134 \cdot 10^{-3}$  kg/mol), an average suction pressure of 10 bar and at rotational speeds between 9,000 and 16,000 rpm.

Throttling of the compressor is done by means of a butterfly valve (Wouter Witzel DN 300). The parallel control valve (Masoneilan Varipack 28000) is used for more precise adjustments of the mass flow rate. Further details for the control valve can be found in Appendix C. The return piping contains a measurement section with a flow straightener,



**Figure 3.1** / Scheme of compressor test rig A with sensor locations:  $N$  = rotational speed,  $p$  = pressure,  $T$  = temperature,  $\dot{m}$  = mass flow,  $u_r$  = valve opening.

an orifice flow meter with a diameter ratio of 0.533, and a gas cooler. We point out that the configuration with a closed circuit is not common for industrial applications where the inlet and discharge side of the compressor are physically decoupled. However, with the combination of a throttle, return piping, and cooler, the conditions of the gas at the compressor inlet were kept approximately constant and thereby effectively decoupling the inlet and discharge side of the system.

On the compressor shaft a shrouded impeller is mounted that contains 17 blades and has hub and exit diameters of 145 mm and 284 mm, respectively. The constant width diffuser is equipped with 10 low solidity vanes and has inlet and exit diameters of 287 mm and 450.9 mm, respectively. The complete aerodynamic package, consisting of the impeller, diffuser, inlet and return channels, is installed in a wide casing that offers the flexibility to test a variety of assemblies and to install additional measurement equipment, see also Figures 3.2 and 3.3. For the presented configuration, the wide casing results in reasonably large additional volumes before and after the compressor that should be taken into account. The most important characteristics of the installation are summarized in Table 3.1. In the next section we will come back to the selection of the values for the system parameters.

### Measurement equipment

The compression installation is equipped with numerous temperature probes (J-type thermocouple) and static pressure transducers (Rosemount) to determine the steady-state performance of the compressor, see Figure 3.1. The steady mass flow is measured with an orifice mass flow meter that uses a combination of static (differential) pressure and temperature measurements in accordance with ISO 5167-2 (2003). Accuracy of the mass

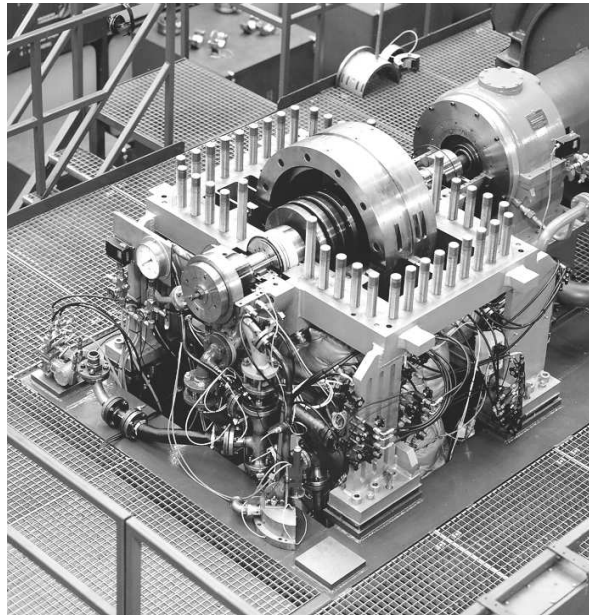


Figure 3.2 / Photograph of the centrifugal compressor with top casing removed.

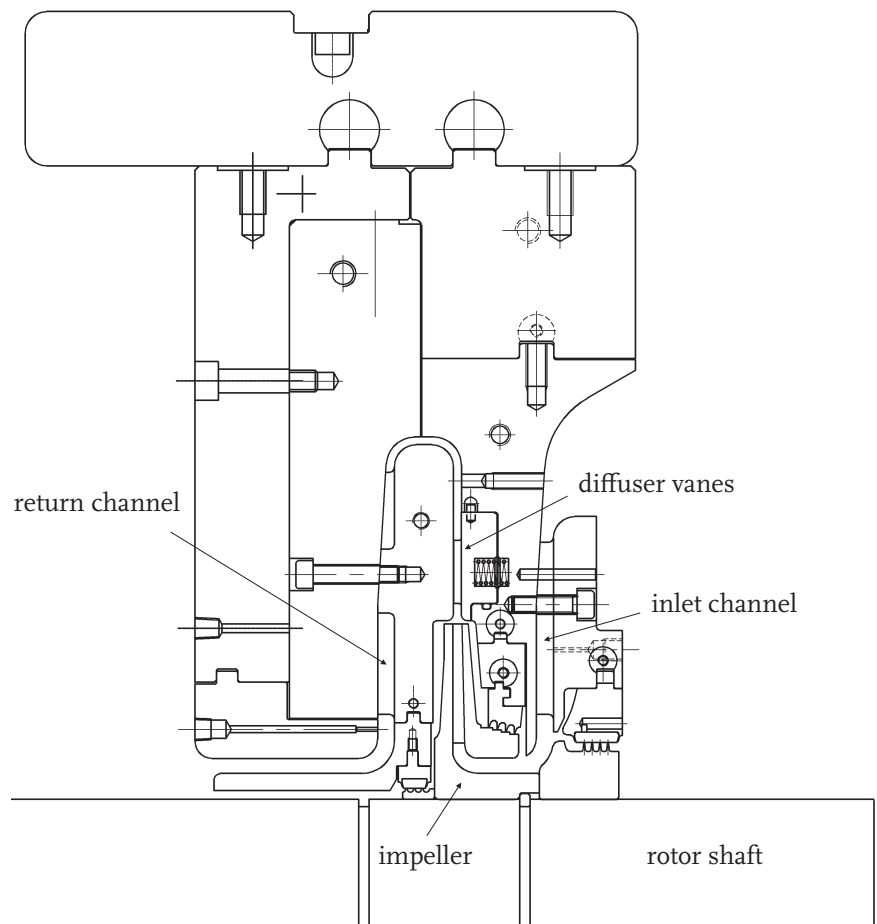


Figure 3.3 / Aerodynamic components of the centrifugal compressor.

**Table 3.1** / Parameters of compressor test rig A.

| Element  | Parameter                         | Value  | Unit           |
|----------|-----------------------------------|--------|----------------|
| Impeller | number of blades                  | 17     | -              |
|          | inducer diam. at hub $d_{i,h}$    | 0.145  | m              |
|          | inducer diam. at shroud $d_{i,s}$ | 0.1592 | m              |
|          | exit diameter $d_e$               | 0.284  | m              |
|          | exit width                        | 0.0043 | m              |
| Diffuser | number of vanes                   | 10     | -              |
|          | inlet diameter                    | 0.287  | m              |
|          | outlet diameter                   | 0.450  | m              |
|          | width                             | 0.0036 | m              |
| Valves   | throttle duct area $A_t$          | 0.071  | m <sup>2</sup> |
|          | throttle duct length $L_t$        | 0.080  | m              |
|          | control valve area $A_r$          | 0.002  | m <sup>2</sup> |
|          | control valve length $L_r$        | 0.003  | m              |
| System   | compressor duct area $A_c$        | 0.0034 | m <sup>2</sup> |
|          | compressor duct length $L_c$      | 0.300  | m              |
|          | suction volume $V_1$              | 3.1    | m <sup>3</sup> |
|          | suction piping length $L_1$       | 29     | m              |
|          | discharge volume $V_2$            | 0.32   | m <sup>3</sup> |
|          | discharge piping length $L_2$     | 2.7    | m              |

flow measurements is  $\pm 3.5\%$ . The rotational speed of the impeller is measured through a pulse counter with a resolution of 1 pulse per revolution. A dedicated data-acquisition and control system is used for operating the installation, converting and recording sensor outputs, and for online monitoring. The sample time for all static measurements is 5 s.

Additional dynamic pressure transducers (Kulite) are installed in the suction and discharge pipes to measure the pressure rise fluctuations during experiments. Accuracy of these gauge pressure transducers is  $\pm 5.8\%$  with respect to a full range of 0–10 bar. The natural frequency of the pressure transducers is 175 kHz.

For data-acquisition a stackable system (SigLab) with a maximum of 16 input and 8 output channels, anti-aliasing filters and A/D convertors is used. This system is also used to operate the control valve during the experiments. All the connected temperature and dynamic pressure signals are measured at a sampling rate of 1.28 kHz. Reliable transient mass flow measurements are not available.

### 3.3 Lumped parameter model

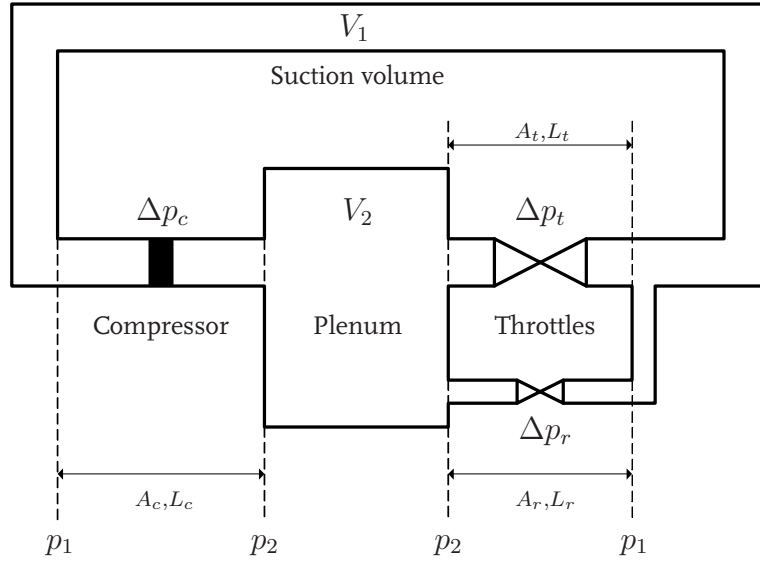
In this section we discuss the development of a dynamic model for the transient behavior of the compression system under study. First we introduce the equations for the nonlinear lumped parameter model, analogous to the original Greitzer model that we discussed in Section 2.2. Then, we explain how the various (geometric) model parameters can be obtained. Finally, we address the nonlinear compressor and throttle characteristics in detail and we will discuss how appropriate representations for these characteristics are obtained.

#### 3.3.1 Model equations

In order to describe the dynamic behavior of the compression system we used a lumped parameter model, following the geometry of Figure 3.4. The difference between this model and the original Greitzer model in Figure 2.2 is that the throttle exits are now connected to the compressor inlet through a suction volume. The suction volume represents the combined volume of the return piping and the gas cooler, see also Figure 3.1.

Following the same approach and using the same assumptions as discussed in Section 2.2 we apply the principles of mass and momentum conversation to the model in Figure 3.4, yielding

$$\frac{L_c}{A_c} \frac{d\dot{m}_c}{dt} = \Delta p_c(\dot{m}_c, N) - \Delta p \quad (3.1)$$



**Figure 3.4** / Schematic representation of the lumped parameter model for test rig A.

$$\frac{V_1}{c_1^2} \frac{dp_1}{dt} = \dot{m}_t(\Delta p, u_t) + \dot{m}_r(\Delta p, u_r) - \dot{m}_c \quad (3.2)$$

$$\frac{V_2}{c_2^2} \frac{dp_2}{dt} = \dot{m}_c - \dot{m}_t(\Delta p, u_t) - \dot{m}_r(\Delta p, u_r) \quad (3.3)$$

where  $\Delta p = p_2 - p_1$  denotes the pressure difference between the suction and discharge volumes. Note that we included the dependency of the compressor characteristic  $\Delta p_c$  on the impeller speed  $N$ . Furthermore, we have indicated that both the throttle and control valve characteristics,  $\dot{m}_t$  and  $\dot{m}_r$ , depend on their respective valve opening  $u_t$  and  $u_r$ .

Substituting the parameters from Tables 3.1 and 3.2 in the expression for the dimensionless parameter  $G$  from Equation (2.21) yields  $G \approx 0.01$ . Hence, the inertial effects in the throttle duct are neglected in the above model, see also Section 2.2.3. Inertial effects in the control valve are neglected since the control valve capacity is small ( $< 10\%$ ) in comparison with the capacity of the main throttle.

Additionally, in contrast to the original Greitzer model, we neglect the time lag associated with rotating stall development, following the arguments of Mazzawy (1981); Willems (2000). Therefore, we also omit the time lag in the compressor response, described by Equation (2.12), from the current model. We will come back to this omission in the next section.

Furthermore, note that the resulting model contains an additional mass balance for the suction side volume. However, it can easily be seen that the mass balances in Equation (3.2) and (3.3) are not independent because the total mass within the system is constant. Therefore, we can combine Equations (3.2) and (3.3) to obtain a single differential equation for the pressure difference  $\Delta p$  over the two volumes.

Making use of the scaling factors from Section 2.3.2 and rewriting the impulse balance and combined mass balance then gives

$$\frac{d\phi_c}{d\xi} = B (\Psi_c(\phi_c) - \psi) \quad (3.4)$$

$$\frac{d\psi}{d\xi} = \frac{F}{B} (\phi_c - \phi_t(\psi, u_t) - \phi_r(\psi, u_r)) \quad (3.5)$$

with  $\psi$  denoting the dimensionless pressure difference. Note that next to the Greitzer stability parameter  $B$ , a new dimensionless parameter  $F$  is introduced. The parameter  $F$  is defined as

$$F = 1 + \frac{Z_1 T_1 V_2}{Z_2 T_2 V_1} \quad (3.6)$$

where we have used the definition for the speed of sound  $c_i^2 = \gamma_i Z_i R T_i$ . Furthermore, we assumed that  $\gamma_1 = \gamma_2$  and that the temperatures  $T_1$  and  $T_2$  in both volumes are constant. The parameter  $F$  accounts for the coupling between the discharge and suction side volumes. When  $V_1$  becomes infinitely large, the original Greitzer model is obtained.

### 3.3.2 Model parameters

In order to describe the dynamic behavior of the centrifugal compression system under study, appropriate values must be assigned to the various model parameters. The geometric parameters are already given in Table 3.1. The value for the compressor duct area  $A_c$  is chosen equal to the frontal area of the impeller eye  $\frac{1}{4}\pi(d_{i,s}^2 - d_{i,h}^2)$ . The compressor duct length  $L_c$  is used as a tuning parameter in the model to match simulation results with experimental data, following a similar approach as Willems (2000). In the next section we show that good results are obtained with the tuned value for  $L_c$  from Table 3.1. The other parameters are obtained directly from the dimensions of the test rig. In Chapter 6 we will address the selection of appropriate model parameters in more detail.

Furthermore, the full-dimensional mass balance contains the sonic velocities in both volumes  $V_1$  and  $V_2$  where the sonic velocity is defined as  $c = \sqrt{\gamma Z R T}$ , see also Appendix A. The gas properties  $\gamma$  and  $Z$  depend on both pressure and temperature so either a reference value or the momentary pressure and temperature are required to calculate those properties. Since temperature variations are not included in the model, it makes sense to select constant temperature values  $T_1$  and  $T_2$  in both volumes, for example from averaged (steady-state) measurements. The same approach can be followed to select appropriate reference pressures. When both a pressure and temperature are available, the aforementioned gas properties are obtained from gas table data provided by Sychev et al. (1987), using a spline interpolation when required.

We remark that the use of a constant reference pressure is not always the most logical choice. When the compression system operates in surge large pressure oscillations

occur, in particular in the discharge volume. In such a case, a more realistic choice would be to use the momentary pressure value within the discharge volume. Since the suction and discharge pressure cannot be calculated directly from Equations (3.2) and (3.3) an additional equation is needed to solve for  $p_1$  and  $p_2$ . This equation is obtained by combining the mass balance for both volumes ( $\Delta m_1 = -\Delta m_2$ ) with the ideal gas law (Shavit and Gutfinger, 1995) and using steady-state data to determine the total mass inside the entire system.

Another way to determine the momentary plenum pressure is to select a constant reference pressure  $p_1$  and adding this value to the calculated pressure difference  $\Delta p$  to obtain a value for  $p_2$ . The assumption of a constant suction pressure  $p_1$  can be justified by noting that  $V_1 \approx 10V_2$ . We will use this approach to estimate  $p_2$  in the model equations for the valve characteristic that will be discussed in Section 3.3.4.

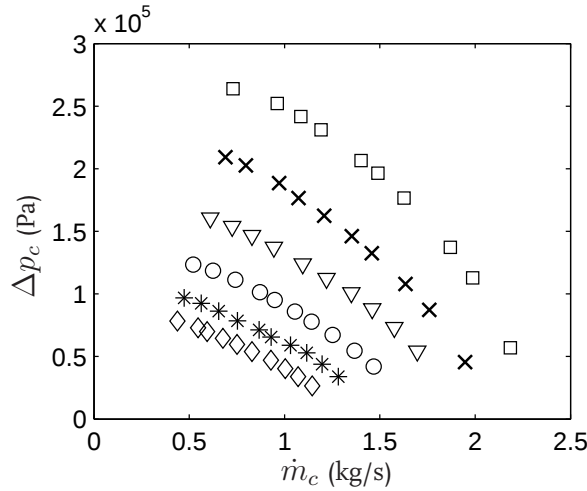
Finally, the aerodynamic scaling factors that were introduced in Section 2.3.2 contain two other parameters,  $U_e$  and  $\rho$ . The impeller tip speed follows directly from  $U_e = \frac{\pi d_e N}{60}$ , with  $d_e$  the impeller exit diameter. The reference density  $\rho$  is obtained from the mentioned gas table data, using appropriate reference values for  $p_1$  and  $T_1$  that can be obtained from steady-state measurements. The remaining model parameters are the compressor and throttle characteristics and they will be discussed in more detail below.

### 3.3.3 Compressor characteristic

An important element in the derived model is the compressor characteristic  $\Delta p_c(\dot{m}_c, N)$ . Greitzer (1976a) used a relaxation equation to obtain  $\Delta p_c$  from the steady-state compressor characteristic, see also Section 2.2.4. As stated before, we have omitted this relaxation equation in the model and hence we assume that  $\Delta p_c = \Delta p_{c,ss}$ .

The stable part of the steady-state compressor characteristic can be measured for different rotational speeds by throttling down the compressor towards the surge line and measure the mass flow and pressure difference while keeping the speed constant. To measure the true steady-state performance of the compressor, time between measurements must be sufficiently long to allow the system to reach a thermodynamic equilibrium. The measured compressor map for the system under study is shown in Figure 3.5. Note that, in contrast to Greitzer (1976b), the compressor map shows no evidence of rotating stall. The absence of rotating stall was confirmed by pressure measurements inside the machine, justifying the approach to leave out the relaxation equation.

The next step is to develop an analytical or numerical representation for the term  $\Delta p_c$ . To completely specify the behavior of the compressor, the characteristic must be defined for both the stable and unstable regime. However, steady-state measurements cannot be performed in the unstable flow regime so an approximation is required.



**Figure 3.5** / Steady-state compressor map of test rig A;  $\diamond$  7,806 rpm,  $*$  8,914 rpm,  $\circ$  10,052 rpm,  $\nabla$  11,484 rpm,  $\times$  13,369 rpm,  $\square$  15,038 rpm.

Hansen et al. (1981) described the compressor characteristic with three different polynomial approximations: a cubic polynomial for the stable regime, fitted on available steady-state data, a quadratic polynomial for the negative flow regime, fitted on data from reversed flow experiments with externally supplied compressed air, and finally a cubic polynomial for the unstable positive flow regime, connecting the other parts of the curve.

Justified by numerical and experimental results, Moore and Greitzer (1986) suggested a single cubic polynomial to describe the entire compressor map. This approach has been successfully applied by, for example, Meuleman et al. (1998) and also by Pinsley et al. (1991); Badmus et al. (1996) who used similar polynomial approximations. In contrast, Botros (1994b) and Macdougall and Elder (1983) combined data fitting with results from the polytropic analysis of centrifugal compressors by Schultz (1962) to describe the compressor characteristic.

Gravdahl and Egeland (1999a) introduced an analytical method to approximate the compressor characteristic on the basis of energy considerations and geometry of the compressor. Their method is based on modifying the ideal characteristic by including incidence and friction losses. Experimental validation and further refinements of this approach were presented by Gravdahl et al. (2004).

Essentially, all the mentioned methods describe the same structure for the compressor characteristic, namely a quadratic curve for stable flows and a quadratic resistance curve for negative flows that are connected through a continuous curve for unstable positive flows. A requirement for the unstable positive flow part of the curve is that it has a positive slope as pointed out by Greitzer (1976a). Furthermore, we remark that the existence of a secondary or stalled curve can introduce discontinuous jumps in pressure rise, even though the steady-state characteristic is assumed continuous for unstable flows. Finally,

we point out that the choking effect at high mass flows causes the quadratic curve to fall off steeply beyond the choking mass flow.

Given its successful application and the simple calculations involved, we select the cubic polynomial from [Moore and Greitzer \(1986\)](#) to describe the term  $\Delta p_c$  in the model for test rig A. This polynomial is of the following form

$$\Psi_c(\phi_c) = \Psi_c(0) + H(N) \left[ 1 + \frac{3}{2} \left( \frac{\phi_c}{W(N)} - 1 \right) - \frac{1}{2} \left( \frac{\phi_c}{W(N)} - 1 \right)^3 \right] \quad (3.7)$$

The parameters  $\Psi_c(0)$ ,  $H$ , and  $W$  can be determined from steady-state measurements of the compressor characteristic and subsequently interpolated by polynomials in  $N$  ([Willemis, 2000](#)). However, applying this procedure to the measured speed lines, which are rather steep near the surge line, resulted in characteristics with a non-positive slope for all  $\phi_c$ . This would imply that the compression system is stable for any given mass flow, which is obviously not the case. Therefore, we use different approximations for the stable and unstable parts of the characteristics, yielding piecewise continuous compressor characteristics. Details are given in [Appendix C](#).

The resulting compressor characteristics are shown in [Figure 3.6](#) together with the measured steady-state data. The curves are also depicted in full dimensions in [Figure 3.7](#). Comparing these figures clearly illustrates the reduced dependency on rotational speed as an effect of the applied aerodynamic scaling. However, given the high operating speeds—the Mach number at the impeller tip ranges from 0.32 to 0.62—aerodynamic scaling does not entirely remove the speed dependency. Finally, differences between data and approximations, in particular around the top of the curves, are caused by the speed variations encountered during the measurements on the individual speed lines.

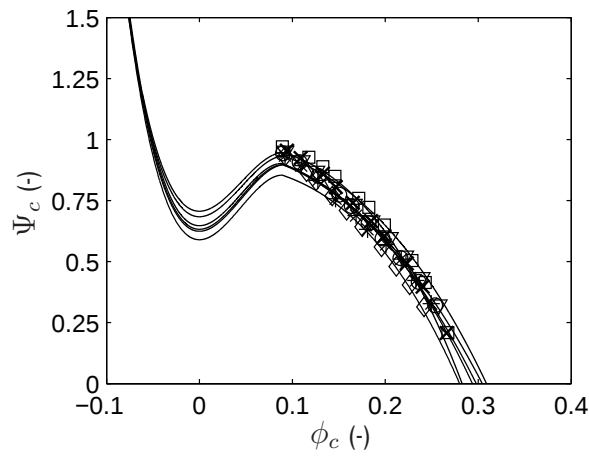
### 3.3.4 Throttle characteristic

In addition to the compressor characteristic, the dynamic model also includes two valve characteristics that describes the mass flows leaving the plenum as a function of the opening  $u$  and the pressure difference  $\Delta p$  of the throttle and control valve, respectively.

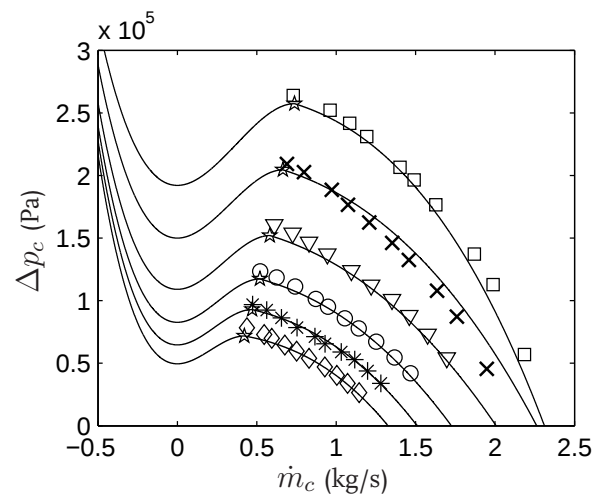
[Greitzer \(1976a\)](#) and many others have used a simple quadratic characteristic to describe the resistance or load in the compression system. The general relation between the mass flow through and pressure difference over a flow restriction follows from Bernoulli's law, yielding  $\dot{m}_t = A(u_t) \sqrt{2\rho \Delta p}$ . Here,  $A(u_t)$  denotes the cross-sectional area at the valve discharge plane, which is a function of the valve opening  $u_t$ .

Often this expression is further generalized into the form

$$\dot{m}_t = K u_t \sqrt{\Delta p} \quad (3.8)$$



**Figure 3.6** / Data and approximation for the compressor map of test rig A;  $\diamond$  7,806 rpm, \* 8,914 rpm,  $\circ$  10,052 rpm,  $\nabla$  11,484 rpm,  $\times$  13,369 rpm,  $\square$  15,038 rpm,  $\star$  = points on surge line, — = polynomial approximation.



**Figure 3.7** / Compressor map of test rig A in full dimensions; annotation as in Figure 3.6.

assuming that the mass flow is proportional to the valve opening and density is constant. The constant of proportionality  $K$  is usually called the valve coefficient.

Kurz and White (2004) used a more involved relation, taking into account compressibility, turbulence and friction effects as prescribed in an industrial standard. Valve manufacturers normally provide expressions for the valve flow characteristics in line with industrial standards, but the notation and terminology are not unambiguous.

We choose to express the throttle and control valve characteristics in accordance with the latest version of the industrial standard IEC 60534-2-1 (1998). The equations provided are based on the general form of Equation (3.8) but various correction factors are included to account for deviations from ideal test conditions during actual operation. Most importantly, a distinction is made between normal and choked flow conditions. For turbulent gaseous flow through a valve, the following expressions for the mass flow rate through the valve are given below. We denote flow conditions upstream of the valve with the subscript 2 and the pressure drop with  $\Delta p = p_2 - p_1$ , following the notation used in the compressor model.

For normal (non-choked) flow:

$$\dot{m}_t = 0.0316 K_t(u_t) Y \sqrt{\rho_2 \Delta p} \quad \text{when} \quad \frac{\Delta p}{p_2} < \frac{\gamma_2}{1.40} x_T \quad (3.9)$$

and for choked flow

$$\dot{m}_t = 0.667 \cdot 0.0316 \sqrt{\frac{\gamma_2}{1.40} x_T \rho_2 p_2} \quad \text{when} \quad \frac{\Delta p}{p_2} \geq \frac{\gamma_2}{1.40} x_T \quad (3.10)$$

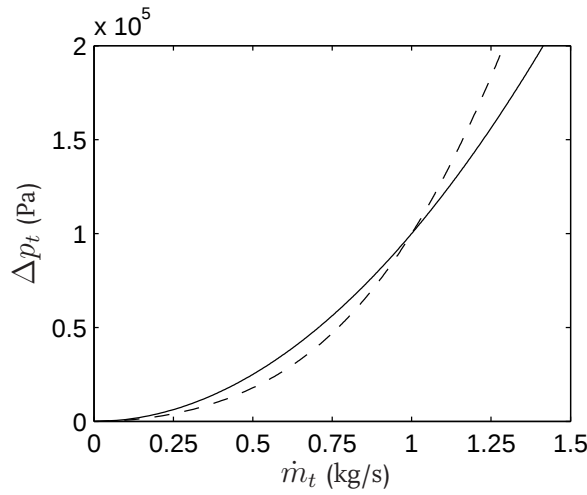
where  $K_t(u_t)$  denotes the valve flow coefficient. Normally, values for  $K_t$  as function of the valve opening  $u_t$  are provided by the valve manufacturer. Otherwise, when the valve mass flow and the corresponding pressure difference are known, the above equations can be used to determine  $K_t$  for a specific (fixed) valve opening. Further details can be found in Appendix C. The above equations and the different constants and parameters are usually presented in full dimensions. Hence, we choose to apply aerodynamic scaling of the resulting mass flow value afterwards.

The throttle characteristic  $\phi_t$  and control valve characteristic  $\phi_r$  for test rig A are defined by the scaled equivalents of Equations (3.9) and (3.10) for test rig A. In Section 3.3.2 we already discussed how to obtain appropriate density and upstream pressure values. Specific parameter values for the control valve are given in Table 3.1. For the installed throttle valve, values for  $K_t$  and  $x_{T,t}$  are not available, nor is an accurate throttle position measurement. From IEC 60534-2-1 (1998) we obtained a reasonable estimate  $x_{T,t} = 0.8$  for the installed butterfly valve. However, the flow coefficient  $K_t$  must be determined from experimental data as discussed above.

Before we proceed with validating the dynamic model for the compression system, it is interesting to see the difference between the generalized approximation in Equation (3.8)

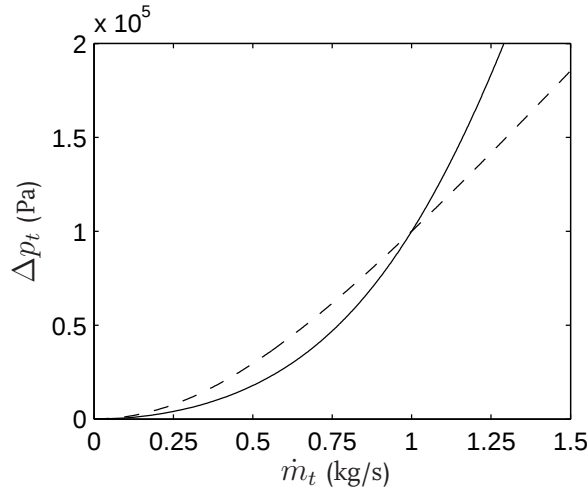
**Table 3.2** / Parameters of installed valves in test rig A.

| Valve    | Parameter                           | Value | Unit                  |
|----------|-------------------------------------|-------|-----------------------|
| Throttle | valve duct area $A_t$               | 0.071 | $\text{m}^2$          |
|          | valve duct length $L_t$             | 0.080 | m                     |
|          | press. diff. ratio factor $x_{T,t}$ | 0.8   | -                     |
| Control  | valve duct area $A_r$               | 0.002 | $\text{m}^2$          |
|          | valve duct length $L_r$             | 0.003 | m                     |
|          | valve flow coefficient $K_r$        | 1.99  | $\text{m}^3/\text{h}$ |
|          | press. diff. ratio factor $x_{T,r}$ | 0.64  | -                     |

**Figure 3.8** / Comparison between different valve characteristics for non-choked flow; — generalized model, -- IEC standard model.

and the more involved expression that we adopt here. For that purpose, we have calculated the resulting characteristics with both models, using arbitrary, full dimensional reference values  $\dot{m}_t = 1 \text{ kg/s}$ ,  $\Delta p = 1 \cdot 10^5 \text{ Pa}$ ,  $p_1 = 1 \cdot 10^5 \text{ Pa}$ , and  $T_2 = 340 \text{ K}$  to calculate  $K_t$  and the constant density  $\rho_2$ . The resulting characteristics are shown in Figure 3.8. From this figure we clearly see the differences between the two models at low and high  $\Delta p$  values that, for off-design conditions, can influence the overall accuracy of the model for the centrifugal compression system. In Section 3.4 we will investigate this matter in more detail.

Finally, we investigate the effect of using either a constant density  $\rho_2(T_2)$  or using the density  $\rho_2(p_2, T_2)$  that corresponds to the momentary upstream (plenum) pressure  $p_2$ . The resulting throttle characteristics for both cases are shown in Figure 3.9. The difference between the two cases appear to be significant. In the next section we will discuss the influence of density variations on the overall model accuracy.



**Figure 3.9** / Effect of upstream density on IEC valve characteristic for non-choked flow;  
— constant density, — — variable density.

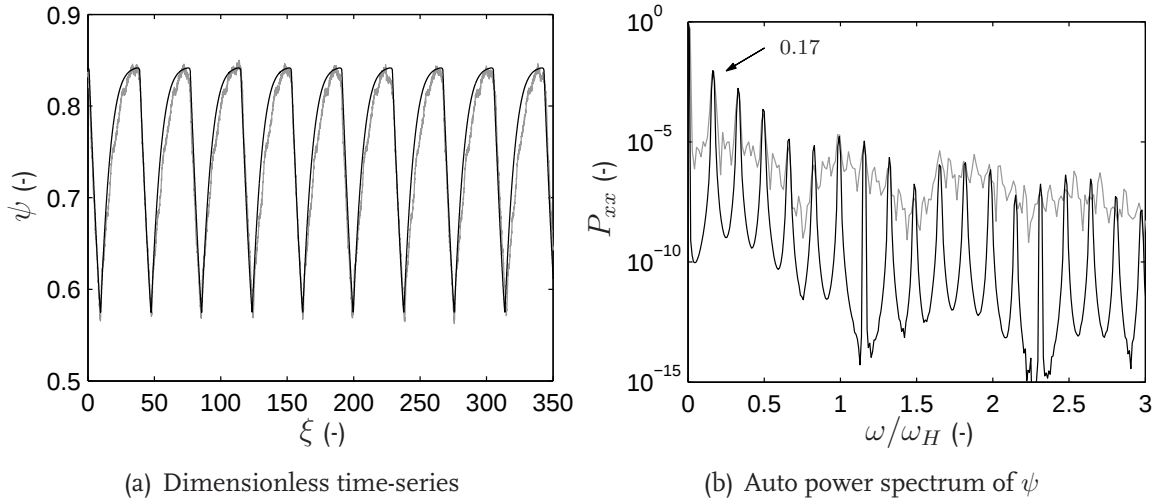
## 3.4 Model identification and validation

In the previous section we discussed the dynamic model for test rig A. Now we will present the results that were obtained from comparing surge simulations with measurement data. The measured and simulated surge oscillations are compared in a qualitative manner by judging the amplitude, frequency and overall shape of both pressure difference signals. Based on the results we tune the model and we propose some adjustments to improve the match between simulations and experimental data. Furthermore, we provide data from the conducted experiments to investigate if all the model assumptions as mentioned in Section 2.2 are valid. These results allow us to validate the model for the surge dynamics of the actual system.

### 3.4.1 Experimental results

In order to make a proper comparison of simulated and measured surge transients, we obtain the model parameters and initial values for each simulation from actual test conditions. Temperature and suction pressure values are obtained by averaging the recorded signals over the entire measurement period of 6.4 s. Ambient pressure, impeller speed, and stationary mass flow values are obtained from the standard measurement equipment of the test rig. To realize a reproducible measurements, a stable operating point near the surge line is selected by throttling down the large throttle valve. Subsequently, the system is brought into surge by closing the small control valve.

However, due to small changes in process conditions, leakage and calibration errors the stable operating point varies slightly in between and during the measurements. Hence,

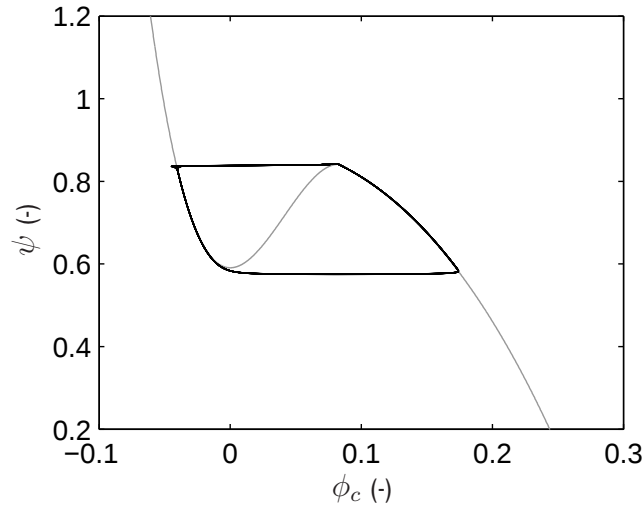


**Figure 3.10** / Pressure measurement (gray) and simulation result (black) during surge;  
 $N = 9,730$  rpm,  $\phi_{c0} = 0.0829$ ,  $u_r = 0$ ,  $B = 3.50$ ,  $F = 1.1$ ,  $\omega_H = 69$  rad/s.

prior to each simulation some fine tuning of the initial mass flow value in the model is required. Similarly, variations in rotor speed, both during surge experiments and during the measurements of the steady-state compressor characteristics, required some fine tuning of the fixed rotor speed value. The resulting parameter values and initial conditions for all simulations presented here are summarized in Table 3.3.

The validation result for a typical experiment at  $N = 9,730$  rpm is shown in Figure 3.10. The time-series plot reveals a good qualitative and quantitative match between the measured and simulated surge oscillations. Moreover, the surge frequency and the first 9 harmonics are accurately predicted by the model as can be seen from the plot of the auto power spectrum. However, we point out that the spectrum of the measured signal is higher than that of the simulated signal in between the harmonics of the surge oscillation. This difference is most likely caused by the significant flow noise (e.g. turbulence) and, to a lesser extend, measurement noise in the measured signal. The surge frequency observed during this experiment, in physical values, is 1.8 Hz. We point out that we will use the term surge frequency throughout this thesis to denote the inverse of the period time of the surge cycle, even though the oscillation is not sinusoidal.

Furthermore, the measured pressure difference reveals some small fluctuations around the top of each surge cycle. Most likely these fluctuations are caused by traveling waves in the suction piping and gas cooler. Given the fact that the spectrum of the dimensionless pressure difference shows no clear indication of standing waves and the amplitude of the pressure fluctuations is small, a more detailed investigation was not considered necessary. In Chapter 4 we will discuss a different situation in which pressure waves in the piping system are no longer negligible.



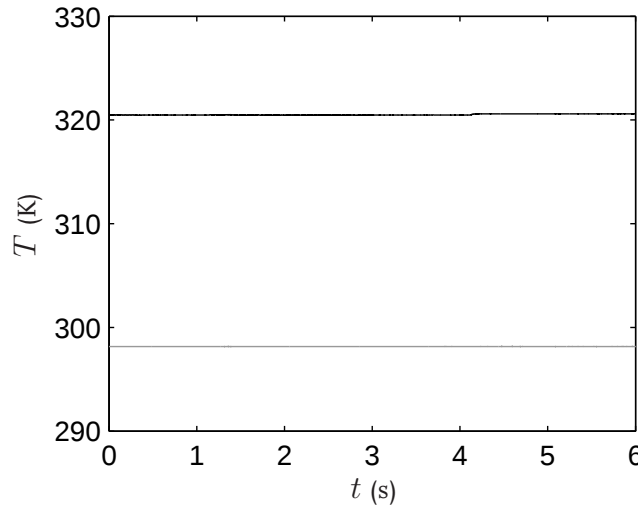
**Figure 3.11** / Compressor characteristic (gray) and simulated limit cycle (black) during surge;  $N = 9,730$  rpm,  $\phi_{c0} = 0.0829$ ,  $u_r = 0$ ,  $B = 3.50$ ,  $F = 1.1$ ,  $\omega_H = 69$  rad/s.

To illustrate the limit cycle that is characteristic for compressor surge, the dimensionless pressure difference is plotted as function of the dimensionless mass flow in Figure 3.11. We recall that no unsteady mass flow measurements are available so only simulated results are shown. This figure shows that the simulation predicts that back flow occurs during each surge cycle. Based on the sound of the compressor and the pressure readouts from sensors inside the compressor, test engineers confirmed that back flow indeed occurred during the experiments.

The shape of the limit cycle oscillation is in agreement with the large value of  $B$  for the investigated test rig (Greitzer, 1976a). In Chapter 5 we will discuss the effect of  $B$  on the shape of the surge oscillation in more detail.

We remark that Greitzer (1976a) argued that for a relaxation type of surge oscillation, the first order time lag in the compressor response is inadequate given the very abrupt changes in mass flow that characterize this type of oscillation. Hence, for a compression system with a large value of  $B$  it is appropriate to neglect the difference between steady-state and unsteady compressor response. Given the good overall match between measured and simulated surge oscillations, we will not pursue a more detailed description of the compressor response during surge transients. However, the discussion of (Hansen et al., 1981; Mazzawy, 1981) and the work by Meuleman (2002) indicate that a conclusive statement on using only the steady-state compressor characteristic cannot be made at this point.

To continue our discussion on the validity of the developed model we recall that temperature variations are not included in the model. The measured suction and discharge temperatures are shown in Figure 3.12. The maximum temperature variations in these



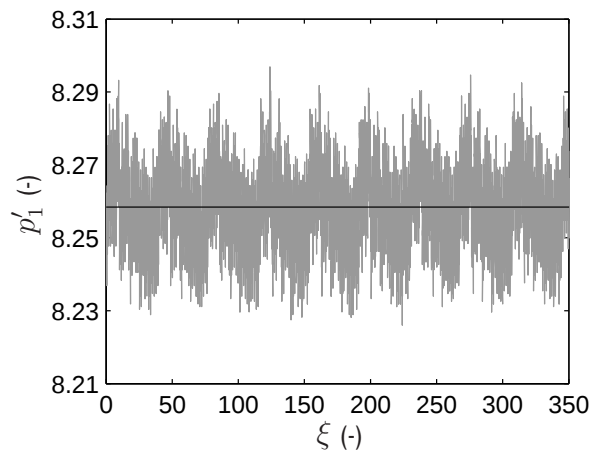
**Figure 3.12** / Measured suction temperature (gray) and discharge pressure (black) during surge;  $N = 9,730$  rpm,  $\phi_{c0} = 0.0829$ ,  $u_r = 0$ ,  $B = 3.50$ ,  $F = 1.1$ ,  $\omega_H = 69$  rad/s.

data are  $\Delta T_1 < 0.017$  K and  $\Delta T_2 < 0.16$  K. These results justify the assumption that both temperatures  $T_1$  and  $T_2$  have a negligible effect of  $< 0.03\%$  on the gas density and hence they can be considered to be constant.

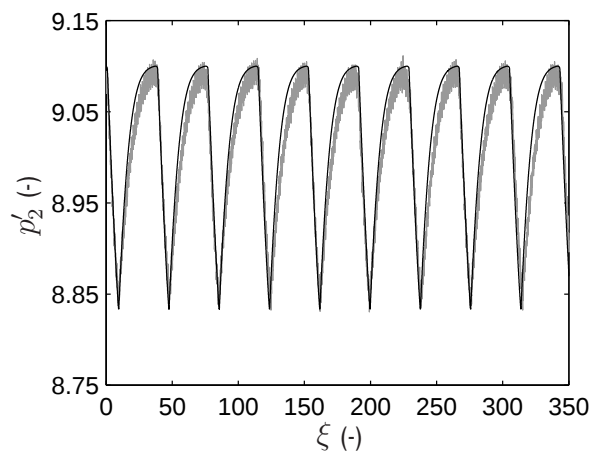
Furthermore, in Section 3.3.4 we introduced the assumption of a constant suction pressure  $p_1$  such that a value for the discharge pressure can be obtained by adding the simulated value for  $\Delta p$  to the assumed value for  $p_1$ . In Figure 3.13 the measured suction pressure signal is shown (in dimensionless form for comparison with other results). Although this pressure is obviously not constant, the variation is small with respect to the variation in the pressure difference. Hence, we conclude that it is acceptable to use the averaged suction pressure as a constant value for  $p_1$  in the simulation model. This conclusion is further supported by the good agreement between the measured discharge pressure and the calculated value from the simulated pressure difference and constant suction pressure, depicted in Figure 3.14.

Additionally, we remark that fluctuations of the suction pressure are inherent for a compressor experiencing intermittent normal and reverse flow. However, in addition to the earlier conclusion that the effect of pressure waves in the suction piping is small, the low amplitude of the suction pressure fluctuations justify the assumption to lump the entire return piping and gas cooler into a single suction side volume. Hence, modeling the different piping sections and the gas cooler as individual components (e.g. Botros, 1994b), is not necessary to adequately describe the surge transients in the test rig.

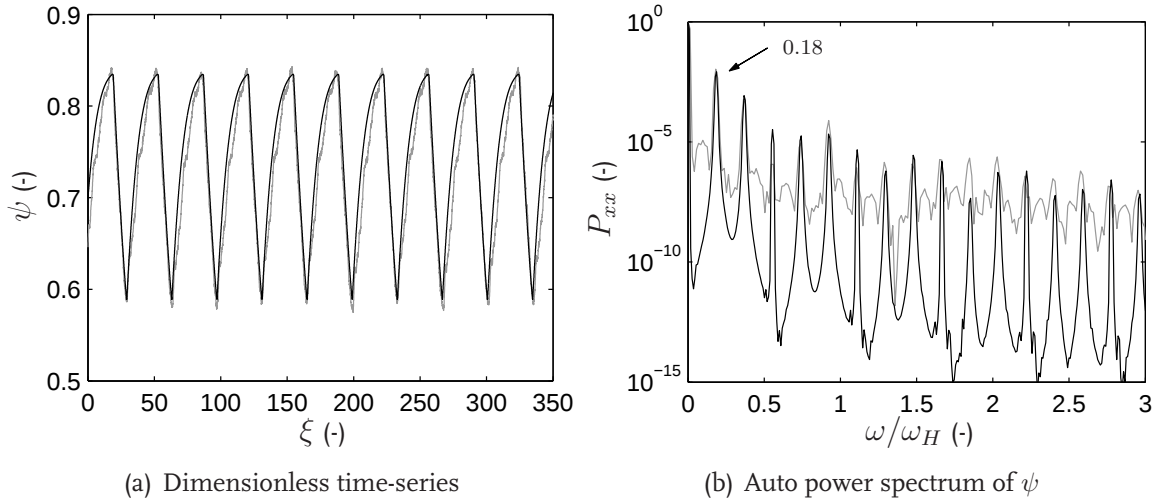
The model identification presented so far is based on surge measurements at a relatively low speed. In Figure 3.15 the results for a measurement at a higher rotor speed of 11,900 rpm are shown. From this figure we see that the model accurately describes the surge



**Figure 3.13** / Measured suction pressure (gray) and averaged value (black) in dimensionless form;  $N = 9,730$  rpm,  $\phi_{c0} = 0.0829$ ,  $u_r = 0$ ,  $B = 3.50$ ,  $F = 1.1$ ,  $\omega_H = 69$  rad/s.



**Figure 3.14** / Discharge pressure measurement (gray) and simulation result (black) with constant suction pressure;  $N = 9,730$  rpm,  $\phi_{c0} = 0.0829$ ,  $u_r = 0$ ,  $B = 3.50$ ,  $F = 1.1$ ,  $\omega_H = 69$  rad/s.



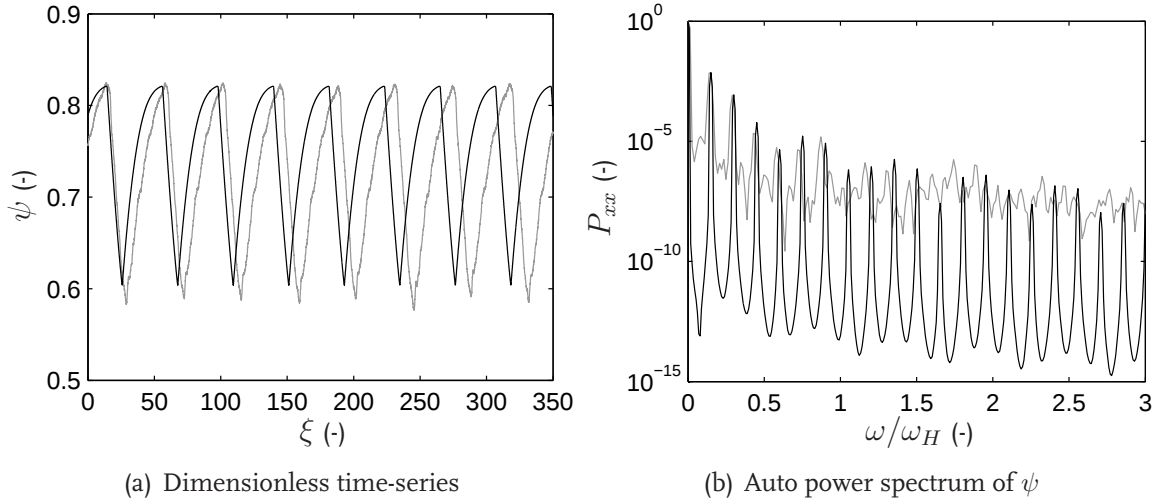
**Figure 3.15** / Pressure measurement (gray) and simulation result (black) during surge;  
 $N = 11,900$  rpm,  $\phi_{c0} = 0.0774$ ,  $u_r = 0$ ,  $B = 4.21$ ,  $F = 1.1$ ,  $\omega_H = 70$  rad/s.

transients at this speed. Note that the surge frequency is slightly higher at this speed, namely 2 Hz. This higher frequency is caused by the unintentional selection of a smaller throttle valve opening and hence an operating point at a lower mass flow value, see also Chapter 5.

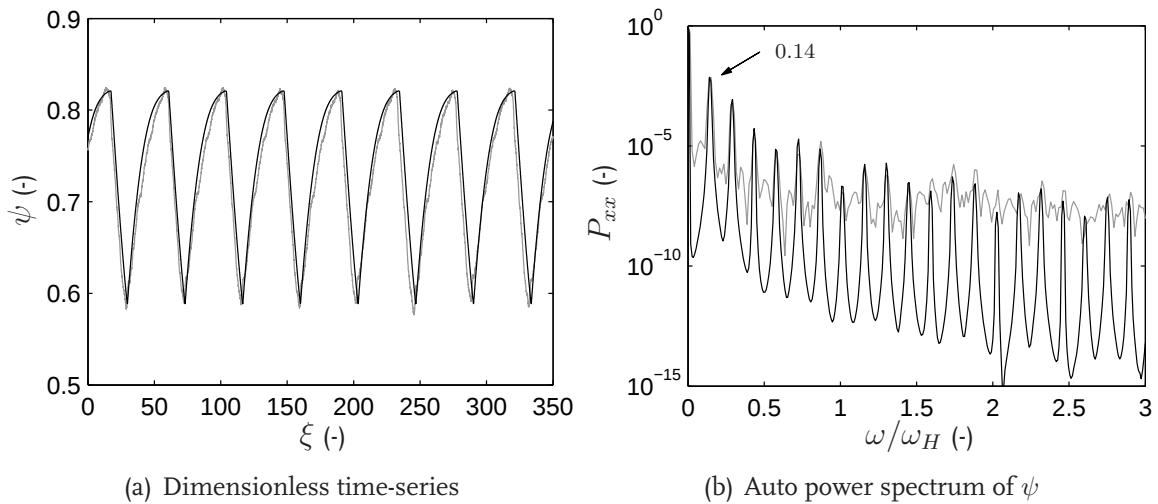
The results in Figure 3.16 for  $N = 15,200$  rpm reveal a much larger difference between the measured and simulated pressure difference. Besides the incorrect surge frequency, the minimum value of the pressure difference is clearly overestimated in the model. From Figure 3.11 we see that the minimum value of  $\psi$  occurs around zero mass flow after the plenum is emptied. Hence, the minimum value of  $\psi$  is determined by the location of the valley point of the steady-state characteristic.

Following the approach suggested by Willems (2000, pp. 29), we adjust the value  $\Psi_c(0)$  to improve the match between measured and simulated data. We remark that the original value for  $\Psi_c(0)$  is determined analytically, see also Appendix C. However, the behavior of the compressor around zero mass flow is not at all well understood (see Greitzer, 1976b; Meuleman, 2002) and hence the calculated value for  $\Psi_c(0)$  is considered to be rather inaccurate, justifying the reduction of approximately 2.5% to obtain the result depicted in Figure 3.17. Both the amplitude and frequency of the surge oscillation are accurately predicted after the modification of the compressor curve for  $N = 15,200$  rpm. The surge frequency during this particular measurement was 1.7 Hz.

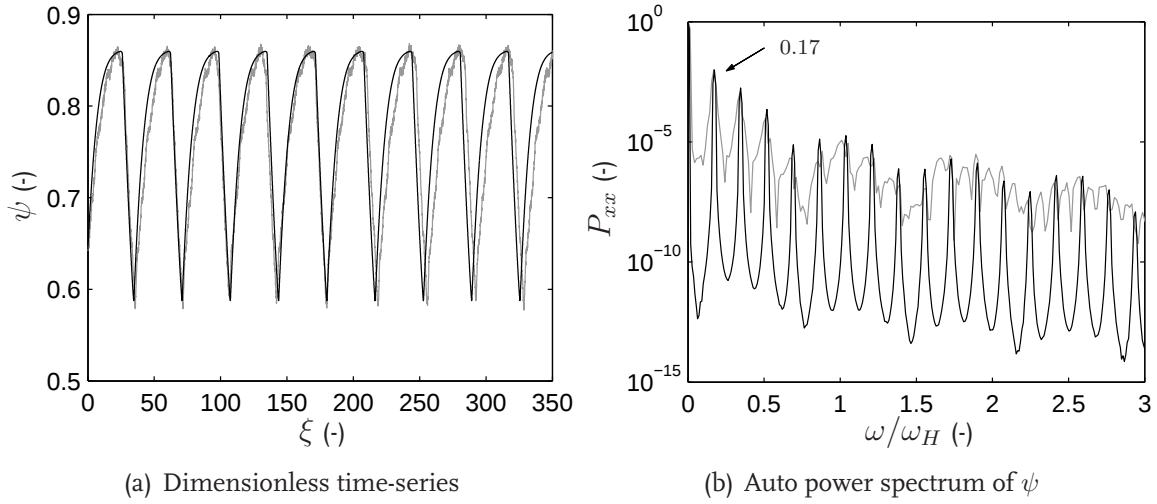
To complete the validation of the dynamic model for test rig A, including the adjustment of the compressor curve valley point at high speeds, we compared the simulation results with entirely new data from additional experiments. The results for two surge measurements at  $N = 9,750$  and  $N = 15,200$  rpm are shown in Figure 3.18 and 3.19, respectively.



**Figure 3.16** / Pressure measurement (gray) and simulation result (black) during surge;  $N = 15,200$  rpm,  $\phi_{c0} = 0.0807$ ,  $u_r = 0$ ,  $B = 5.31$ ,  $F = 1.1$ ,  $\omega_H = 71$  rad/s.



**Figure 3.17** / Pressure measurement (gray) and simulation result (black) during surge after lowering  $\Psi_c(0)$  with 2.5%;  $N = 15,200$  rpm,  $\phi_{c0} = 0.0807$ ,  $u_r = 0$ ,  $B = 5.31$ ,  $F = 1.1$ ,  $\omega_H = 71$  rad/s.



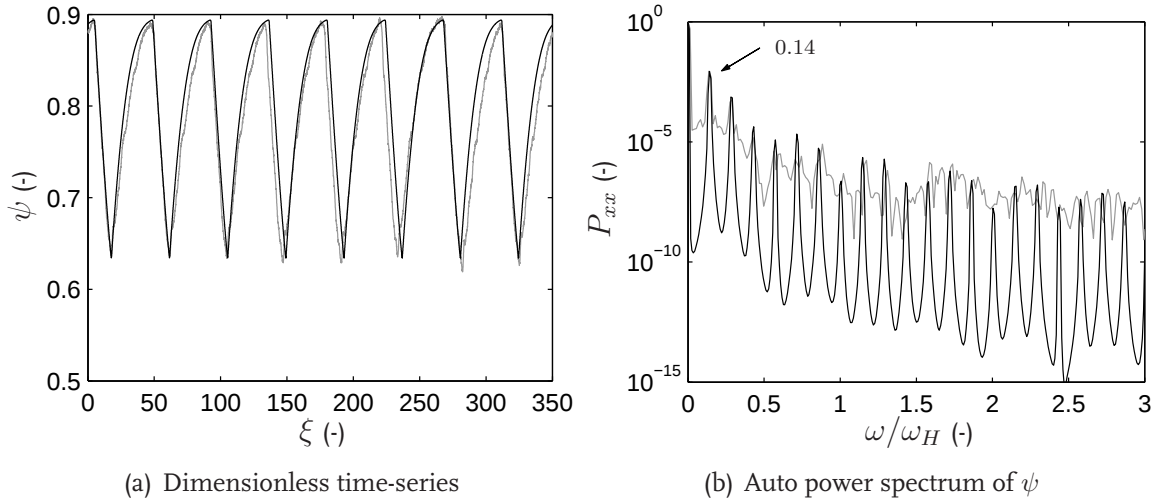
**Figure 3.18** / Pressure measurement (gray) and simulation result (black) during surge;  
 $N = 9,750$  rpm,  $\phi_{c0} = 0.0920$ ,  $u_r = 0$ ,  $B = 3.50$ ,  $F = 1.1$ ,  $\omega_H = 69$  rad/s.

**Table 3.3** / Initial conditions used for model validation of test rig A.

| Parameter                            | Value at rotor speed (rpm) |                    |                    | Unit              |
|--------------------------------------|----------------------------|--------------------|--------------------|-------------------|
|                                      | 9,730                      | 11,900             | 15,200             |                   |
| ambient pressure $p_{\text{amb}}$    | $1.015 \cdot 10^5$         | $1.015 \cdot 10^5$ | $1.015 \cdot 10^5$ | Pa                |
| average suction pressure $\bar{p}_1$ | $1.081 \cdot 10^6$         | $1.066 \cdot 10^6$ | $1.003 \cdot 10^6$ | Pa                |
| suction temperature $T_1$            | 298.1                      | 299.1              | 294.6              | K                 |
| discharge temperature $T_2$          | 320.5                      | 331.1              | 339.1              | K                 |
| throttle valve coefficient $K_t$     | 15.88                      | 14.87              | 15.38              | m <sup>3</sup> /h |

The small difference between the measured and simulated surge frequency in Figure 3.18 is caused by a slight mismatch in the actual and assumed initial operating points. These results illustrate that the developed model is capable of describing the surge behavior of test rig A at different operating conditions.

Based on the results presented here we conclude that the developed lumped parameter model adequately describes the dynamic behavior of the investigated centrifugal compression system. We will finish with an investigation of the effect of using a constant discharge pressure to determine the density and speed of sound in the discharge volume. A further analysis of the model, the importance of the various parameters and characteristics, and the relevant compressor dynamics that it must describe, is provided in Chapter 5.



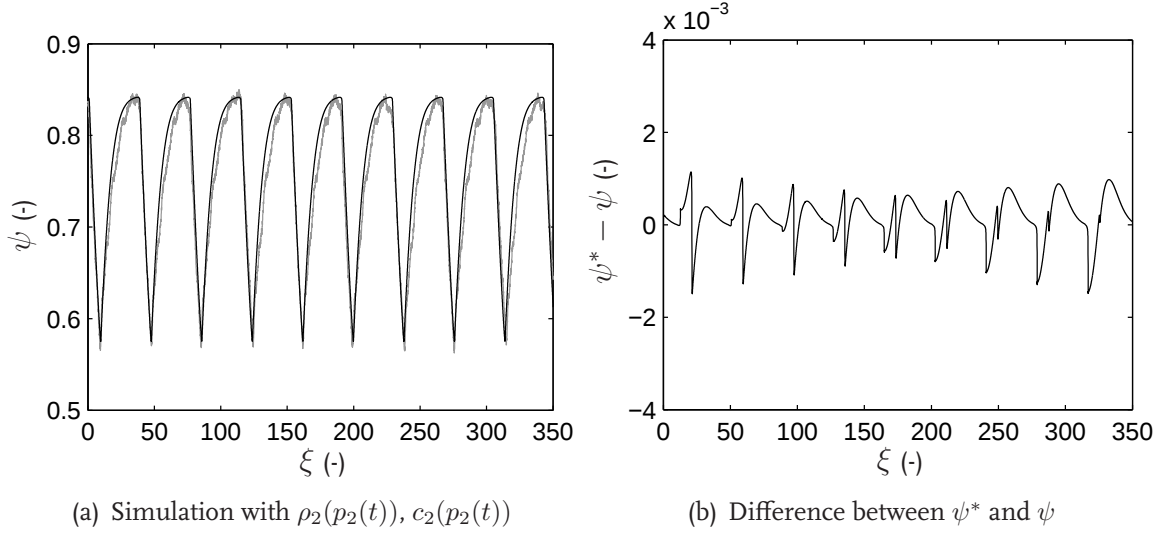
**Figure 3.19** / Pressure measurement (gray) and simulation result (black) during surge;  $N = 15,200$  rpm,  $\phi_{c0} = 0.0903$ ,  $u_r = 0$ ,  $B = 5.29$ ,  $F = 1.1$ ,  $\omega_H = 71$  rad/s.

### 3.4.2 Time varying gas properties

As mentioned in Sections 3.3.2 and 3.3.4, we use constant temperature and pressure values to calculate the various gas properties like the density and speed of sound. We also argued that this approach is not always the most logical one. To investigate the effect of these variations we perform a simulation with a model that includes the pressure dependency of the gas density and speed of sound. Note that this will result in time varying parameters  $\rho_2(p_2(t), T_2)$  and  $c_2(p_2(t), T_2)$ .

We repeat the simulation for  $N = 9,730$  rpm that was discussed above, but now with the density and speed of sound modeled as functions of the momentary discharge pressure  $p_2(t)$ . The result is depicted in Figure 3.20, again showing a good agreement with the measured surge oscillations. The calculated pressure difference with time varying density and speed of sound parameters is denoted by  $\psi^*$  to distinguish it from the value obtained with constant values for the density and speed of sound.

For comparison we have included a plot of  $\psi^* - \psi$  to illustrate the small difference ( $< 0.2\%$ ) between the two calculations. The difference is growing in time due to a physically negligible difference in the period time of the oscillations so the two signals slowly get out of phase, yielding a numerically larger difference. From these results we conclude that the effect of using time varying density and speed of sound values on the calculated pressure rise  $\psi$  is negligible and hence constant values can be used.



**Figure 3.20** / Pressure measurement (gray) and simulation result (black) during surge with gas properties as function of discharge pressure (left figure) and difference with previous result for constant gas properties (right figure);  $N = 9,730$  rpm,  $\phi_{c0} = 0.0829$ ,  $u_r = 0$ ,  $B = 3.51$ ,  $F = 1.1$ ,  $\omega_H = 69$  rad/s.

### 3.5 Discussion

In this chapter we have treated the development of a model for a large centrifugal compression system. After a detailed discussion of the different parameters and characteristics in this model, we discussed the results from the extensive experimental identification and model validation of this lumped parameter model. These results confirm that the developed model has a large value of  $B$  in comparison with many experimental setups described in literature (e.g. Greitzer, 1976b; Hansen et al., 1981; Willems, 2000; Arnulfi et al., 2006). In the terminology from Greitzer (1976b) the large value of  $B$  implies that the surge transients of the compressor test rig are characterized by the filling and emptying of the plenum volume on the one hand and abrupt flow reversals on the other. The importance of the stability parameter  $B$  for describing the surge dynamics of the investigated compressor test rig will be discussed in Chapter 5.

The fact that the studied compression system operates in a closed circuit leads to the introduction of a new model parameter that describes the relative effect of the coupled volumes on the pressure fluctuations. Measurements of the suction pressure during surge revealed only small variations so we concluded that a lumped volume is adequate to describe the suction piping of the test rig. Furthermore, the small variations justified the use of a single time-averaged value of the suction pressure in the simulation model. Moreover, we illustrated that neglecting the pressure dependency of gas density and the speed of sound hardly has an effect on the model predictions.

To include the compressor characteristics in the model, we adopted the common approach to fit the measured steady-state data with cubic polynomials. Given the trends in the measured data, a piecewise continuous fit of these cubic polynomials is required to approximate the unstable and stable regimes. The applied fitting resulted in an approximation of the entire compressor map as a function of compressor speed and mass flow. Comparison of simulation results with measurement data showed that the calculated compressor response at zero mass flow requires some tuning at high compressor rotational speeds. Hence, a certain level of uncertainty for the unstable part of the compressor characteristic remains when analytical compressor maps (e.g. Gravdahl and Egeland, 1999a; Gravdahl et al., 2004) are used.

In contrast to most of the available literature, we described the throttle characteristics in the model in accordance with the industrial standard that is used by valve manufacturers to specify throttle and control valves. Although the exact shape of the throttle characteristic did not appear to influence the model predictions significantly, we showed that different valve characteristics can have different slopes at the same values of mass flow and pressure drop. The possible effect of this difference on the dynamic behavior of the compression system in the vicinity of its operating point will be investigated in Chapter 5.

Referring back to Section 3.1 most of the available literature dealt with either the experimental validation of compressor models on laboratory setups or the application of more complex models to describe large compression systems. The presented validation results for the industrial scale test rig A illustrate that the Greitzer model is suitable for describing the surge dynamics of such a centrifugal compression system, despite the numerous simplifications and assumptions that were addressed in Chapter 2. In the following chapter we will discuss the effect of piping acoustics on the surge transients in another test rig, a phenomenon for which we concluded that it is of minor influence in the test rig studied here.



# Dynamic compressor model including piping acoustics<sup>1</sup>

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**Abstract** / In this chapter experimental evidence is provided for the presence of acoustic pulsations in another centrifugal compression system that was investigated. Aeroacoustic theory is applied to model the dynamics of the piping system that is connected to the compressor. Furthermore, appropriate boundary conditions are selected to combine the transmission line and lumped parameter compressor models. Simulation results illustrate the qualitative improvements of the model. The chapter ends with a discussion on the benefits and shortcomings of the new dynamic model for a centrifugal compression system.

## 4.1 Introduction

In the industrial compression systems that are studied in this thesis, the outlet is usually connected to a long discharge line rather than to a plenum volume. The strong influence of piping systems on transient response and flow stability of centrifugal compressors was addressed by Sparks (1983). The main conclusion from this work was that surge is a system instability whose onset is determined by system acoustic damping and whose frequency is determined largely by attached piping.

The effect of shock waves—a violent acoustic phenomenon—that occur during the first milliseconds of a surge cycle on the dynamic behavior of high-speed axial compressors was addressed by Cargill and Freeman (1991). Feulner et al. (1996) briefly mentioned the similarity with acoustic ducts in their discussion on the distributed character of compressible flows. Furthermore, Jungowski et al. (1996) investigated active and passive control for systems with various inlet and discharge piping configurations. However, their study on piping system dynamics was rather limited since they only included the piping

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<sup>1</sup>This chapter is based on Van Helvoirt and De Jager (2007).

capacitance in a modified Greitzer lumped parameter model. Nelson et al. (2000) experimentally confirmed the presence of a significant acoustic mode in an axi-centrifugal compressor of a helicopter engine. They introduced acoustics into a compressor surge model by using a transmission matrix approach but the resulting model gave a poor quantitative match with the measured surge and acoustic modes.

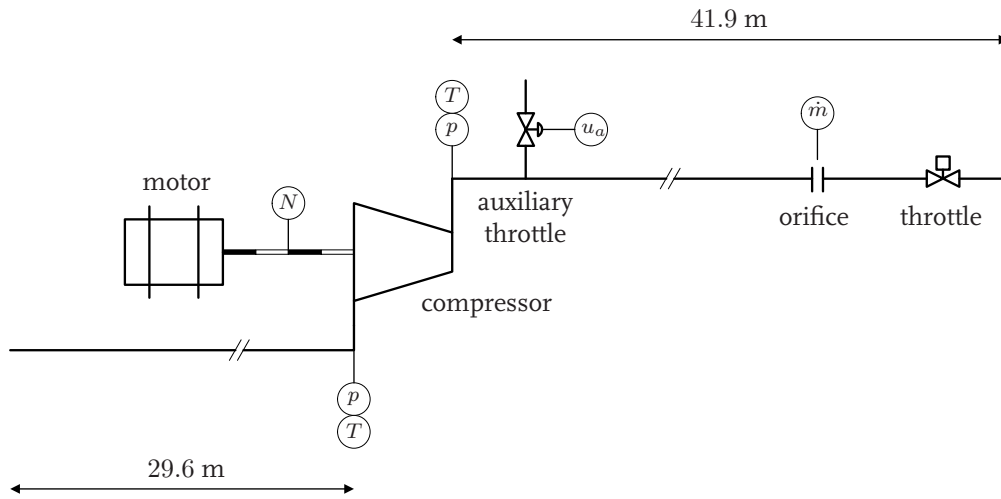
The study of flow pulsations in pipelines has become common practice in industry, for a great deal originating from the pulsation problems arising from the use of reciprocating compressors (e.g. Eijk et al., 1996; Eijk and Egas, 2001; Egas, 2001). Research on acoustics, fluid-structure interactions and transients in fluid transmission lines is a mature field as indicated by the many available publications and textbooks (e.g. Goodson and Leonard, 1972; Stecki and Davis, 1986; Munjal, 1987; Howe, 1998; Kinsler et al., 1999). Unfortunately, hardly any quantitative results are available from literature on the coupling between piping acoustics and the dynamic behavior of axial and centrifugal compressors.

In this chapter we will address the influence of the piping system on the transient behavior of a full-scale centrifugal compression system. We will discuss the application of the Greitzer lumped parameter model to model an industrial compressor test rig. Secondly, we will provide experimental evidence for the presence of acoustic pulsations in the compression system under study. The discussion of the experimental setup and the dynamic model for this system form the starting point for the subsequent sections.

The main objective of this chapter is to present an extension of the basic compressor model that accounts for the relevant piping system dynamics. Hence, we proceed with a brief introduction of acoustic terminology and the underlying physics, after which we propose a model structure for compressor piping systems that is based on previous developments for fluid transmission lines. Then we discuss how this model can be connected to the lumped parameter model via the choice of appropriate boundary conditions and model parameters. Free-response measurement and simulation results will be presented to show that the combined model provides a more detailed description of the dynamic behavior of the entire centrifugal compression system.

## 4.2 Centrifugal compression system

All experiments described in this chapter were conducted on a different full-scale centrifugal compressor test rig than the one discussed in Chapter 3. Although both compressors have similar flow characteristics, the connected piping systems of the two installations differ significantly. In this section we introduce the main components and instrumentation of the installation under consideration and we address the main differences with the installation discussed earlier. Furthermore, we introduce the equations for



**Figure 4.1** / Scheme of compressor test rig B with sensor locations:  $N$  = rotational speed,  $p$  = pressure,  $T$  = temperature,  $\dot{m}$  = mass flow,  $u_a$  = valve opening.

the nonlinear lumped parameter model, similar to the original Greitzer model. Details of the derivation are omitted since those have been treated extensively in previous chapters. We identify and validate the model by comparing simulation results with data from actual surge measurements. These results are also used to illustrate the need for a model extension to account for piping system dynamics in order to improve the agreement between simulations and measured data.

### 4.2.1 Experimental setup

The system under study in this chapter is a second single-stage, centrifugal compressor rig that is normally used to test industrial compressors for the oil and gas industry. Throughout this thesis we will denote the system discussed here as *Test rig B*. The whole installation is schematized in Figure 4.1. The compressor is driven by an 1 MW electric motor and typical rotational speeds of the compressor lie between 14,000 and 21,000 rpm. The impeller that is mounted on the shaft has 17 blades and an outer diameter of 0.3 m.

The compressor is connected to atmosphere through long suction and discharge piping. The discharge piping contains a measurement section with a flow straightener, and an orifice flow meter with a diameter ratio of 0.392. Accuracy of the mass flow measurements is  $\pm 3.5\%$ , according to ISO 5167-2 (2003). Throttling of the compressor is done by means of a large butterfly valve located at the end of the discharge line. The auxiliary throttle valve directly behind the compressor has not been used since the capacity of this valve ( $K_r = 6.4 \text{ m}^3/\text{h}$ ) was too small to have a significant influence on the flow through the compressor. The overall dimensions of the installation, relevant for the dynamic model, are summarized in Table 4.1.

**Table 4.1** / Parameters of compressor test rig B.

| Parameter                     | Value  | Unit           |
|-------------------------------|--------|----------------|
| compressor duct length $L_c$  | 10.6   | m              |
| compressor duct area $A_c$    | 0.0255 | m <sup>2</sup> |
| suction volume $V_1$          | 3.57   | m <sup>3</sup> |
| suction piping length $L_1$   | 29.6   | m              |
| discharge volume $V_2$        | 9.82   | m <sup>3</sup> |
| discharge piping length $L_2$ | 41.9   | m              |

The compression installation is equipped with numerous temperature probes (J-type thermocouple) and static pressure transducers (Rosemount) to determine the steady-state performance of the compressor. Additional dynamic total pressure probes (Kulite) were installed in the suction and discharge pipes to measure the pressure rise fluctuations during experiments, see also Figure 4.1.

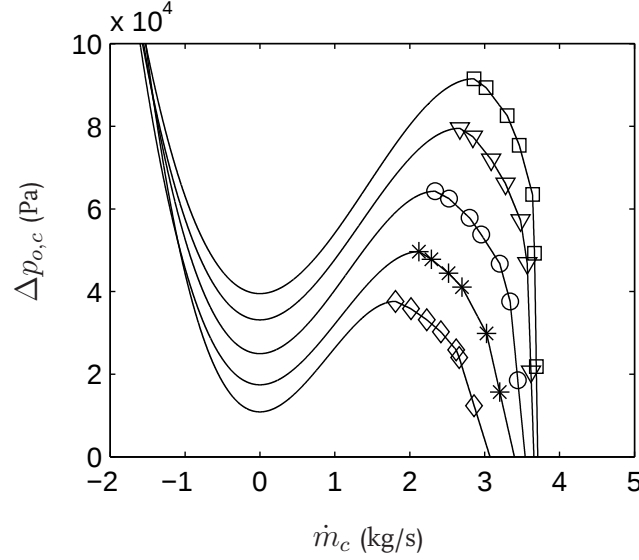
The 3 suction side and 2 discharge side probes are located approximately 1 m upstream and downstream of the compressor, oriented along the circumference at 0°, 90°, and 180°, respectively. Accuracy of the probes is  $\pm 5.8\%$ , with respect to a full range of 0–3.5 bar differential and ambient pressure as reference. The natural frequency of the pressure transducers is 300 kHz. Suction and discharge pressures are obtained by averaging the readings from the individual probes at the corresponding location. It is not exactly known to what extent the accuracy of the averaged pressure values is improved.

For data-acquisition a stackable measurement system (Siglab) with a total of 16 input channels, anti-aliasing filters and A/D convertors was used. All signals were measured at a sampling rate of 256 Hz.

### 4.2.2 Compressor model

In order to describe the dynamic behavior of the centrifugal compression system (test rig B) we used the Greitzer model as discussed in Chapter 2. Because the modeling approach was similar to that for test rig A, see Chapter 3, details of the model development are omitted here. Where needed, differences with the modeling approach for test rig A are highlighted.

The structure of the applied model is identical to that depicted in Figure 2.2. The resulting model equations are



**Figure 4.2** / Data and approximation for the compressor map of test rig B;  $\diamond$  13,783 rpm,  $*$  15,723 rpm,  $\circ$  17,637 rpm,  $\nabla$  19,512 rpm,  $\square$  20,855 rpm,  $-$  = spline-polynomial approximation.

$$\frac{L_c}{A_c} \frac{d\dot{m}_c}{dt} = \Delta p_{o,c}(\dot{m}_c, N) - \Delta p_o \quad (4.1)$$

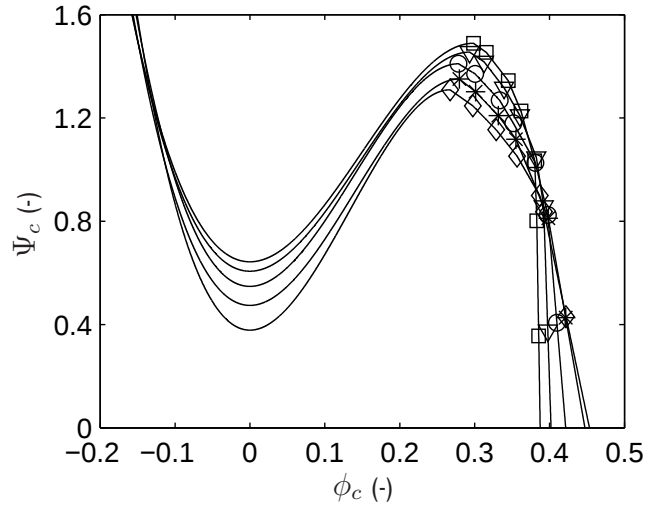
$$\frac{V_2}{c_2^2} \frac{dp_{o,2}}{dt} = \dot{m}_c - \dot{m}_t(\Delta p_o, u_t) - \dot{m}_r(\Delta p_o, u_r) \quad (4.2)$$

with  $\Delta p_o = p_{o,2} - p_{o,1}$ . The compressor characteristic  $\Delta p_{o,c}(\dot{m}_c, N)$  for the investigated test rig is shown in Figure 4.2. Note that we have again neglected the throttle duct dynamics and omitted the relaxation equation in the model.

In contrast to the polynomial approximations used for test rig A, the stable part of the compressor curves are now approximated with spline functions, see also Appendix C. We use spline functions because they are more suited to capture the profound choking effect—an almost vertical compressor characteristic—at high mass flows. The numerical implementation of the spline fit guarantees that the curves line up with the polynomial approximation for the unstable part at the measured surge points. However, expressing the dependency of the spline parameters on rotational speed with a quadratic polynomial in  $N$  is not straightforward and therefore omitted. Hence, compressor characteristics are only available at the speeds for which measurements were performed.

Although the test rig B is similar to test rig A, there are some important differences that we briefly address here. In summary, the main differences between the two test rigs are

Test rig B is an open system with both inlet and outlet connected to atmosphere, while test rig A is a recirculating system where inlet and outlet are



**Figure 4.3** / Compressor map of test rig B in dimensionless form.

connected through a gas cooler. Hence, the compressed medium in test rig B is air ( $28.0134 \cdot 10^{-3}$  kg/mol) that enters the system at atmospheric pressure.

Test rig B has a significantly larger plenum volume due to the long (41.9 m) discharge piping. Moreover, the length of the discharge piping gives the system a more profound distributed character and it introduces aeroacoustic phenomena that are the main topic of this chapter.

Test rig B is equipped with total pressure probes instead of the normal dynamic pressure sensors in test rig A. Hence, the model equations for test rig B contain stagnation pressures as discussed in Section 2.3.1.

Finally, we recall that it is common practice to reformulate the differential equations in non-dimensional form. However, to simplify the connection of the acoustic pipeline model that will be discussed later on, we will use their full-dimensional form throughout this chapter. Hence, measurement and simulation results will also be presented in full-dimensional form. However, some important results, will be repeated in dimensionless form to enable the easy comparison with results from other chapters. The dimensionless equivalent of the compressor map is shown in Figure 4.3.

### 4.2.3 Model identification and validation

We obtained parameter values for the developed compressor model by comparing simulation results with data from actual surge measurements. Subsequently, we verified whether the model describes the surge dynamics of the investigated test rig. In Chapter 6 we will discuss the results from forced-response experiments. At the start of each measurement the compressor was brought into surge by closing the throttle valve, after which

the dynamic pressure oscillations were measured for 128 s. The temperature and pressure data were also used to initialize the simulation model. Gas properties are obtained from gas table data provided by Baehr and Schmier (1961); Davis (1992); Mohr and Taylor (2005), using a spline interpolation when required.

To improve the agreement between measurement and simulation we selected the compressor duct length, the valley point of the compressor characteristic, and the throttle valve opening as tuning parameters, see also Chapter 3. We used a bisection algorithm to determine the valley point that resulted in a correct prediction of the measured pressure amplitude during surge. Subsequently, we performed a series of calculations with different throttle openings and we selected the throttle opening that gave the best match with the measured surge frequency. Prior to these calculations we confirmed that the valley point and throttle opening can be determined independently from each other.

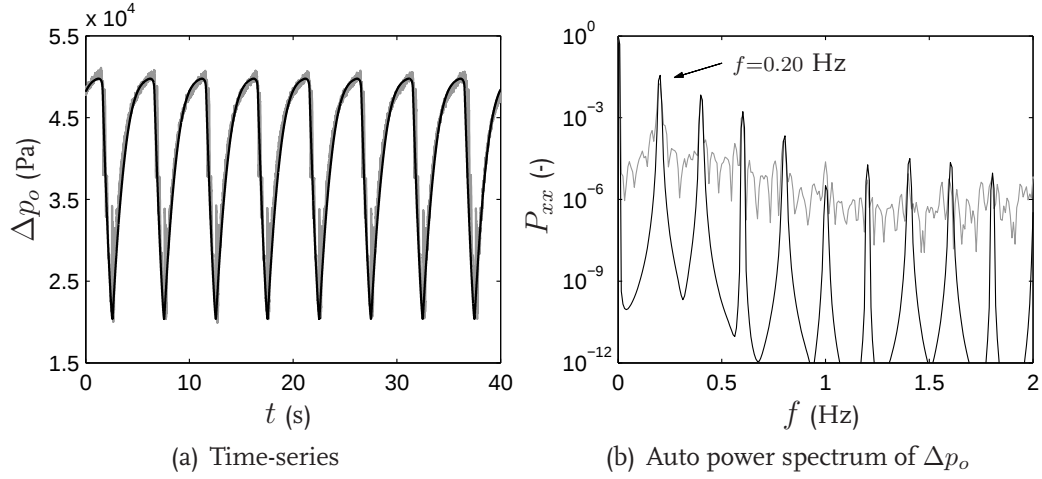
The above procedure was carried out for different values of  $L_c$  and we selected the smallest compressor duct length that gave satisfactory results. We point out that the obtained value for  $L_c$  is significantly larger than the one used in the model for test rig A. In Chapter 6 we will discuss the selection of a proper value for  $L_c$  in more detail.

The final results for two particular measurements are shown in Figures 4.4 and 4.5, both revealing a good agreement between the measured pressure oscillations and the outcome of the tuned simulation model. However, we point out that the power spectral densities of the measurements are higher than those of the simulation results in between the harmonic peaks of the surge oscillation. This is caused by the significant flow noise (e.g. turbulence) and some measurement noise that are present in the experimental system, which are not included in the simulation model.

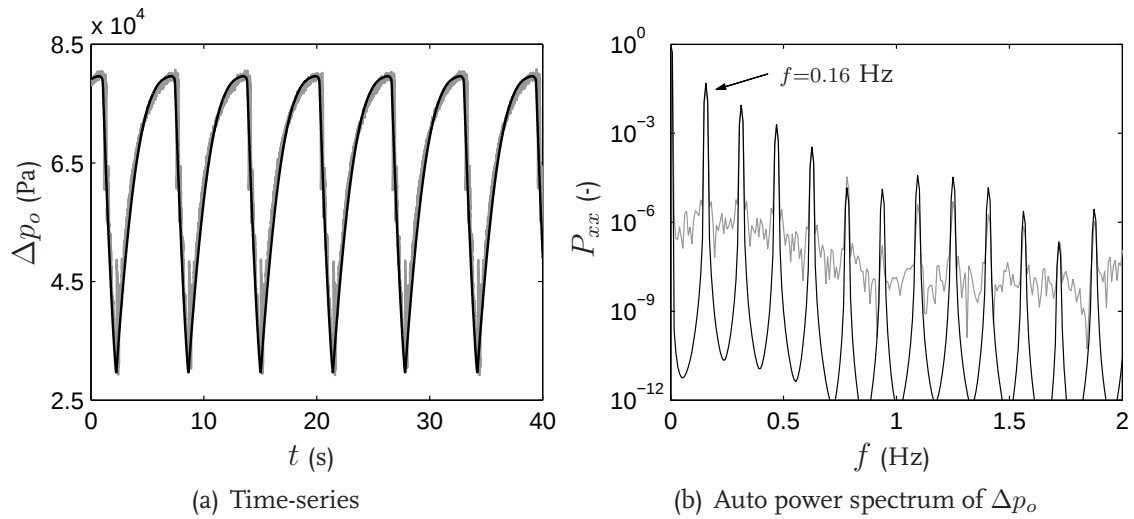
More importantly, zooming in on one surge cycle reveals some differences between the measured and simulated time-series as can be seen in Figure 4.6. In the first place, we remark that the measured pressure appears to increase faster than the simulated pressure after reaching its minimum value. A possible explanation for this difference is the influence of rotor speed variations during surge.

Qualitative observations during the surge experiments indicated that the compressor slows down during the negative flow phase of each surge cycle. However, no quantitative data of compressor speed and drive torque is available to investigate the effect of rotor speed variations in more detail, for example by including rotor dynamics in the Greitzer model as proposed by (Fink et al., 1992).

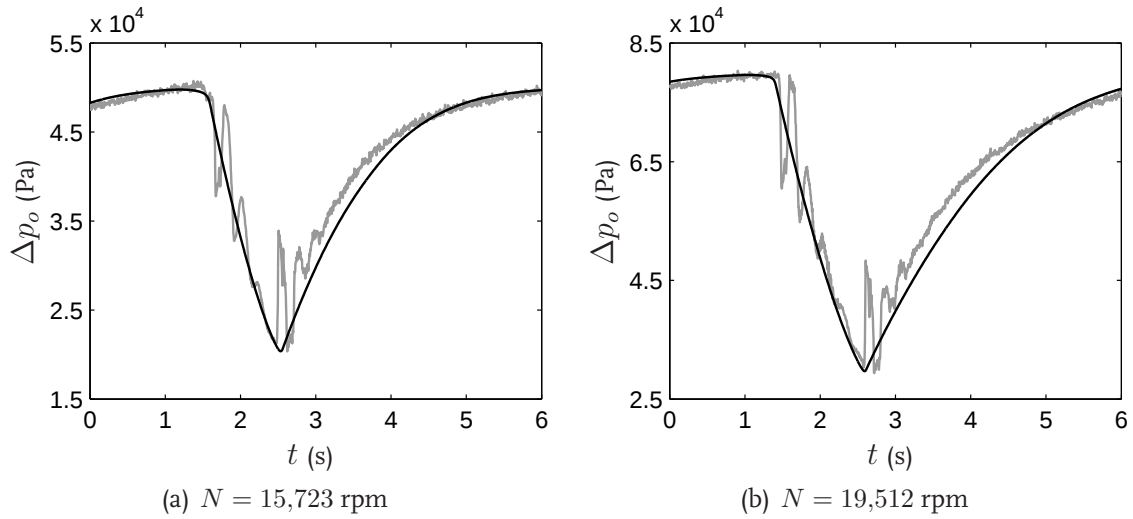
Secondly, in Figure 4.6 we observe various rapid pressure transients of decreasing amplitude in the measured signals that are not captured by the simulation model. See Figure 4.7 for the dimensionless equivalent of these results. From the simulated surge limit cycle in Figure 4.8 we can see that the mentioned pressure transients occur after each flow reversal.



**Figure 4.4** / Pressure measurement (gray) and simulation result of the tuned Greitzer model (black) during surge;  $N = 15,723$  rpm,  $\dot{m}_{c0} = 2.07$  kg/s,  $u_r = 0$ ,  $L_c = 10.6$  m.



**Figure 4.5** / Pressure measurement (gray) and simulation result of the tuned Greitzer model (black) during surge;  $N = 19,512$  rpm,  $\dot{m}_{c0} = 2.62$  kg/s,  $u_r = 0$ ,  $L_c = 10.6$  m.



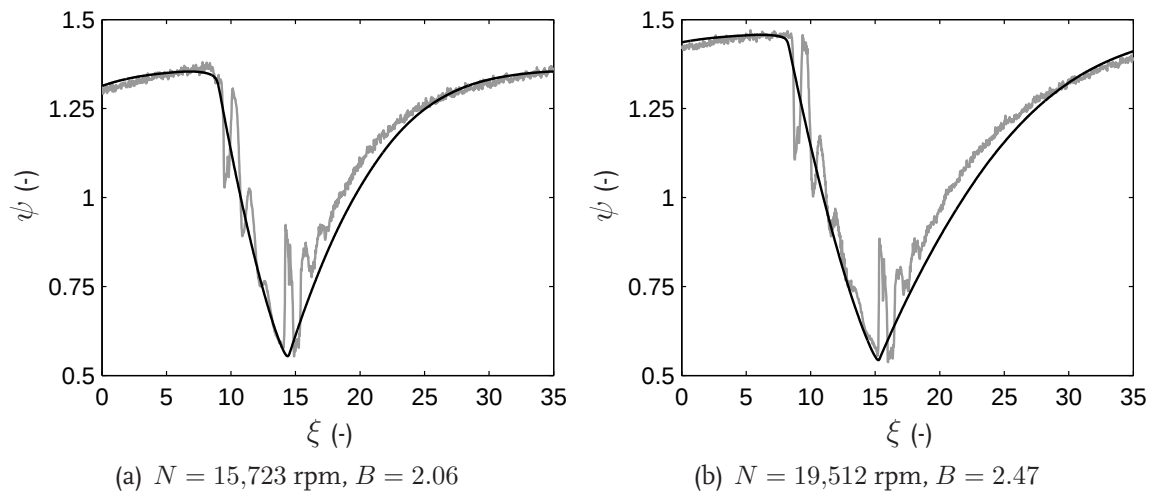
**Figure 4.6** / Close-up of pressure measurements (gray) and simulation results of the tuned Greitzer model (black) at two rotational speeds.

Close inspection of the rapid transients reveals a resemblance of the signal shape with that of a pressure response in a fluid transmission line subjected to a stepwise change in flow (Goodson and Leonard, 1972). Furthermore, when a correction ( $c \pm u$ ) is applied for the speed of sound and mean flow velocity, the duration of the fast transients appears to be almost independent (relative difference  $< 1.5\%$ ) of compressor rotational speed and surge frequency. Finally, transients as observed in Figure 4.6 were not visible in measurements on test rig A that has a much shorter discharge line of 1.45 m.

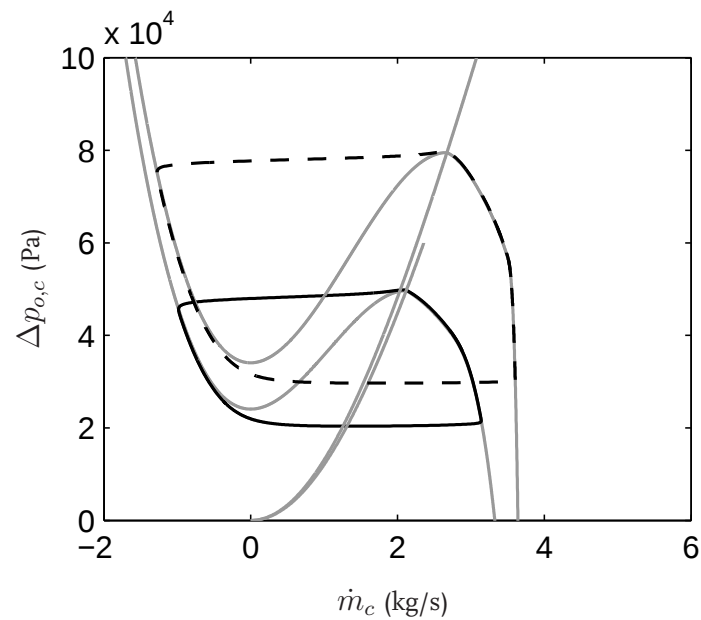
Based on the above findings we conclude that the damped pressure oscillations, occurring after each flow reversal, are the result of acoustic waves traveling back and forth in the discharge piping. Therefore, we will now discuss how the lumped parameter model can be extended to take into account the acoustic phenomena in the piping system.

### 4.3 Piping system acoustics

The study of flow and pulsations in piping systems is a field on its own with applications ranging from process plants to musical instruments. One of the many textbooks on flow-structure interactions is the one by (Howe, 1998). Over the years, high-order finite element, boundary element and finite difference methods have been developed to solve the governing equations. An attractive simplification follows from the fact that the wave equation is linear and hence that it is usually appropriate to limit the analysis to that of harmonic perturbations. In that case, a linear frequency domain model is sufficient for describing the sound propagation and this approach is extensively used in the literature (e.g. Munjal, 1987; Hirschberg et al., 1995). However, coupling these linear system



**Figure 4.7** / Dimensionless close-up of pressure measurements (gray) and simulation results of the tuned Greitzer model (black) at two rotational speeds;  $u_r = 0$ .



**Figure 4.8** / Simulated limit cycles during surge at  $N = 15,723$  rpm (solid line) and  $N = 19,512$  rpm (dashed line).

descriptions to a, in many cases nonlinear, source description is not always straightforward as discussed by [Albertson et al. \(2006\)](#).

Waves propagate through a piping system with finite speed as we discussed in the previous section. To cover this aspect of acoustics, so-called transmission line models or derivatives have been proposed in literature. Originating from the field of power systems (e.g. [Dommel, 1969](#)), this type of model is used in many fields of applications like hydraulics ([Goodson and Leonard, 1972](#); [Stecki and Davis, 1986](#)), communication networks ([Matick, 1995](#)), and music synthesis ([Smith, 1992](#)).

Of equal importance as the wave propagation through the pipes, is the behavior of the fluid at the pipe boundaries. To completely describe the piping system acoustics, the wave propagation model must be augmented with proper boundary conditions that describe the, possibly nonlinear, coupling between the pipe and adjacent system components or the environment.

In this section we will propose an acoustic model for the discharge piping to account for the acoustic pressure fluctuations behind the compressor. After discussing the wave propagation model we address the selection of appropriate boundary conditions. These boundary conditions will enable the coupling of the pipeline model to the Greitzer lumped parameter model for the compression system under study. For notational simplicity we omit the stagnation subscript  $o$  from all pressure variables in this section.

### 4.3.1 Dynamic model for piping system

The model for the compressor discharge piping is based on the pipeline model developed by [Krus et al. \(1994\)](#). This model was derived by using the method of characteristics and the authors showed that it captures the essential dynamics of a finite wave propagation speed and distributed frequency-dependent friction. Benefits of the model are its robustness, straightforward implementation, and computational efficiency [Krus et al. \(1994\)](#).

The main assumptions for the pipeline model are

1. acoustic perturbations of  $p, T, \rho, c$  are small compared to their undisturbed values;
2. disturbances propagate isentropically along the transmission line;
3. thermal effects are negligible;
4. surge frequency is below the duct's cut-off frequency.

The first assumption implies that a linear model is sufficient to describe the propagation of acoustic waves. The second assumption allows the introduction of the isentropic speed of sound into the state equation. The third assumption implies that the energy equation

can be ignored. Finally, the fourth assumption indicates that the flow in the pipeline can be considered to be one-dimensional. In other words, only planar acoustic waves will propagate through the pipeline. See the review paper by [Stecki and Davis \(1986\)](#) for an extensive overview of the various assumptions used in transmission line modeling.

As mentioned, the pipeline model is based on the method of characteristics that uses the fact that pressure and flow are related to each other by the following characteristic variables

$$\Phi(x, t) = p(x, t) + Z_0 q(x, t) \quad \Psi(x, t) = p(x, t) - Z_0 q(x, t) \quad (4.3)$$

where  $\Phi$  and  $\Psi$  are associated with right and left traveling waves, respectively. Here, the right traveling wave is defined as moving from the compressor outlet ( $x = 0$ ) to the end of the pipe ( $x = L$ ). The parameter  $Z_0 = \rho_0 c_0 / A$  represents the acoustic impedance of a filled pipe with cross-sectional area  $A$ .

The boundary conditions at the ends of the line are described by adjacent components where the flow is typically a function of the pressure. Using Equation (4.3), we get the following set of general equations that has to be solved at the left end ( $x = 0$ ) of the line

$$q_i(t) = f(p_i(t)) \quad (4.4)$$

$$p_i(t) = Z_0 q_i(t) + \Psi_i(t) \quad (4.5)$$

and, similarly, for the right end ( $x = L$ ) of the line

$$q_j(t) = f(p_j(t)) \quad (4.6)$$

$$p_j(t) = Z_0 q_j(t) + \Phi_j(t) \quad (4.7)$$

where  $q = uA$  represents the volume flow in the line. The general indices  $i, j$  are used to denote the pipeline boundaries. Note that we have used the convention that flows entering the line are positive.

According to [Krus et al. \(1994\)](#), in the frequency domain a transmission line can be described by the following equation

$$\begin{bmatrix} \cosh(\tau \sqrt{F(s)}) & -\frac{1}{Z_0 \sqrt{F(s)}} \sinh(\tau \sqrt{F(s)}) \\ -Z_0 \sqrt{F(s)} \sinh(\tau \sqrt{F(s)}) & \cosh(\tau \sqrt{F(s)}) \end{bmatrix} \begin{bmatrix} Q_i(s) \\ P_i(s) \end{bmatrix} = \begin{bmatrix} -Q_j(s) \\ P_j(s) \end{bmatrix} \quad (4.8)$$

where  $F(s)$  is a frequency-dependent friction factor. The terms  $Q(s)$  and  $P(s)$  are the Laplace<sup>2</sup> transforms of the volume flow  $q$  and pressure  $p$ , respectively. The time delay

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<sup>2</sup>Pierre Simon Laplace (1749–1827) was a French mathematician who worked on probability, differential equations and the application of mathematics in astronomy and physics.

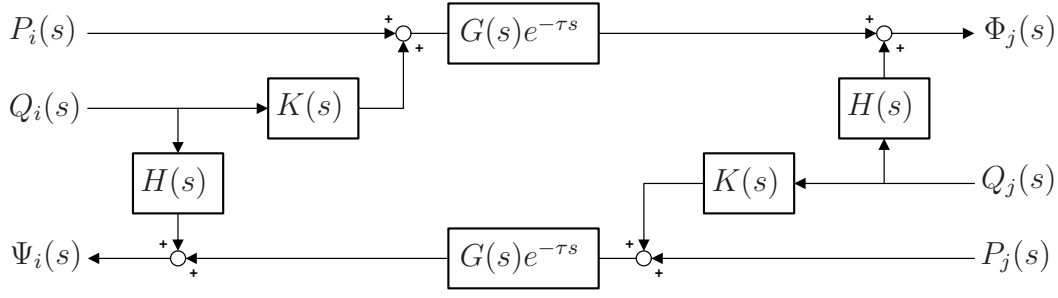


Figure 4.9 / Acoustic model for discharge line.

$\tau = L/c_0$  corresponds to the time needed for a wave to travel through the pipe. A detailed derivation is provided in Appendix B.

In the case of a uniformly distributed resistance,  $F(s)$  is given by

$$F(s) = \frac{R}{Z_0 \tau s} + 1 \quad (4.9)$$

where  $R$  represents the total resistance in the line. In the case of frequency dependent friction, the expression for  $F(s)$  is much more complicated (Krus et al., 1994).

By using Equations (4.4)–(4.7) we can rewrite Equation (4.8), yielding

$$\Psi_i(s) = e^{-\tau s \sqrt{F(s)}} \left( P_j(s) + Z_0 \sqrt{F(s)} Q_j(s) \right) + Z_0 \left( \sqrt{F(s)} - 1 \right) Q_i(s) \quad (4.10)$$

$$\Phi_j(s) = e^{-\tau s \sqrt{F(s)}} \left( P_i(s) + Z_0 \sqrt{F(s)} Q_i(s) \right) + Z_0 \left( \sqrt{F(s)} - 1 \right) Q_j(s) \quad (4.11)$$

In order to enable inverse transformation of the above model into the time domain, some simplifications are required. For that purpose Krus et al. (1994) used the block diagram for the derived pipeline model as shown in Figure 4.9.

The frequency region of interest lies around  $1/2\tau$ , corresponding to the time period in which a wave can travel through the entire pipeline and back. By approximating the asymptotes of Equations (4.10) and (4.11) and matching the steady-state pressure drop in the line, the following rational transfer functions were obtained

$$H(s) = \frac{R}{\kappa \tau s + 1} \quad (4.12)$$

$$K(s) = Z_0 \quad (4.13)$$

$$G(s) = \frac{s/\omega_n + 1}{(s/\omega_l + 1)(s/\omega_d + 1)} \quad (4.14)$$

where  $\omega_d = 1/\kappa\tau$  and  $\omega_n = \omega_d e^{R/2Z_0}$ . A suitable value  $\kappa$  was found by numerical experiments, yielding  $\kappa = 1.25$  (Krus et al., 1994). The value of  $\omega_l$  can be used to tune the low-pass filter part of Equation (4.14) in order to get the correct attenuation at a particular

frequency, for example to match the damping of a higher harmonic. This low-pass filter has been introduced to circumvent the use of the complex and unpractical expression for  $F(s)$  in the case of frequency dependent friction. Modeling the acoustic damping due to the friction in the system will be discussed in detail later on.

We point out that the introduced rational transfer functions are only a low-order approximation of the irrational representation of Equation (4.8). Higher-order approximations might improve the accuracy of the model at the cost of increased complexity. The focus for this thesis is to include those acoustic effects that have the largest impact on the overall system behavior during surge. The experimental data presented in Section 4.2.3 indicate that the dominant acoustic phenomenon is associated with the traveling waves through the discharge piping and hence a low-order approximation is considered to be sufficient (Whitmore and Moes, 1991). Higher order approximation techniques for the irrational transmission line model are discussed by, for example, Yang and Tobler (1991).

To summarize, an approximation of a discharge pipeline with distributed, frequency-dependent resistance is defined by Figure 4.9 and Equations (4.12), (4.13) and (4.14). We can obtain a time domain solution ( $q_i(t)$ ,  $p_i(t)$ ,  $q_j(t)$ ,  $p_j(t)$ ) by choosing an appropriate numerical implementation. For example, we can use inverse Laplace transformation of inputs and outputs or convert the filters into difference equations via a bilinear transformation.

### 4.3.2 Piping boundary conditions

Finally, the acoustic model for the discharge piping must be connected to the dynamic model of the compression system by selecting appropriate boundary conditions at each end of the pipeline (Krus et al., 1994; Albertson et al., 2006). Generally speaking, boundary conditions are associated with a change of the acoustic impedance. At any point in the system where the acoustic impedance changes, an incident wave will be (partially) reflected. This property will be used to select boundary conditions that enable us to couple the acoustic model of the discharge piping to the lumped parameter model, given by Equations (4.1) and (4.2).

For that purpose it is convenient to use the simplified two-port representation of the Greitzer model given in Figure 4.10. From this figure we see that the input  $q_i(t)$  for the line model can be obtained directly from the impulse balance of the adjacent section of the compressor model. More specifically,  $q_i(t) = q_c(t)$  is calculated by dividing  $\dot{m}_c(t)$ , provided by Equation (4.1), with  $\rho_2$ . Note that in this notation the required transformation between time and frequency domain is omitted. The line input  $p_i(t)$  at the left boundary is calculated via Equation (4.5). Note that this pressure represents the pressure downstream of the compressor, so now  $\Delta p(t) = p_i(t) - p_1$  with  $p_i(t) = p_c(t)$  has to be used in Equation (4.1).

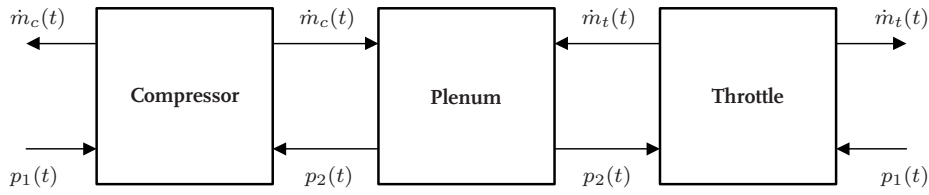


Figure 4.10 / Two-port representation of the Greitzer model.

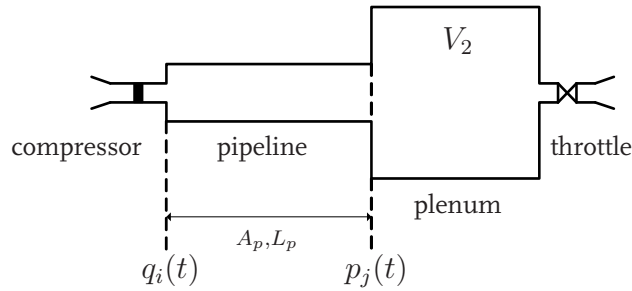


Figure 4.11 / Schematic representation of a pipeline-plenum boundary condition.

Selecting a second prescribed flow boundary condition at the other end of the line would only allow the trivial solution  $q_i(t) = q_j(t)$ ,  $p_i(t) = p_j(t)$ . Therefore, the prescribed flow boundary at the entrance of the discharge line model dictates a prescribed pressure (or a linear combination of pressure and flow) boundary condition at the other end of the line. A suitable boundary of this kind is for example a point where the pipeline is connected to a vessel or a pipe with a larger diameter, i.e., a plenum volume. Such a boundary is schematically depicted in Figure 4.11.

The prescribed pressure for such a boundary condition is provided by the plenum mass balance, yielding  $p_j(t) = p_2(t)$  with  $p_2(t)$  from Equation (4.2). The flow variable  $q_j(t) = q_p(t)$  at the right boundary is then calculated via Equation (4.7) and the corresponding mass flow  $\dot{m}_p(t) (= q_p(t)\rho_2)$  replaces  $\dot{m}_c(t)$  in Equation (4.2). Note the sign convention in Equation (4.8) that all flows entering the line are positive.

When physically plausible, another option to obtain a non-trivial system is to place a flow restriction with a known flow-pressure relation (e.g. a throttle) between the end of the line and a constant pressure reservoir. In that case a plenum volume is not needed and Equations (4.6) and (4.7) can be combined to provide the variables  $q_j(t)$  and  $p_j(t)$  at the line boundary.

Based on numerical experiments and practical experience we neglect the effect of acoustic perturbations on throttle mass flow, i.e. acoustic effects behind a plenum volume or pipe with a sufficiently large diameter do not need to be modeled. Hence, in our case the compressor and plenum volume from Figure 2.2 provide the boundary conditions for the acoustic model of the discharge line. A two-port representation of the proposed coupling

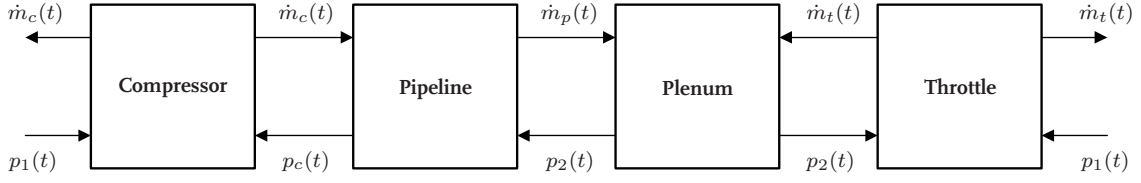


Figure 4.12 / Two-port representation of the combined Greitzer-pipeline model.

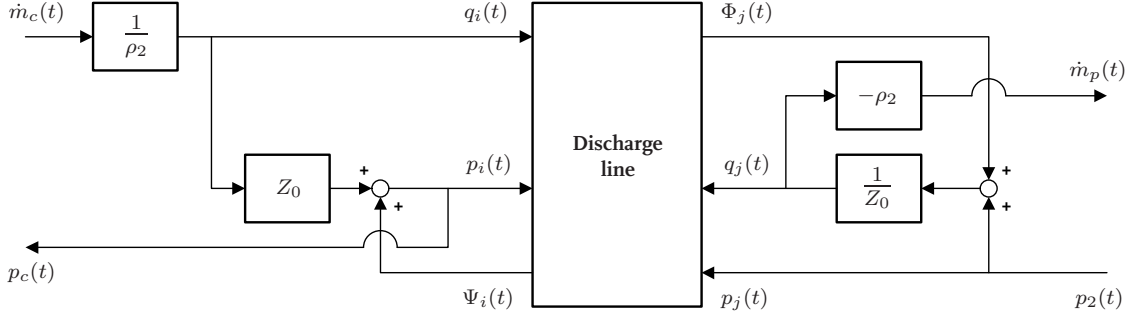


Figure 4.13 / Two-port representation of the pipeline model boundary conditions.

between the Greitzer model and the discharge line model is given in Figure 4.12 while Figure 4.13 shows the pipeline model boundary conditions in more detail. In the next section we will address the implementation of the developed model and we will discuss the simulation results that were obtained.

## 4.4 Numerical results

The numerical implementation of the proposed pipeline model was done in accordance with Figure 4.12. Details of the pipeline model and the coupling with the original Greitzer model are shown in Figures 4.9 and 4.13. In order to carry out simulations for the centrifugal compression system under study, the parameters for the pipeline model must be set to appropriate values.

An important parameters that determines the damping of acoustic transients is the resistance or friction coefficient  $R$ . For turbulent flows the viscous friction coefficient  $R$  is usually estimated by using Blasius'<sup>3</sup> empirical law (Whitmore, 1988)

$$R = 0.1582 \text{Re}^{\frac{3}{4}} \frac{\mu}{D^2} \quad (4.15)$$

where  $Re = \rho_0 u_0 D / \mu$  is the Reynolds number and  $\mu$  denotes the dynamic viscosity of

<sup>3</sup>Paul Richard Heinrich Blasius (1873–1970) was a German fluid dynamics engineer who is best known for his mathematical work on steady two-dimensional boundary layers. In 1911 he developed the expression for the flow resistance in smooth pipes.

**Table 4.2** / Simulation parameters of the combined Greitzer-pipeline model.

| Parameter                   | Value  | Unit           |
|-----------------------------|--------|----------------|
| pipeline length $L_p$       | 15.4   | m              |
| pipeline area $A_p$         | 0.0707 | m <sup>2</sup> |
| plenum volume $V_2$         | 7.62   | m <sup>3</sup> |
| pipe resistance $R$         | 0.674  | -              |
| numerical constant $\kappa$ | 1.25   | m <sup>3</sup> |

the medium. With the above formula we calculated an average value of  $R$  by using the various flow conditions that were encountered in the compression system under study, yielding  $\bar{R} \approx 0.674$ .

The cut-off frequency  $\omega_l$  of the low-pass filter in  $G(s)$  was initially set to  $2/\tau = 2c_0/L_p$  during simulations, neglecting the influence of the mean flow velocity  $u_0$ . The influence of the parameter  $\omega_l$  on the acoustic damping properties of the model will be discussed in more detail below. Values for the density and speed of sound are obtained from gas property tables (Baehr and Schwier, 1961), using pressure and temperature measurements at the start of an experiment.

In order to define the time delays and the acoustic impedance, also values for  $L_p$  and  $A_p$  are needed. Initial simulations showed that the dominant acoustic effects are captured correctly when the pipeline-plenum boundary is placed at the first diameter change of the discharge piping, see also Figure 4.14. An identification procedure was followed to determine the precise pipeline length  $L_p$  and plenum volume  $V_2$ , similar to the method that we used for the original Greitzer model of the test rig. For physical consistency we set the value for  $L_c$  equal to the one for  $L_p$ . The resulting simulation parameters are summarized in Table 4.2. Finally, we remark that all identification and tuning procedures were carried out for one particular set of measurement data. Afterwards we validated the obtained parameters by comparing simulation results with different sets of measurement data.

Before presenting the simulation results we will also discuss the assumption that the pipeline dynamics can be described by a linear model for planar waves. The linearity assumption is justified when acoustic perturbations  $\tilde{p}$ ,  $\tilde{T}$ ,  $\tilde{\rho}$ , and  $\tilde{c}$  are small compared to their undisturbed values  $p_{o,0}$ ,  $T_0$ ,  $\rho_0$ , and  $c_0$ . Measurement data indicates that  $\tilde{p}/p_{o,0} \approx 0.1$ ,  $\tilde{\rho}/\rho_0 \approx 0.1$ ,  $\tilde{T}/T_0 \approx 0.02$ , and  $\tilde{c}/c_0 \approx 0.02$ . Furthermore, we remark that the cut-off frequency of a round pipe  $\omega_b = 1.84c/r$  with radius  $r = 0.3$  m is well above the surge frequencies encountered in the compression system under study. Hence, the linearity and planar wave assumptions for the pipeline model are valid for all operating conditions of the compression system under study.

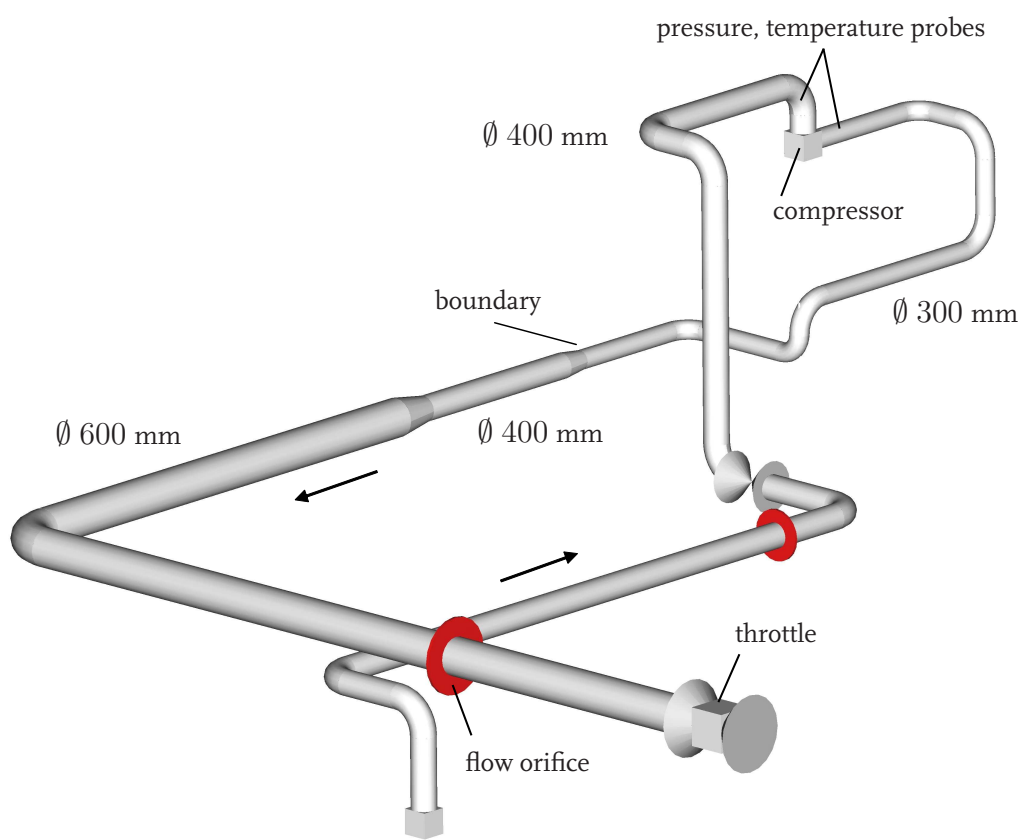
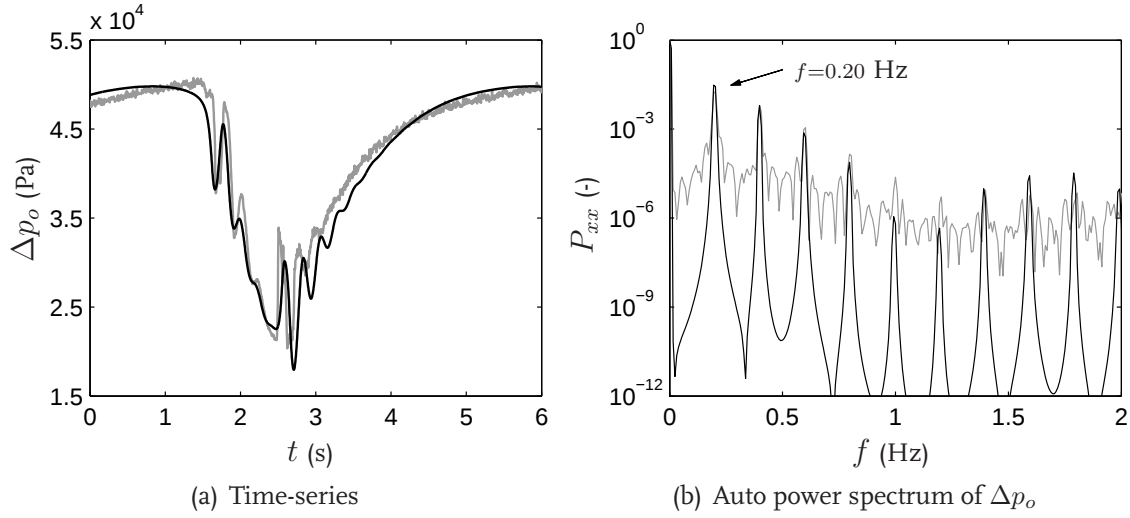


Figure 4.14 / Piping layout of the centrifugal compressor test rig.

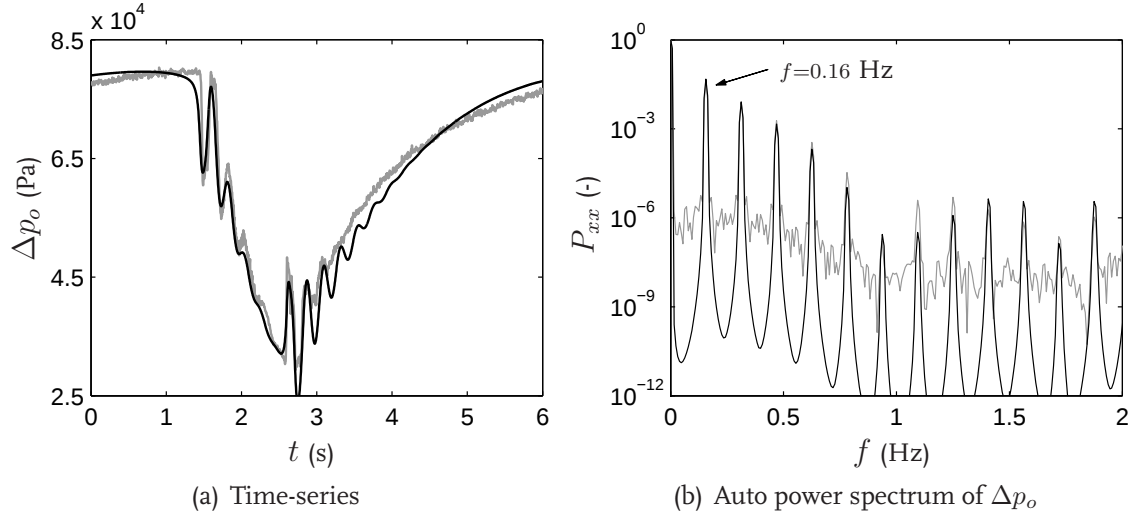


**Figure 4.15** / Pressure measurement (gray) and simulation result after tuning of the combined Greitzer-pipeline model (black);  $N = 15,723$  rpm,  $\dot{m}_{c0} = 2.06$  kg/s,  $u_r = 0$ ,  $L_c = 10.6$  m,  $\omega_l = 2/\tau$ .

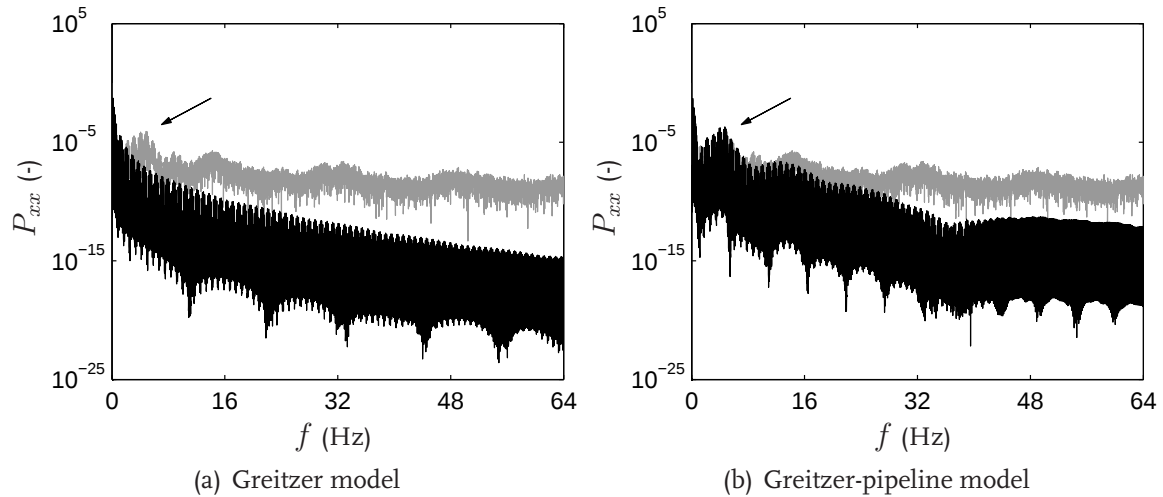
The simulation results for two specific measurements are shown in Figures 4.15 and 4.16. These results indicate that the pipeline model extension has introduced the necessary dynamics to describe the observed pressure transients after flow reversals that arise when the compression system operates in surge. The contribution of the pipeline model extension also becomes clear from Figures 4.17 and 4.18. From these figures we see that the extended model, in contrast to the original Greitzer model, also describes the signal components around 4 Hz that are associated with traveling waves in the discharge piping. The period time of the transients is captured well by the extended model, although numerical tests learned that precise tuning of the model is required to predict the correct frequency of the slow surge oscillations and pipe system pulsations. In particular, the values for  $L_p$  and  $V_2$  appear to have a large influence on the frequency of the various oscillations and transients.

Equivalently, we can say that the value of the stability parameter  $B$  has a significant influence on the surge behavior and acoustic reflections. At this point we remark that we used a rather high value for  $L_c$ , yielding a value for  $B$  between 1.5 and 1.8. An incorrect (too low) value for  $B$  might explain the fact that the simulated pressure drop at the start of a new surge cycle is less steep than the measured pressure drop. In Chapter 5 we will address the effect of  $B$  and the associated physical parameters on the surge oscillations and transients in more detail.

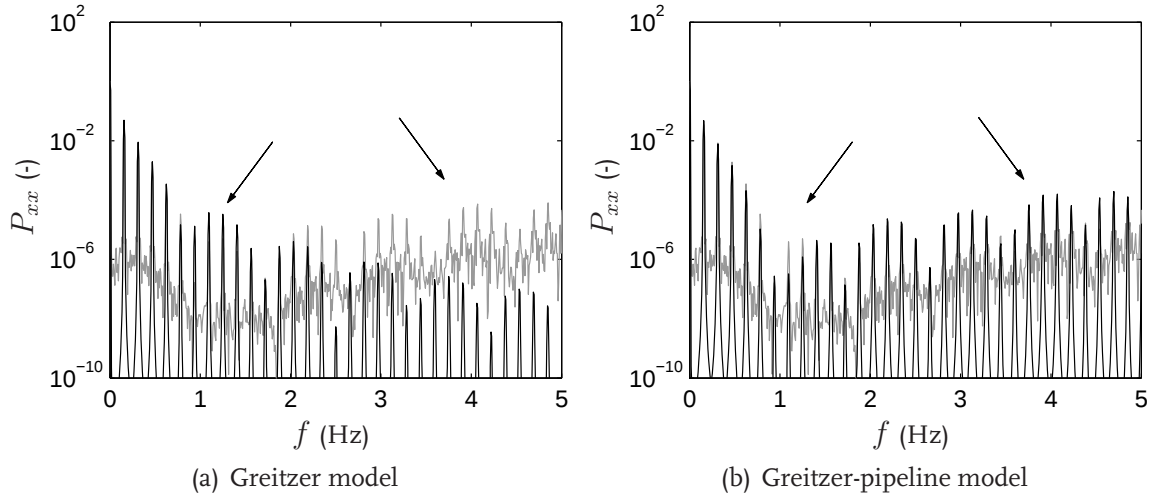
In Figures 4.15 and 4.16 we observe that the amplitudes of the pipeline transients are predicted by the extended model with limited accuracy. Both at 15,723 rpm and 19,512 rpm the measured transients during back-flow (between 1–2 s) have a larger amplitude



**Figure 4.16** / Pressure measurement (gray) and simulation result after tuning of the combined Greitzer-pipeline model (black);  $N = 19,512$  rpm,  $\dot{m}_{c0} = 2.62$  kg/s,  $u_r = 0$ ,  $L_c = 15.4$ ,  $\omega_l = 2/\tau$ .



**Figure 4.17** / Auto power spectra of measurement (gray) and simulation result (black) of the original and extended models;  $N = 19,512$  rpm,  $\omega_l = 2/\tau$ .



**Figure 4.18** / Close-up of auto power spectra in region of interest of measurement (gray) and simulation result (black) of the original and extended models;  $N = 19,512$  rpm,  $\omega_l = 2/\tau$ .

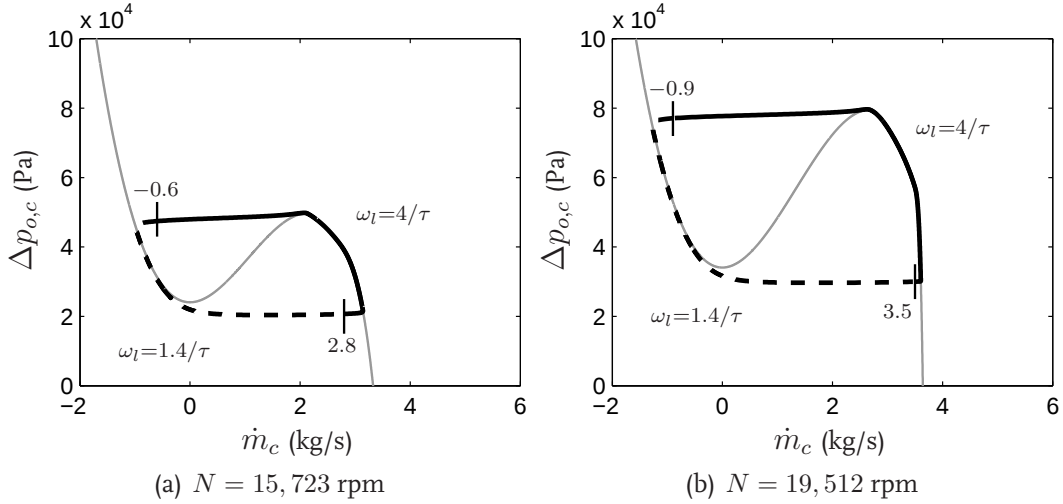
than the simulated ones, see Figures 4.15 and 4.16. Furthermore, at 19,512 rpm the damping is clearly too low during the positive flow phase of the surge cycle.

By comparing Figures 4.15(a) and 4.16(a) with Figure 4.8, we see that damping of the transients appears to depend on the momentary pressure and flow velocity in the pipeline. The dependency of acoustic damping on flow conditions (density) was already mentioned by Whitmore and Moes (1991). Given the above observations we modified the pipeline model by making the density  $\rho_0$  and speed of sound  $c_0$  a function of the momentary pressure  $p_{o,c}(t)$ . This modification also caused the related model parameters ( $R$ ,  $\tau$ ,  $Z_0$ ,  $\omega_n$ ,  $\omega_d$ , and  $\omega_l$ ) to become a function of  $p_{o,c}(t)$ .

Simulations with this modified model showed some improvements in the damping properties of the model in comparison with experimental data. However, a mismatch between the damping of measured and simulated transients remained.

The model parameter that directly influences the damping is the cut-off frequency  $\omega_l$ . Hence, we made another modification to the pipeline model by applying two different values for  $\omega_l$  during the periods of negative (low velocity) and positive (high velocity) flow. This rather ad-hoc approach is based on the intuitive reasoning that the flow direction has an effect on the amount of damping at the compressor boundary. The values for  $\omega_l$  were determined by manual tuning and we used a relay to switch between the two values. The switching behavior is illustrated in Figure 4.19, showing the switching instants and the corresponding values for  $\omega_l$ .

The results from the simulations with the modified pipeline model are shown in Figures 4.20 and 4.21. The dimensionless equivalents are depicted in Figure 4.22. We point



**Figure 4.19** / Modeled switching behavior to select the value of cut-off frequency  $\omega_l$ .

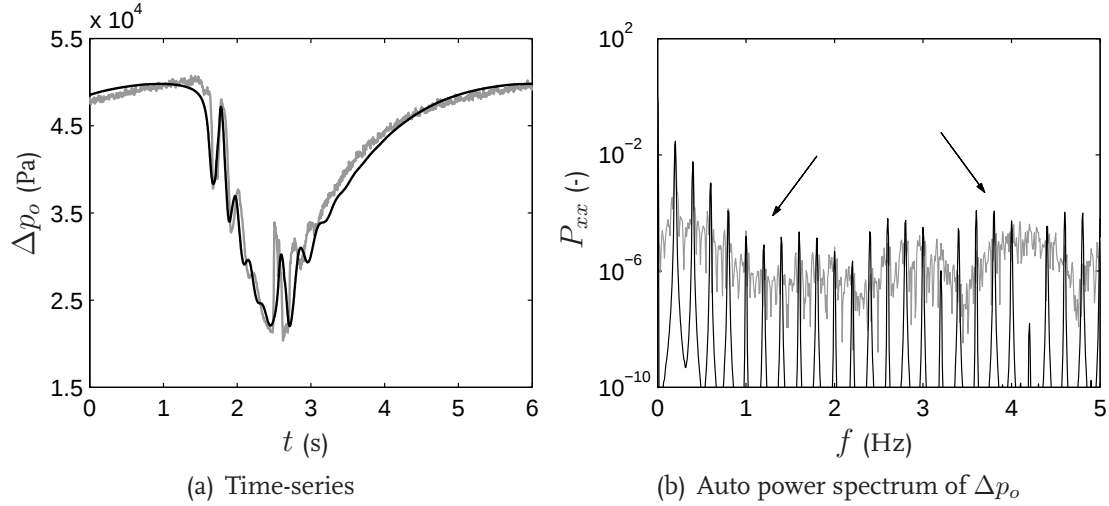
out that including the pressure dependency of the model parameters and the switching law for  $\omega_l$ , has made the pipeline model highly nonlinear.

When comparing Figure 4.20 with Figure 4.15 we observe that the nonlinear pipeline model has resulted in a slightly better match between the measurement and simulation. However, despite the introduced pressure dependency and the fact that damping is altered via the selection of two different values for  $\omega_l$ , the amplitudes of the simulated pressure transients during back-flow are still too low. A possible explanation for this mismatch is that the effect of rotor speed variations is not captured in the model for the compression system. In a variable speed system the compressor characteristic is a function of rotor speed and this will influence the pressure and mass flow transients during surge (e.g. Fink et al., 1992; Gravdahl and Egeland, 1999a).

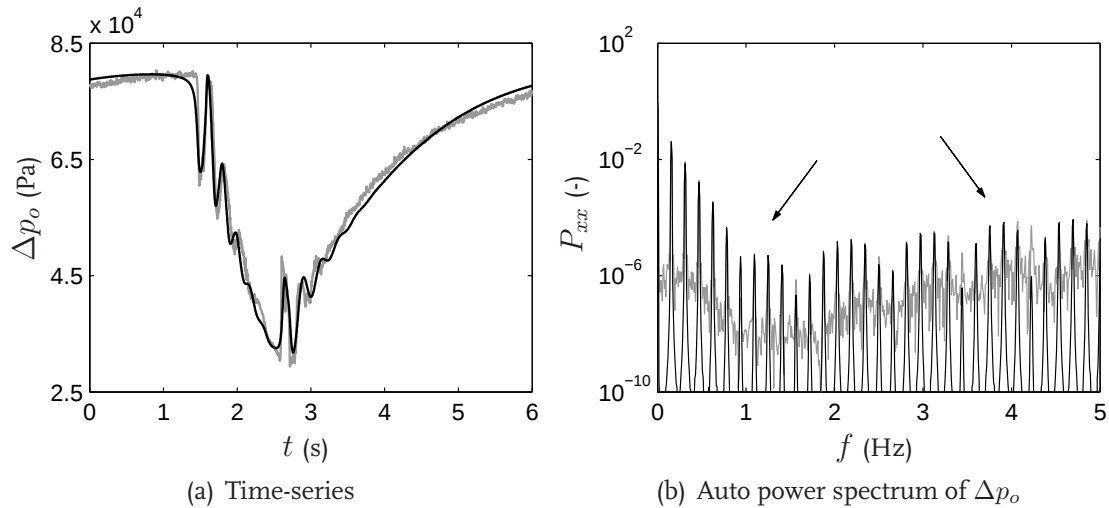
Other possible explanations for the observed differences between measurements and simulations are that the selected value for  $\omega_l$  is incorrect or the simple switching method is not capable of describing the variations in acoustic damping that occur during back-flow. Furthermore, the mismatch can be caused by the inadequate modeling of friction elements like piping bends or by estimation errors in the compressor curve.

Furthermore, from the auto power spectrum in Figure 4.20 we see that one of the acoustic resonances in the measurement is not present in the simulated signal. On the other hand, the prediction for the amplitude of the peak around 1 Hz is improved in comparison with the pipeline model with constant parameters. An explanation for these differences in the predicted auto power spectra is not available yet.

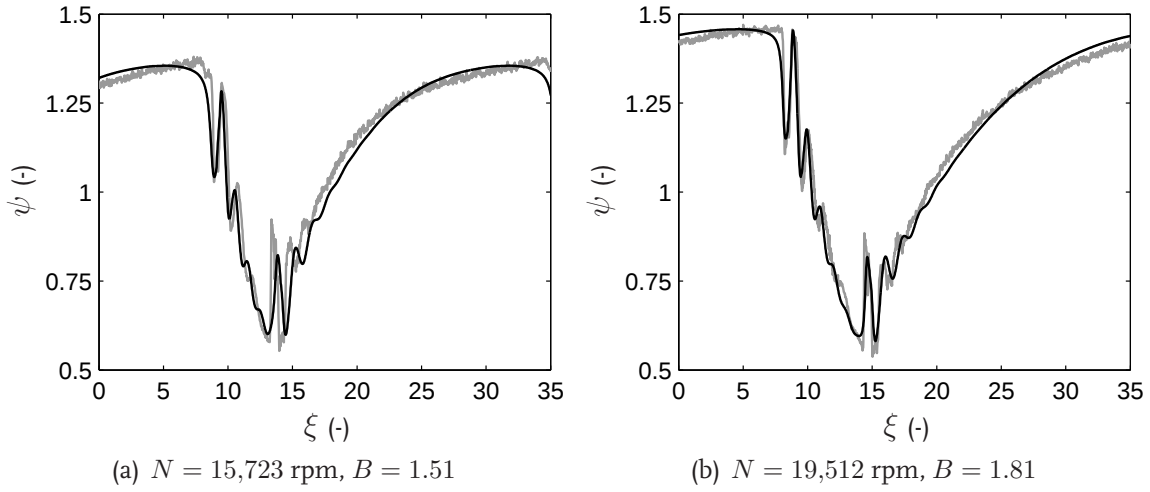
For the experiment at  $N = 19,512$  rpm the modified model resulted in a larger improvement as can be seen in Figure 4.21. The amplitudes of the acoustic transients are accurately described by the modified pipeline model. However, since the cut-off frequency in



**Figure 4.20** / Pressure measurement (gray) and simulation result after tuning of the combined Greitzer-pipeline model with velocity-dependent damping (black);  $N = 15,723$  rpm,  $\dot{m}_{c0} = 2.05$  kg/s,  $u_r = 0$ ,  $L_c = 15.4$  m,  $\omega_l = \{1.4/\tau, 4/\tau\}$ .



**Figure 4.21** / Pressure measurement (gray) and simulation result after tuning of the combined Greitzer-pipeline model with velocity-dependent damping (black);  $N = 19,512$  rpm,  $\dot{m}_{c0} = 2.61$  kg/s,  $u_r = 0$ ,  $L_c = 15.4$  m,  $\omega_l = \{1.4/\tau, 4/\tau\}$ .



**Figure 4.22** / Dimensionless plot of pressure measurements (gray) and simulation result after tuning of the combined Greitzer-pipeline model with velocity-dependent damping (black).

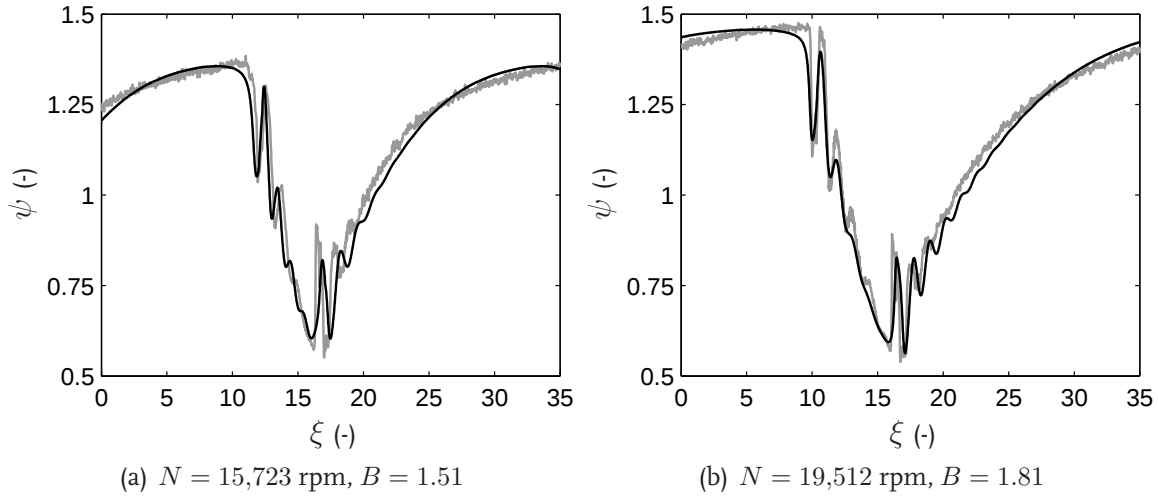
the positive flow phase is relatively low (31.7 Hz), the simulated signal starts to lag behind the measured signal due to the phase shift introduced by the low-pass filter. We point out that for this speed the amplitude of the resonance around 4 Hz is accurately predicted in contrast to the test at  $N = 15,723$  rpm.

To complete the validation of the dynamic model including piping acoustics for test rig B, including the velocity-dependent damping, we compared the simulation results with entirely new data from additional experiments. The results for two surge measurements at  $N = 15,750$  and  $N = 19,200$  rpm are shown in Figure 4.23. Despite the differences between the measured and simulated pressure signals we conclude that the developed model is capable of describing the surge behavior of test rig B with reasonable accuracy at different operating conditions.

## 4.5 Discussion

In this chapter we have discussed the modeling of the dynamic behavior of large centrifugal compression systems. We applied the well-known Greitzer lumped parameter model to describe surge transients of a centrifugal compressor test rig. Comparison with experimental data showed that, after tuning the appropriate parameters, this model describes the surge oscillations that occur at low mass flows with reasonable accuracy.

However, we have argued that the dynamics of a long pipe system can have a profound influence on the shape of the surge oscillations in a compression system. Experimental evidence is presented for the presence of acoustic waves in the discharge piping of



**Figure 4.23** / Dimensionless plot of validation result for the combined Greitzer-pipeline model with velocity-dependent damping; pressure measurements (gray) and simulation result (black).

the centrifugal compression system under study. The amplitude and frequency of these pipe system transients are such that a detailed study of their effect on the overall system dynamics is justified.

The main contribution of this chapter is the developed aero-acoustic model for a compressor discharge line that enables the study of pipe system transients in more detail. The first benefit of the proposed model is its relative simplicity while it describes the relevant acoustic phenomena in a pipeline. We made it plausible that the linearity and planar wave assumptions on which the pipeline model is based, hold for all encountered operating conditions. Secondly, the modular port-structure of the model enables a direct coupling to the dynamic Greitzer model. The good agreement between simulation results and actual surge measurements indicates that the developed model indeed captures the essential dynamics of both the compressor and the pipe system.

However, the introduced transfer function approximations contain various parameters that require tuning when the model is applied to a specific compression system. Despite the qualitatively good results that were obtained with the applied tuning approach, it seems worthwhile to investigate how these model parameters can be determined in a more structured manner, for example via experimental identification methods. In this context it is also relevant to investigate which model parameters are critical for the accuracy of the overall model and how their uncertainty can be sufficiently reduced. In Chapter 5 we briefly come back to this issue when we discuss the results of a limited sensitivity analysis for the various acoustic model parameters.

Next to the parameters in the pipeline model, the selection of proper values for the compressor duct length and the plenum volume requires further attention. The values ob-

tained from tuning the model on experimental data are very high in comparison with values found for test rig A. The role of these parameters on both the surge dynamics and the acoustic transients will be investigated in Chapter 5.

Furthermore, we argued that the quality of the resulting model critically depends on the proper selection of boundary conditions. In this paper a rather pragmatic approach was used to select appropriate boundary conditions, partially depending on engineering experience. A more systematic approach for the coupling of a pipeline model to a lumped parameter compressor model and the positioning of boundary conditions might increase the applicability of the proposed model structure to other compression systems.

Finally, the numerical results indicate that damping of pipe system transients depends on the local flow conditions in the pipeline. For that reason we made the model parameters a function of the momentary pressure  $p_c(t)$ . Furthermore, we introduced a relay nonlinearity in the model to switch between two different cut-off frequencies of a low-pass filter, effectively changing the attenuation of acoustic transients. With this relatively simple modification of the pipeline model the damping was improved and the model describes the dynamic behavior of test rig B during surge with reasonable accuracy. However, the obtained results illustrated that the transfer function approximations in the pipeline model are not really suited to capture all damping effects of acoustic transients during surge. More research is required in order to develop a dynamic model that adequately describes all acoustic effects in full-scale compression systems. In particular the effect of rotor speed variations on system dynamics in general and acoustic transients in particular deserves further attention.

# Analysis of compressor dynamics

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**Abstract** / In this chapter the dynamic behavior of the compression system and the associated physics are discussed. First, the surge mechanism and the effect of the various model parameters on the surge oscillations is discussed. Then, linearized compressor models are introduced and the effect of the model parameters on the local dynamics are addressed. Finally, the results are summarized and the relevance of the different model parameters for predicting capabilities of the relevant dynamics is discussed.

## 5.1 Introduction

In the previous chapters we have discussed the model development and validation for the dynamic behavior of centrifugal compression systems. The presented results show that the Greitzer model and its derivatives are capable of describing surge transients in the investigated test rigs. Furthermore, the possible effects of varying gas properties, shapes of the compressor and load characteristics, and acoustic filter settings have been addressed in more or less detail. However, the questions on what the relevant dynamics are for surge control design and how these are influenced by different model parameters remain to be answered.

Greitzer (1976a) not only developed the equations for his successful compression system model, he also compared experimental results with theoretical predictions (Greitzer, 1976b) and discussed the underlying physics and the influence of various model parameters. However, the author paid most attention to the effect of the plenum volume on the encountered dynamics. In particular we point out that the effective compressor duct length received much less attention even though it is of equal importance for determining the value of the important stability parameter.

After the pioneering work by Greitzer (1976a,b), the model and the underlying principles have been exploited in numerous studies. However, hardly any report exists in literature

on parameter studies and dynamic analyses of the Greitzer model or one of its many derivatives.

Macdougall and Elder (1983) investigated the accuracy and parameter influence of a somewhat different dynamic model for a centrifugal compression system. The authors concluded that the accuracy of the geometry and compressor characteristic are of major importance when developing a model for an actual compression system. Elder and Gill (1985) presented a study of the various factors affecting surge in centrifugal compression systems. Next to discussing the effect of component geometry on the flow patterns and hence surge, they also concluded that the slope of the compressor curve is of major importance for the stability of a centrifugal compression system. Willems (2000) conducted a sensitivity analysis of the Greitzer model for a small laboratory scale compression system. His conclusions on the effect of the compressor duct length and compressor characteristic on the period time and amplitude, respectively of the surge oscillations are in line with the findings of Greitzer (1976a,b).

We now recall that one of our research goals is to develop a model for industrial scale compression systems that accurately describe those dynamics that are relevant for surge control design. The studies mentioned above all discussed small scale compression systems and neither of them paid equal attention to the sensitivity of all model parameters in relation with both the global and local dynamic behavior. The general idea behind active surge control is to suppress flow instabilities in an early stage of development. Hence, for adequate control design a compression system model must describe the local dynamic behavior around the desired operating point. Under the assumptions from Chapter 2, this implies that small transients in mass flow and pressure must be modeled correctly.

We point out that the adopted lumped parameter approach implies that only a limited number of modes<sup>1</sup> or resonances are represented by the model. Deciding on the minimum number of required modes in a linearized compression system model to enable the design of stabilizing controllers is far from trivial. Examples exist in literature that, next to the fundamental surge mode, higher order (acoustic) modes had to be included in order to succeed in designing a stabilizing control system (Weigl et al., 1998; Nelson et al., 2000).

Furthermore, it is beneficial when the model describes the dynamic behavior of the system during surge where we point out that surge is a nonlinear phenomenon. The first reason is because such a dynamic model makes it possible to design and evaluate a control system that is capable of bringing the system out of its surge mode. Secondly, surge cycle measurements are the only data available for model validation in the unstable regime. Hence, for a model that is capable of describing the nonlinear surge phenomenon, at least some means of experimental validation exist, see also Chapters 3 and 4.

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<sup>1</sup>The notion of modes in a dynamic system will be explained in more detail in Section 5.3.

In this chapter we will address the issues mentioned above. More specifically, we will discuss the stability and surge dynamics of turbocompressors and we illustrate how these are captured in the developed models. The main contribution of this chapter is an extensive parameter study that reveals the influence of different model parameters on the model predictions of turbocompressor dynamics, including those that are characteristic for the industrial scale test rigs under study. First of all, the results will confirm or extend the available qualitative insights in the relevant compressor dynamics. Secondly, the quantitative information from this study will indicate which parameters have the largest influence on the predicting capabilities of the dynamic models for the centrifugal compression systems under study. Finally, the results from our study will reveal how and to what extent the characteristics of various auxiliary components like valves and pipes influence the dynamics of the studied compression systems.

## 5.2 Nonlinear compressor dynamics

In this section we will discuss how the stability and surge dynamics of turbocompressors are captured by the developed nonlinear models. After an explanation of the surge behavior we will address the effect of the various model parameters on the predicted dynamic behavior. The insights gained from this analysis will be helpful in a qualitative assessment of the model accuracy. Furthermore, the results form the starting point for further work on improving the accuracy of the most relevant model parameters.

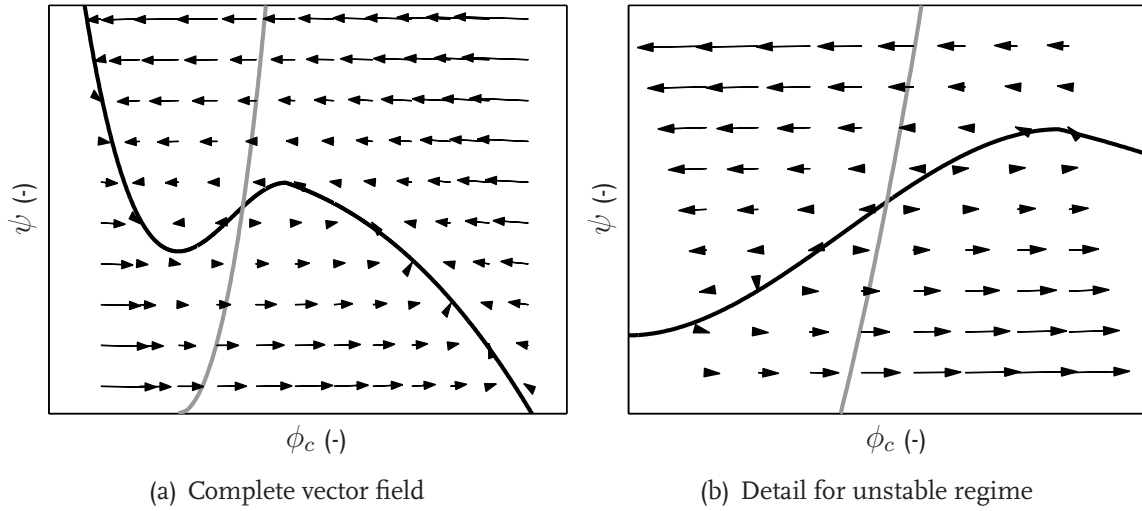
### 5.2.1 Surge cycle

As mentioned in previous chapters, deep surge is a limit cycle oscillation of the compression system that occurs when the intersection point of the compressor and load characteristics is located in the unstable regime. The apparent instability of such an operating point is best illustrated by a vector plot. Such a plot shows, for arbitrary points  $(\phi_c, \psi)$ , the rate and direction of change as described by the differential equations from a general Greitzer model

$$\frac{d\phi_c}{d\xi} = B (\Psi_c(\phi_c) - \psi) \quad (5.1)$$

$$\frac{d\psi}{d\xi} = \frac{F}{B} (\phi_c - \phi_t(\psi, u_t)) \quad (5.2)$$

where  $\phi_t(\psi, u_t)$  denotes a general load characteristic that represents all throttle and control valves in the system. Note that the parameter  $B$  can be regarded as a scaling factor of the  $\phi_c$  and  $\psi$  components of the velocity vectors. Note that the standard Greitzer model equations are obtained by selecting  $F = 1$ , see also Section 3.3.1.



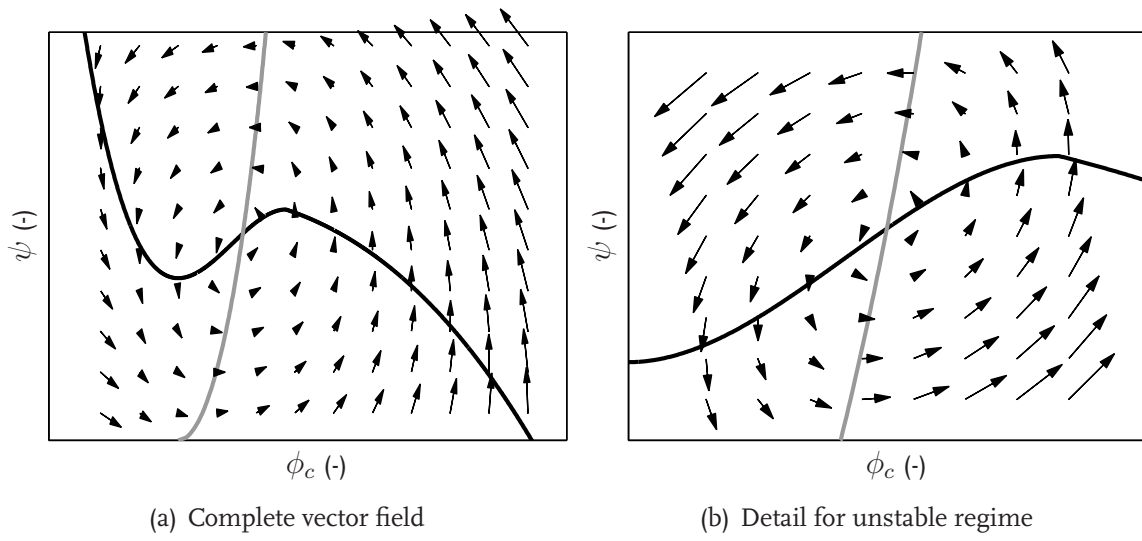
**Figure 5.1** / Vector field with compressor curve (black) and load curve (gray) to illustrate compressor behavior;  $B = 1$ ,  $F = 1$ .

The resulting vector plots for two different values of  $B$  are shown in Figures 5.1 and 5.2. We remark that some additional scaling of the vector lengths is applied to improve the clarity of the plots. The vector plot for a large value of  $B$  in Figure 5.1 clearly shows that the vectors point away from the positive slope part and towards the negative slope parts of the compressor curve. For a smaller value of  $B$  the vectors appear to focus more around the intersection of the compressor and load curves as can be seen in Figure 5.2.

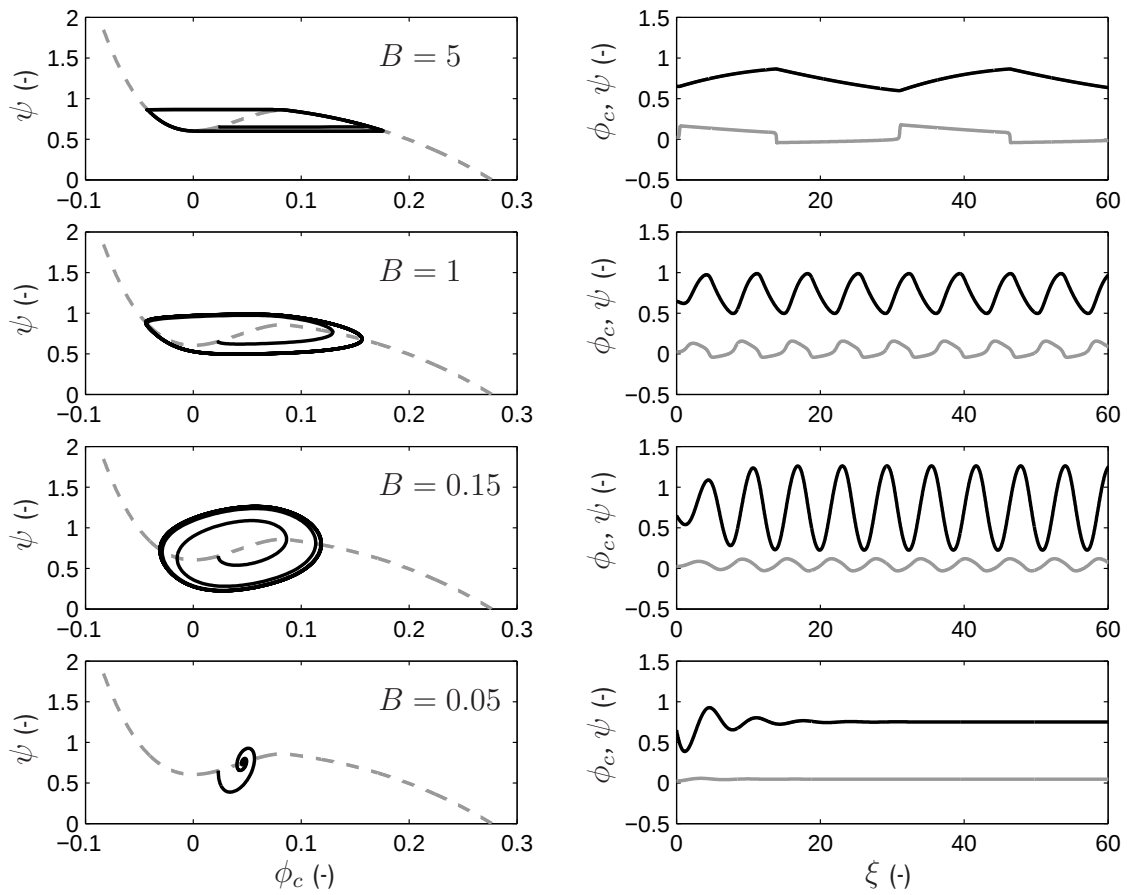
From these plots we conclude that operating points on the positive slope part of the compressor curve are unstable and that a larger value of  $B$  results in a faster development of deep surge. Hence, the parameter  $B$  is often called the stability parameter. In Section 5.3 we will make a more precise statement on the stability of the steady-state operating points. Finally, we point out that the vector fields are in line with the results shown in Figure 5.3 that show the surge behavior for different values of the stability parameter  $B$ .

Greitzer (1976a) identified two different types of surge oscillations, quasi-sinusoidal or relaxation, occurring at low and high values of  $B$ , respectively. The shape of the simulated limit cycles in Figures 3.11 and 4.8 from previous chapters indicate that both the investigated test rigs exhibit surge oscillations of the relaxation type (see Khalil, 2000, pp. 58) in which two different time scales can be distinguished. The long time scale is associated with the pressure building up (or discharging during back flow) in the plenum while the short time scale of the cycle is associated with an abrupt flow reversal at almost constant pressure. See also the upper part of Figure 5.3

At this point we repeat the statement by Greitzer (1976a) that “surge oscillations are possible only when the mechanical energy input from the compressor is greater during the oscillatory flow than during a (mean) steady flow, and this can occur only if the character-



**Figure 5.2** / Vector field with compressor curve (black) and load curve (gray) to illustrate compressor behavior;  $B = 0.15$ ,  $F = 1$ .



**Figure 5.3** / Effect of  $B$ -parameter on the shape of the surge limit cycle; left figures:  $\Psi_c(\phi)$  (dashed gray), right figures:  $\phi_c$  (gray),  $\psi$  (black).

istic is somewhere positively sloped so that high mass flow and high mechanical energy input per unit mass go together". Note that a sustained periodic oscillation requires that the energy input over a cycle is balanced by the total dissipation over this cycle.

Finally, we point out that the positive sloped compressor curve is the result of an accumulation of various losses that occur inside the machine at low mass flows (Gravdahl and Egeland, 1999a). Details on the relevant loss terms and the associated flow phenomena are discussed by, for example, Cumpsty (1989); Cohen et al. (1996); Whitfield and Baines (1990) and the references therein.

### 5.2.2 Parameter analysis

In the above we already addressed the effect of the stability parameter  $B$  on the surge behavior. Now, we will discuss the effect of various parameters in the Greitzer model on the shape and period time of the predicted limit cycle during deep surge. For that purpose, we use the model developed for test rig A and vary the parameters  $B$ ,  $F$ , and the slope of the compressor characteristic. This slope is varied indirectly by repeating the calculations for different operating points. As a measure of the limit cycle shape we will use the amplitudes of the dimensionless mass flow and pressure difference during a surge cycle. For comparison, we will also show how the value for  $B$  affects the predicted surge cycle for test rig B. The effect of the model parameters that are related to the piping acoustics in test rig B will be discussed separately.

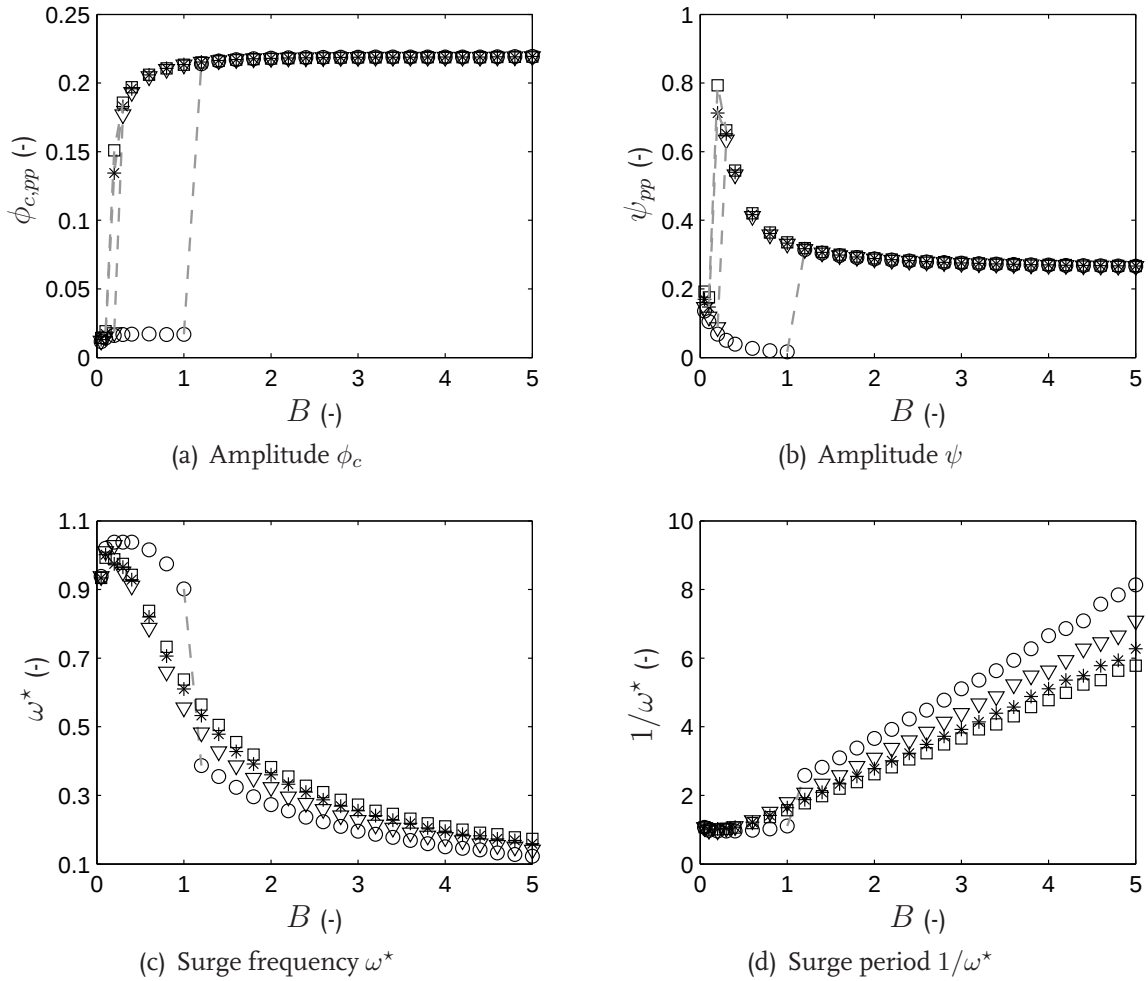
#### Effect of varying $B$

The effect that varying  $B$  (between 0.05 and 5) has on the surge limit cycle of test rig A at different operating points is illustrated in Figure 5.4. The most relevant observations from this figure are summarized as follows.

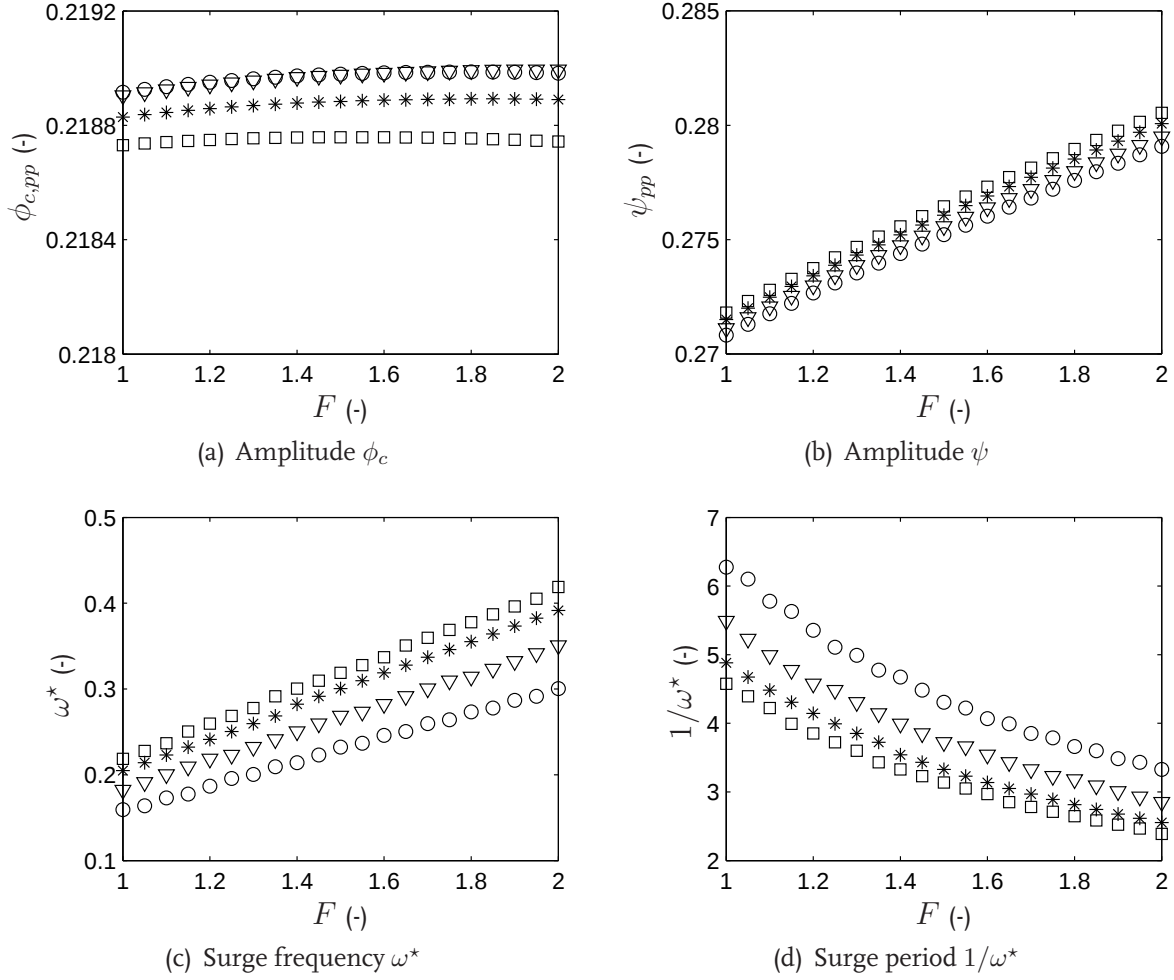
Firstly, the results show that the transition from stable to classic and then deep surge occurs within a small range of low  $B$  values and is rather abrupt. However, for operating points just left of the surge line this transition occurs at higher values for  $B$ .

Secondly, the amplitude of  $\psi$  has a maximum for  $B$  between 0.15 and 0.25, which is in agreement with Figure 5.3. Given the shape and period time of the limit cycle we call this behavior classic surge, following the terminology from De Jager (1995).

Thirdly, the period time of the deep surge oscillations depends linearly on the value of  $B$ . Furthermore, the slope of the compressor curve at the operating point determines the rate at which the surge frequency increases for increasing values of  $B$  and a larger positive slope results in a slightly higher surge frequency.



**Figure 5.4** / Effect of varying  $B$  on shape and period time of surge limit cycle for different operating points of test rig A;  $\circ$  :  $0.98\phi_c^*$ ,  $\nabla$  :  $0.95\phi_c^*$ ,  $*$  :  $0.90\phi_c^*$ ,  $\square$  :  $0.85\phi_c^*$ ,  $N = 9,730$  rpm,  $u_r = 0$ ,  $F = 1.1$ ,  $\omega_H = 69$  rad/s, transition to deep surge (dashed gray).



**Figure 5.5 /** Effect of varying  $F$  on shape and period time of surge limit cycle for different operating points of test rig A;  $\circ$  :  $0.98\phi_c^*$ ,  $\nabla$  :  $0.95\phi_c^*$ ,  $*$  :  $0.90\phi_c^*$ ,  $\square$  :  $0.85\phi_c^*$ ,  $N = 9,730$  rpm,  $B = 3.50$ ,  $\omega_H = 69$  rad/s.

The different observations for different operating points indicate that the slope of the compressor curve influences the stability of the compression system. However, this influence is less profound than the influence of the parameter  $B$  for the studied range of operating points. In Section 5.3 we will address the effect of the operating point and the corresponding slope of the compressor curve on the local dynamics of the system. Finally, we remark that similar results are obtained when varying  $B$  in the model for test rig B.

### Effect of varying $F$

The effect that varying  $F$  (between 1 and 2) has on the surge limit cycle of test rig A at different operating points is illustrated in Figure 5.4. The most relevant observations from this figure are summarized as follows.

Firstly, the results show that variations of  $F$  have a limited effect on the shape and frequency of the surge oscillations. The different effect on the amplitudes of  $\phi_c$  and  $\psi$ , respectively, follows from the fact that  $F$  appears only in the mass balance Equation (5.2).

Secondly, the effect of  $F$  on the period time of the limit cycle oscillations is lower than that of  $B$ . However, we point out that the results in Figure 5.5 are obtained with a high value of  $B$  (3.50) so the resulting variations of  $F/B$  are relatively small.

By combining the results from Figures 5.4 and 5.5 we see that variations of  $1/B$  and  $F$  influence the surge frequency in the same manner. Hence, for large values of  $B$ , we can conclude that the surge frequency is mainly determined by Equation (5.2), which is in line with the observation that the long time scale of the limit cycle is associated with the filling and emptying of the plenum.

### Effect of varying acoustic model parameters

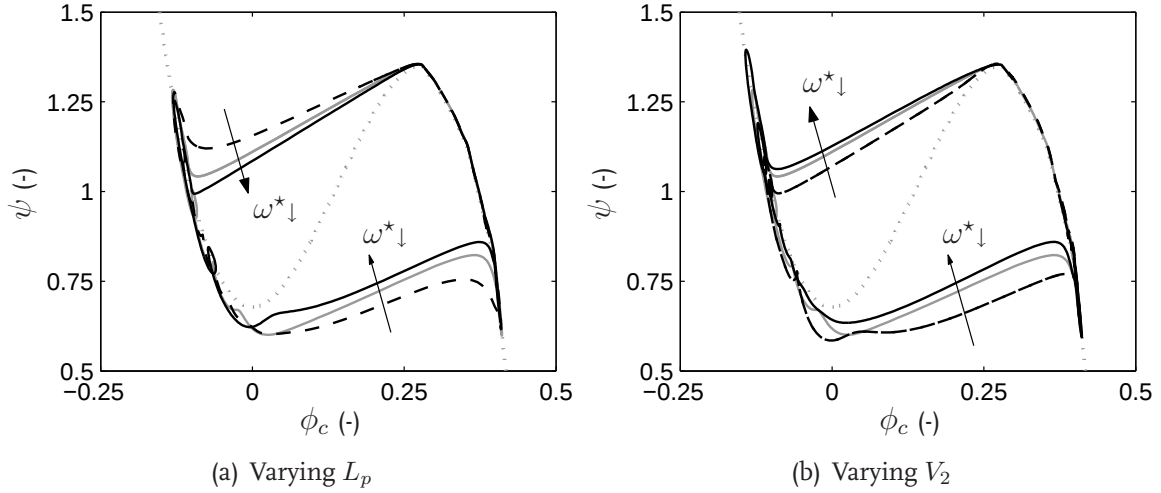
In Chapter 4 we already discussed the effect of various parameters in the acoustic model on the predicted surge behavior. Here, we will first focus on the effect of the pipe length  $L_p$  and the plenum volume  $V_2$ . The effect of varying these parameters on the surge cycle is illustrated in Figure 5.6.

These results show that increasing either  $L_p$  or  $V_2$  results in a lower surge frequency. This observation can be explained by noting that both a larger  $L_p$  or  $V_2$  represent an increase in the capacity of the discharge system, resulting in longer filling and emptying times.

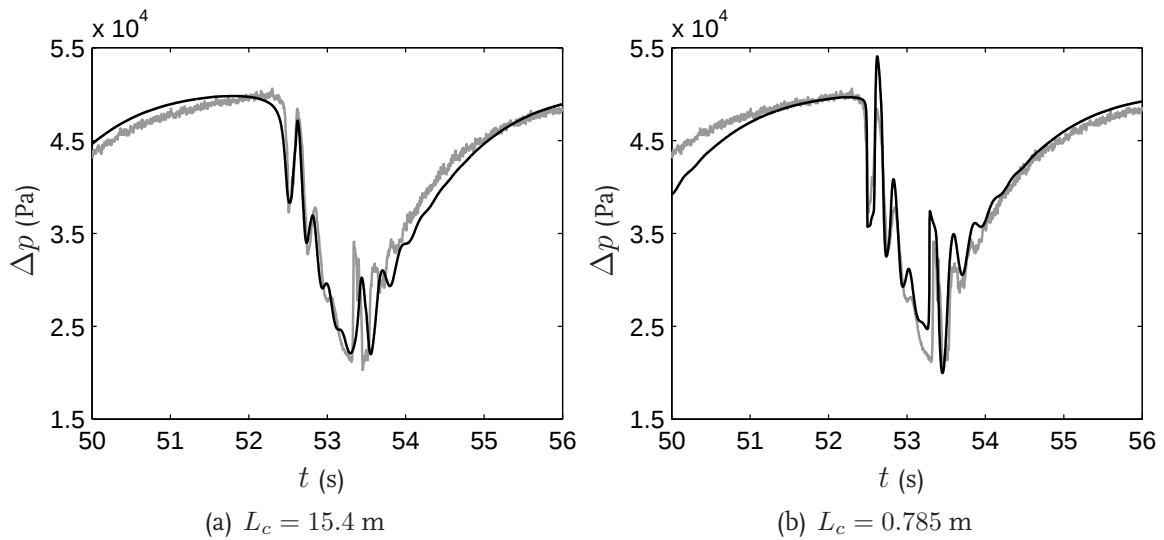
We point out that variations of the two parameters have a different effect on the shape of the limit cycle. A detailed physical explanation for these observations cannot be given at this point.

It is also interesting to study the effect of the compressor duct length  $L_c$  on the surge cycle predictions of the acoustic model. In Section 4.4 we already mentioned that the value of  $L_c$  might be chosen too large, given the difference in the measured and simulated pressure drops at each new surge cycle.

In Figure 5.7 we compare an earlier simulation result with a new simulation in which a much lower value for  $L_c$  is used. The results clearly show that the lower value for  $L_c$  results in a much steeper pressure drop at the start of a new cycle that is similar to the measured pressure drop. However, the amplitudes of the acoustic reflections are overestimated in Figure 5.7b. These results indicate that both the selection of an appropriate value for  $L_c$  and the description of the piping acoustics require further attention in order to improve the overall match between the model predictions and measurement data.



**Figure 5.6** / Effect of varying acoustic duct properties on surge limit cycle of test rig B; compressor curve (dotted gray), nominal surge cycle (solid gray),  $2\times$  nominal (solid black),  $0.5\times$  nominal (dashed black),  $N = 15,723$  rpm,  $\phi_{c0} = 0.27$ ,  $u_r = 0$ ,  $B = 1.51$ ,  $\omega_H = 5.3$  rad/s.



**Figure 5.7** / Effect of varying  $L_c$  on the initial pressure drop during surge in test rig B;  $N = 15,723$  rpm,  $\dot{m}_{c0} = 2.05$  kg/s,  $u_r = 0$ ,  $\omega_l = \{1.4/\tau, 4/\tau\}$ .

## 5.3 Linear compressor dynamics

So far we have investigated the effect of the various model parameters, in particular that of the stability parameter  $B$ , on the amplitude and frequency of the surge limit cycle. As mentioned before, the idea of active surge control is to increase the aerodynamic damping of flow instabilities and to suppress destabilizing disturbances in an early stage where amplitudes are still small (Epstein et al., 1989). The underlying assumption is that the initial phases of those instabilities can be modeled by linear theory.

Hence, we will now take a closer look at the effect of the model parameters on the local dynamic behavior. For that purpose we will linearize the Greitzer model from Equations (5.1) and (5.2) around a specific operating point  $(\phi_{c0}, \psi_0, u_{t0})$ , yielding

$$\begin{bmatrix} \dot{\tilde{\phi}}_c \\ \dot{\tilde{\psi}} \end{bmatrix} = \begin{bmatrix} B \frac{\partial \Psi_c}{\partial \phi_c} & -B \\ \frac{F}{B} & -\frac{F}{B} \frac{1}{\partial \Psi_t} \end{bmatrix} \begin{bmatrix} \tilde{\phi}_c \\ \tilde{\psi} \end{bmatrix} \quad (5.3)$$

where, with a slight abuse of notation,

$$\frac{\partial \Psi_c}{\partial \phi_c} = \left. \frac{\partial \Psi_c(\phi_c)}{\partial \phi_c} \right|_{(\phi_{c0})} \quad (5.4)$$

$$\frac{1}{\partial \Psi_t} = \left. \frac{\partial \phi_t(\psi, u_t)}{\partial \psi} \right|_{(\psi_0, u_{t0})} \quad (5.5)$$

represent the slopes of the compressor and load characteristics, respectively. We recall from that the developed fits for the compressor curves are not differentiable at the top. In practice, this discontinuity in  $\partial \Psi_c$  is not causing problems because we do not need to linearize the model exactly around the top of the compressor curve. We stress that the obtained linear models are only valid in the immediate vicinity of the selected operating points.

A general form of the linearized system above is  $\dot{\underline{x}} = \underline{A}\underline{x}$  and the eigenvalues  $\lambda$  of this system are given by the roots  $s$  of the characteristic equation  $\det(s\underline{I} - \underline{A}) = 0$ . The eigenvalues (modes) and their related eigenvectors (modal vectors) describe the stability and response of the system around a specific operating point. Details can be found in systems and control textbooks (e.g., Franklin et al., 1994; Khalil, 2000) and the specific case of the Greitzer model is described by Gravdahl and Egeland (1999b, pp. 199–201).

It is worthwhile to investigate how the modes of the linearized Greitzer model are related to the physical mechanisms that play a role in a compression system. For that purpose we recall that the eigenvector  $\underline{v}_i$ , corresponding to the eigenvalue  $\lambda_i$ , satisfies  $\underline{A}\underline{v}_i = \lambda_i \underline{v}_i$ . The resulting analytical expressions for the eigenvectors of the Greitzer model are rather involved and therefore omitted here. However, numerically we can show that, for distinct real eigenvalues and a large value of  $B$ , the direction of the eigenvector

- associated with the slow eigenvalue is almost parallel to the compressor curve,

- associated with the fast eigenvalue is almost horizontal,

while for smaller values of  $B$  the two eigenvectors become more aligned. Note that the slow or dominant eigenvalue is the one nearest to the imaginary axis.

At this point we recall the comments from Section 5.2.1 on the different time scales in limit cycle oscillations of the relaxation type. By combining that observation with the findings presented here we can state that, for large  $B$  values, the dominant mode is related to the filling and emptying of the plenum while the fast mode is associated with the acceleration and deceleration of the flow inside the compressor. With this in mind, we will now discuss the influence of the compressor operating point and various model parameters on the modes of the investigated compression systems.

### Effect of varying the operating point

Figure 5.8 shows the eigenvalues of the linearized model for test rig A around different operating points, a so-called root locus plot. The operating points were varied in the range from  $0.85\phi_c^*$  to  $1.15\phi_c^*$ . From this figure we see that the operating points to the left of the surge line yield eigenvalues with  $\text{Re}(\lambda) > 0$ , confirming that these operating points are unstable. The exact conditions for which  $\text{Re}(\lambda) > 0$  can be obtained from the characteristic equation by using the Routh<sup>2</sup>-Hurwitz<sup>3</sup> stability criterion (Franklin et al., 1994). The resulting conditions depend on the values for  $B$ ,  $F$  and the slopes of the compressor and throttle curves. However, the stability boundary practically coincides with the top of the compressor curve.

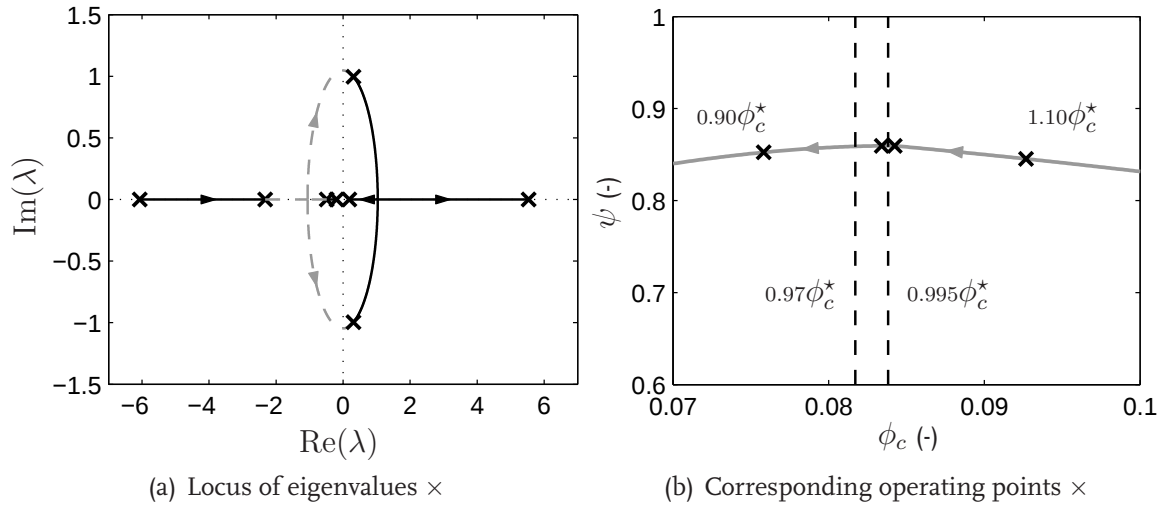
In Figure 5.8 we have indicated that a part of the root locus does not exist for test rig A. This is due to the discontinuous slope at the top of the compressor curve, see also Chapter 3. Finally, we point out that the region of mass flows for which the system has complex conjugate eigenvalues, indicating an oscillatory response, is small. This implies that the possible operating range increase by using the one-sided control strategy proposed by Willems et al. (2002) is small for test rig A.

We now show the eigenvalue locations for the linearized system model, including piping acoustics and time-varying parameters, for test rig B. Due to the complexity of this model an analytical expression is not available and a numerical linearization is obtained instead. The resulting eigenvalues for operating points in the range of  $0.81\phi_c^*$  and  $1.15\phi_c^*$  are shown in Figure 5.9.

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<sup>2</sup>Edward John Routh (1831–1907) was a British mathematician. Routh was a superb teacher and author of several influential publications like 'A treatise on the stability of a given state of motion, particularly steady motion' for which he received the Adams Prize in 1877.

<sup>3</sup>Adolf Hurwitz (1859–1919) was a German mathematician. In 1895 he published the paper 'Über die Bedingungen, unter welchen eine Gleichung nur Wurzeln mit negativen reellen Theilen besitzt' in Mathematische Annalen.



**Figure 5.8** / Influence of compressor operating point on local dynamics of test rig A; non-existing part of locus (dashed gray), region with complex conjugate eigenvalues (dashed black), compressor curve (gray),  $N = 9,730$  rpm,  $u_r = 0$ ,  $B = 3.50$ ,  $F = 1.1$ ,  $\omega_H = 69$  rad/s.

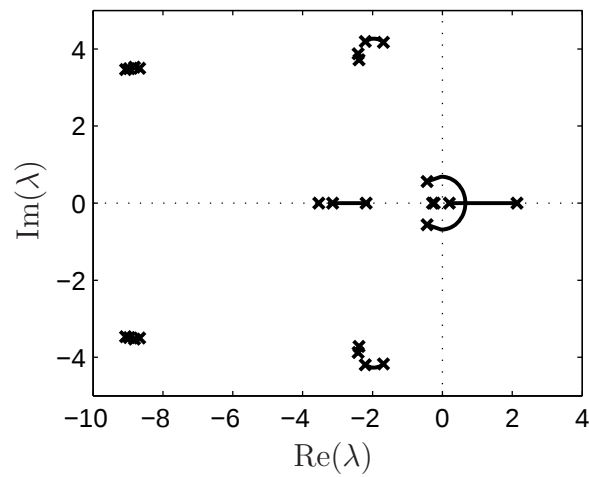
Qualitatively, the loci of the eigenvalues associated with the momentum and mass balance are similar to those of test rig A. However, the model for the piping acoustics has introduced several other eigenmodes. In Figure 5.9 we can see that these acoustic modes are located in the vicinity of the modes associated with the compressor duct and plenum volume. Hence, from Figure 5.9 we conclude that the dynamics of the piping system are not negligible in test rig B.

### Effect of varying $B$

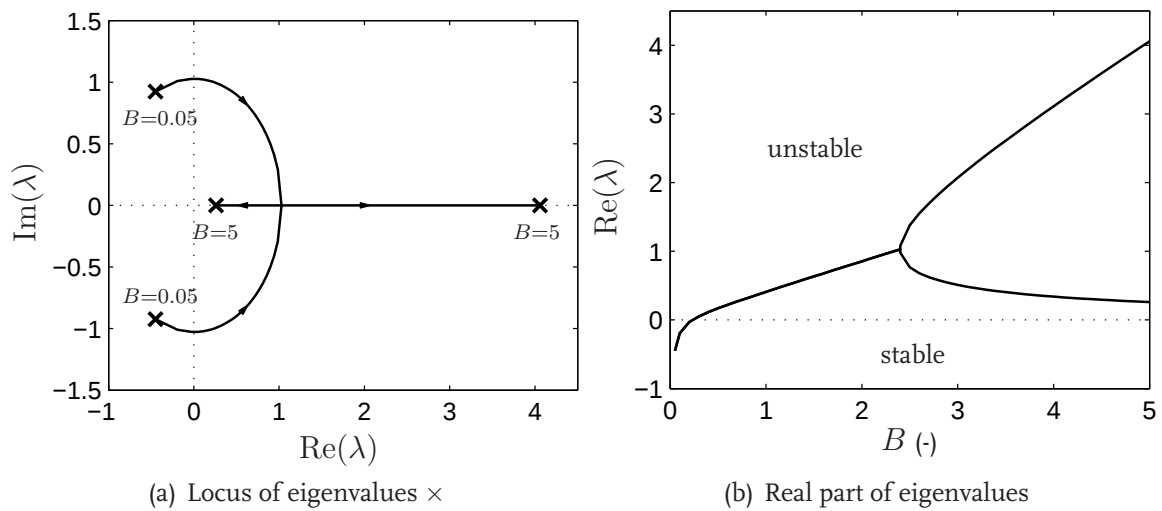
The effect that varying  $B$  (between 0.05 and 5) has on the eigenvalue locations for a specific operating point of test rig A is illustrated in Figure 5.10. These results show that for very small values of  $B$  the operating point at 95% of the surge mass flow is stable, see also Figure 5.3. Furthermore, the plot of  $\text{Re}(\lambda)$  as a function of  $B$  shows that for the system has two distinct real eigenvalues when  $B > 2.4$ .

### Effect of varying $\partial\Psi_t$

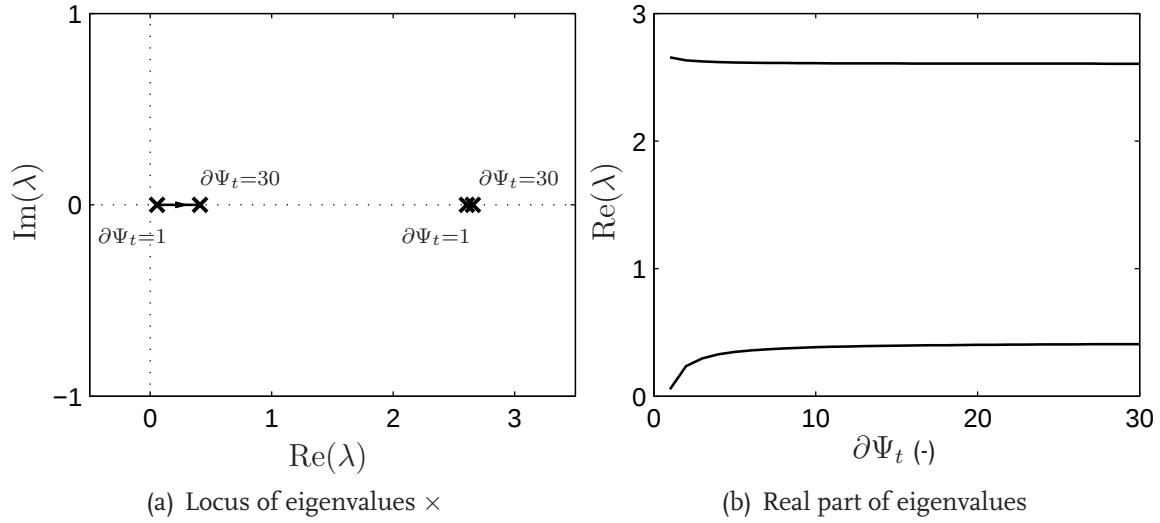
The effect that varying  $\partial\Psi_t$  (between 1 and 30) has on the eigenvalue locations for a specific operating point of test rig A is illustrated in Figure 5.11. These results show that the slope of the load characteristic has no significant effect on the eigenvalue location. Only for very small values of  $\partial\Psi_t$  the eigenvalues are located further apart. From Figure 5.12 we see that this is the case for a very flat load characteristic. A throttle valve usually results in



**Figure 5.9** / Root locus as function of compressor operating point for test rig B;  $N = 15,723$  rpm,  $u_r = 0$ ,  $B = 1.51$ ,  $\omega_H = 5.3$  rad/s.



**Figure 5.10** / Effect of varying  $B$  on local dynamics of test rig A;  $N = 9,730$  rpm,  $\phi_{c0} = 0.95\phi_c^*$ ,  $u_r = 0$ ,  $F = 1.1$ ,  $\omega_H = 69$  rad/s.



**Figure 5.11** / Effect of varying  $\partial\Psi_t$  on local dynamics of test rig A;  $N = 9,730$  rpm,  $\phi_{c0} = 0.95\phi_c^*$ ,  $u_r = 0$ ,  $B = 3.50$ ,  $F = 1.1$ ,  $\omega_H = 69$  rad/s.

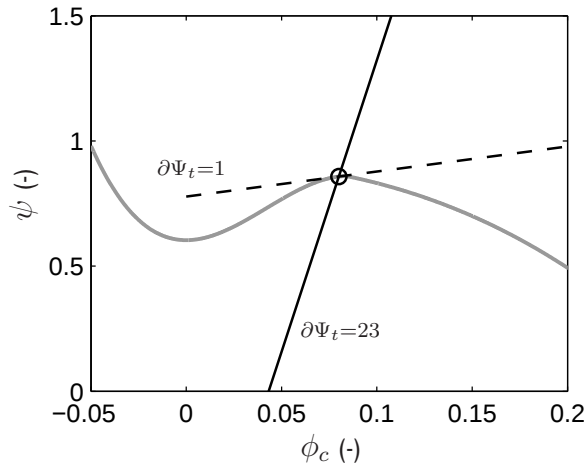
a much steeper characteristic, see also Section 3.3.4. However, flat load characteristic can occur in, for example, gasification processes, transportation pipelines and gas re-injection applications where the discharge volumes are very large.

## 5.4 Discussion

In this chapter we have addressed the surge dynamics of a turbocompressor and we discussed why this unstable operating mode occurs at low mass flows. Subsequently, we investigated how the surge dynamics are captured in the Greitzer model. From our analysis we confirmed the conclusion that the stability parameter  $B$  in the dimensionless Greitzer model plays a crucial role in describing the stability and surge behavior of a turbocompressor. A similar conclusion can be drawn for the influence of the operating point and the associated slope of the compressor characteristic of the compressor.

More specifically, the parameter analysis for the nonlinear Greitzer model showed that variations of  $B$  have the most profound effect on the shape and frequency of the surge limit cycle. Furthermore, we showed that throttling down the compressor towards lower mass flows after surge initiation results in higher surge frequencies.

The parameter analysis of the model for the compressor dynamics and piping acoustics of test rig B showed that both the plenum volume and pipe length affect the shape and frequency of the surge limit cycle. Hence, the boundary condition at the end of the pipeline must be selected with care as we already mentioned in Chapter 4.



**Figure 5.12** / Linear throttle curves with different slopes for test rig A.

More importantly, the compressor duct length  $L_c$  appears to play a large role in determining the speed at which the pressure drops in the initial phase of a new surge cycle. The presented simulation results show that the selection of  $L_c$  requires more attention, as well as the model for the acoustic reflections inside the discharge piping.

The presented linear analysis showed that the relevant physics, i.e., the filling and emptying of the plenum and the flow accelerations and decelerations of the flow in the compressor duct, are associated with the two eigenvalues of the linearized Greitzer model. Again, we concluded that the value of  $B$  has a large effect on the modes of the compression system. We also showed that the local dynamics are relatively insensitive for variations in the slope of the load curve. This finding simplifies the issue of choosing an appropriate description of the valve characteristic that was raised in Chapter 3. However, we remark that the local dynamics of a compression system change when the load characteristic becomes extremely flat, a situation that might occur in some practical compressor applications.

Finally, the linear analysis for test rig B illustrated that the frequencies of the acoustic modes lie in the same range as the compressor and plenum modes. Hence, the effect of the piping acoustics must be taken into account when studying or modifying the dynamic behavior of a compression system with long discharge pipelines.

In previous chapters we found that the Greitzer model and derivatives thereof are capable of describing the dynamic behavior of the compression systems under study. From the above we conclude that the slope of the compressor curve has a large effect on the local dynamics. Unfortunately, the unstable part of the compressor characteristic is unknown and it cannot be measured directly in the investigated test rigs, see also Section 3.3.3. Moreover, the capability of the compressor model to predict the relevant dynamics critically depends on knowledge of the stability parameter  $B$ . In the next chapter we will address two methods that can be used to obtain an appropriate value for this parameter.

# Stability parameter identification<sup>1</sup>

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**Abstract** / In this chapter two methods are proposed to obtain an estimate of the important stability parameter. The first method is based on an approximation of the hydraulic inductance from the internal compressor geometry. The second method uses step response data in combination with an approximate realization algorithm to identify the linear dynamics of a centrifugal compression system. Where possible, results from both methods are compared and validated with additional experimental data.

## 6.1 Introduction

The stability parameter of the lumped parameter model plays an important role in describing the dynamics of a centrifugal compression system. In Chapter 3 and 4 we showed that a good agreement between simulated and measured surge transients can be obtained by tuning the compressor duct length and thereby the dimensionless stability parameter. The analysis presented in Chapter 5 confirmed the importance of the stability parameter and the shape of the compressor characteristic. The latter is addressed in Chapter 3 and Appendix C and therefore we will now focus on the stability parameter.

The mentioned tuning approach has been used by, for example, [Arnulfi et al. \(1999\)](#) and [Willems \(2000\)](#). However, tuning the compressor duct length  $L_c$  while assuming that the cross-sectional area  $A_c$  and plenum volume  $V_p$  are known, does not automatically lead to an accurate estimate for the stability parameter  $B$  ([Meuleman, 2002](#); [Van Helvoirt et al., 2004](#)). In particular for compression systems with a large value for  $B$  the effect of  $L_c$  on the dominant dynamics is small. In this chapter we will therefore propose two identification or approximation methods to determine, either directly or indirectly, a value for the important parameter  $B$ .

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<sup>1</sup>Parts of this chapter are based on [Van Helvoirt et al. \(2004\)](#), and [Van Helvoirt et al. \(2005b\)](#).

The idea of applying identification techniques to obtain a model for the relevant dynamics of a compression system or to estimate particular model parameters is not new. Paduano et al. (1993b) and Paduano et al. (1994) propose both open- and closed-loop identification methods to obtain a multi-input multi-output (MIMO) model for the stall dynamics of an axial compression system equipped with variable inlet guide vanes. Forced response experiments are carried out to generate the required data set for their transfer function estimate and instrumental variable identification schemes. Similar approaches are reported by Haynes et al. (1994) who use control valves instead of inlet guide vanes, and by Weigl et al. (1998); Nelson et al. (2000) who use sinusoidal air injection.

We point out that the mentioned methods from literature result in linear dynamic models. The drawback of the above methods is the complicated actuator hardware that is required to carry out the forced response experiments. An advantage is that, besides the model structure, relatively limited a priori knowledge of the system is required in order to obtain the linear dynamic model.

In the type of centrifugal compression systems that are considered in this thesis, means for actuation are limited by both the limited options to modify the industrial test rigs and the availability of high speed and high capacity actuators. The main contributions of this chapter are two methods to determine values for the stability parameter  $B$  of an industrial scale centrifugal compression systems. The first method is based on the geometric approximation of the hydraulic inductance  $L_c/A_c$  from which the compressor duct length and hence the stability parameter can be determined. The second one is a forced response identification method that uses an approximate realization algorithm to estimate the linear dynamics from step response experiments. Additionally, these forced response data allows us to perform an additional validation of the developed nonlinear model for one of the investigated test rigs.

Both methods to estimate  $B$  assume that other required parameters are available or can be obtained by other means. Furthermore, combining the outcomes from both methods with the previous results that were obtained by tuning the dynamic compressor model provides information on the accuracy and uncertainty of the parameter of interest.

We will first address the approximation of the hydraulic inductance from the compressor geometry. Then we will discuss the choice for the approximate realization algorithm from the vast amount of identification methods available. Subsequently, we will discuss the acquisition of the required step response data, the realization algorithm and we will present the results of the parameter estimation procedure. The chapter is concluded with a discussion on the benefits and shortcomings of the presented methods.

## 6.2 Hydraulic inductance approximation

From Chapter 2 we recall that the stability parameter  $B$  is defined as

$$B = \frac{U_e}{2c_2} \sqrt{\frac{V_2}{A_c L_c}} \quad (6.1)$$

with  $L_c$  the effective compressor duct length and  $A_c$  a reference area. These parameters were introduced to account for the fluid dynamics in the compressor and connecting ducts in the lumped parameter model, see also Chapter 2. The term  $L_c/A_c$  in the momentum balance is usually referred to as the hydraulic inductance. This term is a simplification of the following integral

$$\int_{\substack{\text{actual} \\ \text{ducting}}} \frac{dl}{A(l)} = \left( \frac{L_c}{A_c} \right)_{\text{model}} \quad (6.2)$$

It is common practice to select an appropriate reference area  $A_c$  and tune  $L_c$  in the Greitzer model to obtain a good match between measured and simulated surge oscillations, see also Chapters 3 and 4. Hence, the stability parameter  $B$  is indirectly determined through a tuning procedure and as a consequence the value for  $B$  can be affected by model and measurement uncertainties.

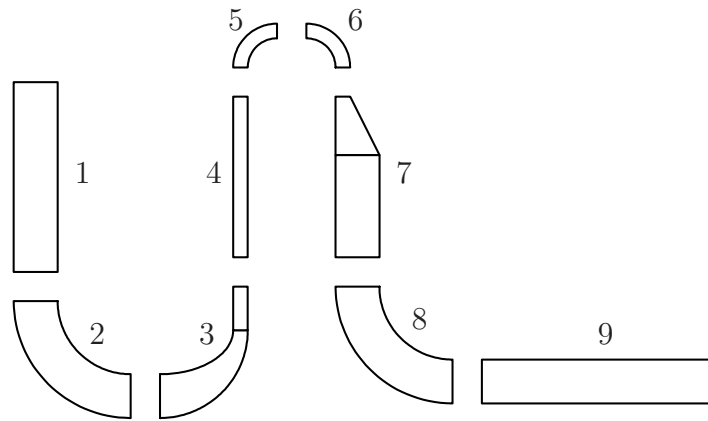
In order to avoid the tuning procedure for  $L_c$  and potential inaccuracies in the stability parameter, we propose to calculate the integral of the duct area  $A(l)$  along the flow path  $l$  directly. Note that using this procedure to determine  $B$  requires that the other parameters,  $U_e$ ,  $c_2$ , and  $V_2$  must be known with sufficient accuracy.

We point out that the internals of the studied centrifugal compressors typically have a complex three-dimensional geometry, see for example Figure 3.3. Hence, we divide the total duct into several sections and apply appropriate assumptions to solve the integral for each of those sections.

### 6.2.1 Compressor duct division

Following the geometry of a typical compressor design, we suggest a partitioning of the total duct into various elements as presented in Figure 6.1 and Table 6.1. We point out that the impeller is subdivided into a curved duct and a radial duct.

The crucial assumption for this partitioning is that the flow only has significant kinetic energy in the ducting between the inlet and exit channel of the compressor, see also Figure 3.3. This assumption seems justifiable given the large diameters of the inlet and outlet chambers before and after the respective ducts. We will now present the analytical approximations of the integral in Equation (6.2) for each of the simplified geometries.



**Figure 6.1** / Division of compressor ducting into elements with simple geometry.

**Table 6.1** / Description of compressor duct division.

| Number | Description    | Geometry              |
|--------|----------------|-----------------------|
| 1      | inlet channel  | radial duct           |
| 2      | inlet bend     | curved duct, type A   |
| 3      | impeller       | curved duct, type B   |
| 4      | diffuser       | radial duct           |
| 5      | diffuser bend  | curved duct, type C   |
| 6      | diffuser bend  | curved duct, type D   |
| 7      | return channel | diverging radial duct |
| 8      | return bend    | curved duct, type A   |
| 9      | exit channel   | axial duct            |

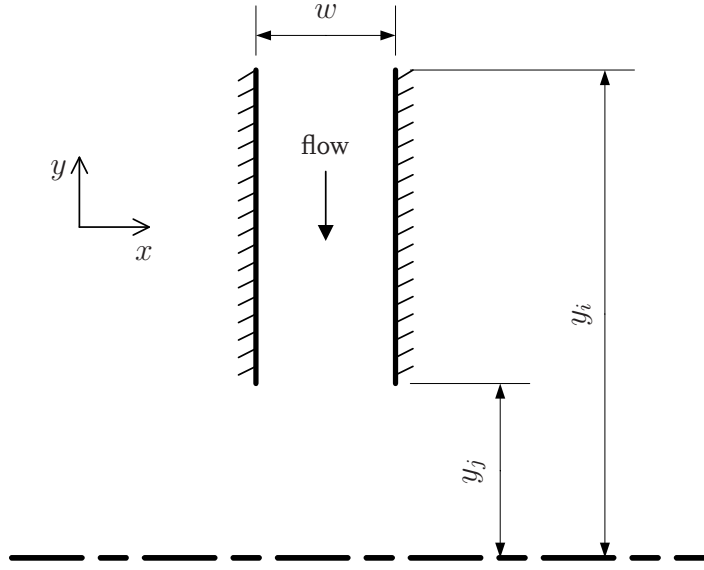


Figure 6.2 / Radial duct geometry.

### Straight radial ducts

Following the geometry from Figure 6.2 the following expression for the hydraulic inductance is obtained

$$\int_{\mathcal{L}} \frac{dl}{A(l)} = \int_{y_i}^{y_j} \frac{dy}{2\pi w y} = \begin{cases} \frac{1}{2\pi w} \ln \left( \frac{y_j}{y_i} \right) & \text{for } y_j > y_i > 0 \\ -\frac{1}{2\pi w} \ln \left( \frac{y_j}{y_i} \right) & \text{for } y_i > y_j > 0 \end{cases} \quad (6.3)$$

where we assume that  $dl = dy$ , requiring that the flow through the duct is strictly radial and uniform.

### Diverging radial ducts

Following the geometry from Figure 6.3 the following expression for the hydraulic inductance is obtained

$$\int_{\mathcal{L}} \frac{dl}{A(l)} = \int_{y_i}^{y_j} \frac{dy}{2\pi w(y)y} = \begin{cases} \frac{1}{2\pi b} \ln \left( \frac{y_j(ay_i+b)}{y_i(ay_j+b)} \right) & \text{for } y_j w(y_i) > y_i w(y_j) > 0 \\ -\frac{1}{2\pi b} \ln \left( \frac{y_j(ay_i+b)}{y_i(ay_j+b)} \right) & \text{for } y_i w(y_j) > y_j w(y_i) > 0 \end{cases} \quad (6.4)$$

where we assume that  $dl = dy$ , requiring that the flow through the duct is strictly radial and uniform. Furthermore, we assume a linearly increasing or decreasing width of the duct, yielding the following expression for  $w(y)$

$$w(y) = ay + b = \underbrace{\frac{w_j - w_i}{y_j - y_i}}_a y + \underbrace{w_i - \frac{w_j - w_i}{y_j - y_i} y_i}_b \quad (6.5)$$

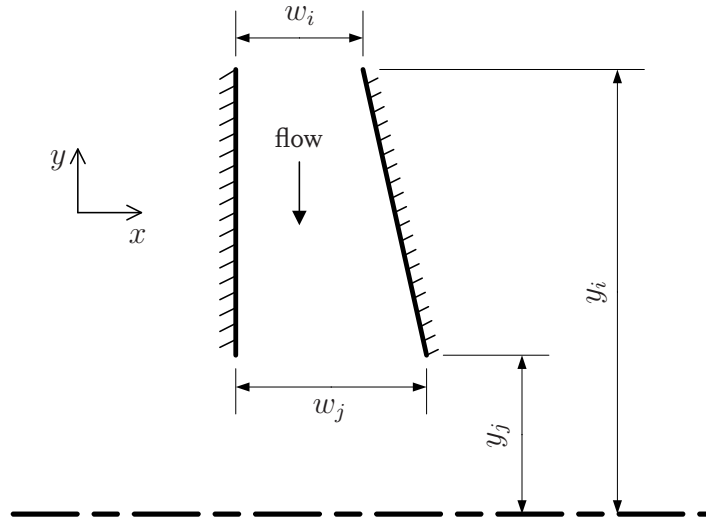


Figure 6.3 / Diverging radial duct geometry.

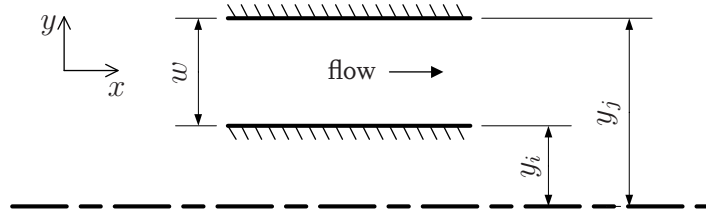


Figure 6.4 / Axial duct geometry.

### Straight axial ducts

Following the geometry from Figure 6.4 the following expression for the hydraulic inductance is obtained

$$\int_{\mathcal{L}} \frac{dl}{A(l)} = \int_{x_i}^{x_j} \frac{dx}{\pi(y_i + y_j)w} = \frac{|x_j - x_i|}{\pi(y_i + y_j)w} \quad (6.6)$$

where we assume that  $dl = dx$ , requiring that the flow through the duct is strictly axial and uniform.

### Curved ducts

For the various bends in the compressor duct we make use of an auxiliary geometry as shown in Figure 6.5. From this figure we see that the cross sectional area of the duct at an angle  $\theta$  is given by the lateral area of the frustum  $A(\theta) = \pi w(y_i(\theta) + y_j(\theta))$  with  $y_k = h - R_k \sin \theta$ ,  $k = 1, 2$ .

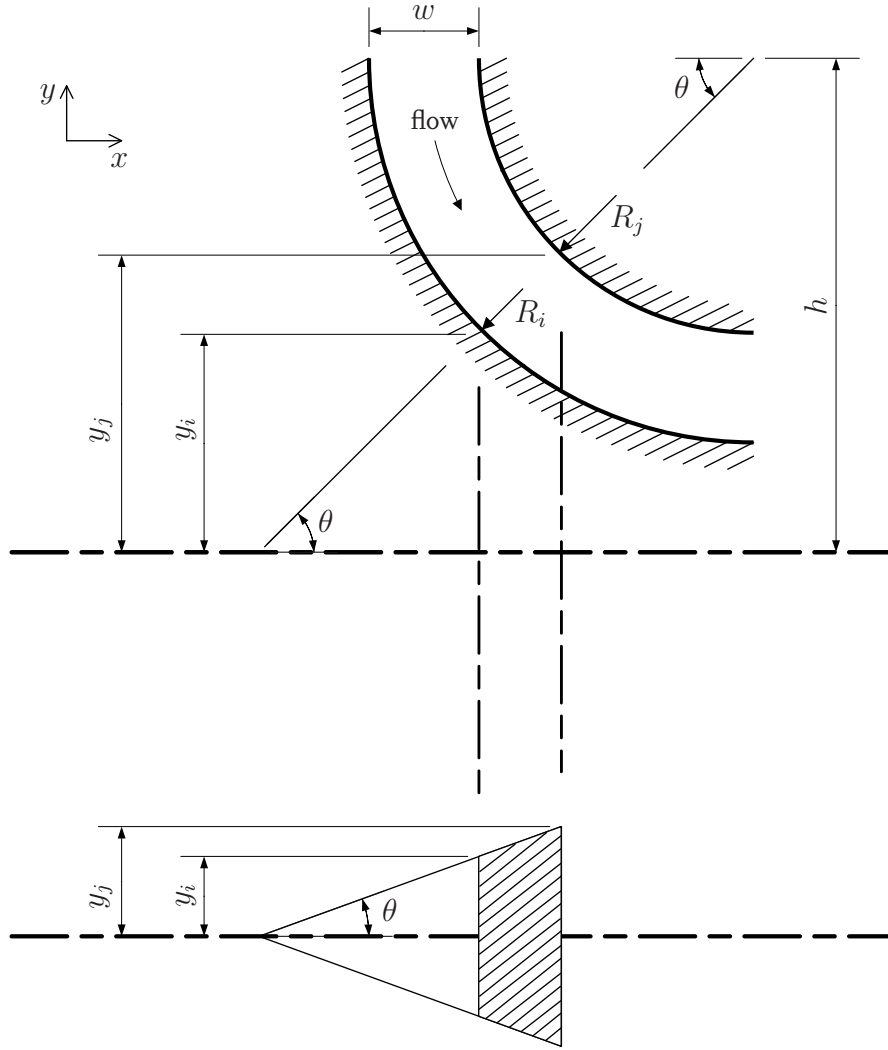


Figure 6.5 / Curved duct geometry, type A.

Following the geometry from Figure 6.5 the following expression for the hydraulic inductance is obtained

$$\int_{\mathcal{L}} \frac{dl}{A(l)} = \int_0^{\phi} \frac{\frac{1}{2}(R_i + R_j)}{A(\theta)} d\theta = \frac{R_i + R_j}{\pi w \sqrt{4h^2 - (R_i + R_j)^2}} \tan^{-1} \left( \frac{-R_i - R_j + 2h \tan(\frac{\theta}{2})}{\sqrt{4h^2 - (R_i + R_j)^2}} \right) \quad (6.7)$$

where we assume that  $dl = \frac{1}{2}(R_i + R_j)d\theta$ , requiring that the flow is uniform and that it follows the centerline of the curved duct from Figure 6.5. Note that  $\theta$  denotes the angle in rad at the end of the curved duct.

Analogous to this approach, expressions for the curved ducts of type B, C, and D can be obtained. However, note that the resulting expressions are different due to the start and end position of the ducts relative to the flow.

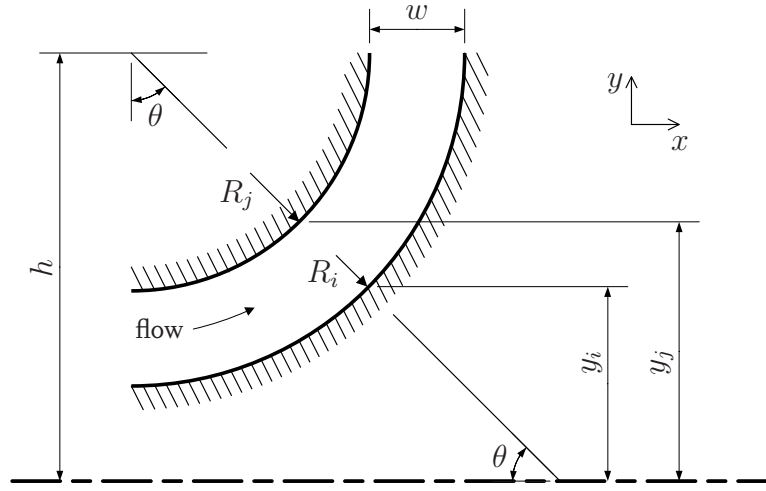


Figure 6.6 / Curved duct geometry, type B.

Following the geometry from Figure 6.6 the following expression for the hydraulic inductance for a curved duct of type B is obtained

$$\int_{\mathcal{L}} \frac{dl}{A(l)} = \frac{R_i + R_j}{\pi w \sqrt{(R_i + R_j)^2 - 4h^2}} \tanh^{-1} \left( \frac{(R_i + R_j + 2h) \tan(\frac{\theta}{2})}{\sqrt{(R_i + R_j)^2 - 4h^2}} \right) \quad (6.8)$$

using  $y_k = h - R_k \cos \theta$ , while the hydraulic inductance for a curved duct of type C (see Figure 6.7) is given by

$$\int_{\mathcal{L}} \frac{dl}{A(l)} = \frac{R_i + R_j}{\pi w \sqrt{4h^2 - (R_i + R_j)^2}} \tanh^{-1} \left( \frac{R_i + R_j + 2h \tan(\frac{\theta}{2})}{\sqrt{4h^2 - (R_i + R_j)^2}} \right) \quad (6.9)$$

using  $y_k = h + R_k \sin \theta$ . Finally, the hydraulic inductance for a curved duct of type D (see Figure 6.8) is given by

$$\int_{\mathcal{L}} \frac{dl}{A(l)} = \frac{R_i + R_j}{\pi w \sqrt{(R_i + R_j)^2 - 4h^2}} \tanh^{-1} \left( \frac{(R_i + R_j - 2h) \tan(\frac{\theta}{2})}{\sqrt{(R_i + R_j)^2 - 4h^2}} \right) \quad (6.10)$$

using  $y_k = h + R_k \cos \theta$ .

### 6.2.2 Discussion on hydraulic inductance calculations

In the above we have introduced analytical expressions for the hydraulic inductance of each element of the compressor duct. These geometric approximations are subsequently used to calculate the hydraulic inductance for the compressor ducts in both test rig A and B. Dimensions of the compressor internals are obtained from construction drawings of the respective installations.

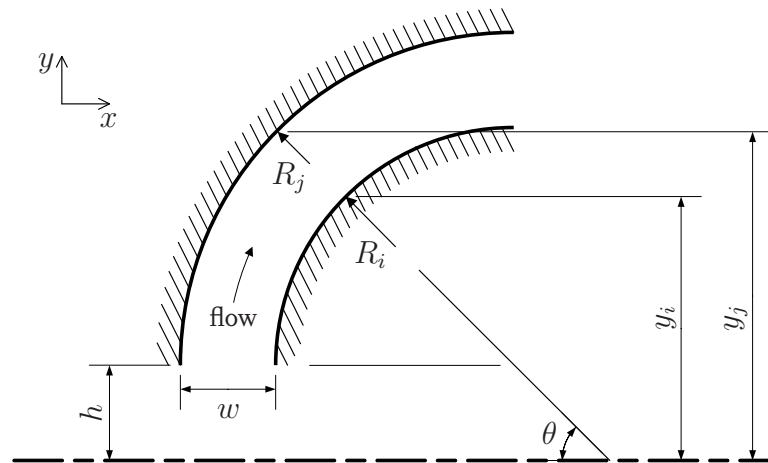


Figure 6.7 / Curved duct geometry, type C.

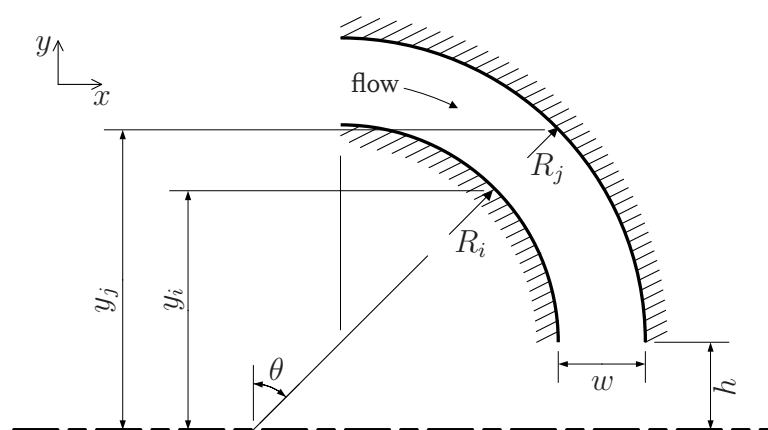


Figure 6.8 / Curved duct geometry, type D.

**Table 6.2** / Calculated hydraulic inductances for test rig A and B.

| Number | Duct type      | $L_c/A_c$  |            |
|--------|----------------|------------|------------|
|        |                | Test rig A | Test rig B |
| 1      | inlet channel  | 15.0       | 3.55       |
| 2      | inlet bend     | 7.38       | 2.11       |
| 3      | impeller bend  | 7.38       | 2.11       |
|        | impeller duct  | 16.6       | 1.10       |
| 4      | diffuser       | 18.0       | 5.37       |
| 5      | diffuser bend  | 3.68       | 1.38       |
| 6      | diffuser bend  | 3.68       | 1.38       |
| 7      | return channel | 20.4       | 4.29       |
| 8      | return bend    | 7.38       | 1.86       |
| 9      | exit channel   | 23.9       | 7.69       |
|        | total channel  | 123        | 30.8       |

**Table 6.3** / Comparison of tuned and calculated values of  $L_c$  and  $B$ .

| Test rig | $L_c$ |            | $B$       |            |
|----------|-------|------------|-----------|------------|
|          | tuned | calculated | tuned     | calculated |
| A        | 0.300 | 0.413      | 3.50–5.31 | 2.98–4.52  |
| B        | 15.4  | 0.787      | 1.51–1.81 | 6.67–7.98  |

The results of the calculations are given in Table 6.2. The values for the hydraulic inductance  $L_c/A_c$  of the entire ducts are obtained by summing the values for the individual components.

In Table 6.2 we can see that the calculated values for test rig A are much higher than those for test rig B. This difference can be explained by noting that the average cross-sectional area  $A$  is much smaller in test rig A, resulting in a larger hydraulic inductance.

To compare these results with the tuned values of  $L_c$  for the two test rigs, we multiply the hydraulic inductance with the same reference areas  $A_c$  as used in Chapters 3 and 4. Moreover, we calculate the corresponding range for the value of the stability parameter  $B$ , using Equation (6.1). Note that we use the same rotational speeds at which the various surge experiments were conducted. The results are given in Table 6.3.

The calculated compressor duct length of test rig A is in the same order of magnitude as the tuned value. In contrast, for test rig B the calculated value of  $L_c$  is an order of magnitude smaller than the tuned value. However, in Chapter 4 we already mentioned that the value  $L_c = 15.4$  might be an overestimation. In Chapter 5 we presented the result

of a simulation with a value for  $L_c$  similar to the calculated ones for test rig B in Table 6.3, illustrating that such a low  $L_c$  value yields a better prediction of the initial pressure drop in a surge cycle.

We point out that the suggested geometries in Figure 6.1 are simplifications of the actual compressor geometry. In particular, representing the impeller by a 90 deg bend and a radial duct results in a drastic simplification of the impeller geometry with its curved blades. A more detailed (numerical) calculation of the integral in Equation (6.2) could provide more insight in the validity of the geometric simplifications. However, the relatively contribution of the impeller to the total inductance value is 10–20%. Therefore it is unlikely that a more accurate integration will, for example, reduce the difference between the calculated and tuned compressor duct lengths for test rig B.

To conclude, the results as discussed here indicate that the proposed approximation returns plausible values for the hydraulic inductance and, indirectly,  $B$ . In Section 6.4 we will come back to the validity of the calculations presented here. Now, we will first discuss another method to estimate the stability parameter  $B$  that is based on realization theory and makes use of forced response measurements.

## 6.3 Model identification

System identification deals with developing mathematical models of dynamic systems based on observed data from the system. The field of system identification is very diverse as becomes clear from, for example, the important work by Box and Jenkins (1970) on time-series analysis, the state-of-the-art survey by Åström and Eykhoff (1971), and the classic text books by Eykhoff (1974); Söderström and Stoica (1989); Ljung (1999). Other relevant references are the books by Van Overschee and De Moor (1996) and Schoukens and Pintelon (2001) that focus on subspace and frequency domain identification, respectively.

In general, constructing a model from data requires a data set, a set of candidate models, and a rule by which candidate models can be assessed (Ljung, 1999). If the outcome of the system identification procedure is not satisfactory, adjustments of one or more basic ingredients is required and the procedure has to be repeated. Let us now consider each of the above elements in relation with our objective to identify the stability parameter  $B$ .

With respect to the data set we remark that the means of actuation in the compression systems under study are limited. The type of input-output data that is easiest to obtain is a step response since this only requires a stepwise change of the control valve and measuring the resulting pressure transients. For the selection of a candidate model we use the results from previous chapters where we showed that the dynamic behavior of the compression systems can be described by a low order, nonlinear, lumped parameter

model. We point out that the stability parameter  $B$  also has a strong influence on the behavior of the linearized model. Therefore, we assume that a linear model for the local dynamics around a specific operating point is sufficient to obtain an estimate for the stability parameter. With respect to the assessment of candidate models we point out that our modeling efforts have resulted in a reasonably accurate dynamic model and we gained valuable insights in the dynamic behavior of the compressor test rigs. We can exploit these qualitative and quantitative insights to assess the results of an identification procedure in a heuristic manner.

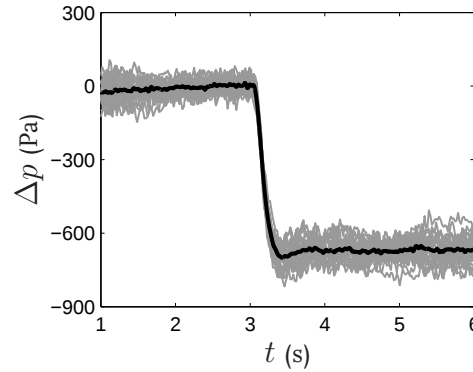
One of the many ways (e.g., [Ljung, 1999](#)) to obtain a linear model from transient data is an approach based on approximate realization theory. Approximate realization algorithms have been successfully applied for system identification ([Van Helmont et al., 1990](#); [Hesseling, 2002](#); [Klaassen et al., 2004](#)) and experience with these algorithms was readily available. Therefore, we have adopted a similar approach in our research to obtain an estimate of the stability parameter  $B$ . Beneficial characteristics of the used approximate realization algorithm are its straightforward implementation and computational simplicity, while providing enough flexibility and insight in order to tune the algorithm. In Section 6.4 we will briefly reflect on our choice for this specific identification method.

In the remainder of this section we will subsequently address the acquisition of step response data sets, the approximate realization algorithm and stability parameter estimation, and finally the identification results and validation thereof. In the discussion of the method and results we will concentrate on test rig A since no usable step response data is available for test rig B.

### 6.3.1 Step response measurements

Step response data from the compression system can be obtained by applying a step input to the control valve and measuring the resulting pressure rise over a certain period of time. Before the step is applied, the compression system must be in a steady-state operating point to avoid that other transients influence the measurements. Moreover, the size of the applied step must be large enough to result in a measurable response. However, the perturbation must remain small to keep the operating point of the compressor close to its initial value. Otherwise, linear approximations of the system dynamics are invalid.

For test rig A we conducted 10 different experiments at various stable operating points, rotational speeds and with different step sizes. The step signal was generated at the 0–10 V output channel of the data acquisition system and electronically converted to a 4–20 mA current. Before the step input was applied, the compressor was brought to a stable operating point. The measurement time was 6.4 s in which 8192 samples of the signals  $p_1$ ,  $p_2$ ,  $T_1$ , and  $T_2$  were collected.



**Figure 6.9** / Filtered step response data (gray) and mean value (black);  $N = 9,432$  rpm,  $\dot{m}_{c,0} = 1.13$  kg/s,  $u_{r,0} = 0.409$ ,  $\Delta u_r = 0.287$ ,  $L_c = 0.30$ .

For each experiment we performed 25 measurements such that the effect of measurement noise on the results could be reduced through averaging. Measurement and flow noise were further reduced through zero-phase filtering of the data with a 5<sup>th</sup> order, low-pass Butterworth filter with a cut-off frequency between 11–17 Hz. We point out that we have chosen a very low filter cut-off frequency to remove the significant amount of 50 Hz pollution and flow noise inside the compression system. However, since the filter cut-off frequency is around the bandwidth of the used control valve, filtering is not believed to remove any important dynamic information from the raw data.

Analysis of the filtered data from all experiments showed that the smallest variance between different measurements, indicating a good signal-to-noise ratio, was obtained with an initial valve opening  $u_{r,0}$  of 40.9% and a step size  $\Delta u_r$  of 28.7%. Therefore, from now on we will only present results obtained with these settings, unless stated otherwise. Results of the step response measurements for one operating condition and with the selected step input from 40.9% to 69.6% are shown in Figure 6.9. We remark that we shifted the measured signals to obtain an initial value around zero. This shifting was done by subtracting the average value of 1000 samples ( $\approx 0.8$  s) prior to the step input from the measured data. Note that relative large differences exist between the 25 measurements. These differences are mainly caused by low frequent flow noise and variations of the operating conditions between subsequent measurements.

The goal of the identification method that we will discuss below is to obtain a linear model of the compression system that can be used to estimate the stability parameter  $B$ . The input of this linear model is the valve opening  $u_r$ , see also Section 5.3. Hence, including the dynamics of the control valve actuator, sensors and the electronics in the data set for the identification method is not desirable. We experimentally confirmed that the dynamic response of the voltage to current converter is almost ideal up to 10 kHz. Based on specifications of the manufacturer we considered the dynamic response of the sensors to be ideal up to at least 100 kHz.

However, a detailed analysis of the control valve dynamics revealed that the valve response to a step input is too slow to be neglected. Furthermore, we concluded that the valve has a considerable time delay of 37 samples at a sampling frequency of 1.28 kHz. In addition, we found that an additional time delay of 16 samples is present in the anti-aliasing filters of the each channel of the data-acquisition system. A detailed investigation of the valve dynamics and time delay is given by [Van Helvoirt et al. \(2005a\)](#).

The effect of the time delay in the valve actuator and data-acquisition system was compensated for by removing the first 53 samples from each step response time series. In order to remove the effect of the valve dynamics we designed a pre-filter that represents the inverse of the actual valve dynamics. The valve dynamics are given by

$$H_r(s) = e^{-\frac{53}{1280}s} \frac{1.552 \cdot 10^5}{(s + 30.79)(s^2 + 60.32s + 5042)} \quad (6.11)$$

We remark that this model is slightly different from the original approximation for the valve dynamics that was given by [Van Helvoirt et al. \(2005a\)](#), since we replaced the (non-minimum phase) zeros in the transfer function by a static gain to assure a stable inverse. Through step response simulations we verified that this further simplification of the valve dynamics did not lead to noticeable different responses.

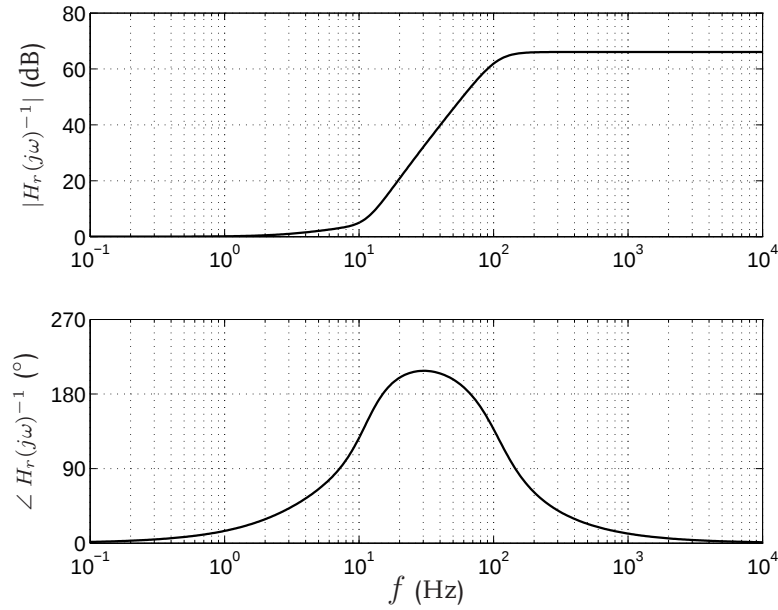
The pre-filter can be obtained by taking the inverse of Equation (6.11). However, to obtain a proper inverse we added an additional 3<sup>rd</sup> order, low-pass Butterworth filter with a cut-off frequency of 108 Hz. The cut-off frequency was chosen such that it will not compromise the accuracy of the inverse in the region of interest ( $< 20$  Hz). The Bode<sup>2</sup> diagram of the resulting pre-filter is shown in Figure 6.10.

So far we have addressed the generation of a set of input-output data from which a linear model of the compression system can be obtained. In summary, the pre-processing of the data consists of the following steps:

- conduct step response experiments;
- calculate ensemble average of raw data;
- apply low-pass filter to reduce noise;
- remove initial steady-state offset from data;
- shift time series to remove effect of time delays;
- filter data with approximate inverse of valve dynamics.

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<sup>2</sup>Hendrik Wade Bode (1905–1982) was an American scientist and pioneer of modern control theory and electronic telecommunications. In 1945 he published the classic book 'Network Analysis and Amplifier Feedback Design' and one of his important contributions is the well-known Bode's sensitivity integral.



**Figure 6.10** / Bode diagram of the pre-filter to compensate for non-ideal valve dynamics.

Now, we will address the method that will be used to calculate this linear model and subsequently estimate the stability parameter.

### 6.3.2 Parameter identification with approximate realizations

The term realization refers to a state-space representation of a given input-output mapping. When the input-output data set is of finite dimensions or when the data are corrupted with noise we speak of a partial or approximate realization. Realization theory has been under development since the 1960s (Kalman, 1963; Gilbert, 1963) and a well-known result is the algorithm by Ho and Kalman (1965). A recent overview of developments in the field of realization theory is provided by De Schutter (2000).

A straightforward way to obtain an approximate realization for a discrete-time LTI system is through the construction of a Hankel<sup>3</sup> matrix from a finite sequence of measured Markov parameters. Note that for a stable discrete-time LTI system the Markov parameters  $G_i$  represent the impulse response of the system. An approximate realization of appropriate order  $\rho$ , is then calculated from the singular value decomposition (SVD) of this Hankel matrix (Kung, 1978). The algorithm reads

---

<sup>3</sup>Hermann Hankel (1839–1873) was a German mathematician who studied and worked with, among others, Möbius, Riemann, Weierstrass and Kronecker. He is known for the Hankel transformation, which occurs in the study of functions that only depend on the distance from the origin.

1. Construct the Hankel matrix  $\underline{H}$  according to

$$\underline{H}_{i,j} = \begin{cases} G_{i+j-1} & \text{for } i+j \leq n+1 \\ 0 & \text{for } i+j > n+1 \end{cases}$$

from the step response sequence  $\{S_k\}_{k=0}^n$ , using  $S_k = \sum_{i=1}^k G_i$  with  $i = 1, 2, \dots$

2. Compute the SVD  $\underline{H}_r = \underline{U}\underline{\Sigma}\underline{V}^T$  of the matrix<sup>4</sup>  $\underline{H}_r = \underline{H}(1:r, 1:r)$  with  $r = \lfloor n/2 \rfloor$
3. Construct the matrices  $\underline{U}_\rho = \underline{U}(:, 1:\rho)$ ,  $\underline{V}_\rho = \underline{V}(:, 1:\rho)$ , and  $\underline{\Sigma}_\rho = \underline{\Sigma}(1:\rho, 1:\rho)$
4. Construct the matrices  $\underline{H}_a = \underline{H}(2:r+1, 1:r)$ ,  $\underline{H}_b = \underline{H}(1:r, 1)$ ,  $\underline{H}_c = \underline{H}(1, 1:r)$ , and  $\underline{H}_d = S_0$
5. Construct the system matrices of the realization

$$\begin{aligned} \hat{\underline{A}} &= \underline{\Sigma}_\rho^{-\frac{1}{2}} \underline{U}_\rho^T \underline{H}_a \underline{V}_\rho \underline{\Sigma}_\rho^{-\frac{1}{2}} \\ \hat{\underline{B}} &= \underline{\Sigma}_\rho^{-\frac{1}{2}} \underline{U}_\rho^T \underline{H}_b \\ \hat{\underline{C}} &= \underline{H}_c \underline{V}_\rho \underline{\Sigma}_\rho^{-\frac{1}{2}} \\ \hat{\underline{D}} &= \underline{H}_d \end{aligned}$$

The selection of a value for the order  $\rho$  of the realization determines the singular values that are considered to be contributing to the order of the approximate realization. Note that when  $\rho$  is taken too large, the noise present in the Markov parameters is modeled as a part of the realization.

The presented algorithm can be used to obtain a state-space description  $(\hat{\underline{A}}, \hat{\underline{B}}, \hat{\underline{C}}, \hat{\underline{D}})$  for the linear dynamics of a system from step response data. We now return to the compression system and the problem of determining the stability parameter  $B$ . When an approximate realization of the linear compression system is available, the eigenvalues(s) of  $\hat{\underline{A}}$  will approximate the eigenvalues of the system matrix for the actual system. An estimate  $\hat{B}$  for the actual stability parameter  $B$  is then easily obtained from the analytical solution of the characteristic equation  $\det(\hat{\lambda}\underline{I} - \hat{\underline{A}}) = 0$ , with  $\hat{\lambda}$  an eigenvalue of  $\hat{\underline{A}}$ .

The general analytical expression for  $B$  as a function of the eigenvalues of the linearized Greitzer model from Equation (5.3) is given by

$$B = \frac{F(1 - \frac{\partial \Psi_c}{\partial \Psi_t}) + \lambda^2 + \sqrt{F^2(1 - \frac{\partial \Psi_c}{\partial \Psi_t})^2 + 2F\lambda^2(1 + \frac{\partial \Psi_c}{\partial \Psi_t}) + \lambda^4}}{2\partial \Psi_c \lambda} \quad (6.12)$$

---

<sup>4</sup>The used Matlab-like notation  $(:)$  is a shorthand for an entire row or column of a matrix. Similarly,  $(i:j)$  indicates consecutive rows or columns.

with  $\partial\Psi_c$  and  $\partial\Psi_t$  denoting the slopes of the compressor and throttle characteristics, respectively. An estimate  $\hat{B}$  can be obtained from Equation (6.12) by substituting the appropriate eigenvalue  $\hat{\lambda}$ . Note that this approach requires sufficiently accurate knowledge of the other parameters at the selected operating point in order to obtain estimates for  $B$  with the desired level of accuracy.

Finally, we point out that in general  $\hat{A}$  can have multiple eigenvalues. Selection of the appropriate eigenvalue will be addressed in the next section where we discuss the results of the proposed identification method.

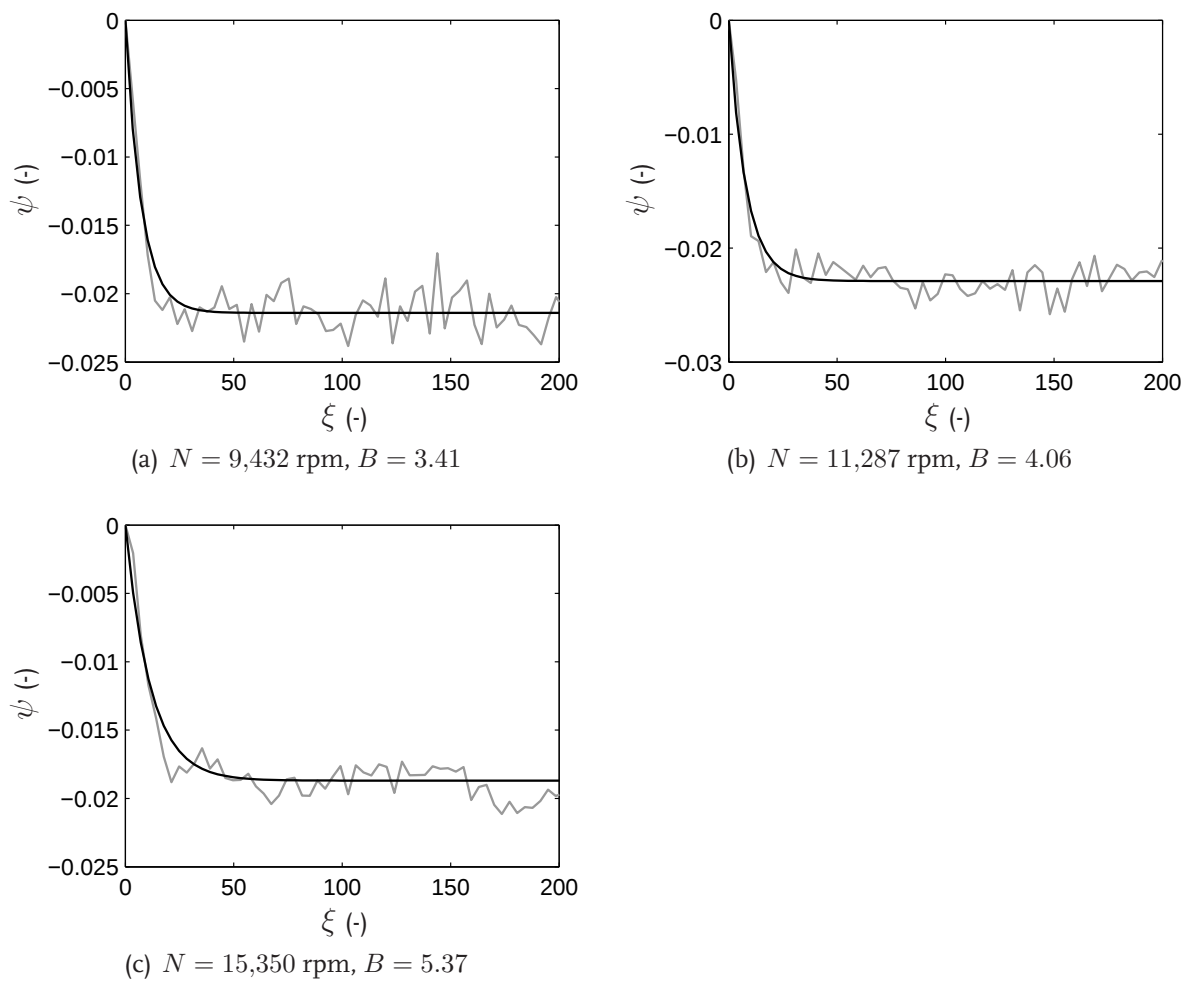
### 6.3.3 Results and validation

With the above procedure and the available step response data we can calculate an estimate for the stability parameter  $B$ . The first step is to provide the approximate realization algorithm with appropriate input-output data from the step response experiment.

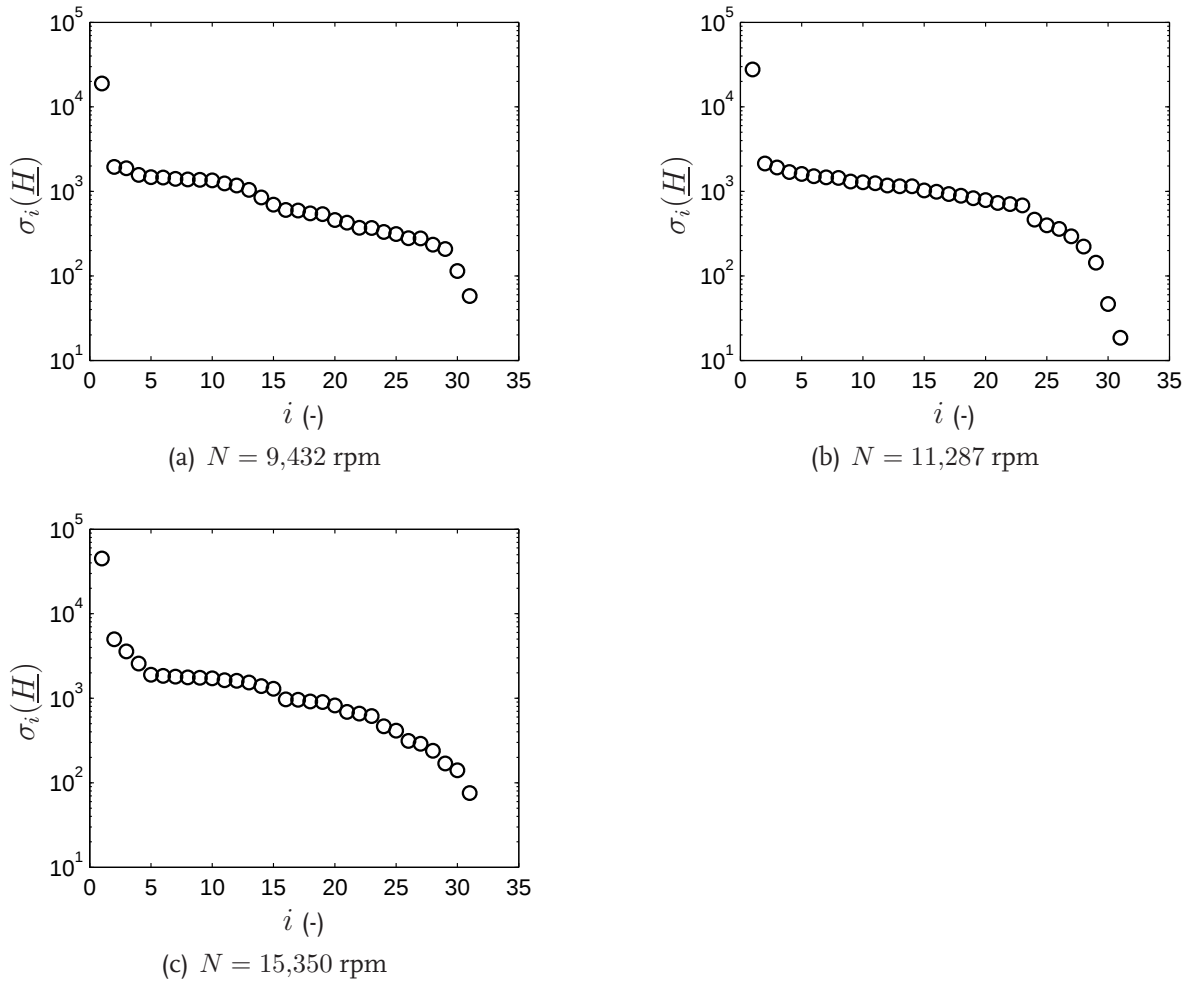
Prior to this step a choice must be made for the length of the sequence that is fed into the algorithm or, more specifically, the number of data points  $n$  and the (sub)sampling frequency  $f_{s,x}$  for the data have to be selected. Jointly,  $n$  and  $f_{s,x}$  must assure that the resulting step response sequence  $\{S_k\}_{k=0}^n$  represents both the transient and steady-state part of the response equally well. At the same time the step response sequence must be kept small enough to avoid excessive computational times due to the large size of the Hankel matrix  $\underline{H}$ . For the test rig A case, we determined through trial and error that using  $n = 63$  samples from a time interval of 3.1 s ( $f_{s,x} = 20$  Hz) gives reliable results with an acceptable computational time ( $< 0.15$  s) on a 3.2 GHz processor.

The most important step in the algorithm is to select the order  $\rho$  of the realization. Usually, a trade-off between relevant dynamics and noise is made with the help of the SVD, as proposed by, for example, Kung (1978). However, from previous chapters we know that the second-order Greitzer model is capable of describing the essential dynamics of the compression systems under study. Furthermore, the linear analysis from Chapter 5 showed that, for relatively large  $B$  values, the system has two distinct eigenmodes with the one associated with filling and emptying of the plenum clearly dominates the other mode. Therefore, despite the fact that the developed nonlinear model is of second order, we choose to construct first order approximate realizations from the step response data. A further justification for this choice will be given below.

In Figure 6.11 the results of the approximate realization algorithm are shown for particular measurements during three experiments at different rotational speeds. Visual inspection of these plots shows that the transient pressure decrease and the steady-state value of the measured step responses are reproduced by the approximated first order LTI systems with reasonable accuracy.



**Figure 6.11** / Measured step responses (gray) and step responses of the approximated LTI models (black);  $\phi_{c,0} = 0.18 \dots 0.20$ ,  $u_{r,0} = 0.409$ ,  $\Delta u_r = 0.287$ ,  $F = 1.1$ ,  $\omega_H = 69 \dots 71$  rad/s.



**Figure 6.12** / Singular values of the step response Hankel matrices.

To verify if the choice to use  $\rho = 1$  in the algorithm is valid, we refer to the plots of the singular values  $\sigma_i$  of the Hankel matrices for the same cases as in Figure 6.12. These plots clearly show that there is one dominant singular value, indicating that the first order approximation is indeed sufficient to describe the measured step responses.

After the verification that the first order approximations accurately describe the measured step response data, we can proceed with calculating estimates for the stability parameter. For each experiment we calculate the eigenvalue  $\hat{\lambda}$  of  $\hat{A}$  for all 25 approximate realizations and use Equation (6.12) to obtain estimates of  $B$ . Finally, we calculate the average value for  $\hat{B}$  for each rotational speed. The resulting estimates are given in Table 6.4. For comparison we have also included the values for  $B$  that were found through the geometric calculation of the hydraulic inductance, see Section 6.2.2, and through tuning of the nonlinear model, see Chapter 3, where the different rotational speeds during the step response experiments has been accounted for.

**Table 6.4** / Estimated stability parameter  $\hat{B}$  for test rig A.

| N rpm  | $\hat{B}$ |            |       |
|--------|-----------|------------|-------|
|        | estimated | calculated | tuned |
| 9,432  | 2.99      | 2.48       | 3.41  |
| 11,287 | 3.87      | 2.95       | 4.06  |
| 15,350 | 4.21      | 3.90       | 5.37  |

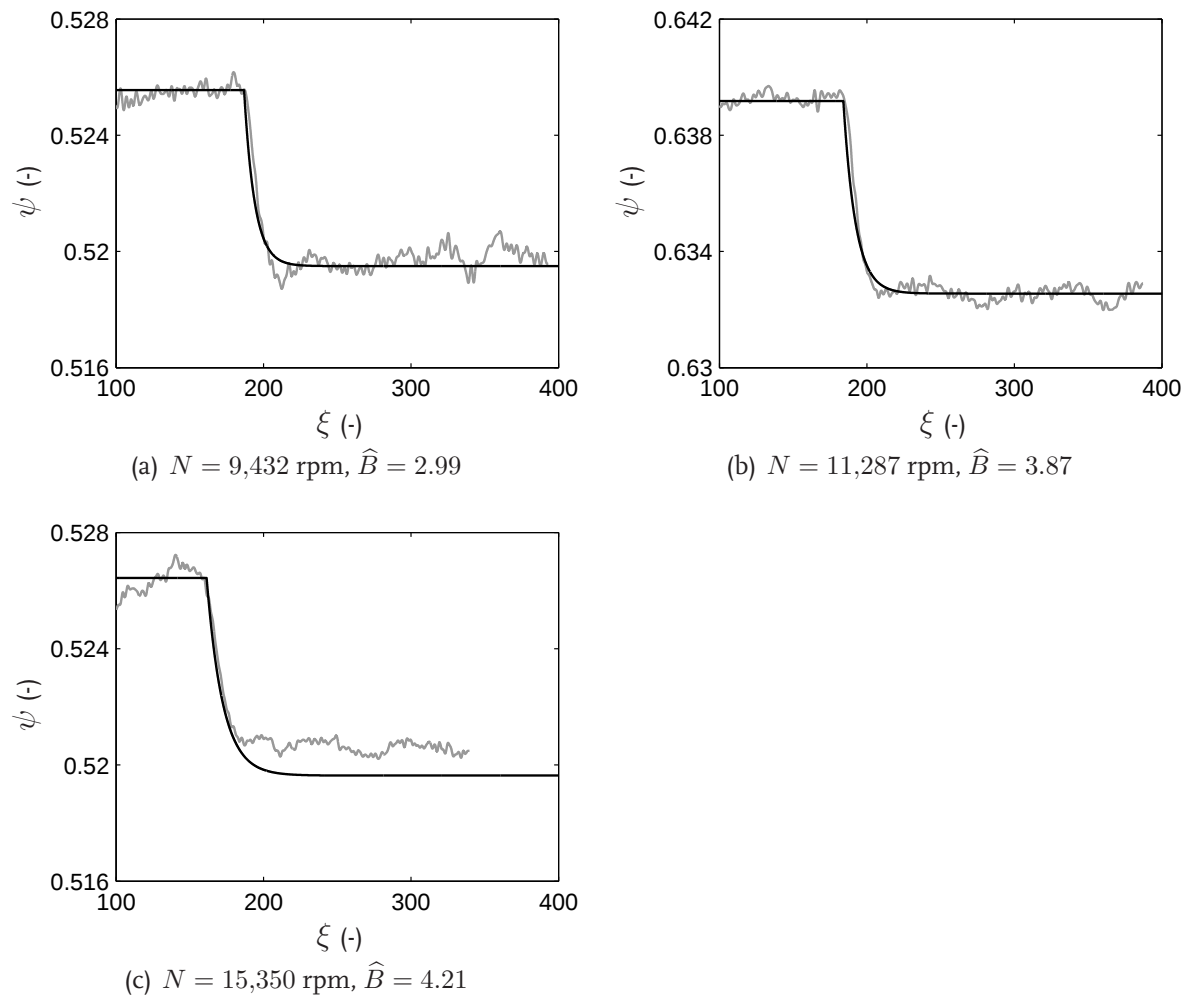
From the results in Table 6.4 we see that the approximate realization method yields estimates for  $B$  that are in the same range as we found earlier from tuning and calculating the hydraulic inductance. At this point we remark that the identification method presented in this section is relatively sensitive to measurement noise and hence a large number of step response measurements is required. Furthermore, the value for  $\hat{B}$  appears to be sensitive to the number of samples  $n$  that is used to construct the Hankel matrix in the approximate realization algorithm.

With the results of the parameter estimation method available, it is interesting to use the obtained values in the nonlinear Greitzer model, see Chapter 3, and compare the measured step response data with outcomes from the simulation model. The results for three other step responses at the same rotational speeds are shown in Figure 6.13. From these plots we see that for both  $N = 9,432$  and  $N = 11,287$  rpm the simulated step response are in good agreement with the experimental data. However, for  $N = 15,350$  rpm the steady-state pressure drop that results from opening the control valve is overestimated. A possible explanation for this discrepancy is the inaccuracy of the compressor curve  $\Psi_c(\phi_c)$  at  $N = 15,350$  rpm. For this rotational speed the approximation of  $\Psi_c(\phi_c)$  is obtained by extrapolating the measured steady-state compressor characteristic, see also Section 3.3.3.

Nevertheless, given the good overall agreement between the three methods (tuning the model on surge data, approximating the hydraulic inductance, and using the approximate realization algorithm) we conclude that all methods provide a reasonable estimate. By combining the results from the three methods it is therefore possible to obtain a quantitative value for the stability parameter  $B$  and its associated uncertainty for test rig A.

## 6.4 Discussion

In this chapter we have addressed the question of how to obtain an estimate for the important stability parameter in the dynamic model for industrial compression systems. We have proposed two methods other than model tuning to solve the stability parameter estimation problem within the constraints inherent to industrial compression systems.



**Figure 6.13** / Measured (gray) and simulated step responses (black) using the corresponding estimates  $\hat{B}$ ;  $\phi_{c,0} = 0.18 \dots 0.20$ ,  $u_{r,0} = 0.409$ ,  $\Delta u_r = 0.287$ ,  $F = 1.1$ ,  $\omega_H = 61 \dots 53$  rad/s.

The first method comprises an approximation for the hydraulic inductance from the internal compressor geometry. By comparing the outcomes of this method with values found earlier from a model tuning approach we conclude that the approximation provides a plausible estimate for the compressor duct length and hence the stability parameter  $B$ .

However, the complex 3D geometry of an industrial compressor has been greatly simplified in order to obtain an easily solvable integral. A more detailed calculation, for example using a 3D geometric model, should provide more insight in the effect of the introduced simplifications and the accuracy of the end result.

Furthermore, some rather strict assumptions on the flow pattern in the various parts of the compressor have been introduced. It is known from literature (e.g. Whitfield and Baines, 1990) that the actual flow patterns, in particular in the impeller and diffuser, are complex and by no means one dimensional. Again, more detailed calculations, for example using CFD models, can be applied to assess the accuracy of the approximations presented here.

The second method is an identification method that uses an approximate realization algorithm to obtain a linear dynamic model for the compression system from measured step response data. In this method we made use of the fact that the response of the investigated compression systems is dominated by the dynamics of the plenum. Hence, we were able to describe the step response data with a first order model, making it easy to determine an estimate for the stability parameter from the eigenvalue of this model.

However, both the selection of the appropriate model order, the number of used data points and their associated effective sampling frequency influence the outcome of the approximate realization algorithm. Furthermore, calculating an estimate for  $B$  from the eigenvalue of the identified model requires accurate knowledge of the plenum and suction volumes and the compressor and throttle characteristics. Similarly, when using the hydraulic inductance approximation knowledge of the plenum volume, compressor speed, and sonic velocity are required to determine an estimate for  $B$ .

From the discussion so far it has become clear that the approximate realization method can provide a reasonable estimate of the local system dynamics but the algorithm appears to be rather sensitive for the choice of various 'tuning' variables. Furthermore, the identification method uses step response data that can be easily obtained from experiments on the compressor test rigs. However, using other excitation signals like sinusoids or a pseudo-random binary sequence could yield input-output data that contains more information on the dynamics of the investigated system. Hence, applying more advanced identification techniques like subspace or prediction error methods, in combination with excitation signal could improve the estimates of the local compression system dynamics, see for example (Van den Bremer et al., 2006).

**Table 6.5** / Estimated values and uncertainties for the stability parameter.

| N rpm  | Test rig A |            | Test rig B |          |
|--------|------------|------------|------------|----------|
|        | $\hat{B}$  | accuracy   | $\hat{B}$  | accuracy |
| 9,500  | 2.9        | $\pm 17\%$ |            |          |
| 11,000 | 3.6        | $\pm 18\%$ |            |          |
| 15,000 | 4.5        | $\pm 19\%$ | 6.7        | —        |
| 19,000 |            |            | 8.0        | —        |

Since no usable step response data is available for test rig B, we cannot investigate if the identification method provides plausible values for the stability parameter in test rig B. Moreover, it is not precisely known what the effect of the piping acoustics will be on the identification result once step response data becomes available. However, based on the agreement between the results from both estimation methods for test rig A and the findings from Figure 5.7 it is reasonable to assume that the hydraulic inductance approximation for test rig B has provided a reasonable estimate for the stability parameter.

To conclude, for the methods presented in this chapter we cannot precisely determine the accuracy of the resulting values for the stability parameter  $B$ . However, the methods provide  $B$  values in the same ranges for test rig A as were found through model tuning. Therefore, we combine the results from all three methods to obtain the estimates for the stability parameter  $B$  of test rig A as summarized in Table 6.5. Note that the rotational speeds are rounded values.

For test rig B we have used the values for  $B$  from Table 6.2 since these values appear to be more reliable than the tuned values. However, the large difference between the tuned and calculated values and the absence of results from the approximate realization method it is not possible to give an indication of the accuracy for the presented estimates of the stability parameter for test rig B.

So far, we have addressed the modeling and identification of the relevant dynamics in industrial scale centrifugal compression systems. These results and gained insights have been used to make a first step in the design and implementation of an active surge control system. This part of our research will be discussed in the next chapter.



# Surge control design and evaluation<sup>1</sup>

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**Abstract** / In this chapter the design of an active surge controller is discussed. After a review of the available literature the choice for a straightforward LQG controller is substantiated. Subsequently, closed-loop simulation results are presented that provide rather strict actuator requirements. The design of an actuator prototype that meets these requirements is discussed. Finally, results from closed-loop experiments are presented and possible explanations for the unsuccessful tests are given. The chapter ends with a discussion on the gained insights concerning the design and implementation of active surge control on an industrial scale centrifugal compression system.

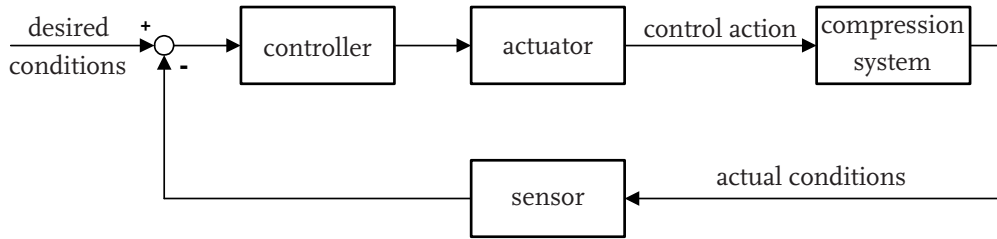
## 7.1 Introduction

Despite the progress in the field and the potential impact on industrial compressor operability, full-scale applications of active surge control have not been realized yet. Various survey papers (e.g. [Greitzer, 1998](#); [Gu et al., 1999](#); [Paduano et al., 2001](#)) show that various experimental surge control studies have been carried out on different laboratory scale systems. An overview of past studies is given by [Willems and De Jager \(1999\)](#) and more recently, [Willems \(2000\)](#); [Nelson et al. \(2000\)](#); [Spakovszky \(2004\)](#); [Arnulfi et al. \(2006\)](#) presented results from their surge control experiments.

In this chapter we will focus on the barriers for industrial scale surge control in centrifugal compression systems. Our aim is to build up experience with the design and implementation of a surge controller on an industrial scale setup. Hence, we will adopt a straightforward control structure and pragmatic design procedure that allow us to gain insight in the theoretical and practical problems in industrial scale surge control. At this point we remark that all simulations, controller designs and experiments presented in this chapter are conducted on test rig B only. Test rig A was not available for testing anymore during this part of our research.

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<sup>1</sup>This chapter is partially based on [Van Helvoirt et al. \(2007\)](#).



**Figure 7.1** / Schematic active surge control system.

One critical technological barrier for surge control is put up by the limited actuation capabilities as stated by [Paduano et al. \(2001\)](#); [Van Helvoirt et al. \(2006\)](#). The absence of an adequate actuator will hamper the implementation and experimental evaluation of active surge control. Hence, one of the contributions of this chapter is the specification and design of a high-speed control valve. We will discuss how the actuator specifications are obtained from closed-loop simulations and we will present test results to show that the designed actuator meets these specifications.

A second contribution of this chapter is the thorough discussion of the results from various closed-loop experiments on centrifugal compressor test rig B. Although we were not able to achieve actual stabilization of the compression system with the implemented control system, the results provide valuable insights in the problems associated with active surge control under realistic conditions.

In the following sections we will first address the actuator selection and controller synthesis for an active surge control system and we will present various closed-loop simulation results. Secondly, we will discuss the actuator design and the evaluation thereof in detail. Then we discuss the experimental implementation and evaluation of the control design on the test rig. Finally, we will discuss the results from this chapter where specific attention is paid to the unsuccessful control experiments and the issues that remain to be solved in order to achieve active stabilization on an industrial scale test rig.

## 7.2 Surge control design

An active surge control system comprises sensor(s) that provide information about the momentary flow conditions, actuator(s) to influence those conditions, and a controller that determines appropriate control action after comparing measured data with the desired flow conditions. A schematic representation of a general active surge control system is given in Figure 7.1. In this section each of the three elements of the control system will be addressed where we will use the results from previous chapters where appropriate.

### 7.2.1 Sensor and actuator selection

An important aspect of surge control design is the selection of the sensor(s) and actuator(s). The type and location of sensors and actuators must be chosen such that it becomes possible to measure and influence the behavior of the compression system in an adequate manner. Furthermore, the choice for specific sensors and actuators and their location in the system can influence the structure of the controller itself. Systematic studies of sensor and actuator selection for surge control can be found in [Simon et al. \(1993\)](#); [Van de Wal et al. \(2002\)](#).

A typical compression system can be equipped with a variety of pressure, temperature and mass flow sensors. However, dynamic mass flow measurements are usually not available as mentioned in Chapter 3. Furthermore, temperature sensors are not useful for our purpose since we have not included temperature effects directly in the dynamic model of the compression systems under study. Hence, dynamic pressure measurements will be used as input for the surge control system.

Roughly, the following means of actuation can be distinguished within the literature: control valves, air injectors, and variable geometries like movable plenum walls and inlet guide vanes. A more recent idea is to use the electric drive of a compressor as an actuator, see for example [Gravdahl et al. \(2002\)](#). Other actuators like loudspeakers, heaters and fuel regulators (for turbine-powered compressors) appear to be less suitable for large systems and therefore we will not discuss them here.

Surge control tests with a movable wall actuator are discussed in [Gysling et al. \(1991\)](#); [Arnulfi et al. \(2006\)](#). Variable inlet guide vanes are mainly used in industry to optimize the performance and efficiency of turbocompressors during stable operation (e.g. [Simon et al., 1987](#)). The main drawback of variable geometries for active surge control in full-scale installations is that these actuators are complex, bulky and expensive.

Air injectors have proved to be highly effective actuators for surge suppression as well as for stabilization of rotating stall [Nelson et al. \(2000\)](#); [Spakovszky \(2004\)](#). Their drawback is the drastic modifications of the compressor internals that are required for installation, in particular for multi-stage machines. Furthermore, practical experience with air injectors is limited and their reliability in industrial applications is still to be proven.

A close-coupled valve (located directly behind the compressor) yields the best surge control performance according to [Simon et al. \(1993\)](#); [Van de Wal et al. \(2002\)](#). Successful surge control experiments with control valve actuation, both with close-coupled and plenum bleed valves, are reported in [Pinsley et al. \(1991\)](#); [Badmus et al. \(1995b\)](#); [Jungowski et al. \(1996\)](#); [Willems \(2000\)](#). Control valves are standard equipment in industry (e.g. in conventional surge avoidance systems) and installation of an active surge control valve is relatively easy.

Hence, we selected a control valve as actuator for the surge control system. Given the limited possibilities to modify the test rig, a plenum bleed valve will be installed directly behind the compressor discharge. An additional benefit of using a plenum bleed valve is that the developed dynamic model does not need to be changed to account for the effect of the actuator. This will become clear below when we discuss the controller design.

According to the overviews by [Willems and De Jager \(1999\)](#); [Willems \(2000\)](#) a plenum pressure measurement is a widely used sensor for active surge control experiments. Furthermore, a similar argument holds for the sensor location as for the actuator and therefore we select a dynamic pressure measurement directly behind the compressor discharge as input for the control system. We point out that using a plenum pressure sensor as control input yields a non-minimum phase (NMP) closed-loop system. We briefly come back to the disadvantage of this choice later on.

After the selection of the sensor and actuator for the control system we proceeded with the synthesis of the controller itself. This step in the design will be discussed next.

### 7.2.2 Controller design

As mentioned in Chapter 1 the basic idea for active control of aerodynamic flow instabilities has been introduced by [Epstein et al. \(1989\)](#). Their approach is based on the assumption that the compression system can be stabilized with a linear controller that suppresses the small amplitude perturbations and disturbances before they develop into surge or rotating stall. The synthesis of linear controllers and their application to experimental setups shows promising results ([Ffowcs Williams and Huang, 1989](#); [Pinsley et al., 1991](#); [Jungowski et al., 1996](#); [Arnulfi et al., 2006](#)).

However, a drawback of linear controllers is the limited operating region in which these controllers are valid. Hence, stabilization can only be achieved when the perturbed system or, ideally, the entire surge limit cycle is contained in the domain of attraction of the desired equilibrium. An adequate domain of attraction can be obtained by, for example, gain scheduling, linear parameter varying or nonlinear controllers. In the literature various nonlinear surge controllers have been proposed (e.g. [Badmus et al., 1996](#); [Wang et al., 2000](#); [Bøhagen and Gravdahl, 2005](#); [Chaturvedi and Bhat, 2006](#)). An overview of the developments in this field is given by [Gu et al. \(1999\)](#). However, up to date the practical experience and experimental results with nonlinear surge controllers are limited.

Given the promising results with linear feedback control (e.g. [Willems, 2000](#)), the lack of experimental results for more advanced nonlinear controllers, and our focus on acquiring insight rather than controller performance, we will follow a linear control design approach. Before selecting a particular design method it is interesting to investigate the linearization of the compression system model around an unstable operating point.

For the moment we will focus on the Greitzer model for test rig B given by Equations (4.1) and (4.2). For the actual controller design we will include the model for the piping acoustics to get a more accurate representation of the dynamics that need to be stabilized, see also Chapter 4. Since the aerodynamic scaling of the dynamic model including piping acoustics is rather involved, we will use the full-dimensional form of the dynamic equations throughout this chapter. Some simulation results will be repeated in dimensionless form to allow for a comparison with results from other chapters.

The linearized Greitzer model for test rig B around a specific operating point  $(\dot{m}_{c0}, p_{20}, u_{t0}, u_{r0})$  is given by

$$\begin{bmatrix} \frac{d\tilde{m}_c}{dt} \\ \frac{d\tilde{p}_2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{A_c}{L_c} \partial p_c & -\frac{A_c}{L_c} \\ \frac{c_2^2}{V_2} & -\frac{c_2^2}{V_2} \partial \dot{m}_t \end{bmatrix} \begin{bmatrix} \tilde{m}_c \\ \tilde{p}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{c_2^2}{V_2} \partial \dot{m}_r \end{bmatrix} \tilde{u}_r \quad (7.1)$$

where, again with a slight abuse of notation,

$$\partial p_c = \left. \frac{\partial \Delta p_c(\dot{m}_c)}{\partial \dot{m}_c} \right|_{(\dot{m}_{c0})} \quad (7.2)$$

$$\frac{1}{\partial \dot{m}_t} = \left. \frac{\partial (\dot{m}_t(p_2, u_t) + \dot{m}_r(p_2, u_r))}{\partial p_2} \right|_{(p_{20}, u_{t0}, u_{r0})} \quad (7.3)$$

$$\partial \dot{m}_r = \left. \frac{\partial \dot{m}_r(p_2, u_r)}{\partial u_r} \right|_{(p_{20}, u_{r0})} \quad (7.4)$$

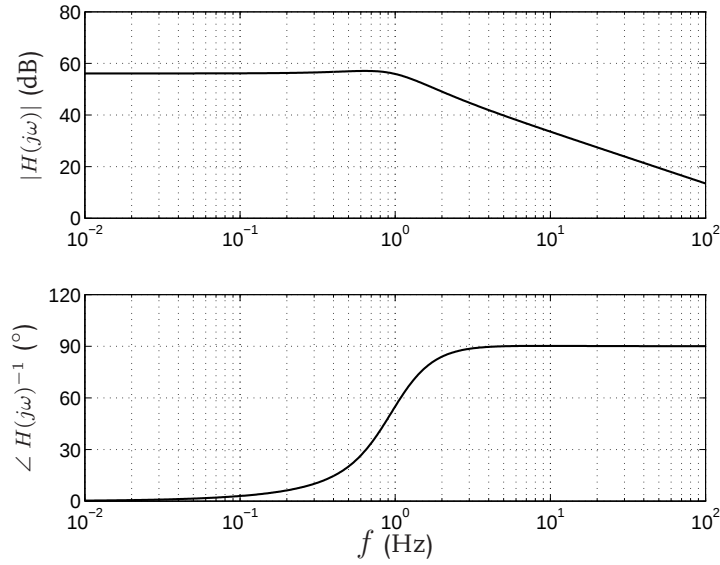
Note that, in contrast with the linearization from Chapter 5, we have included the effect of the control valve opening  $u_r$  as input to the system in Equation (7.1). The above linearization can be written as  $\dot{\tilde{x}} = \underline{A}\tilde{x} + \underline{B}\tilde{u}_r$  with the state vector  $\tilde{x}$ , system matrix  $\underline{A}$ , and input matrix  $\underline{B}$ . With the plenum pressure as measured variable we obtain the following output equation

$$\tilde{y} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{m}_c \\ \tilde{p}_2 \end{bmatrix} \quad (7.5)$$

or  $\tilde{y} = \underline{C}\tilde{x} + \underline{D}\tilde{u}_r$  with output (vector)  $\tilde{y}$ , output matrix  $\underline{C} = \begin{bmatrix} 0 & 1 \end{bmatrix}$ , and the direct feed-through matrix  $\underline{D} = 0$  in our case. With the relation  $H(s) = \underline{C}(s\underline{I} - \underline{A})^{-1}\underline{B}$  we obtain the following transfer function from the plenum pressure input to the control valve opening output

$$H(s) = \frac{-\frac{c_2^2}{V_2} \partial \dot{m}_r \left( s - \frac{A_c}{L_c} \partial p_2 \right)}{s^2 + \left( \frac{c_2^2}{V_2} \partial \dot{m}_t - \frac{A_c}{L_c} \partial p_2 \right) s + \frac{c_2^2 A_c}{V_2 L_c} (1 - \partial p_2 \partial \dot{m}_t)} \quad (7.6)$$

For a specific unstable operating point the corresponding Bode diagram is shown in Figure 7.2.



**Figure 7.2** / Bode diagram of the linearized Greitzer model for test rig B;  $N = 15,860$  rpm,  $\dot{m}_{c0} = 0.95\dot{m}_c^* = 2.14$  kg/s,  $u_{r0} = 0.5$ ,  $L_c = 10.6$  m.

The most important observation from this transfer function is that the system has a zero<sup>2</sup> in the right-half plane. Such a dynamic system is usually called a non-minimum phase (NMP) system and this imposes serious limitations on the achievable performance of the controlled or closed-loop system (Boyd and Barrat, 1991).

In combination with the fact that the uncontrolled system is unstable, the NMP behavior makes it difficult to apply standard loopshaping techniques as became clear from nonlinear closed-loop simulations with some of the resulting designs. Similar problems were encountered during an attempt to design a  $\mathcal{H}_\infty$  surge controller. Moreover, as already mentioned by Willems (2000), only a limited number of experimental results with robust surge controller designs have been reported in literature (Badmus et al., 1995b).

By testing both designs we found that a Linear Quadratic Gaussian (LQG) controller yielded a larger extension of the stable operating regime during closed-loop simulations than static output feedback controllers designed with the root locus technique that was used by Willems (2000). Furthermore, the design of a stabilizing LQG controller is straightforward and provides some means of tuning the closed-loop such that the amplitude of the control input is minimized, and thereby realize a large domain of attraction (Willems, 2000, pp. 47).

<sup>2</sup>The roots of the numerator of a transfer function are called the zeros of the system.

### LQG design

Given the arguments above, we proceeded with designing an LQG controller for the linearization of the entire compressor model for test rig B, including piping acoustics, around a specific unstable operating point. As we already mentioned in Chapter 5 we have to use a numerical routine to obtain a linearization of the dynamic model.

We recall that the LQG framework assumes that all the states of the dynamic system are available as input to the controller. As pointed out earlier, for the compression system under study the only measurable state of the resulting linear state-space model  $(\underline{A}, \underline{B}, \underline{C}, \underline{D})$  is the plenum pressure  $p_c(t)$ . Accurate dynamic mass flow measurements are not available and there exist no straightforward relations between the six internal states of the transmission line model and measurable variables.

The reference signal  $r(t)$  for the controller is the discharge pressure  $p_{c0}(t)$  that corresponds to the selected operating point. The operating point is specified as a percentage of the mass flow ( $\dot{m}_c^*(t)$ ) at the surge line. The nominal control valve opening  $u_{r0}$  is set to 0.5 to avoid problems with actuator saturation as much as possible. More efficient strategies that minimize the nominal bleed flow, for example the one-sided control strategy proposed by Willems (2000); Willems et al. (2002) that uses  $u_{r0} = 0$ , will not be considered in this thesis.

Initially, we use a standard LQ regulator to obtain a state-feedback law  $u_r = -\underline{K}\underline{x}$  that minimizes the quadratic cost function

$$J(u_r) = \int_0^{\infty} (\underline{x}^T \underline{Q} \underline{x} + u_r^T R u_r) dt \quad (7.7)$$

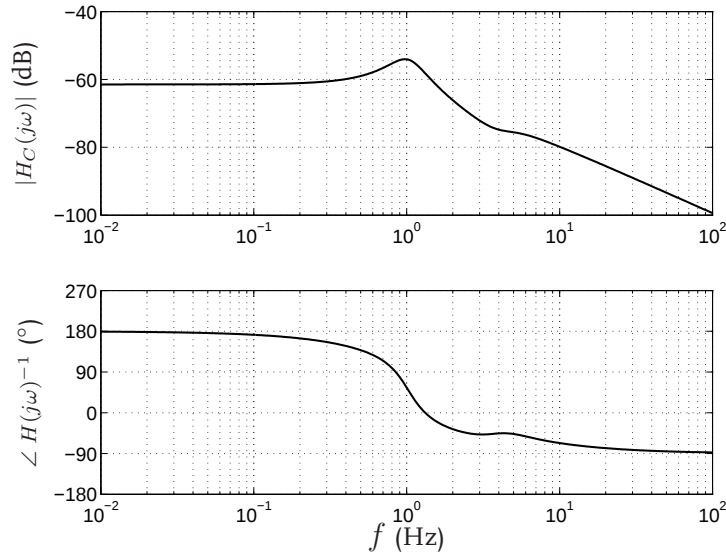
with the weights  $R$  and  $\underline{Q}$  as design parameters. In this section we will omit the perturbed variable notation  $\tilde{\cdot}$  for the sake of clarity.

In order to have access to the unmeasurable states, a Kalman filter

$$\hat{\underline{x}} = \underline{A}\hat{\underline{x}} + \underline{B}u_r + L(y - \underline{C}\hat{\underline{x}} - \underline{D}u_r) \quad (7.8)$$

with filter gain  $L$  is designed. This filter provides an estimate  $\hat{\underline{x}}(t)$  such that  $u_r = -\underline{K}\hat{\underline{x}}$  remains optimal. The covariances  $\mathbb{E}(ww^T) = \underline{W}$ ,  $\mathbb{E}(vv^T) = V$  of the process noise  $w$  and measurement noise  $v$ , respectively, are considered to be design parameters.

The weight  $\underline{Q}$  was set to  $\underline{I}$  and we used  $R$  to tune the controller. We remark that the right-half plane zero present in the linearized system model imposes a lower bound on the achievable bandwidth. Hence, further reduction of  $R$  after reaching this bandwidth is not useful. Fortunately, minimizing control effort requires a large  $R$ . Similar arguments hold for choosing  $\underline{W} = \underline{I}$  and  $V$  low to achieve the best possible loop transfer recovery. However, noise in the system (variance  $2 \cdot 10^4$  in experiments) requires a minimal value for  $V$  to avoid actuator saturation.



**Figure 7.3** / Bode diagram of the LQG controller;  $R = 5 \cdot 10^7$ ,  $V = 5 \cdot 10^7$ .

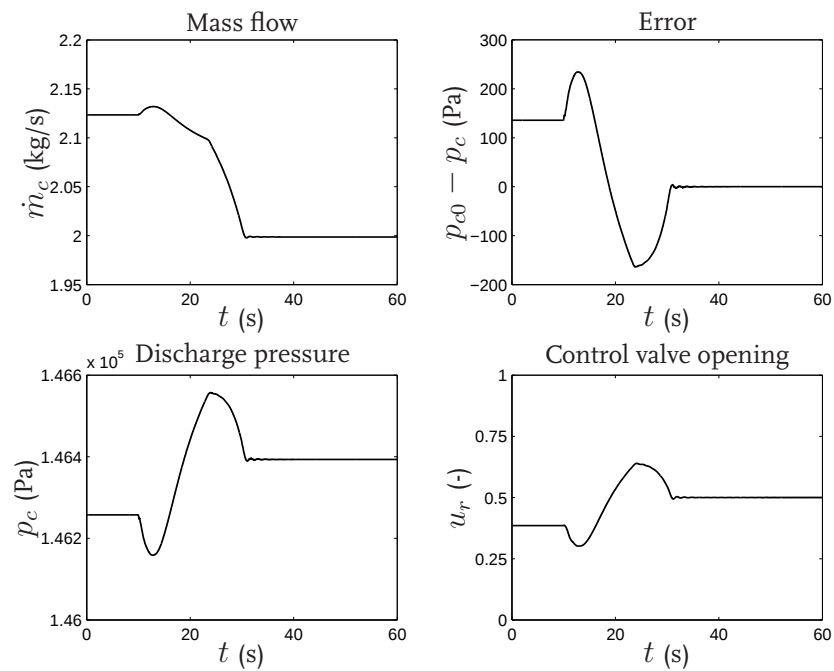
Numerical tuning gave  $R = 5 \cdot 10^7$  and  $V = 5 \cdot 10^7$  as suitable values. The large numerical values are caused by the fact that the mass flow is expressed in kg/s ( $\mathcal{O}(1)$ ) while pressures are in Pa ( $\mathcal{O}(10^5)$ ). The Bode diagram of the LQG controller is shown in Figure 7.3.

### Closed-loop simulations

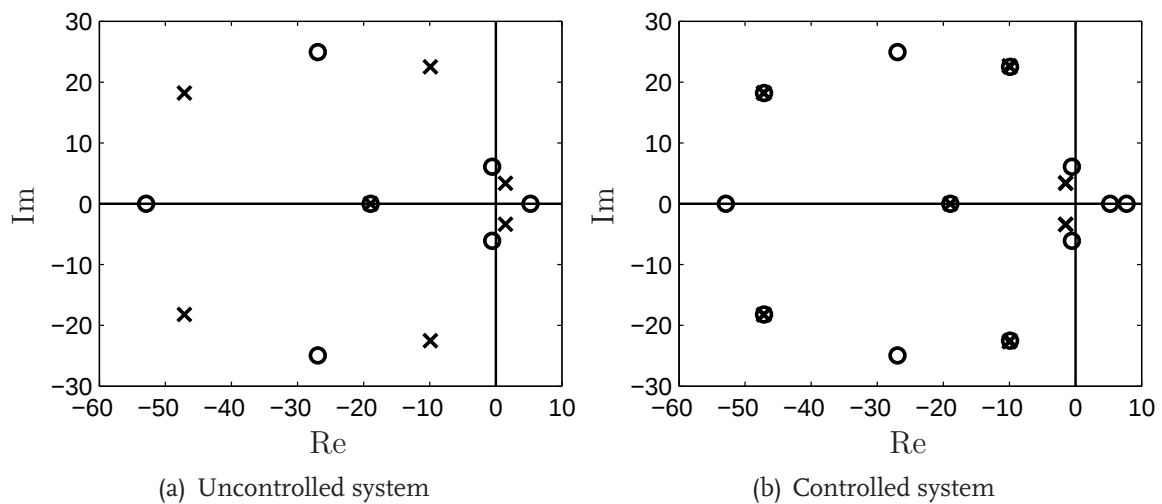
We will now present the results of various closed-loop simulations with the nonlinear model that were done to investigate the performance of the controller. For all simulations discussed in this chapter we used a sampling frequency of 1 kHz. During each simulation the mass flow is throttled down towards the normally unstable operating point by applying a decreasing ramp signal that goes from 1 to 0 during the time interval 10–20 s. The result in Figure 7.4 shows that the controller stabilizes the system for an operating point at 95% of the surge mass flow and a compressor speed of  $N = 15,860$  rpm.

The effect of the LQG controller on the location of the poles and zeros of the linearized compression system model is depicted in Figure 7.5. In the pole-zero map of the uncontrolled system we can distinguish the two unstable modes associated with the compressor dynamics and the different stable acoustic modes, see also Chapter 5. The pole-zero map for the closed-loop system indicates that the LQG controller cancels the acoustic modes while the unstable compressor modes are mirrored in the imaginary axis as expected (see also Willems, 2000, pp. 47).

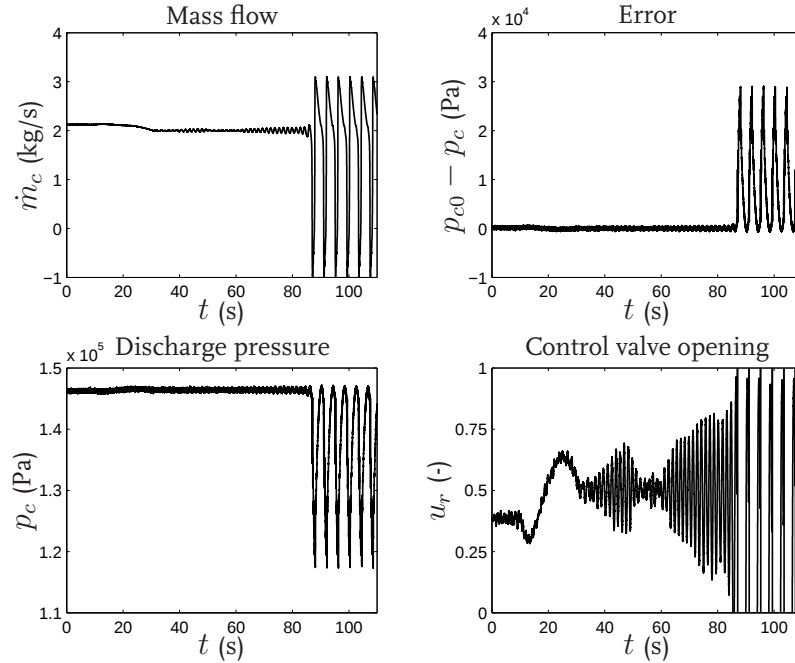
In order to perform a more realistic simulation, we added process noise (variance  $2 \cdot 10^4$ ) to the model of the compression system. Furthermore, we placed a third order low-pass filter with a cut-off frequency  $f_{b,r}$  of 12 Hz and a time delay  $\tau$  of 0.01 s between the controller and compressor model to represent the non-ideal dynamics of the control



**Figure 7.4** / Closed-loop simulation with ideal operating conditions;  $N = 15,860$  rpm,  $\dot{m}_{c0} = 0.95\dot{m}_c^* = 1.99$  kg/s,  $u_{r0} = 0.5$ ,  $L_c = 10.6$  m.



**Figure 7.5** / Effect of the LQG controller on the pole-zero map of the linearized compression system model;  $N = 15,860$  rpm,  $\dot{m}_{c0} = 0.95\dot{m}_c^* = 1.99$  kg/s,  $u_{r0} = 0.5$ ,  $L_c = 10.6$  m,  $R = 5 \cdot 10^7$ ,  $V = 5 \cdot 10^7$ .



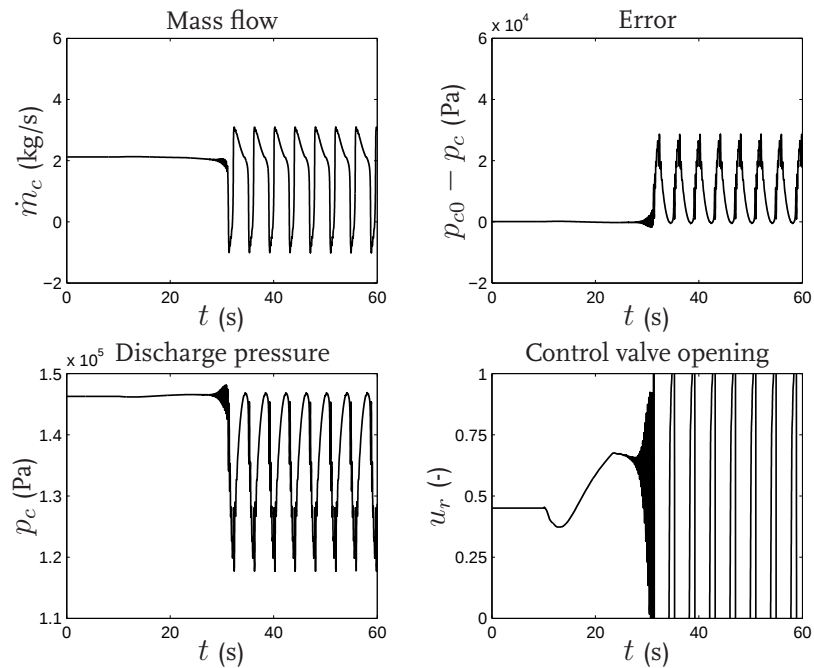
**Figure 7.6** / Closed-loop simulation with simulated process noise, non-ideal actuator dynamics and input delay;  $f_{b,r} = 12$  Hz,  $\tau = 0.01$  s,  $N = 15,860$  rpm,  $\dot{m}_{c0} = 0.95\dot{m}_c^* = 1.99$  kg/s,  $u_{r0} = 0.5$ ,  $L_c = 10.6$  m.

valve, see also (Van Helvoirt et al., 2006). The result in Figure 7.6 shows that this time the controller is not capable to stabilize the system and the compressor enters surge. Additional simulations showed that the failure to stabilize the system can be attributed to the non-ideal dynamics of the control valve. This issue will be discussed below.

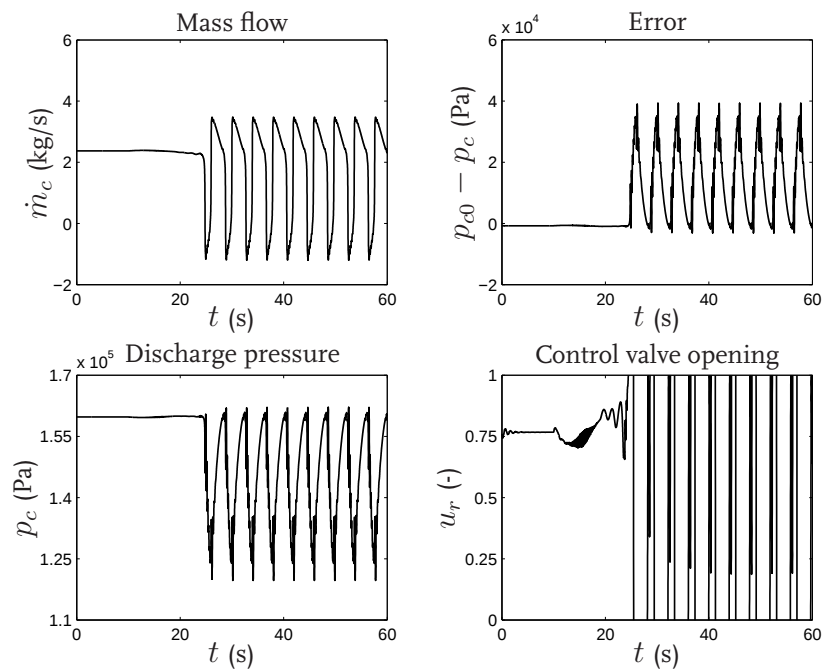
Moreover, we performed simulations for other operating points and rotational speeds to investigate the performance of the designed LQG controller. We point out that the LQG controller was redesigned for each case, using the linearization of the model for the selected operating conditions. From Figures 7.7 we see that the controller does not stabilize the compression system in an operating point at  $0.94\dot{m}_c^*$ . Figure 7.8 shows that the controller also fails to stabilize the compression system at a rotational speed of 17,796 rpm.

It is known from literature (e.g. Willems, 2000) that centrifugal compression systems are more difficult to stabilize at lower mass flows or at higher speeds that yield a larger value for  $B$ . Given our goal to evaluate a surge controller in practice, we will focus on the low speed case for operating points at 95% of the surge mass flow.

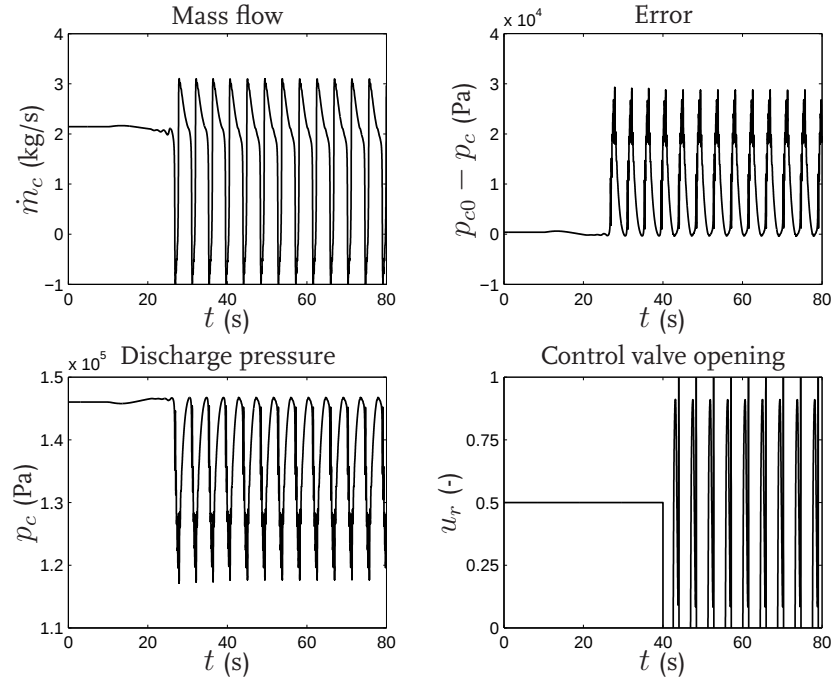
Before we address the actuator dynamics in detail we present some additional simulation results. In Figure 7.9 the result is shown of a closed-loop simulation in which the controller is turned on after 50 s while the compression system is in deep surge. This result shows that the designed controller is not capable to stabilize fully developed surge.



**Figure 7.7** / Closed-loop simulation with ideal, low flow operating conditions;  $N = 15,860$  rpm,  $\dot{m}_{c0} = 0.94\dot{m}_c^* = 1.97$  kg/s,  $u_{r0} = 0.5$ ,  $L_c = 10.6$  m.



**Figure 7.8** / Closed-loop simulation with ideal, high speed operating conditions;  $N = 17,796$  rpm,  $\dot{m}_{c0} = 0.95\dot{m}_c^* = 2.23$  kg/s,  $u_{r0} = 0.5$ ,  $L_c = 10.6$  m.



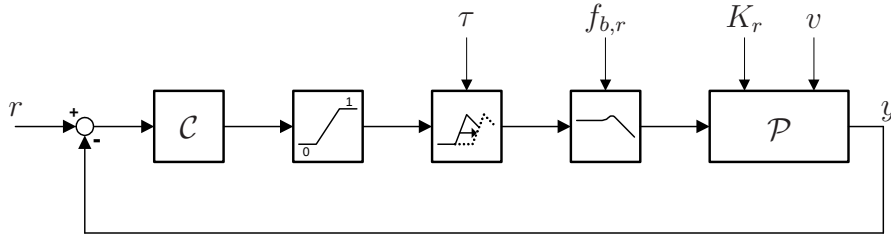
**Figure 7.9** / Closed-loop simulation with controller turned on while compressor is in surge;  $N = 15,860$  rpm,  $\dot{m}_{c0} = 0.95\dot{m}_c^* = 1.99$  kg/s,  $u_{r0} = 0.5$ ,  $L_c = 10.6$  m.

Hence, during the surge control experiments measures must be taken in order to prevent the compression system from entering surge before the surge controller is turned on.

We have also studied the robustness of the designed LQG controller with respect to parameter uncertainties. More specifically, we varied the  $B$  parameter and compressor slope  $\partial\Psi_c$  in the linear model that we used to design the controller. From the subsequent closed-loop simulations with the nonlinear model we found that only deviations of  $\leq 5\%$  between the actual and modeled parameter values are allowed without endangering the stability of the closed-loop system. We come back to the robustness issue when we discuss the experimental results. Now we will first address the issue of non-ideal actuator dynamics.

### 7.3 Surge control actuator

This section deals with the control valve specifications that are required to stabilize the centrifugal compression system under study. First, we discuss the various specifications that are considered and how they are quantified through numerical simulations. Secondly, we will elaborate on the realization of an actuator prototype and present the resulting design. Finally, experimental results are presented to demonstrate that the actuator meets all the imposed requirements.



**Figure 7.10** / Simulation scheme to determine actuator specifications;  $\mathcal{C}$  = controller,  $\mathcal{P}$  = nonlinear model of compression system.

### 7.3.1 Actuator requirements

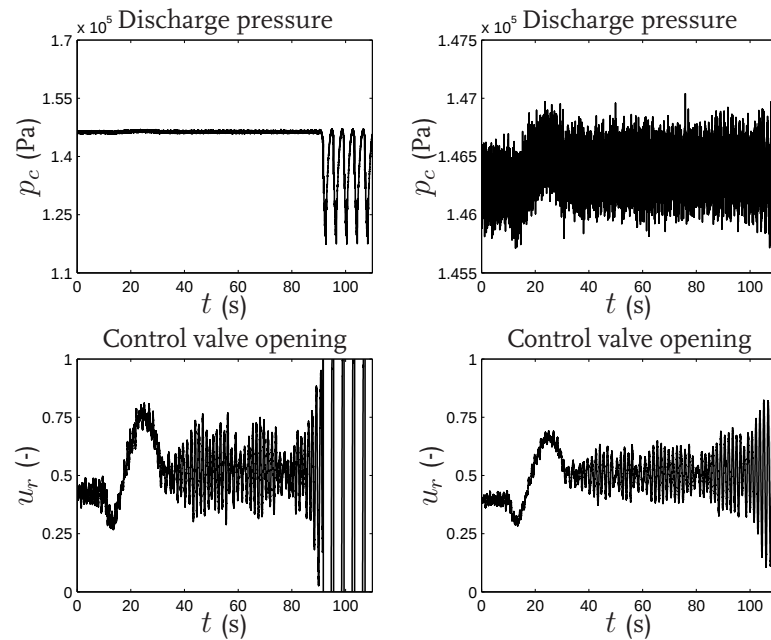
From literature (e.g., [Willems, 2000](#); [Paduano et al., 2001](#)) and our own investigations we know that the bandwidth and, even more important, the valve capacity are important specifications for a surge control valve. The control valve must respond fast enough to commands from the surge controller. Furthermore, changing the valve opening must have a significant influence on the pressure behind the compressor in order to provide adequate damping of surge oscillations. Another well-known issue is the effect of time delays on closed-loop stability. We found that time delays between the control command and the valve response are indeed a serious problem for surge control ([Van Helvoirt et al., 2005a, 2006](#)).

Given the arguments above, we focus on defining quantitative specifications for the valve capacity, bandwidth and the maximum allowable time delay. Other specifications as the life span, power consumption, efficiency and costs, although of practical importance, will not be taken into consideration in this thesis.

We performed numerous closed-loop simulations in the presence of bandlimited white process noise (variance  $2 \cdot 10^4$ ) while subsequently varying the valve capacity, bandwidth, and time delay. A block diagram of the simulation model is shown in Figure 7.10. The valve dynamics are represented by a third order low-pass filter (see also [Van Helvoirt et al., 2005a](#)), representing a mass-spring-damper system with a bandwidth equal to the cut-off frequency of the filter. The valve capacity was varied by modifying the value for  $K_r$ , see [IEC 60534-2-1 \(1998\)](#), of a standard control valve for which an accurate flow characteristic was available. We introduced a single time delay of variable length between the controller and valve position to represent all possible delays.

Some typical results from the simulations are shown in Figure 7.11. This figure shows the effect of varying the capacity of the control valve on the stability of the closed-loop system. The specifications for the control valve actuator that we derived from the results of the various simulations are summarized in Table 7.1.

Additional simulations showed that increasing  $K_r$  well above the specified value relaxes the bandwidth and time delay requirements. However, a larger bandwidth or smaller



**Figure 7.11** / Closed-loop simulation with  $K_r = 30$  (left figures) and  $K_r = 41$  (right figures);  $f_{b,r} = 16$  Hz,  $\tau = 0.006$  s,  $N = 15,860$  rpm,  $\dot{m}_{c0} = 0.95\dot{m}_c^* = 2.23$  kg/s,  $u_{r0} = 0.5$ ,  $L_c = 10.6$  m.

**Table 7.1** / Design specifications for the surge control valve.

| Parameter                         | Value       | Unit                       |
|-----------------------------------|-------------|----------------------------|
| Control valve capacity $K_r$      | $\geq 41$   | $\text{m}^3 \text{h}^{-1}$ |
| Control valve bandwidth $f_{b,r}$ | $\geq 16$   | Hz                         |
| Actuator time delay $\tau$        | $\leq 0.06$ | s                          |

time delay hardly have an effect on the minimum valve capacity that is needed to stabilize the system. Hence, we conclude that the valve capacity is the most critical requirement for the surge control actuator.

### 7.3.2 Actuator design

Given the design specifications obtained from the closed-loop simulations, we first investigated if they could be met with a commercial control valve actuator. Since this appeared not to be the case, we proceeded with designing a custom-made control valve actuator. Details of the development will be omitted here and we will only address the most important design choices.

Firstly, a choice was made to use a commercially available valve with a known flow characteristic. We selected a 50 mm diameter sliding gate valve with a  $K_v$  value of 45 m<sup>3</sup>/h, based on previous experience with this type of valves. Furthermore, this valve has a linear relation between capacity and valve opening, a relatively low moving mass of 0.353 kg, and a short stroke of 8 mm. Details of the control valve characteristics are given in Appendix C.

Secondly, we chose to use an electric servo motor as actuator. In general an electric motor introduces less time delay in comparison with pneumatic actuators. Furthermore, designing an electro-mechanical system is less complicated and it requires less auxiliary equipment than hydraulic and pneumatic actuators. In order to achieve a direct and compact transmission of the motor rotation into a valve translation, an eccentric disc was mounted on the motor shaft and the disc was connected to the valve stem with a connecting rod.

Finally, for the selection of a suitable servo motor, straightforward dynamic calculations were performed where we took into account the required bandwidth, the various masses and inertias, the transmission ratio, conservative estimates of transmission efficiency and valve friction (estimated: 100 N, measured on tensile test bench: 45 N). Through a trial-and-error selection procedure we found a motor that was able to meet the following demands: a peak torque of 2.32 Nm, peak acceleration of 26959 rad/s<sup>2</sup>, maximum speed of 1642 rev/min and a power requirement of 300 W.

The mechanism of the valve actuator and a photograph of the realized prototype are shown in Figure 7.12. Operation of the valve is possible through the communication software of the power amplifier and motor control module, which are not shown here.

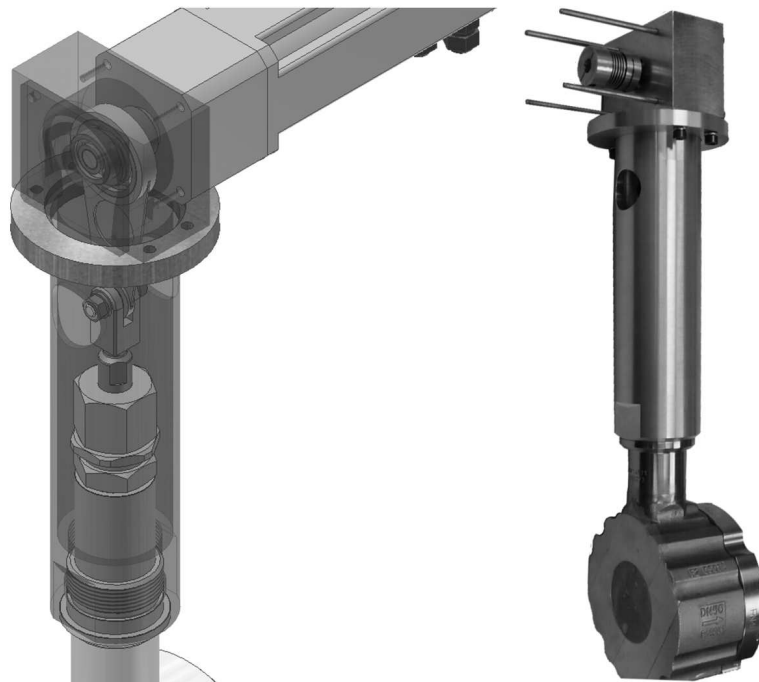


Figure 7.12 / Assembly drawing of design and photograph of final prototype.

### 7.3.3 Design evaluation

After the construction and assembly of the actuator prototype, various tests were conducted to evaluate if the design specifications were met. After installation and calibration of the valve and motor electronics, we first tested the response of the valve with a sequence of step commands of arbitrary size. For all tests we used a sampling frequency of 1 kHz. The results are shown in Figure 7.13 and from this and similar experiments we concluded that the time delay specification was met. In the entire chain "controller board, motor electronics, actuator, encoder, controller board" only one sample time delay, i.e.  $\tau = 0.001$  s, was found. However, we point out that the valve response has significant overshoot at large amplitude steps, indicating the need for fine-tuning of the motor position controller.

Secondly, we made an attempt to measure a transfer function of the actuator dynamics. However, due to the significant amount of friction in the moving parts of the valve (due to rubber seals), a linear transfer function could not be measured. A possible method to tackle this problem is to apply friction compensation. However, we followed a more straightforward approach by applying a sinusoidal position command with an amplitude corresponding to 80 % full stroke and a varying frequency. The result for a 25 Hz sinusoid is shown in Figure 7.14. This measurement indicates that the bandwidth of the system is located around 22 Hz and this result was confirmed by various other step and impulse response experiments.

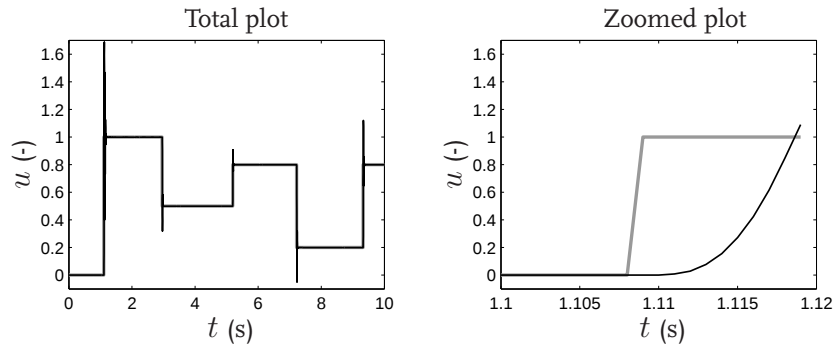


Figure 7.13 / Step reference (gray) and control valve response (black).

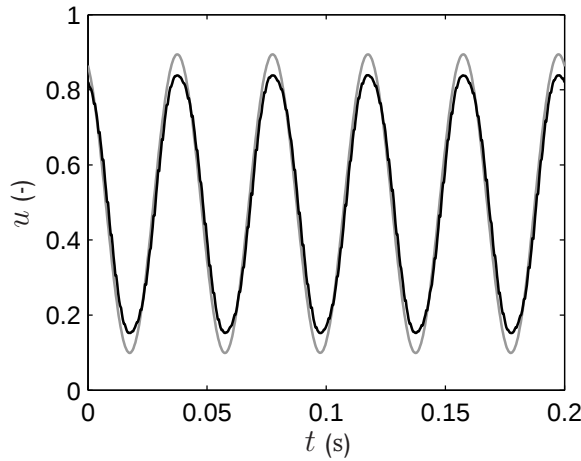


Figure 7.14 / Sine reference (gray) at 25 Hz and control valve response (black).

We point out that a bandwidth of 22 Hz is almost double the speed of commercial control valves of similar size. For example, a standard sliding gate valve with a  $K_r$  value of 6.4 and a conventional pneumatic actuator has a 12 Hz bandwidth and a time delay of 0.01 s, see (Van Helvoirt et al., 2006).

Thirdly, the control valve capacity was measured by the manufacturer of the sliding gate valve body in accordance with the ISO standard IEC 60534-2-1 (1998). After reviewing the measurement data provided by the manufacturer, we found no need to conduct further experiments to verify the specified  $K_r$  value (45 m<sup>3</sup>/h), which is above the required value. However, the point out that the margin between the required and available capacity of the control valve is small.

The control valve was installed in the test rig to conduct further tests at realistic operating conditions and to evaluate the designed surge controller. The results of these experiments will be discussed in the next section.

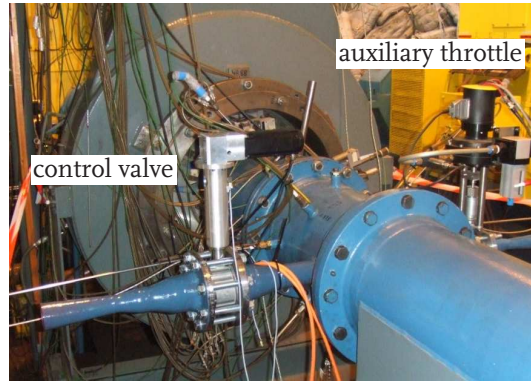


Figure 7.15 / Photograph of the control valve and auxiliary throttle on test rig B.

## 7.4 Surge control experiments

We evaluated the designed surge controller, including the valve actuator prototype, in various closed-loop experiments on test rig B. In this section we will discuss the findings from these experiments.

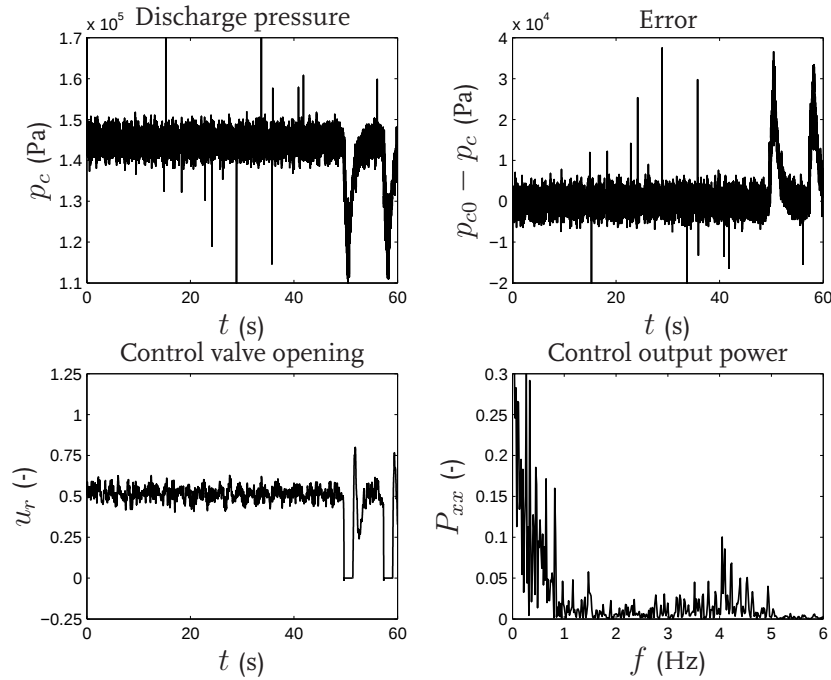
Prior to the closed-loop tests, the control valve was installed in the discharge piping of the test rig, opposite to a new auxiliary throttle valve, see also Figure 7.15. The auxiliary throttle with  $K_a = 28 \text{ m}^3/\text{h}$  allowed us to drive the compression system towards an unstable operating point while the surge controller was already active. In this way we tried to avoid the compression system from going into a surge limit cycle since simulations showed that the LQG controller is not capable of stabilizing the system once it is operating in deep surge.

For the implementation of the surge control algorithm we used a dSpace DS1103 controller board with multiple analog input and output channels and encoder inputs. The dSpace system also allowed for online monitoring and adjustment of controller gain and setpoint, as well as real-time acquisition of data from all relevant sensors. For all closed-loop simulations and experiments a sampling frequency of 1 kHz has been used.

Before starting an experiment we measured the current operating conditions and these data were used to initialize the controller setpoint. However, initial tests showed that this approach in determining the desired discharge pressure setpoint was not adequate due to measurement and model (compressor curve) uncertainties, see also Figure 7.16 and the accompanying discussion below. To circumvent this problem we followed the same approach as Willems (2000) and implemented a bandpass filter in series with the LQG controller. We used the following filter

$$H_{bp}(s) = \frac{\omega_l(s + \omega_n)}{(s + \omega_d)(s + \omega_l)} \quad (7.9)$$

with  $\omega_n = 6.3 \cdot 10^{-6} \text{ rad/s}$  and the pass-band set by  $\omega_d = 0.63 \text{ rad/s}$  and  $\omega_l = 25 \text{ rad/s}$ .

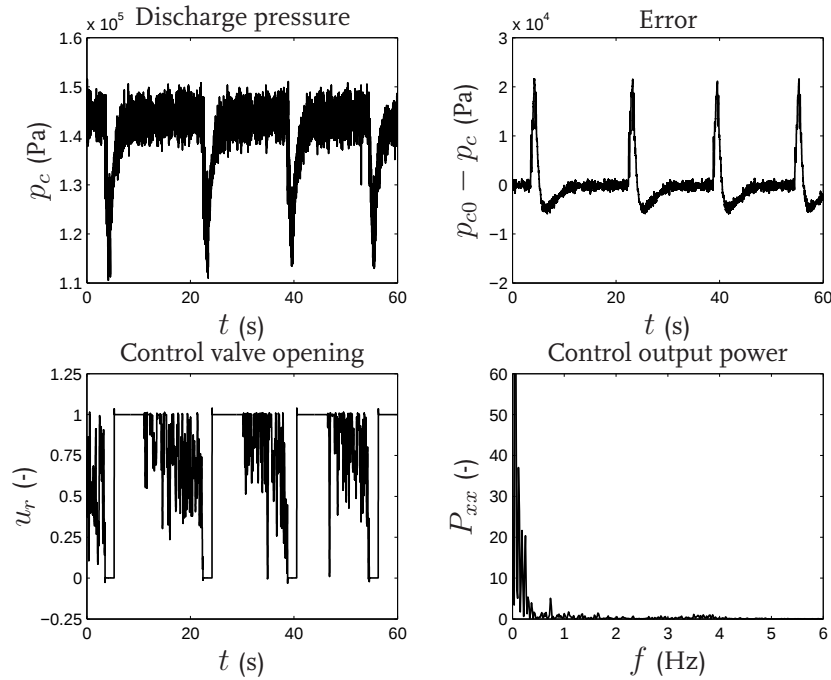


**Figure 7.16** / Closed-loop test result with LQG controller and bandpass filter;  $N = 15,860$  rpm,  $p_{c0} = 1.444 \cdot 10^5$  Pa,  $u_{r0} = 0.5$ ,  $K = 1$ ,  $R = 5 \cdot 10^7$ , and  $V = 5 \cdot 10^7$ .

With this bandpass filter the setpoint for the LQG controller becomes  $r(t) = 0$  since the DC-component of the pressure signal  $y(t)$  is filtered out. Hence, due to the bandpass filter there is no longer the need to know an exact value of  $p_{c0}$  in the desired operating point. Closed-loop simulations with the nonlinear compression system model showed that the system remained stable when using the LQG controller in combination with the selected bandpass filter.

After calibration and initialization of the test equipment we performed numerous closed-loop experiments on test rig B. Next to the initial design values, various other settings of the control parameters ( $R$ ,  $V$ ,  $\omega_l$ ,  $\omega_d$  and the extra controller gain  $K$ ) were tested and in some cases adjusted online. Unfortunately, none of the experiments proved to be successful in the sense that the centrifugal compression system was stabilized to the left of the open-loop surge line.

In Figure 7.16 the results are shown from a typical closed-loop experiment without the bandpass filter but an absolute pressure reference instead. The measured signals indicate a stable situation, despite the two surge cycles at the end of the experiment. Furthermore, the control action is moderate due to the relatively high value for  $R$ . However, turning off the controller and fixing the control valve position to its nominal value of 0.5 showed that the open-loop operating point was stable by itself. Hence, in this experiment the controller merely kept the system operating on the surge line instead of in a normally unstable operating point.



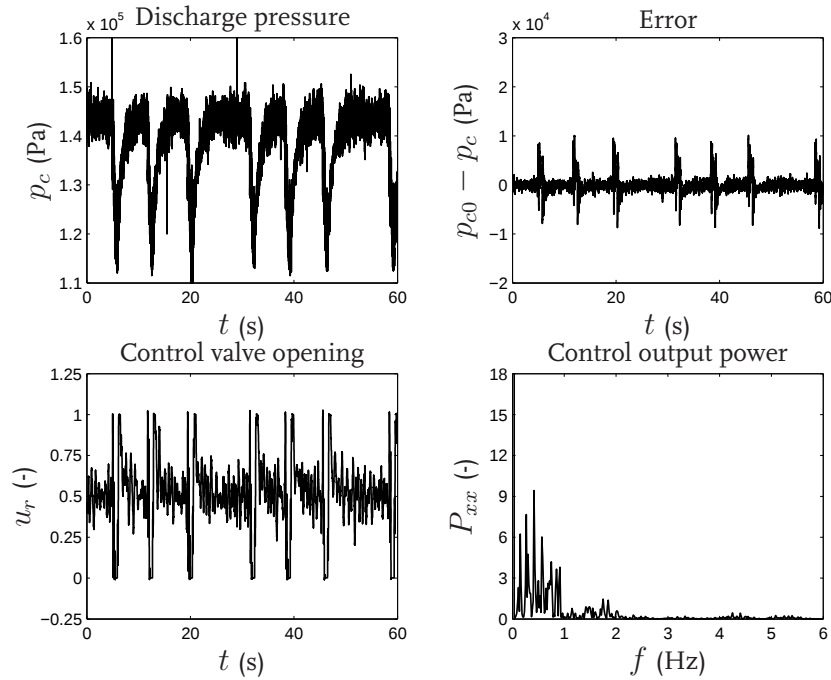
**Figure 7.17** / Closed-loop test result with LQG controller and bandpass filter;  $N = 15,860$  rpm,  $u_{r0} = 0.5$ ,  $K = 4$ ,  $R = 5 \cdot 10^7$ ,  $V = 5 \cdot 10^7$ ,  $\omega_n = 6.3 \cdot 10^{-6}$  rad/s,  $\omega_d = 0.63$  rad/s, and  $\omega_l = 25$  rad/s.

In Figure 7.17 a typical result is shown for an experiment where we did use the bandpass filter and  $p_{c0} = 0$ . The pressure signal clearly reveals several surge cycles that occurred during the experiment. However, the large average control valve opening ( $> 0.5$ ) indicates that the operating point is kept at or to the right of the surge line instead of in the unstable regime.

The result in Figure 7.18 for another experiment shows that opening and closing the control valve hardly affects the pressure when the system goes through a deep surge cycle. However, around 25 and 55 s the start of a new surge cycle appears to be postponed but further fine tuning of the selected controller settings did not yield any improvements. A stability test with the linearized compressor model after the experiments revealed that the applied bandpass filter resulted in an unstable closed-loop model, making it even more difficult to give an explanation for the observations around 25 and 55 s.

Prior to and during the closed-loop experiments we encountered various practical complications. First of all, the targeted unstable operating point must be approached carefully while the controller is active in order to avoid premature surging of the compression system. In practice it appeared to be difficult to control the operating point with the auxiliary throttle.

Secondly, due to model and measurement uncertainties and small variations in process conditions it was difficult to assess whether the compressor was operating exactly at the



**Figure 7.18** / Closed-loop test result with LQG controller and bandpass filter;  $N = 15,860$  rpm,  $u_{r0} = 0.5$ ,  $K = 0.25$ ,  $R = 5 \cdot 10^5$ ,  $V = 5 \cdot 10^7$ ,  $\omega_n = 6.3 \cdot 10^{-6}$  rad/s,  $\omega_d = 12$  rad/s, and  $\omega_l = 126$  rad/s.

design point of the surge controller. Furthermore, the intended operating point is located near the surge line ( $0.95\dot{m}_c^*$ ) so during experiments it was hard to assess if the compressor was stabilized rather than working at an inherently stable operating point.

The most likely reason for the failure to stabilize the compressor system during the tests is the lack of robustness of the closed-loop system for model and measurement uncertainties. We base this conclusion on our experience from the experiments since delicate tuning was already required to obtain the results as presented above. In that respect, including the bandpass filter in the controller already gave better results in comparison with the experiments where a pressure setpoint was used. Furthermore, the presented LQG controllers yielded more prompt actuator commands in response to the error signal than similar controllers that were designed using the Greitzer model (without piping acoustics) for test rig B.

Another possible reason for failure is that the domain of attraction of the controller is too small. Through simulations we already learned that the domain of attraction of the LQG controller is not large enough to stabilize the system once it is operating in surge. However, the significant process noise that is present in the system already causes excursions of the operating point from its design value that might trigger instability. In other words, the designed controller might prove to be inadequate to counteract the process noise in the system. In essence this implies that the small domain of attraction negatively affects the robustness of the surge controller.

Finally, during the experiments we noticed that the flow reversal during the initial phase of a surge cycle occurs very rapidly and violently. Given the large volume of the discharge piping, very rapid and large amplitude control actions are required in order to influence the pressure in the discharge piping sufficiently and thereby prevent surge cycles from occurring. From the results presented above we clearly see that the implemented control system was not capable of keeping the compression system out of surge.

## 7.5 Discussion

In this chapter we have discussed the design of an active surge controller. The goal of this design was to gain insight into the critical barriers for industrial scale surge control.

The first steps in the surge control design were the selection of sensors and actuators and the choice for a specific control strategy. Based on earlier experimental results from literature and the rather straightforward implementation we selected a plenum bleed valve as actuator and a discharge pressure measurement as sensor. According to the literature (e.g. Simon et al., 1993; Van de Wal et al., 2002) there are better choices possible for a sensor/actuator pair but, for example, air injectors and mass flow measurements are difficult or even impossible to implement on the industrial scale setup. Hence, additional efforts to overcome the practical limitations for the application of sophisticated sensors and actuators for active surge controllers are desirable.

Based on promising results with linear controllers in previous work we selected a straightforward LQG controller. This controller is easy to design once a linearized dynamic model of the centrifugal compression system is available and it allows for tuning of the closed-loop response. However, the LQG controller uses a Kalman filter to reconstruct the unmeasurable state(s) of the system so additional dependency on a model with a limited accuracy is introduced into the closed-loop system. Simulation results with the LQG controller showed that the nonlinear dynamic model of the compression system can be stabilized. However, simulations also showed that the achievable extension of the stable regime is limited, that stabilization is only possible at low compressor speeds, and that the controller is not robust for model uncertainties and process variations.

Both from literature and own experience we know that the actuator bandwidth and capacity are a critical barrier for industrial scale surge control (Paduano et al., 2001; Van Helvoirt et al., 2006). In a next step of the design we used the closed-loop simulation model to obtain specifications for such a control valve actuator. Since these specifications were not met by any standard control valve, we designed a new actuator that proved to be adequate in meeting the actuator specifications. A benefit of the applied electro-mechanical actuation principle is that it can be scaled relatively easy in order to increase either the valve size or speed when required.

After implementing the surge control system on the industrial scale test rig closed-loop experiments were conducted. During these tests no stabilization of surge was achieved. Based on the experience during the tests and the analysis of the results afterwards we concluded that the surge controller was not sufficiently robust for model uncertainties and process variations to achieve stabilization.

The poor closed-loop results and the findings from the stability parameter identification as discussed in Chapter 6 might indicate that an incorrect value for the  $B$  parameter has been used during the surge control design. Hence, it is worthwhile to determine the value for  $B$  with more accuracy in order to obtain a more accurate model and subsequently a better surge control design. Similar arguments hold for the exact shape of the compressor characteristic in the unstable regime since the slope of this characteristic appears in the linearized dynamic model and hence it influences the resulting control design.

Furthermore, more attention must be paid to increase the domain of attraction of the surge controller. Implementation of an active surge controller will become easier when the controller is capable of bringing the system out of deep surge. Moreover, the significant amount of process noise leads to rather large deviations of the momentary operating point from the design point. When the domain of attraction of the controller is sufficiently large, these deviations can be suppressed by the controller. However, it is worthwhile to investigate if other ways to reduce the noise in the inputs to the controller, for example through filtering, lead to better results.

In summary, the closed-loop experiments indicate that the robustness and domain of attraction of the implemented surge controller were not sufficient. Moreover, the actuator specifications that were obtained from simulations might be inadequate. Further work is therefore needed to investigate how the robustness and domain of attraction can be increased while taking into account the practical limitations in the selection and specification of sensors and actuators and the inevitable variations in process conditions, typical for industrial scale centrifugal compression systems.

First of all, attention can be paid to further increase the accuracy of the dynamic model for the actual system by improving, for example, the estimates of model parameters, accurately including piping acoustics, and investigating the effect of the position of actuators within the system as well as their speed and capacity. Secondly, more advanced control strategies can be applied to design a robust, stabilizing surge controller. Finally, attention can be paid to design and integrate more advanced means of sensing and actuation into an industrial scale compression system.



# Conclusions and recommendations

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**Abstract** / The main ideas and methods are recapitulated. The conclusions for this thesis are presented. Recommendations for future work are given.

This thesis deals with the modeling and identification for control of surge in industrial scale centrifugal compression systems. The main objectives were to develop a dynamic model for such a compression system, identify the parameters relevant for control design, and determine the barriers for the successful implementation of industrial scale surge control. In this chapter we will present the conclusions with respect to the stated research objectives and we will suggest directions for further research and development.

## 8.1 Conclusions

### Modeling compression system dynamics

In Chapter 2 we have presented a modeling framework for compression system dynamics that is based on the Greitzer model, which was originally developed for low pressure-ratio axial compressors. The adopted lumped parameter approach results in a relatively low order model, which is beneficial from a control design point of view. However, the approach assumes that the model structure can represent the layout of the actual compression system. Furthermore, the model requires knowledge of various geometric parameters and the compressor and throttle characteristics for the entire operating range.

Despite the various limitations of the modeling framework, we used the approach to develop models for the behavior of two different industrial scale centrifugal compressor test rigs. Initially, identification of the model parameters was done in the following manner.

We used steady-state measurements to determine the compressor and throttle characteristics. Since no steady-state mass flow data was available for the unstable flow regime, we

used cubic polynomial or spline approximations to represent the steady-state compressor characteristic over the entire flow regime. For the steady-state pressure rise at zero mass flow we used an analytical approximation. To represent the throttle characteristic we applied an empirical relation in accordance with a common industrial standard.

The geometry of the compression system is represented in the model by the Greitzer stability parameter. We determined a value for  $B$  by selecting fixed values for the plenum volume  $V_p$  and cross-sectional area  $A_c$ , and subsequent tuning of the effective compressor duct length  $L_c$  such that the frequency and pressure amplitude of measured deep surge oscillations are predicted well by the model.

For one of the test rigs, we introduced an additional dimensionless parameter  $F$  to account for the connection between the throttle exit and compressor inlet. We concluded that the large suction piping can be represented by a single volume and that the suction pressure can be regarded as constant.

For the second test rig, we have introduced a modification of the original model to account for the piping acoustics in the long discharge line of the test rig. To represent the piping acoustics we used a relatively simple approximation of a standard transmission line model. We concluded that this is a usable approach as long as the model parameters and boundary conditions are selected with care. The description of the damping of acoustic transients and the effect of rotor speed variations are still open issues.

For both investigated compression systems we made it plausible that the different modeling assumptions are valid. The results presented in Chapter 3 and Chapter 4 show that the developed models can describe the nonlinear behavior of the centrifugal compression systems during deep surge. Both the amplitude, base frequency and several higher order harmonics of the surge oscillations are predicted with reasonable accuracy.

### Identification of model parameters

In Chapter 5 we have investigated the surge dynamics in more detail and we studied the effect of the various model parameters on these dynamics. We concluded, in line with results from literature for small scale setups that the dimensionless stability parameter  $B$  plays a crucial role in describing the stability and surge behavior of industrial scale centrifugal compression systems. Furthermore, the analysis of the linearized model showed that the operating point and the associated slope of the compressor characteristic have a profound influence on the local dynamics around the operating point.

Based on our study of the (linearized) acoustic compression system models we concluded that the piping acoustics have an influence on the overall dynamic behavior of the system. Furthermore, we found that selecting a suitable value for the effective compressor duct length  $L_c$  is important in order to accurately describe both the surge dynamics and the acoustic transients, as well as the coupling between them.

Given the possible ambiguity in the tuned value for the compressor duct length and hence the stability parameter, we developed two new ways to determine accurate values for these parameters. The first method is a direct calculation of the hydraulic inductance, based on an approximation of the internal geometry of a centrifugal compressor. Comparison of the result with the tuned values makes it plausible that this method provides reasonably accurate estimates of the compressor duct length.

The second approach is an identification method for the stability parameter. This method uses approximate realization theory and step response data to obtain a linear model for the compression system. Estimates for the parameters of interest are then calculated from the eigenvalues of this linear approximation, under the assumption that the other geometric parameters are known.

Despite the limited set of validation data, the agreement with results from the other approaches makes it plausible that this identification method gives a reasonable estimate for the stability parameter. Furthermore, the results confirmed our earlier observations that the dynamics of the investigated compression systems are associated with a relatively high value for the stability parameter. However, we point out that the identification of the individual parameters is difficult given the dominant influence of the plenum volume on the system dynamics. Therefore, the accuracy of estimates for the stability parameter is limited by the accuracy of the other parameters that are assumed to be known a priori.

### Surge control design

In Chapter 7 we discussed the design of a surge control strategy with a focus on identifying the critical barriers for the industrial application of active surge control. As a first step we selected a plenum pressure sensor and plenum bleed valve actuator for the surge control system. This choice is based on earlier results from literature while taking into account the practical limitations of the industrial scale test rig. A drawback of the choice for sensor and actuator is that it results in a so-called non-minimum phase model that limits the achievable performance of the control system. Based on the results from our research we cannot conclude whether this limitation of performance has had a negative effect on the stabilizing properties of the investigated surge control strategy.

The second step was the design of an LQG controller, using the modeling and identification results for the compression system under study. The choice for this specific control strategy was based on earlier experimental successes with linear controllers and the straightforward design and tuning capabilities of the LQG controller. A drawback of this choice is the need for a state estimator since not all states, in particular the compressor mass flow, can be measured directly.

Simulations with the nonlinear dynamic model showed that stabilization of the compression system is possible at 95% of the open-loop surge point and low rotational speeds.

However, the simulations already showed that the domain of attraction of the controller is not large enough to achieve stabilization once the compression system is operating in deep surge. Furthermore, robustness for process variations and parameter uncertainties already proved to be small during simulations.

After tuning the controller we performed closed-loop simulations with the nonlinear model to specify the capacity, bandwidth and allowable time delay for the control valve. Since the obtained specifications could not be met by a commercially available valve, we designed a new control valve actuator for that purpose. Evaluation of the resulting prototype showed that the actuator specifications were met. Furthermore, the applied electro-mechanical valve actuator concept allows the development of even larger or faster valves, thereby removing one of the technological barriers for active surge control.

The final step was the implementation and experimental validation of the surge controller on the industrial scale centrifugal compression system. The closed-loop experiments did not result in stabilization of the compression system to the left of the open-loop surge line. The most likely causes for failure are the lack of robustness with respect to model uncertainties, noise and process variations and the small domain of attraction of the implemented controller. However, these findings and the practical experience gained from the closed-loop experiments form a good basis for further research and development. In order to develop any serious interest from compressor industry for active surge control technology, the challenge remains to stabilize industrial scale compression systems below 90% of the original surge line.

## 8.2 Recommendations

### Modeling compression system dynamics

The modeling framework based on the Greitzer model proves to be suitable for describing the surge phenomenon in industrial scale centrifugal compression systems. One drawback of the model is the coarse division in different components that does not necessarily represents the structure of an actual compression system. Furthermore, such a model might indicate if there are other relevant dynamic phenomena (e.g., inside the compressor) that were not detectable with the used sensors rather than being dominated by the plenum dynamics. Hence, it is worthwhile to investigate the applicability of modular models as proposed in literature by, for example, [Elder and Gill \(1985\)](#); [Botros et al. \(1991\)](#); [Badmus et al. \(1995a\)](#) that might provide more flexibility in describing the layout of industrial scale centrifugal compression systems.

A more sophisticated approach is proposed by [Spakovszky \(2000, 2004\)](#) who developed a modular, two-dimensional, compressible flow model for centrifugal compression systems. Benefits of this approach are the inclusion of compressibility effects and the ca-

pability of this model to describe rotating stall. However, given the level of complexity of this model, significant efforts are required to accurately identify the different model parameters involved.

Another drawback of the model that we applied is that it requires accurate knowledge of the compressor characteristic, including parts in the unstable flow regime. It is worthwhile to investigate ways to describe the compressor characteristic more accurately. The analytical derivation based on energy considerations and compressor geometry as proposed by [Gravdahl and Egeland \(1999a\)](#) and the research by [Meuleman \(2002\)](#) seem good starting points.

In this thesis we proposed a modification of the Greitzer model in order to include the aero-acoustic effects in the discharge piping. With respect to the apparent nonlinear damping of acoustic transients, the selection of boundary conditions, and the possible effect of rotor speed variations further improvements can be made. Since many industrial compression systems are connected to long pipelines it is worthwhile to improve the description of aero-acoustic phenomena in present and future compressor models.

### Identification of model parameters

Next to applying the common approach of parameter tuning, we developed two methods for determining an estimate for the important stability parameter. Unfortunately, we only had a limited set of experimental data to evaluate and compare the two methods for both compressor test rigs. Given the promising results mentioned above, it is worthwhile to carry out a more thorough evaluation with data from additional experiments on the studied test rigs and other (multi-stage) compression systems.

Moreover, the development of the novel control valve actuator makes it possible to conduct forced response experiments with, for example, sinusoidal, pseudo-random binary, or bandlimited white noise excitation signals. This makes it possible to use other, more sophisticated, identification techniques (e.g., [Paduano et al., 1993b, 1994](#)) to determine the dynamics of the compression system.

Finally, when stabilization of surge is achieved for the compression system, it is worthwhile to apply closed-loop system identification techniques to validate the accuracy of the dynamic models for the unstable flow regime of the compression system. Furthermore, closed-loop identification can provide valuable insights into the actual shape of the compressor characteristic to the left of the open-loop surge line.

### Surge control design

The implemented LQG surge controller proved to be inadequate for stabilizing one of the centrifugal compression systems under study. An important step towards success-

ful stabilization of surge in industrial compression systems is to increase the robustness of the surge controller with respect to model uncertainties and process variations. Therefore, it is worthwhile to investigate robust control design techniques, see for example (Badmus et al., 1995a; Eveker et al., 1998; Van de Wal et al., 2002). The research presented here has provided useful qualitative and quantitative information on the type and range of possible uncertainties and disturbances for such an approach.

A topic of special attention is the effect of piping acoustics on the behavior of industrial scale compression systems. The developed acoustic model and possible refinements thereof, make it possible to incorporate aeroacoustic disturbances in the design of stabilizing controllers and to study its effectiveness through closed-loop simulations.

Another problem of the implemented surge controller is that its domain of attraction is too small. Further research is needed to determine adequate methods for increasing the domain of attraction, for example by using linear parameter varying or gain scheduling controllers, or by applying nonlinear control techniques (Gu et al., 1999; Wang et al., 2000; Bøhagen and Gravdahl, 2005; Chaturvedi and Bhat, 2006). Another promising control strategy is hybrid model predictive control (Lazar, 2006) that offers the possibility to use multiple dynamic models for different operating points and explicitly take into account actuator constraints.

With respect to actuator requirements the development of a fast control valve actuator is a valuable technological advancement. The electro-mechanical actuation offers the possibility to further increase the valve capacity and speed by improving the mechanical design of the current prototype.

A serious improvement of the possibilities to model, identify, and control industrial scale compression systems becomes possible when a dynamic mass flow sensor is available. Therefore, we recommend to continue research and development activities in this field. Moreover, in relation to the potentially limiting non-minimum phase character of the used model, it should be investigated if other (combinations of) sensors, see (), can be implemented in industrial practice.

In addition to the application of more advanced control strategies and different sensors, it is worthwhile to investigate the effect of using other actuators (Simon et al., 1993; Van de Wal et al., 2002). In particular we mention the application of air injection that proved to be a successful way of actuation for active surge and rotating stall control (Nelson et al., 2000; Spakovszky, 2004). Hence, the development of air injectors for application in industrial scale compression systems deserves further attention. Another promising development is the concept of utilizing the commonly used electric motor of industrial compressors as an actuator for surge control (Gravdahl et al., 2002; Bøhagen and Gravdahl, 2005).

To conclude, the issues that should receive the highest priority towards the successful stabilization of the investigated compression system are: to resolve the remaining uncertainty in the estimate for the stability parameter and the model-based design of a robust control strategy. Resolving these two aspects makes it possible to make a deliberate choice on the further course of action towards stabilization of an industrial scale centrifugal compression system.



## APPENDIX A

# Fluid dynamics<sup>1</sup>

---

**Abstract** / The integral forms of the laws of conservation of mass and momentum are derived. Criteria are provided for the validity of the main assumptions in the Greitzer model.

### A.1 Conservation laws in integral form

Before we address the conservation laws, we first recall Gauss'<sup>2</sup> theorem that is used to relate surface integrals to volume integrals. We consider a tensor field  $\mathbf{F}$  of any order, the region  $V$  within the closed surface  $A$  and the outward unit normal  $\mathbf{n}$  to this surface. Gauss' theorem states that

$$\int_V \nabla \cdot \mathbf{F} dV = \int_A \mathbf{n} \cdot \mathbf{F} dA \quad (\text{A.1})$$

A proof can be found in numerous textbooks, for example that by Marsden et al. (1993). In the specific case where  $\mathbf{F}$  is a vector field as depicted in Figure A.1, the theorem is usually called the *divergence theorem*.

The conservation laws are based on the principle that the net change of a property within an arbitrary region is equal to the net flux of this property into the region. Mathematically, this is expressed by the following general relation

$$\frac{d}{dt} \int_{V(t)} \mathbf{F}(\mathbf{x}, t) dV = \Upsilon(t) - \int_{B(t)} \mathbf{n}(t) \cdot \mathbf{F}(\mathbf{x}, t) dB \quad (\text{A.2})$$

---

<sup>1</sup>Parts of this appendix are based on the fluid dynamics textbook by Kundu (1990).

<sup>2</sup>Carl Friedrich Gauss (1777–1855) was a great German mathematician and scientist. In 'Theoria attractionis corporum, Sphaeroidicorum Ellipticorum Homogeneorum, methoda nova tractata,' published in 1813, Gauss independently rediscovers the divergence theorem. The theorem is rediscovered again by George Green in 1825 and in 1831 by Mikhail Vasilievich Ostrogradsky who provided the first proof. The theorem was discovered for the first time by Joseph Louis Lagrange in 1762.

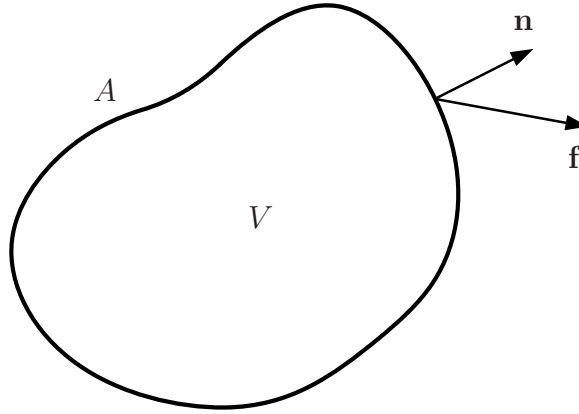


Figure A.1 / Illustration for Gauss' theorem.

with  $\mathbf{F}(\mathbf{x}, t)$  a tensor of any rank,  $V(t)$  an arbitrary region, and  $\Upsilon(t)$  a general source term that describes the generation or annihilation of  $\mathbf{F}(\mathbf{x}, t)$ . Furthermore,  $B(t)$  is the boundary over which  $\mathbf{F}(\mathbf{x}, t)$  can enter or leave the region and the vector  $\mathbf{n}(t)$  is the outward unit normal of this boundary. The conservation principle is illustrated in Figure A.2.

The lefthand side of Equation (A.2) can be rewritten by applying the generalized Leibniz<sup>3</sup> theorem, yielding

$$\frac{d}{dt} \int_{V(t)} \mathbf{F}(\mathbf{x}, t) dV = \int_{V(t)} \frac{\partial \mathbf{F}(\mathbf{x}, t)}{\partial t} dV + \int_{A(t)} \mathbf{F}(\mathbf{x}, t) (\mathbf{n}_A(t) \cdot \mathbf{u}_A(\mathbf{x}, t)) dA \quad (\text{A.3})$$

with  $A(t)$  the boundary surface of the region  $V(t)$  and the vectors  $\mathbf{n}_A(t)$  and  $\mathbf{u}_A(\mathbf{x}, t)$  the unit normal and velocity of this boundary surface, respectively. We point out that in general  $B(t) \subseteq A(t)$ .

For a material volume  $V(t) = \mathcal{V}(t)$  the surface  $\mathcal{A}(t)$  moves with the medium, so that  $\mathbf{u}_A(\mathbf{x}, t) = \mathbf{u}(\mathbf{x}, t)$  and  $\mathcal{B}(t) = \emptyset$ . Substitution in Equations (A.2) and (A.3) gives

$$\frac{d}{dt} \int_{\mathcal{V}(t)} \mathbf{F}(\mathbf{x}, t) d\mathcal{V} = \int_{\mathcal{V}(t)} \frac{\partial \mathbf{F}(\mathbf{x}, t)}{\partial t} d\mathcal{V} + \int_{\mathcal{A}(t)} \mathbf{F}(\mathbf{x}, t) (\mathbf{n}(t) \cdot \mathbf{u}(\mathbf{x}, t)) d\mathcal{A} = \Upsilon(t) \quad (\text{A.4})$$

which is known as the *Reynolds<sup>4</sup> transport theorem*.

<sup>3</sup>Gottfried Wilhelm Leibniz (1646–1716) was a versatile German mathematician, physicist, philosopher and diplomat. In 1684 he published details on his differential calculus in the paper 'Nova methodus pro maximis et minimis'.

<sup>4</sup>Osbourne Reynolds (1842–1912) was an Irish mathematician, physicist, and engineer. In 1883 he published 'An experimental investigation of the circumstances which determine whether the motion of water in parallel channels shall be direct or sinuous and of the law of resistance in parallel channels' in 1833 where he introduced the well-known Reynolds number. The Reynolds transport theorem was most likely named after him in honor of his achievements in fluid dynamics.

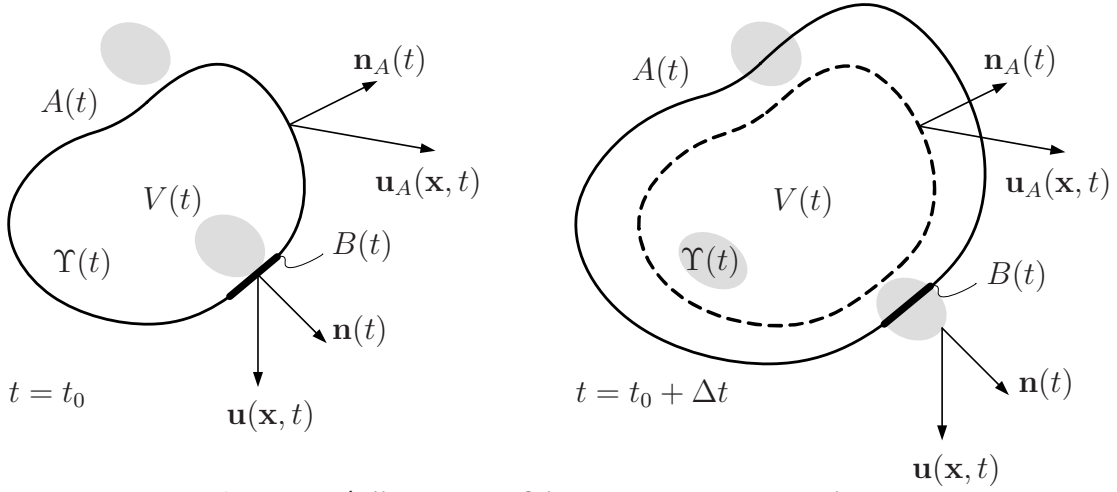


Figure A.2 / Illustration of the conservation principle.

The rate of increase of mass inside a material volume is obtained by substituting  $\mathbf{F}(\mathbf{x}, t) = \rho(\mathbf{x}, t)$  in Equation (A.4), yielding

$$\frac{d}{dt} \int_{\mathcal{V}} \rho d\mathcal{V} = \int_{\mathcal{V}(t)} \frac{\partial \rho}{\partial t} d\mathcal{V} + \int_{\mathcal{A}(t)} \rho(\mathbf{n}(t) \cdot \mathbf{u}(\mathbf{x}, t)) d\mathcal{A} = 0 \quad (\text{A.5})$$

where we have assumed that no mass is generated, hence  $\Upsilon(t) = 0$ . For simplicity we will omit the dependency on  $\mathbf{x}$  and  $t$  in the notation of the variables. The surface integral in Equation (A.5) can be transformed into a volume integral by using the divergence theorem, yielding

$$\int_{\mathcal{V}(t)} \frac{\partial \rho}{\partial t} d\mathcal{V} + \int_{\mathcal{V}(t)} \nabla \cdot (\rho \mathbf{u}) d\mathcal{V} = \int_{\mathcal{V}(t)} \left( \frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \mathbf{u} \right) d\mathcal{V} = 0 \quad (\text{A.6})$$

This equation is the integral form of the law of conservation of mass for a material volume. We now state that an infinitesimal fluid element can be considered as a material volume. Hence, the differential or local form as introduced in Chapter 2 follows directly from Equation (A.6) because the integral holds for *any* material volume. This is only possible when the integrand vanishes at every point, or in other words, when the integrand is equal to zero. In index notation the differential form of the so-called continuity equation reads

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_j)}{\partial x_j} = 0 \quad (\text{A.7})$$

The law of conservation of momentum is derived by replacing  $\mathbf{F}(\mathbf{x}, t)$  with the impulse density  $\rho(\mathbf{x}, t)\mathbf{u}(\mathbf{x}, t)$ . Substitution in the Reynolds transport theorem from Equation (A.4) and using the divergence theorem gives

$$\int_{\mathcal{V}(t)} \left( \frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) \right) d\mathcal{V} = \Upsilon(t) \quad (\text{A.8})$$

where we again omit the dependency on  $\mathbf{x}$  and  $t$  in the notation of the variables. In index notation we easily see that we can rewrite the integrand on the lefthand side of Equation (A.8) as

$$\frac{\partial(\rho u_i)}{\partial t} + \nabla \cdot (\rho u_j u_i) = \rho \frac{\partial u_i}{\partial t} + u_i \frac{\partial \rho}{\partial t} + u_i \frac{\partial(\rho u_j)}{\partial x_j} + \rho u_j \frac{\partial u_i}{\partial x_j} = \rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} \quad (\text{A.9})$$

where we have used Equation (A.7). Note that this result can be written as  $\rho Du_i/Dt$ .

The impulse in  $\mathcal{V}(t)$  changes as a result of a moving boundary. However, the impulse can also be changed by body and surface forces that are working on  $\mathcal{V}(t)$ . Hence,  $\Upsilon(t)$  is in general not equal to zero. By taking gravity (body force) and pressure and stress (surface forces) into account, we obtain the following expression

$$\int_{\mathcal{V}(t)} \rho \frac{D\mathbf{u}}{Dt} = \int_{\mathcal{V}(t)} \rho \mathbf{g} d\mathcal{V} - \int_{\mathcal{A}(t)} p \mathbf{n} d\mathcal{A} + \int_{\mathcal{A}(t)} \mathbf{n} \cdot \mathbf{S} d\mathcal{A} \quad (\text{A.10})$$

where  $\mathbf{g}$  is the acceleration of gravity,  $p$  represents the pressure and  $\mathbf{S}$  denotes the stress tensor. The relation between stress and deformation can be expressed through a so-called constitutive equation. A constitutive equation for a Newtonian fluid, i.e. an isotropic fluid in which stress is linearly related to strain rate and dilatation, is

$$\mathbf{S} = \mu \nabla^2 \mathbf{u} + \left( \kappa + \frac{1}{3} \mu \right) \nabla (\nabla \cdot \mathbf{u}) \quad (\text{A.11})$$

Finally, by applying the divergence theorem we can write Equation (A.10) as

$$\int_{\mathcal{V}(t)} \left( \rho \frac{D\mathbf{u}}{Dt} \right) d\mathcal{V} = \int_{\mathcal{V}(t)} (\rho \mathbf{g} - \nabla p + \nabla \cdot \mathbf{S}) d\mathcal{V} \quad (\text{A.12})$$

which is the integral form of the law of conservation of momentum. We point out that both integrals are carried out over the same material volume, implying that the integrands on the left and righthand side are equal. This equality directly provides the differential form of Equation (A.12). In index notation the differential momentum equation reads

$$\rho \frac{Du_i}{Dt} = \rho g_i - \frac{\partial p}{\partial x_i} + \frac{\partial S_{ji}}{\partial x_j} \quad (\text{A.13})$$

Finally, substitution of Equation (A.11) gives the so-called Navier-Stokes<sup>5</sup> equation that was previously introduced in Chapter 2.

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<sup>5</sup>Claude-Louis Navier (1785–1836) was a French engineer and physicist. Sir George Gabriel Stokes (1819–1903) was an Irish mathematician and physicist. The Navier-Stokes equations are named after these two scientists who made important contributions to the field of fluid dynamics.

## A.2 Speed of sound

Sound can be considered as a small pressure variation that moves through a medium. We will now derive the speed at which these pressure variations or acoustic waves, travel through the medium.

The momentum equation for a pressure variation with infinitesimal amplitude gives

$$\begin{aligned}\rho u du &= -dp \\ \rho u &= -\frac{dp}{du}\end{aligned}\tag{A.14}$$

where we have assumed that the flow within the wave can be considered to be stationary. From the continuity equation we find that  $\rho u = \text{constant}$  and

$$\begin{aligned}d\rho u &= \rho du + u d\rho = 0 \\ du &= -\frac{u}{\rho} d\rho\end{aligned}\tag{A.15}$$

Substitution of Equation (A.15) in (A.14) gives the following expression for the velocity of the acoustic wave

$$u^2 = \frac{dp}{d\rho}\tag{A.16}$$

For an infinitesimal pressure change it is reasonable to assume that this change takes place adiabatically, i.e.  $\delta Q = 0$ . Furthermore, when we assume that viscous friction can be neglected, the pressure change is a reversible process. We now write

$$\begin{aligned}dp &= \left(\frac{\partial p}{\partial \rho}\right)_s d\rho + \left(\frac{\partial p}{\partial s}\right)_\rho ds \\ \frac{dp}{d\rho} &= \left(\frac{\partial p}{\partial \rho}\right)_s\end{aligned}\tag{A.17}$$

where we have used  $ds = 0$  since the process can be considered to be isentropic under the stated assumptions. The subscripts  $s$  and  $\rho$  indicate that the partial derivatives are taken at constant entropy and density, respectively. Combining Equations (A.17) and (A.16) gives the well-known expression for the speed  $c$  of an acoustic wave

$$\left(\frac{\partial p}{\partial \rho}\right)_s \equiv c^2\tag{A.18}$$

In the special case that the medium is an ideal gas, we can further specify the speed of sound by using the equation of state for an ideal gas. From the first law of thermodynamics we find that  $p/\rho^\gamma = \text{constant}$  for an adiabatic, isentropic process in an ideal gas

(e.g. [Shavit and Gutfinger, 1995](#)). Substitution of  $p = a\rho^\gamma$  with  $a$  an arbitrary constant, gives

$$\left(\frac{\partial p}{\partial \rho}\right)_s = \gamma a\rho^{\gamma-1} = \gamma a\rho^\gamma \rho^{-1} = \gamma \frac{p}{\rho}$$

and by using the state equation  $p/\rho = ZRT$  we obtain

$$\left(\frac{\partial p}{\partial \rho}\right)_s \equiv c^2 = \gamma ZRT \quad (\text{A.19})$$

where we have included the compressibility factor  $Z$  to account for small deviations of real gases from the ideal gas law.

### A.3 Incompressible flow assumption

The incompressibility assumption that we have used in large parts of this thesis, is invalid for flows with high Mach numbers. To see this, we consider the one-dimensional continuity equation for a steady flow

$$u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0$$

The incompressibility assumption requires that

$$u \frac{\partial \rho}{\partial x} \ll \rho \frac{\partial u}{\partial x}$$

or that

$$\frac{d\rho}{\rho} \ll \frac{du}{u} \quad (\text{A.20})$$

Pressure changes can be estimated from the relation  $dp \simeq c^2 d\rho$  and from the momentum equation for an inviscid steady flow we find that

$$u du = \frac{dp}{\rho} \quad (\text{A.21})$$

Combining Equation (A.21) with the estimate for  $dp$  gives

$$\frac{d\rho}{\rho} \simeq \frac{u^2}{c^2} \frac{du}{u}$$

We have used  $\simeq$  because the given relations are only approximations when the process is not isentropic. By comparing the result with Equation (A.20) we find that density changes are negligible if

$$\frac{u^2}{c^2} = M^2 \ll 1$$

Hence, the incompressibility assumption is only valid at sufficiently low Mach numbers.

## APPENDIX B

# Aeroacoustics

---

**Abstract /** The linear aeroacoustic wave equation is derived and a distributed parameter model is presented for a general gas transmission line.

### B.1 Aeroacoustic wave equation

The propagation of pressure fluctuations through a medium as sound is described by the wave equation. We will derive the wave equation for a one-dimensional, irrotational and isentropic flow. For the derivation we also assume that the acoustic fluctuations of pressure, density, velocity and entropy are small relative to their undisturbed mean values. Furthermore, we assume that the propagation distances of interest are not too large so it is permissible to neglect attenuation of the sound by viscosity and thermal conduction. Finally, we assume that  $\omega \gg g/c_0$  holds for the wave frequencies so gravitational effects can be neglected, according to [Howe \(1998\)](#).

Under the above assumptions, the one-dimensional laws of conservation of mass, momentum, and energy are

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0 \quad (\text{B.1})$$

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = - \frac{\partial p}{\partial x} \quad (\text{B.2})$$

$$\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} = 0 \quad (\text{B.3})$$

We now consider a moving fluid particle and write

$$dp = (\partial p / \partial \rho)_s d\rho + (\partial p / \partial s)_\rho ds \quad (\text{B.4})$$

Differentiation with respect to time gives

$$\begin{aligned}
 \frac{dp}{dt} &= \left( \frac{\partial p}{\partial \rho} \right)_s \frac{d\rho}{dt} + \left( \frac{\partial p}{\partial s} \right)_\rho \frac{ds}{dt} \\
 &= c^2 \frac{d\rho}{dt} + \left( \frac{\partial p}{\partial s} \right)_\rho \left( \frac{Ds}{Dt} - u \frac{\partial s}{\partial x} \right) \\
 &= c^2 \frac{d\rho}{dt} - u \frac{\partial s}{\partial x} \left( \frac{\partial p}{\partial s} \right)_\rho
 \end{aligned} \tag{B.5}$$

where we have used  $\frac{Ds}{Dt} = 0$ . By dividing Equation (B.4) through  $dx$  and multiplying with  $u$  we obtain

$$u \left( \frac{\partial p}{\partial s} \right)_\rho \frac{\partial s}{\partial x} = u \frac{\partial p}{\partial x} - c^2 u \frac{\partial \rho}{\partial x} \tag{B.6}$$

and after substitution in Equation (B.5) this gives

$$\frac{Dp}{Dt} = c^2 \frac{D\rho}{Dt} \tag{B.7}$$

We now define the following perturbations with respect to the undisturbed mean values

$$\begin{aligned}
 p &= p_0 + \tilde{p} \\
 \rho &= \rho_0 + \tilde{\rho} \\
 c &= c_0 + \tilde{c} \\
 u &= \tilde{u}
 \end{aligned}$$

where we have assumed that the undisturbed medium is at rest, i.e.  $u_0 = 0$ . Substitution in the laws of conservation of mass and momentum, applying a Taylor<sup>1</sup> series expansion and neglecting second and higher order terms, yields

$$\frac{1}{\rho_0} \frac{\partial \tilde{p}}{\partial t} = - \frac{\partial \tilde{u}}{\partial x} \tag{B.8}$$

$$\rho_0 \frac{\partial \tilde{u}}{\partial t} = - \frac{\partial \tilde{p}}{\partial x} \tag{B.9}$$

$$\frac{\partial \tilde{p}}{\partial t} = c_0^2 \frac{\partial \tilde{\rho}}{\partial t} \tag{B.10}$$

and time integration of Equation (B.10) gives the linearized constitutive relation  $\tilde{p} = c_0^2 \tilde{\rho}$ . By subtracting the divergence of Equation (B.9) from the time derivative of Equation (B.8) we can eliminate the velocity, yielding

$$\frac{\partial^2 \tilde{\rho}}{\partial t^2} - \nabla \tilde{p} = 0 \tag{B.11}$$

---

<sup>1</sup>Brook Taylor (1685–1731) was an English mathematician. In 1712 he formulated the famous theorem for approximating a differentiable function near a point, published in his work *Methodus Incrementorum Directa et Inversa* in 1715.

and by using the obtained constitutive relation we obtain the following linear, one-dimensional wave equation for acoustic pressure perturbations

$$\frac{\partial^2 \tilde{p}}{\partial t^2} = c_0^2 \nabla^2 \tilde{p} \quad (\text{B.12})$$

A general solution for the above wave equations was found by d'Alembert.<sup>2</sup> The plane wave solution of Equation (B.12) for a pressure wave, propagating through a stationary medium, reads

$$\tilde{p}(x, t) = p_+(x - c_0 t) + p_-(x + c_0 t) \quad (\text{B.13})$$

where  $p_{\pm}$  denote the pressure waves propagating at speeds  $c_0$  in the positive and negative direction, respectively. The solution for a velocity wave is given by

$$\tilde{u}(x, t) = \frac{1}{\rho_0 c_0} p_+(x - c_0 t) - \frac{1}{\rho_0 c_0} p_-(x + c_0 t) \quad (\text{B.14})$$

The term  $\rho_0 c_0$  is called the specific acoustic impedance of the medium, usually denoted by  $z_0$ . The acoustic impedance  $Z_0$  of a filled pipe is given by  $Z_0 = z_0/A$  where  $A$  denotes the cross-sectional area of the pipe.

## B.2 Transmission line model

A characteristic of a general transmission line is that its inertial, capacitive, and resistive properties are distributed along the length of the line. We will now present the derivation of a distributed parameter model for the transmission line depicted in Figure B.1 that describes the velocity distribution and states of a gas inside the line. As in the previous section, we assume that the flow can be considered to be one-dimensional, irrotational, and isentropic.

The starting point for the derivation are the one-dimensional laws of conservation of mass, momentum, and energy that were introduced before

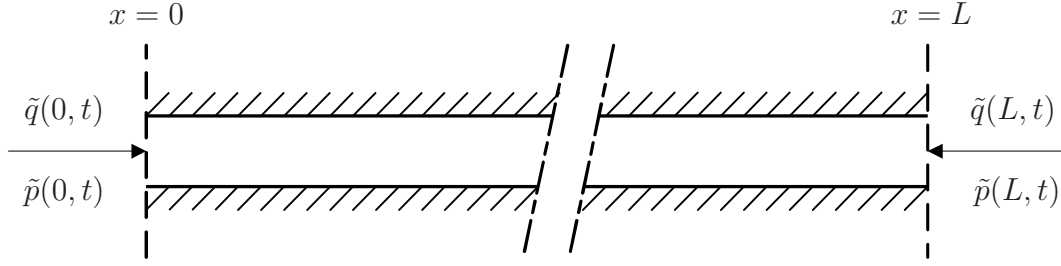
$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0 \quad (\text{B.1})$$

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} - Ru \quad (\text{B.15})$$

$$\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} = 0 \quad (\text{B.3})$$

---

<sup>2</sup>Jean Le Rond d'Alembert (1717–1783) was a French mathematician, physicist, and philosopher. He made a great contribution to the theory on partial differential equations and he is said to be the first to formulate the fundamental theorem of algebra.



**Figure B.1** / Transmission line with general boundary conditions.

However, in the impulse balance an acoustic resistance term  $Ru$  has been added that represents the damping effects due to the viscosity of the gaseous medium, see (Stecki and Davis, 1986; Whitmore, 1988). Following the same procedure as in the previous section, we obtain the following linearized mass and momentum balance

$$\frac{1}{\rho_0} \frac{\partial \tilde{p}}{\partial t} = - \frac{\partial \tilde{u}}{\partial x} \quad (\text{B.8})$$

$$\rho_0 \frac{\partial \tilde{u}}{\partial t} = - \frac{\partial \tilde{p}}{\partial x} - R \tilde{u} \quad (\text{B.I6})$$

$$\frac{\partial \tilde{p}}{\partial t} = c_0^2 \frac{\partial \tilde{p}}{\partial x} \quad (\text{B.I0})$$

and the corresponding wave equation

$$\frac{\partial^2 \tilde{p}(x, t)}{\partial t^2} + \frac{R}{\rho_0} \frac{\partial \tilde{p}(x, t)}{\partial t} = c_0^2 \frac{\partial^2 \tilde{p}(x, t)}{\partial x^2} \quad (\text{B.I7})$$

where we have explicitly noted that the pressure perturbations are a function of both position and time. After specifying boundary and initial conditions and determining values for  $R$  and  $c_0$ , the above wave equation has to be solved in order to completely specify the system.

In general, for the case of arbitrary inputs the boundary value problem cannot be solved analytically. However, for the case of sinusoidal inputs a closed-form solution can be obtained by taking the Laplace transform of Equation (B.I7). When we assume that the system is initially at rest, i.e.  $q(0) = q_0$  and  $p(0) = p_0$ , we obtain

$$\left( s^2 + \frac{R}{\rho_0} s \right) P(x, s) = c_0^2 \frac{\partial^2 P(x, s)}{\partial x^2} \quad (\text{B.I8})$$

The roots of the above equation are given by  $\pm \frac{s\sqrt{\alpha}}{c_0}$  with  $\alpha = 1 + \frac{R}{\rho_0 s}$ . For notational simplicity we omit the dependency on  $s$  of the frequency-dependent friction factor  $\alpha$ .

Integrating Equation (B.I8) with respect to  $x$  now gives

$$P(x, s) = A(s) e^{s(\sqrt{\alpha}/c_0)x} + B(s) e^{-s(\sqrt{\alpha}/c_0)x} \quad (\text{B.I9})$$

and differentiating Equation (B.19)

$$\frac{\partial P(x, s)}{\partial x} = \frac{s\sqrt{\alpha}}{c_0} \left( A(s)e^{s(\sqrt{\alpha}/c_0)x} - B(s)e^{-s(\sqrt{\alpha}/c_0)x} \right) \quad (\text{B.20})$$

Substitution of  $\tilde{u}(x, t) = \tilde{q}(x, t)/A$  in Equation (B.16) and taking the Laplace transform gives, after some manipulations

$$\frac{\rho_0 s}{A} \left( 1 + \frac{R}{\rho_0 s} \right) Q(x, s) = -\frac{\partial P(x, s)}{\partial x} \quad (\text{B.21})$$

where the term  $\rho_0 s \alpha / A$  represents the series impedance per unit length, see for example (Stecki and Davis, 1986).

By using Equation (B.21) we can rewrite Equation (B.20), yielding

$$Q(x, s) = -\frac{1}{Z_0 \sqrt{\alpha}} \left( A(s)e^{s(\sqrt{\alpha}/c_0)x} - B(s)e^{-s(\sqrt{\alpha}/c_0)x} \right) \quad (\text{B.22})$$

The integration constants  $A(s)$  and  $B(s)$  are now obtained by evaluating Equations (B.19) and (B.22) at  $x = 0$ , using the boundary conditions from Figure B.1, yielding

$$\begin{aligned} A(s) &= \frac{1}{2}P(0, s) - \frac{Z_0 \sqrt{\alpha}}{2}Q(0, s) \\ B(s) &= \frac{1}{2}P(0, s) + \frac{Z_0 \sqrt{\alpha}}{2}Q(0, s) \end{aligned}$$

The equations that describe pressure and flow within the transmission line now become

$$P(x, s) = \cosh(s(\sqrt{\alpha}/c_0)x) P(0, s) - Z_0 \sqrt{\alpha} \sinh(s(\sqrt{\alpha}/c_0)x) Q(0, s) \quad (\text{B.23})$$

$$Q(x, s) = \cosh(s(\sqrt{\alpha}/c_0)x) Q(0, s) - \frac{1}{Z_0 \sqrt{\alpha}} \sinh(s(\sqrt{\alpha}/c_0)x) P(0, s) \quad (\text{B.24})$$

Finally, the governing equations describing the transmission line dynamics can be written in the following transfer function matrix form

$$\begin{bmatrix} H_{T,11} & H_{T,12} \\ H_{T,21} & H_{T,22} \end{bmatrix} \begin{bmatrix} Q(0, s) \\ P(0, s) \end{bmatrix} = \begin{bmatrix} -Q(L, s) \\ P(L, s) \end{bmatrix} \quad (\text{B.25})$$

The transfer functions are given by

$$H_{T,11} = \cosh(\tau s \sqrt{\alpha}) \quad (\text{B.26})$$

$$H_{T,12} = -\frac{1}{Z_0 \sqrt{\alpha}} \sinh(\tau s \sqrt{\alpha}) \quad (\text{B.27})$$

$$H_{T,21} = -Z_0 \sqrt{\alpha} \sinh(\tau s \sqrt{\alpha}) \quad (\text{B.28})$$

$$H_{T,22} = \cosh(\tau s \sqrt{\alpha}) \quad (\text{B.29})$$

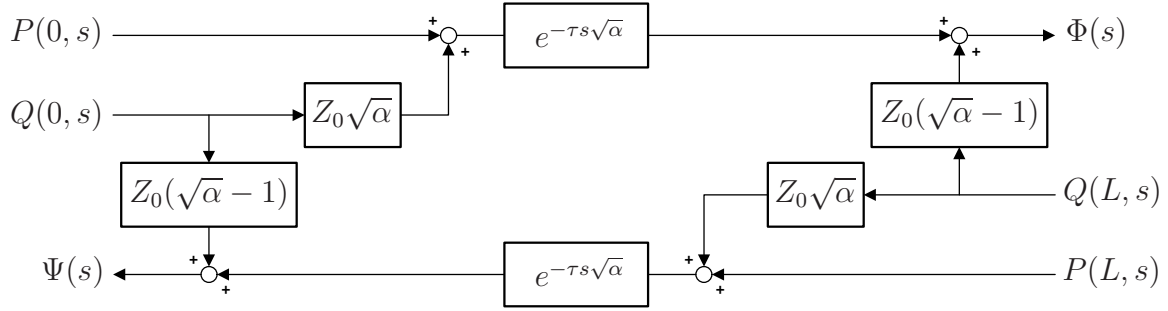


Figure B.2 / Block diagram of transmission line model.

with  $\tau = L/c_0$  denoting the time needed for a wave to travel along the line. We now introduce the characteristic variables  $\Phi(s)$  and  $\Psi(s)$  such that

$$P(L, s) = \Phi(s) + Z_0 Q(L, s) \quad (\text{B.30})$$

$$P(0, s) = \Psi(s) + Z_0 Q(0, s) \quad (\text{B.31})$$

where  $\Phi(s)$  corresponds to the right going wave and  $\Psi(s)$  to the left going wave. In order to obtain an expression for these characteristic variables we first evaluate Equations (B.23) and (B.24) at  $x = L$ . After some manipulations and multiplying by 2 we then obtain

$$\left( e^{\tau s \sqrt{\alpha}} + e^{-\tau s \sqrt{\alpha}} \right) P(0, s) = 2P(L, s) + Z_0 \sqrt{\alpha} Q(0, s) \left( e^{\tau s \sqrt{\alpha}} - e^{-\tau s \sqrt{\alpha}} \right) \quad (\text{B.32})$$

$$\left( e^{\tau s \sqrt{\alpha}} - e^{-\tau s \sqrt{\alpha}} \right) P(0, s) = 2Z_0 \sqrt{\alpha} Q(L, s) + Z_0 \sqrt{\alpha} Q(0, s) \left( e^{\tau s \sqrt{\alpha}} + e^{-\tau s \sqrt{\alpha}} \right) \quad (\text{B.33})$$

Subtracting/adding Equation (B.33) from/to Equation (B.32) yields

$$P(L, s) - Z_0 \sqrt{\alpha} Q(L, s) = \left( P(0, s) + Z_0 \sqrt{\alpha} Q(0, s) \right) e^{-\tau s \sqrt{\alpha}}$$

$$P(0, s) - Z_0 \sqrt{\alpha} Q(0, s) = \left( P(L, s) + Z_0 \sqrt{\alpha} Q(L, s) \right) e^{-\tau s \sqrt{\alpha}}$$

and substitution in Equations (B.30) and (B.31) finally gives

$$\Phi(s) = \left( P(0, s) + Z_0 \sqrt{\alpha(s)} Q(0, s) \right) e^{-\tau s \sqrt{\alpha(s)}} + Z_0 \left( \sqrt{\alpha(s)} - 1 \right) Q(L, s) \quad (\text{B.34})$$

$$\Psi(s) = \left( P(L, s) + Z_0 \sqrt{\alpha(s)} Q(L, s) \right) e^{-\tau s \sqrt{\alpha(s)}} + Z_0 \left( \sqrt{\alpha(s)} - 1 \right) Q(0, s) \quad (\text{B.35})$$

The block diagram of the transmission line model is shown in Figure B.2.

# Approximations for compressor and valve characteristics

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**Abstract** / This appendix provides details on the applied curve fitting to obtain approximations for the compressor characteristics of the investigated test rigs. Furthermore, details on the used valve flow equations are given.

## C.1 Approximation of compressor characteristics

### C.1.1 Compressor characteristics for test rig A

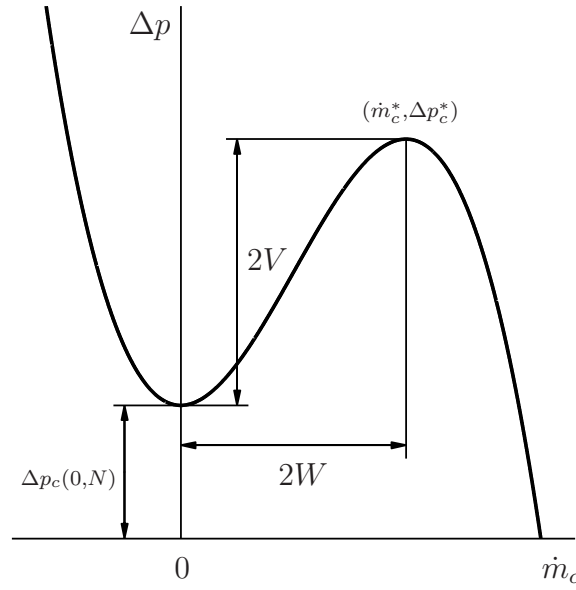
As mentioned in Chapter 3, we selected the cubic polynomial from [Moore and Greitzer \(1986\)](#) to describe the term  $\Delta p_c$  or  $\Psi_c$  in the model for test rig A. The fitting procedure for the polynomial approximations of the compressor characteristic was carried out in full dimensions and the result was scaled afterwards. In full dimensional form the used cubic polynomial is of the following form

$$\Delta p_c(\dot{m}_c, N) = \Delta p_c(0, N) + H(N) \left[ 1 + \frac{3}{2} \left( \frac{\dot{m}_c}{W(N)} - 1 \right) - \frac{1}{2} \left( \frac{\dot{m}_c}{W(N)} - 1 \right)^3 \right] \quad (\text{C.1})$$

The parameters  $\Delta p_c(0, N)$ ,  $H$ , and  $W$  are defined in Figure [C.1](#). Note that we explicitly take into account the dependency of the parameters on the rotational speed of the compressor.

To approximate the measured steady-state data with Equation (C.1), we rewrite the cubic polynomial as

$$\Delta p_c(\dot{m}_c, N) = a_0(N) + a_2(N)\dot{m}_c^2 + a_3(N)\dot{m}_c^3 \quad (\text{C.2})$$



**Figure C.1** / Definitions of parameters in the cubic polynomial;  $(\dot{m}_c^*, \Delta p_c^*)$  = surge point.

with

$$a_0(N) = \Delta p_c(0) \quad (\text{C.3})$$

$$a_2(N) = \frac{3H(N)}{2W^2(N)} \quad (\text{C.4})$$

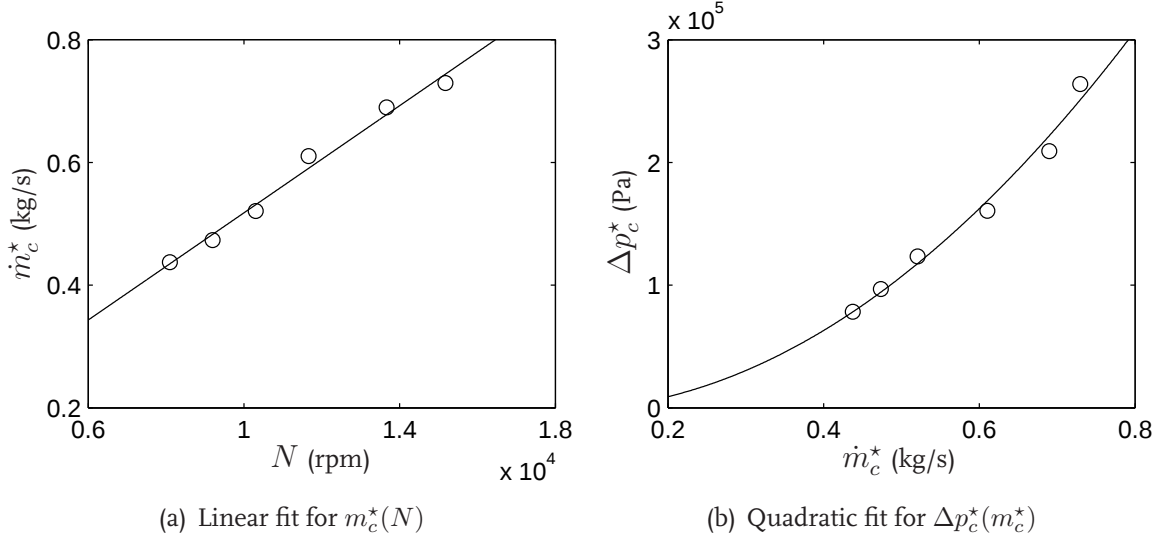
$$a_3(N) = \frac{-H(N)}{2W^3(N)} \quad (\text{C.5})$$

This equation allowed us to determine the parameters  $a_0(N)$ ,  $a_2(N)$ , and  $a_3(N)$  for each compressor characteristic or speed line in a least squares (LS) sense. Subsequently, the dependency of these parameters on the rotational speed was approximated by quadratic polynomials in  $N$ . However, applying the above procedure to the measured speed lines, which are rather steep near the surge line, resulted in characteristics with a non-positive slope for all  $\dot{m}_c$ . This would imply that the compression system would be stable for any given mass flow, which is obviously not the case.

In order to solve this problem, we constructed two separate fits for the stable and unstable parts of each speed line. For this purpose, we first approximated the surge line as function of  $\dot{m}_c$  and  $N$  by determining a linear LS fit of the form  $\dot{m}_c^* = a_0 + a_1 N$  and subsequently a quadratic LS fit of the form  $\Delta p^* = a_0 + a_1 \dot{m}_c^* + a_2 (\dot{m}_c^*)^2$ , see also Figure C.2.

Secondly, to assure that the curves for the unstable and stable parts line up exactly at the point  $(\dot{m}_c^*, \Delta p^*)$ , a constraint was introduced for Equation (C.2), yielding

$$\Delta p_c - \Delta p_c^* = a_2 [\dot{m}_c^2 - (\dot{m}_c^*)^2] + a_3 [\dot{m}_c^3 - (\dot{m}_c^*)^3] \quad (\text{C.6})$$



**Figure C.2** / Polynomial approximation of surge line as function of  $N$  and  $\dot{m}_c$ .

where we left out the dependency on  $N$  in the notation for clarity. The parameters  $a_2(N)$  and  $a_3(N)$  for the stable part of each compressor characteristic were obtained from LS fits of Equation (C.6) on the available steady-state data.

We remark that we have steady-state data available for six different compressor speeds. However, during the measurement of an individual speed line, the rotational speed increased slightly while throttling down towards the surge point so each measurement point was measured at a slightly different rotational speed. To take this variation into account, we repeated the LS fits with different values for  $\dot{m}_c^*(N_i)$  and  $\Delta p_c^*(N_i)$ , corresponding to the different speeds  $N_i$  encountered on each characteristic.

Finally, the obtained values for  $a_2(N)$  and  $a_3(N)$  were fitted with a quadratic polynomial in  $N$  as discussed before. Together with the expression

$$a_0(N) = \Delta p_c^*(N) - a_2(N)\dot{m}_c^*(N) - a_3(N)(\dot{m}_c^*(N))^2 \quad (\text{C.7})$$

these polynomials provide an analytical expression for the stable part of the compressor characteristic as a function of  $N$ .

The approximation in the unstable flow regime was obtained by defining the parameters  $\Delta p_c(0, N)$ ,  $W(N)$  and  $H(N)$  in Equation (3.7) directly. According to [Gravdahl and Egeland \(1999b\)](#) and [Meuleman \(2002\)](#) the valley point of the curve  $\Delta p_c(0, N)$  can be calculated from

$$\Delta p_c(0, N) = \frac{\left( \left( 1 + \frac{\pi^2 (N/60)^2 (d_e^2 - d_i^2)}{2c_p T_1} \right)^{\frac{\gamma}{\gamma-1}} - 1 \right) p_1}{\frac{1}{2} \rho_1 U^2} \quad (\text{C.8})$$

where  $c_p$  is the specific heat at constant pressure and  $d_i^2 = \frac{1}{2}(d_{i,h}^2 + d_{i,c}^2)$  is the average compressor inlet diameter. The dependency of  $\Delta p_c(0)$  on rotational speed was also

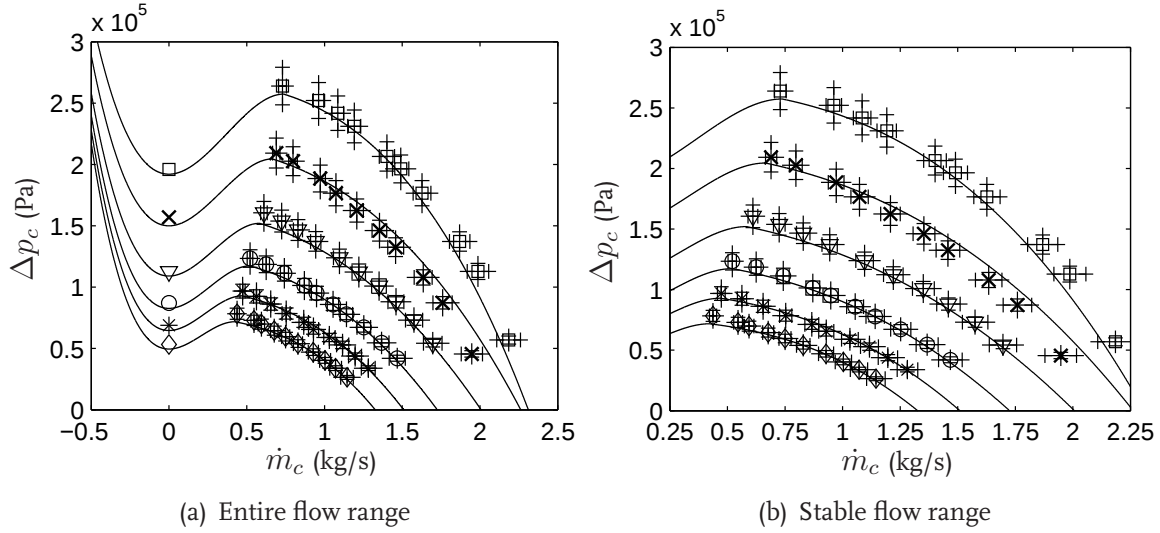
**Table C.1** / Polynomials for compressor map approximation of test rig A.

| Curve    | Polynomials                                                                                                                                                                                   |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Surge    | $\dot{m}_c^*(N) = 8.116939252 \cdot 10^{-2} + 4.367405702 \cdot 10^{-5} N$<br>$\Delta p_c^*(N) = 5.379284127 \cdot 10^2 - 7.065601955 \cdot 10^4 N + 5.673752535 \cdot 10^5 N^2$              |
| Stable   | $a_2(N) = 1.500080502 \cdot 10^5 - 3.218818228 \cdot 10^1 N + 1.452772167 \cdot 10^{-3} N^2$<br>$a_3(N) = -1.607114154 \cdot 10^5 + 2.656858542 \cdot 10^1 N - 1.141629160 \cdot 10^{-3} N^2$ |
| Unstable | $\Delta p_c^*(0, N) = 1.231229177 \cdot 10^4 - 2.982128665 N + 9.932797542 \cdot 10^{-4} N^2$                                                                                                 |

approximated by a quadratic polynomial in  $N$ . We point out that in some cases the calculated value for  $\Delta p_c(0)$  has been changed prior to fitting the polynomial, see Section 3.4.1. The parameters  $W$  and  $H$  were obtained from the surge points  $(\dot{m}_c^*(N), \Delta p_c^*(N))$  at each rotational speed, see also Figure C.1.

Note that the above procedure resulted in piecewise continuous curves for each rotational speed, consisting of a part for  $\dot{m}_c \leq \dot{m}_c^*$  and a part for  $\dot{m}_c > \dot{m}_c^*$ . However, the slope of the curve for unstable flows is zero at the surge point while the curve for stable flows always has a non-positive slope due to the trend in the measured data. Therefore, a discontinuity exists in the derivative of the curves at the surge point. All the relevant polynomial coefficients are provided in Table C.1.

The resulting compressor characteristics are shown in Figure C.3 together with the measured steady-state data and their associated uncertainty. Differences between data and approximations, in particular around the top of the curves, are caused by the speed variations encountered during the measurements on the individual speed lines. However, it can be seen that these variations are well below the uncertainty levels for the mass flow and pressure measurements.



**Figure C.3** / Data with associated uncertainty and approximation for the compressor map of test rig A;  $\diamond$  7,806 rpm,  $*$  8,914 rpm,  $\circ$  10,052 rpm,  $\nabla$  11,484 rpm,  $\times$  13,369 rpm,  $\square$  15,038 rpm,  $*$  = points on surge line,  $-$  = polynomial approximation.

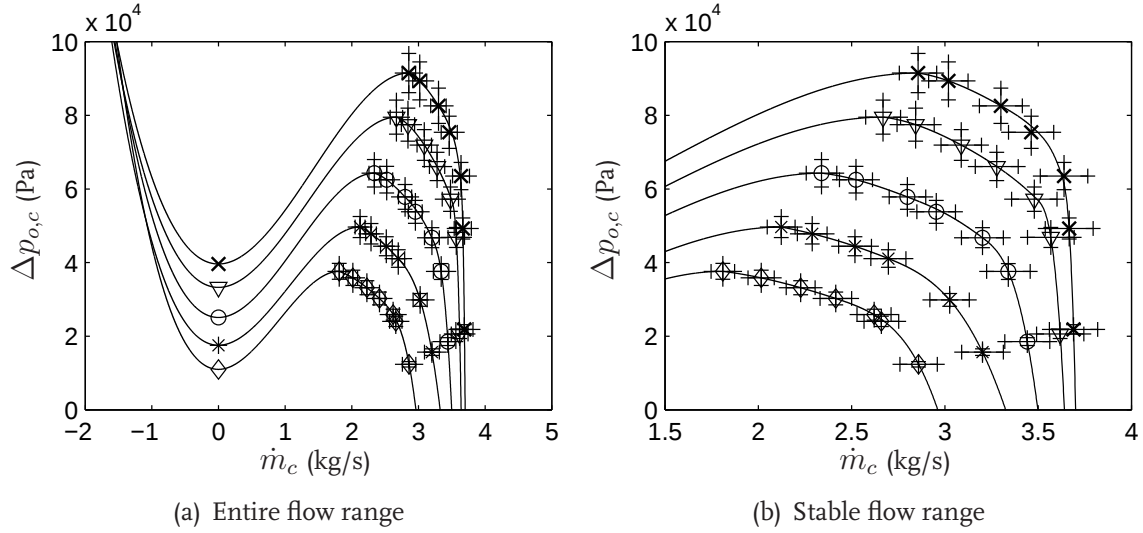
### C.1.2 Compressor characteristics for test rig B

For the approximation of the compressor characteristic for test rig B a similar procedure was followed. However, as mentioned in Section 4.2.2, the stable parts of the compressor curves were approximated with spline functions. To assure spline approximations with a monotonically decreasing second derivative, we manually shifted some of the measured points. These adjustments ( $< 0.1\%$ ) were all well below the uncertainty level of the measurements.

The resulting compressor characteristics are shown in Figure C.4 together with the steady-state data and their associated uncertainty. The adjusted data points are easily identified since they are not located exactly on the line that represents the spline approximation.

## C.2 Throttle characteristics

As mentioned in Chapter 3 we chose to express the throttle and control valve characteristics in accordance with the latest version of the industrial standard IEC 60534-2-1 (1998). For turbulent gaseous flow through a valve, the following expressions for the mass flow rate through the valve are given below.



**Figure C.4** / Data with associated uncertainty and approximation for the compressor map of test rig B;  $\diamond$  13,783 rpm,  $*$  15,723 rpm,  $\circ$  17,637 rpm,  $\nabla$  19,512 rpm,  $\square$  20,855 rpm,  $-$  = spline-polynomial approximation.

For normal (non-choked) flow:

$$\dot{m}_t = 0.0316 K_t(u_t) Y \sqrt{\rho_2 \Delta p} \quad \text{when} \quad \frac{\Delta p}{p_2} < \frac{\gamma_2}{1.40} x_T \quad (\text{C.9})$$

and for choked flow

$$\dot{m}_t = 0.667 \cdot 0.0316 \sqrt{\frac{\gamma_2}{1.40} x_T \rho_2 p_2} \quad \text{when} \quad \frac{\Delta p}{p_2} \geq \frac{\gamma_2}{1.40} x_T \quad (\text{C.10})$$

where  $K_t(u_t)$  denotes the valve flow coefficient,  $\Delta p = p_2 - p_1$ , and the subscript  $_2$  denotes the conditions upstream of the valve. The numerical value 0.0316 (m/s) represents an empirical constant.

The parameter  $Y$  is the so-called expansion factor that accounts for the change in density as the medium passes from the valve inlet to the *vena contracta*<sup>1</sup>. According to IEC 60534-2-1 (1998) the expansion factor can be calculated from

$$Y = 1 - \frac{\Delta p}{3 \frac{\gamma_2}{1.40} x_T p_2} \quad (\text{C.11})$$

For choked flow  $Y = 0.667$  and this value is used in Equation (C.10). From this equation it also becomes clear that, regardless of any further reduction in downstream pressure, the mass flow cannot exceed a certain maximum value once choked conditions are reached.

<sup>1</sup>The *vena contracta* is the location just downstream of the valve orifice where the jet stream area is minimal.

Choking can play a role in determining the correct capacity of a control valve, in particular when the valve is installed in such a way that the downstream pressure is not constant. A physical explanation of the choking phenomenon can be found in textbooks (e.g. Kundu, 1990) that deal with compressible flows.

The parameter  $x_T$ , which is a function of the valve opening  $u_t$ , is also related to the occurrence of choking. The so-called pressure differential ratio factor defines the pressure differential ratio ( $\Delta p/p_2$ ) at which choking occurs. This value is usually available from the valve manufacturer or it can be determined experimentally.

### C.2.1 Valve data

Here we provide the flow characteristics data for the various valves in the investigated compressor test rigs. The valve installed in test rig A was a DN20 Masoneilan Varipack 28000 control valve. The available flow characteristics data for the installed configuration are given in Table C.2. In test rig B both the auxiliary throttle and the valve body of the control valve prototype were so-called sliding gate valves. The data of the DN50 Schubert & Salzer GS8036 auxiliary throttle are given in Table C.3. Finally, the flow characteristics data of the DN50 GS8036 Schubert & Salzer control valve body are given in Table C.4.

**Table C.2** / Flow characteristics of Varipack 28000 control valve.

| Valve opening (%) | $K_r$ m <sup>3</sup> /h | $x_{T,r}$ (-) |
|-------------------|-------------------------|---------------|
| 100               | 1.99                    | 0.64          |

**Table C.3** / Flow characteristics of GS8o36 auxiliary throttle valve.

| Valve opening (%) | $K_a \text{ m}^3/\text{h}$ | $x_{T,a} (-)$ |
|-------------------|----------------------------|---------------|
| 0                 | 0                          | 0.816         |
| 5                 | 0.678273522                | 0.798         |
| 10                | 1.561786007                | 0.780         |
| 15                | 2.577361176                | 0.761         |
| 20                | 3.678030218                | 0.742         |
| 25                | 4.83863841                 | 0.722         |
| 30                | 6.051451719                | 0.702         |
| 35                | 7.321763414                | 0.682         |
| 40                | 8.66350068                 | 0.661         |
| 45                | 10.09483122                | 0.640         |
| 50                | 11.63376988                | 0.618         |
| 55                | 13.29378522                | 0.596         |
| 60                | 15.0794062                 | 0.574         |
| 65                | 16.9818287                 | 0.551         |
| 70                | 18.97452219                | 0.528         |
| 75                | 21.00883632                | 0.504         |
| 80                | 23.00960754                | 0.480         |
| 85                | 24.87076569                | 0.455         |
| 90                | 26.45094062                | 0.430         |
| 95                | 27.56906882                | 0.405         |
| 100               | 28                         | 0.379         |

**Table C.4** / Flow characteristics of GS8036 control valve.

| Valve opening (%) | $K_r \text{ m}^3/\text{h}$ | $x_{T,r} (-)$ |
|-------------------|----------------------------|---------------|
| 0                 | 0                          | 0.790         |
| 5                 | 0.988650964                | 0.775         |
| 10                | 2.213128911                | 0.759         |
| 15                | 3.57521685                 | 0.740         |
| 20                | 5.020496294                | 0.720         |
| 25                | 6.530604258                | 0.698         |
| 30                | 8.115490259                | 0.674         |
| 35                | 9.805673321                | 0.649         |
| 40                | 11.64449897                | 0.622         |
| 45                | 13.68039625                | 0.593         |
| 50                | 15.95913469                | 0.562         |
| 55                | 18.51608134                | 0.529         |
| 60                | 21.36845776                | 0.495         |
| 65                | 24.50759702                | 0.459         |
| 70                | 27.89120069                | 0.421         |
| 75                | 31.43559586                | 0.381         |
| 80                | 35.00799212                | 0.340         |
| 85                | 38.41873859                | 0.296         |
| 90                | 41.41358088                | 0.251         |
| 95                | 43.66591813                | 0.205         |
| 100               | 44.76906                   | 0.156         |



# Bibliography

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- ALBERTSON, F., BODÉN, H., and GILBERT, J. (2006). Comparison of different methods to couple non-linear source descriptions in the time domain to linear system descriptions in the frequency domain—Application to a simple valveless one-cylinder cold engine. *J. Sound and Vibration*, **291**, 963–985.
- ANDERSON, L. (2005). Compressing the Yamal pipeline. *World Pipelines*, February, 53–56.
- ARNULFI, G. L., GIANNATTASIO, P., GIUSTO, C., MASSARDO, A. F., MICHELI, D., and PINAMONTI, P. (1999). Multistage centrifugal compressor surge analysis: Part II—Numerical simulation and dynamic control parameter evaluation. *ASME J. Turbomachinery*, **121**, 312–320.
- ARNULFI, G. L., GIANNATTASIO, P., MICHELI, D., and PINAMONTI, P. (2001). An innovative device for passive control of surge in industrial compression systems. *ASME J. Turbomachinery*, **123**, 473–482.
- ARNULFI, G. L., BLANCHINI, F., GIANNATTASIO, P., MICHELI, D., and PINAMONTI, P. (2006). Extensive study on the control of centrifugal compressor surge. *Proc. IMechE, Part A: J. Power and Energy*, **220**, 289–304.
- ÅSTRÖM, K. J. and EYKHOFF, P. (1971). System identification—A survey. *Automatica*, **7**, 123–162.
- BADMUS, O. O., EVEKER, K. M., and NETT, C. N. (1995a). Control-oriented high-frequency turbomachinery modeling: General one-dimensional model development. *ASME J. Turbomachinery*, **117**, 320–335.
- BADMUS, O. O., CHOWDHURY, S., EVEKER, K. M., and NETT, C. N. (1995b). Control-oriented high-frequency turbomachinery modeling: Single-stage compression system one-dimensional model. *ASME J. Turbomachinery*, **117**, 47–61.
- BADMUS, O. O., CHOWDHURY, S., and NETT, C. N. (1996). Nonlinear control of surge in axial compression systems. *Automatica*, **32**, 59–70.
- BAEHR, H. D. and SCHWIER, K. (1961). *Die thermodynamischen Eigenschaften der Luft*. Springer-Verlag, Berlin.
- BLANCHINI, F., GIANNATTASIO, P., MICHELI, D., and PINAMONTI, P. (2002). Experimental evaluation of a high-gain controller for compressor surge suppression. *ASME J. Turbomachinery*, **124**, 27–35.
- BØHAGEN, B. and GRAVDAHL, J. T. (2005). Active control of compression systems using drive torque; a backstepping approach. In *Proc. 44th IEEE Conf. on Decision and Control*, 2493–2498, Seville, Spain.
- BOTROS, K. K. (1994a). Developments in centrifugal compressor surge control—A technology assessment. *ASME J. Turbomachinery*, **116**, 240–249.
- BOTROS, K. K. (1994b). Transient phenomena in compressor stations during surge. *ASME J. Engineering for Gas Turbines and Power*, **116**, 133–142.
- BOTROS, K. K., CAMPBELL, P. J., and MAH, B. B. (1991). Dynamic simulation of compressor station operation including centrifugal compressor and gas turbine. *ASME J. Engineering for Gas Turbines and Power*, **113**, 300–311.
- BOX, G. E. P. and JENKINS, G. M. (1970). *Time Series Analysis, forecasting and control*. Holden-Day, San Francisco.
- BOYD, S. and BARRAT, C. (1991). *Linear Controller Design: Limits of Performance*. Prentice-Hall, Englewood Cliffs.

- VAN DEN BREMER, W. A. P., MARECHAL, T. M. A., and VUGTS, R. P. A. (2006). Linear identification of nonlinear systems. Technical Report DCT 2006.35, Technische Universiteit Eindhoven.
- CARGILL, A. M. and FREEMAN, C. (1991). High-speed compressor surge with application to active control. *ASME J. Turbomachinery*, **113**, 303–311.
- CHATURVEDI, N. A. and BHAT, S. P. (2006). Output-feedback semiglobal stabilization of stall dynamics for preventing hysteresis and surge in axial-flow compressors. *IEEE Trans. Control Systems Technology*, **14**, 301–307.
- CHELLINI, R. (2002). Centrifugal compressors designed for LNG unloading terminals. *Compressor Tech Two*, July-August, 56–58.
- CHERNYSHENKO, S. I. and PRIVALOV, A. V. (2004). Internal degrees of freedom of an actuator disk model. *J. Propulsion and Power*, **20**, 155–163.
- COHEN, H., ROGERS, G. F. C., and SARAVANAMUTTOO, H. I. H. (1996). *Gas Turbine Theory*. Addison Wesley Longman Limited, Harlow, fourth edition.
- CUMPSTY, N. A. (1989). *Compressor Aerodynamics*. Longman Scientific & Technical, New York.
- CUMPSTY, N. A. and GREITZER, E. M. (2004). Ideas and methods of turbomachinery aerodynamics: A historical view. *J. Propulsion and Power*, **20**, 15–26.
- DAVIS, R. S. (1992). Equation for the determination of the density of moist air (1981/91). *Metrologia*, **29**, 67–70.
- DAY, I. J. (1993). Active suppression of rotating stall and surge in axial compressors. *ASME J. Turbomachinery*, **115**, 40–47.
- DAY, I. J. (1994). Axial compressor performance during surge. *J. Propulsion and Power*, **10**, 329–336.
- DAY, I. J. and FREEMAN, C. (1994). The unstable behavior of low and high-speed compressors. *ASME J. Turbomachinery*, **116**, 194–201.
- DOMMEL, H. W. (1969). Digital computer solution of electromagnetic transients in single- and multiphase networks. *IEEE Trans. Power Apparatus and Systems*, **88**, 388–399.
- DREMIN, I. M., FURLETOV, V. I., IVANOV, O. V., NECHITAIOLO, V. A., and TERZIEV, V. G. (2002). Precursors of stall and surge processes in gas turbines revealed by wavelet analysis. *Control Engineering Practice*, **10**, 599–604.
- DUKLE, N. and NARAYANAN, K. (2003). Validating anti-surge control systems. *PTQ Magazine*, July-September. 5 pages.
- EGAS, G. (2001). PULSIM, a powerful tool to prevent vibrations in pipe systems'. In *Proc. 1st European Forum on Materials and Products for Noise and Vibration Control in Machinery and Transportation*, Senlis, France. 13 pages.
- EIJK, A. and EGAS, G. (2001). Effective combination of on-site measurements and simulations for a reciprocating compressor system. In *Proc. 2nd EFRC Symposium*, The Hague, The Netherlands. 9 pages.
- EIJK, A., EGAS, G., and SMEULERS, J. P. M. (1996). Cost-effective and detailed modelling of compressor manifold vibrations. In *Proc. ASME Pressure Vessels and Conf.*, 415–425, Montreal, Canada.
- EISENBERG, B. and VOSS, H. (2006). Realization of large scale ASU turbo compressor machine trains in mega process plants. In *Proc. IMechE 9th European Fluid Machinery Congress*, 353–366, The Hague, The Netherlands.
- ELDER, R. L. and GILL, M. E. (1985). A discussion of the factors affecting surge in centrifugal compressors. *ASME J. Engineering for Gas Turbines and Power*, **107**, 499–506.
- EMMONS, H. W., PAERSON, C. E., and GRANT, H. P. (1955). Compressor surge and stall propagation. *Trans. ASME*, **77**, 455–469.
- EPSTEIN, A. H., FLOWCS WILLIAMS, J. E., and GREITZER, E. M. (1986). Active suppression of aerodynamic instabilities in turbomachines. In *Proc. 10th AIAA Aeroacoustics Conf.*, Seattle, WA, USA. Paper 86–1994.

- EPSTEIN, A. H., FLOWCS WILLIAMS, J. E., and GREITZER, E. M. (1989). Active suppression of aerodynamic instabilities in turbomachines. *J. Propulsion and Power*, **5**, 204–211.
- EVEKER, K. M., GYSLING, D. L., NETT, C. N., and SHARMA, O. P. (1998). Integrated control of rotating stall and surge in high-speed multi-stage compression systems. *ASME J. Turbomachinery*, **120**, 440–445.
- EYKHOFF, P. (1974). *System Identification*. Wiley, London.
- FARMER, R. (2005). Unocal gas storage showcase for 18,000 hp compression package. *Gas Turbine World*, March–April, 24–26.
- FEULNER, M. R., HENDRICKS, G. J., and PADUANO, J. D. (1996). Modeling for control of rotating stall in high-speed multistage axial compressors. *ASME J. Turbomachinery*, **118**, 1–10.
- FFOWCS WILLIAMS, J. E. and HUANG, X. Y. (1989). Active stabilization of compressor surge. *J. Fluid Mechanics*, **204**, 245–262.
- FINK, D. A., CUMPSTY, N. A., and GREITZER, E. M. (1992). Surge dynamics in a free-spool centrifugal compressor system. *ASME J. Turbomachinery*, **114**, 321–332.
- FRANKLIN, G. F., POWEL, J. D., and EMAMI-NAEINI, A. (1994). *Feedback Control of Dynamic Systems*. Addison-Wesley, Reading, third edition.
- FREEMAN, C., WILSON, A. G., DAY, I. J., and SWINBANKS, M. A. (1998). Surge dynamics in a free-spool centrifugal compressor system. *ASME J. Turbomachinery*, **114**, 637–647.
- GILBERT, E. G. (1963). Controllability and observability in multivariable control systems. *SIAM J. Control, Series A*, **2**, 128–151.
- GOODSON, R. E. and LEONARD, R. G. (1972). A survey of modeling techniques for fluid line transients. *ASME J. Basic Engineering*, **94**, 474–482.
- GRAVDAHL, J. T. and EGELAND, O. (1997). A Moore-Greitzer axial compressor model with spool dynamics. In *Proc. 36th IEEE Conf. on Decision and Control*, 4714–4719, San Diego, CA, USA.
- GRAVDAHL, J. T. and EGELAND, O. (1999a). Centrifugal compressor surge and speed control. *IEEE Trans. Control Systems Technology*, **7**, 567–579.
- GRAVDAHL, J. T. and EGELAND, O. (1999b). *Compressor Surge and Rotating Stall: Modeling and Control*. Springer-Verlag, London.
- GRAVDAHL, J. T., EGELAND, O., and VATLAND, S. O. (2002). Drive torque actuation in active surge control of centrifugal compressors. *Automatica*, **38**, 1881–1893.
- GRAVDAHL, J. T., WILLEMS, F., JAGER, B. DE, and EGELAND, O. (2004). Modeling of surge in free-spool centrifugal compressors: Experimental validation. *J. Propulsion and Power*, **20**, 849–857.
- GREITZER, E. M. (1976a). Surge and rotating stall in axial flow compressors: Part I—Theoretical compression system model. *ASME J. Engineering for Power*, **98**, 190–198.
- GREITZER, E. M. (1976b). Surge and rotating stall in axial flow compressors: Part II—Experimental results and comparison with theory. *ASME J. Engineering for Power*, **98**, 199–217.
- GREITZER, E. M. (1981). The stability of pumping systems—The 1980 Freeman Scholar Lecture. *ASME J. Fluids Engineering*, **103**, 193–242.
- GREITZER, E. M. (1998). Smart jet engines: Case history of a multidisciplinary research program. *JSME Int. J. Series B*, **41**, 90–102.
- GREITZER, E. M. and MOORE, F. K. (1986). A theory of post-stall transient in axial compression systems: Part II—Application. *ASME J. Engineering for Gas Turbines and Power*, **108**, 231–239.
- GU, G., SPARKS, A., and BANDA, S. S. (1999). An overview of rotating stall and surge control for axial flow compressors. *IEEE Trans. Control Systems Technology*, **7**, 639–647.
- GYSLING, D. L., DUGUNDJI, J., GREITZER, E. M., and EPSTEIN, A. H. (1991). Dynamic control of centrifugal compressor surge using tailored structures. *ASME J. Turbomachinery*, **113**, 710–722.
- HANSEN, K. E., JØRGENSEN, P., and LARSEN, P. S. (1981). Experimental and theoretical study of surge in a small centrifugal compressor. *ASME J. Fluids Engineering*, **103**, 391–395.
- HAYNES, J. M., HENDRICKS, G. J., and EPSTEIN, A. H. (1994). Active stabilization of rotating stall in a three-stage axial compressor. *ASME J. Turbomachinery*, **116**, 226–239.

- VAN HELMONT, J. B., VAN DER WEIDEN, A. J. J., and ANNEVELD, H. (1990). Design of optimal controllers for a coal fired Benson boiler based on a modified approximate realization algorithm. *Application of Multivariable System Techniques*, 313–320.
- VAN HELVOIRT, J. and DE JAGER, B. (2007). Dynamic model including piping acoustics of a centrifugal compression system. *J. Sound and Vibration*, **302**, 361–378. doi:10.1016/j.jsv.2006.12.003.
- VAN HELVOIRT, J., DE JAGER, B., STEINBUCH, M., and SMEULERS, J. (2004). Stability parameter identification for a centrifugal compression system. In *Proc. 43rd IEEE Conf. on Decision and Control*, 3400–3405, Atlantis, Paradise Island, Bahamas.
- VAN HELVOIRT, J., BOSGRA, O., DE JAGER, B., and STEINBUCH, M. (2005a). Approximate realization of valve dynamics with time delay. In *Proc. 16th IFAC World Congress*, Prague, Czech Republic. 6 pages.
- VAN HELVOIRT, J., DE JAGER, B., STEINBUCH, M., and SMEULERS, J. (2005b). Modeling and identification of centrifugal compressor dynamics with approximate realizations. In *Proc. 2005 IEEE Conf. Control Applications*, 1441–1447, Toronto, Canada.
- VAN HELVOIRT, J., DE JAGER, B., STEINBUCH, M., and SMEULERS, J. (2006). Practical issues in model-based surge control for centrifugal compressors. In *Proc. IMechE 9th European Fluid Machinery Congress*, 285–295, The Hague, The Netherlands.
- VAN HELVOIRT, J., DE JAGER, B., STEINBUCH, M., and ROSIELLE, N. (2007). A high-speed valve for surge control in a centrifugal compression system. In *Proc. 2007 IEEE Multi-conf. on Systems and Control*, Singapore. Accepted for publication.
- HESSELING, R. J. (2002). *Active Restraint Systems, Feedback Control of Occupant Motion*. Ph.D. thesis, Technische Universiteit Eindhoven, Eindhoven, The Netherlands.
- HIRSCHBERG, A., KERGOMARD, J., and WEINREICH, G., eds. (1995). *Mechanics of Musical Instruments. CISM courses and lectures 355*. Springer-Verlag, Wien.
- HO, B. L. and KALMAN, R. E. (1965). Effective construction of linear state-variable models from input/output data. In *Proc. 3th Annual Allerton Conf. on Circuit and System Theory*, 449–459, Monticello, IL, USA.
- HOWE, M. S. (1998). *Acoustics of Fluid-Structure Interactions*. Cambridge University Press, Cambridge.
- HUNT, J. (2006). Re-injecting gases. *Turbomachinery International*, September-October, 22–25.
- HUNZIKER, R. and GYARMATHY, G. (1994). The operational stability of a centrifugal compressor and its dependence on the characteristics of the subcomponents. *ASME J. Turbomachinery*, **116**, 250–259.
- IEC 60534–2–1 (1998). Part 2–1: Flow capacity—Sizing equations for fluid flow under installed conditions. In *Industrial-process control valves*. International Electrotechnical Commission, Geneva, first edition.
- ISO 5167–2 (2003). Part 2: Orifice plates. In *Measurement of fluid flow by means of pressure differential devices inserted in circular-cross section conduits running full*. International Organization for Standardization, Geneva, second edition.
- ISHII, H. and KASHIWABARA, Y. (1996). Study on surge and rotating stall in axial compressors. *JSME Int. J. Series B*, **39**, 621–631.
- DE JAGER, B. (1995). Rotating stall and surge control: A survey. In *Proc. 34th IEEE Conf. on Decision and Control*, 1857–1862, New Orleans, LA, USA.
- JUNGOWSKI, W. M., WEISS, M. H., and PRICE, G. R. (1996). Pressure oscillations occurring in a centrifugal compressor system with and without passive and active surge control. *ASME J. Turbomachinery*, **118**, 29–40.
- KALMAN, R. E. (1963). Mathematical description of linear dynamical systems. *SIAM J. Control, Series A*, **1**, 152–192.
- KALYANARAMAN, K. and HALEY, N. (2006). Subsea compression systems. *Turbomachinery International*, November-December, 16–18.
- KHALIL, H. K. (2000). *Nonlinear Systems*. Pearson Educational, Upper Saddle River, third edition.
- KINSLER, L. E., FREY, A. R., COPPENS, A. B., and SANDERS, J. V. (1999). *Fundamentals of Acoustics*. John Wiley & Sons, Inc., New York, fourth edition.

- KLAASSEN, T. W. G. L., BONSEN, B., VAN DE MEERAKKER, K. G. O., VROEMEN, B. G., and STEINBUCH, M. (2004). Control-oriented identification of an electromechanically actuated metal V-belt CVT. In *Proc. 2004 Int. Continuously variable and hybrid transmission congress*, Davis, CA, USA. Paper 04CVT-13.
- KRUS, P., WEDDFELT, K., and PALMBERG, J. (1994). Fast pipeline models for simulation of hydraulic systems. *ASME J. Dynamic Systems, Measurements, and Control*, **116**, 132–136.
- KUNDU, P. K. (1990). *Fluid Mechanics*. Academic Press, Inc., San Diego.
- KUNG, S. (1978). A new identification and model reduction algorithm via singular value decompositions. In *Proc. 12th Asilomar Conf. on Circuits, Systems, and Computers*, 705–714, Pacific Grove, CA, USA.
- KURZ, R. and WHITE, R. C. (2004). Surge avoidance in gas compression systems. *ASME J. Turbomachinery*, **126**, 501–506.
- LAZAR, M. (2006). *Model Predictive Control of Hybrid Systems: Stability and Robustness*. Ph.D. thesis, Technische Universiteit Eindhoven, Eindhoven, The Netherlands.
- LJEVAR, S. (2007). *Rotating Stall in Wide Vaneless Diffusers*. Ph.D. thesis, Technische Universiteit Eindhoven, Eindhoven, The Netherlands.
- LJUNG, L. (1999). *System Identification, Theory for the User*. Prentice Hall PTR, Upper Saddle River, second edition.
- LONGLEY, J. (1994). A review of nonsteady models for compressor stability. *ASME J. Turbomachinery*, **116**, 202–215.
- MACDOUGAL, I. and ELDER, R. L. (1983). Simulation of centrifugal compressor transient performance for process plant applications. *ASME J. Engineering for Power*, **105**, 885–890.
- MARKOPOULOS, N., NEUMEIER, Y., PRASAD, J. V. R., and ZINN, B. T. (1999). An extended analytical model for compressor rotating stall and surge. AIAA Paper 99–2124.
- MARSDEN, J. E., TROMBA, A. J., and WEINSTEIN, A. (1993). *Basic Multivariable Calculus*. Springer-Verlag, New York.
- MATICK, R. E. (1995). *Transmission Lines for Digital and Communication Networks*. IEEE Press, Piscataway.
- MAZZAWY, R. S. (1980). Surge-induced structural loads in gas turbines. *ASME J. Engineering for Power*, **102**, 162–168.
- MAZZAWY, R. S. (1981). Discussion of Hansen et al. (1981). *ASME J. Fluids Engineering*, **103**, 395.
- MEULEMAN, C., WILLEMS, F., LANGE, R. DE, and JAGER, B. DE (1998). Surge in a low-speed radial compressor. In *Proc. ASME Turbo Expo 1998*, Stockholm, Sweden. Paper 98-GT-426.
- MEULEMAN, C., LANGE, R. DE, and STEENHOVEN, A. VAN (1999). Surge dynamics in a centrifugal compressor system. In *Proc. 3rd European Conf. on Turbomachinery—Fluid Dynamics and Thermodynamics (ImechE)*, 895–904, London, UK.
- MEULEMAN, C. H. J. (2002). *Measurement and Unsteady Flow Modelling of Centrifugal Compressor Surge*. Ph.D. thesis, Technische Universiteit Eindhoven, Eindhoven, The Netherlands.
- MITCHELL, J. V. (2006). A new era for oil prices. Royal Institute of International Affairs. <http://www.chathamhouse.org.uk/pdf/research/sdp/Oilprices.pdf>.
- MOHR, P. J. and TAYLOR, B. N. (2005). CODATA recommended values of the fundamental physical constants: 2002. *Reviews of Modern Physics*, **77**, 1–107.
- MOORE, F. K. and GREITZER, E. M. (1986). A theory of post stall transient in axial compression systems: Part I—Development of equations. *ASME J. Engineering for Gas Turbines and Power*, **108**, 68–76.
- MUNJAL, M. J. (1987). *Acoustics of Ducts and Mufflers*. Wiley-Interscience, Chichester.
- NAKAGAWA, K., FUJIWARA, M., NISHIOKA, T., S., TANAKA, and KASHIWABARA, Y. (1994). Experimental and numerical analysis of active suppression of centrifugal compressor surge by suction-side valve control. *JSME Int. J. Series B*, **37**, 878–885.
- NELSON, E. B., D., PADUANO J., and EPSTEIN, A. H. (2000). Active stabilization of surge in an axial-centrifugal turboshaft engine. *ASME J. Turbomachinery*, **122**, 485–493.

- OLIVA, S. M. and NETT, C. M. (1991). A general nonlinear dynamical analysis of a second-order, one-dimensional, theoretical compression system model. In *Proc. 1991 American Control Conf.*, 3158–3165, Boston, MA, USA.
- VAN OVERSCHEE, P. and DE MOOR, B. (1996). *Subspace Identification for Linear Systems: Theory, Implementation, Applications*. Kluwer Academic Publishers, Dordrecht.
- PADUANO, J. D., EPSTEIN, A. H., VALAVANI, L., LONGLEY, J. P., GREITZER, E. M., and GUENETTE, G. R. (1993a). Active control of rotating stall in a low-speed axial compressor. *ASME J. Turbomachinery*, **115**, 48–56.
- PADUANO, J. D., VALAVANI, L., and EPSTEIN, A. H. (1993b). Parameter identification of compressor dynamics during closed-loop operation. *ASME J. Dynamic Systems, Measurements, and Control*, **115**, 694–703.
- PADUANO, J. D., VALAVANI, L., EPSTEIN, A. H., GREITZER, E. M., and GUENETTE, G. R. (1994). Modeling for control of rotating stall. *Automatica*, **30**, 1357–1373.
- PADUANO, J. D., GREITZER, E. M., and EPSTEIN, A. H. (2001). Compression system stability and active control. *Ann. Rev. Fluid Mechanics*, **33**, 491–517.
- PINSLEY, J. E., GUENETTE, G. R., EPSTEIN, A. H., and GREITZER, E. M. (1991). Active stabilization of centrifugal compressor surge. *ASME J. Turbomachinery*, **113**, 723–732.
- ROSE, M. G., IRMLER, K., SCHLEER, M., STAHLCKER, D., and ABHARI, R. S. (2003). Classic surge in a centrifugal compressor. In *Proc. ASME Turbo Expo 2003*, Atlanta, GA, USA. Paper GT2003–38476.
- SCHEIDLER, S. G. and FOTTNER, L. (2006). Experimental operating range extension of a twin-spool turbofan engine by active stability control measures. *ASME J. Engineering for Gas Turbines and Power*, **128**, 20–28.
- SCHOUKENS, J. and PINTELON, R. (2001). *System Identification: A Frequency Domain Approach*. IEEE Press, New York.
- SCHULTZ, J. M. (1962). The polytropic analysis of centrifugal compressors. *ASME J. Engineering for Power*, **84**, 69–82.
- DE SCHUTTER, B. (2000). Minimal state-space realization in linear system theory: An overview. *J. Computational and Applied Mathematics*, **121**, 331–354. Special issue on Numerical Analysis in the 20th Century—Vol. I: Approximation theory.
- SHAVIT, A. and GUTFINGER, C. (1995). *Thermodynamics*. Prentice Hall International, Ltd., Hertfordshire.
- SHELLEY, S. (2006). The challenge of mega syngas plants. *Turbomachinery International Magazine*, March–April, 8–11.
- SIMON, H., WALLMANN, T., and MONK, T. (1987). Improvements in performance characteristics of single-stage and multi-stage centrifugal compressors by simultaneous adjustments of inlet guide vanes and diffuser vanes. *ASME J. Turbomachinery*, **109**, 41–47.
- SIMON, J. S., VALAVANI, L., EPSTEIN, A. H., and GREITZER, E. M. (1993). Evaluation of approaches to active compressor surge stabilization. *ASME J. Turbomachinery*, **115**, 57–67.
- SMITH, J. O. III (1992). Physical modeling using digital waveguides. *Computer Music Journal*, **16**, 74–91. Special issue on Physical Modeling of Musical Instruments, Part I.
- SÖDERSTRÖM, T. and STOICA, P. (1989). *System Identification*. Prentice Hall International, Ltd., Hertfordshire.
- SPAKOVSKY, Z. S. (2000). *Applications of Axial and Radial Compressor Dynamic System Modeling*. Ph.D. thesis, Massachusetts Institute of Technology, Boston, MA, USA.
- SPAKOVSKY, Z. S. (2004). Backward traveling rotating stall waves in centrifugal compressors. *ASME J. Turbomachinery*, **126**, 1–12.
- SPARKS, C. R. (1983). On the transient interaction of centrifugal compressors and their piping system. *ASME J. Engineering for Power*, **105**, 891–901.
- STAROSELSKY, N. and LADIN, L. (1979). Improved surge control for centrifugal compressors. *Chemical Engineering*, **86**, 175–184.

- STECKI, J. S. and DAVIS, D. C. (1986). Fluid transmission lines—distributed parameter models, Part I: a review of the state of the art. *Proc. IMechE*, **200**, 215–228.
- STEIN, N., NIAZI, S., and SANKAR, L. N. (2000). Numerical analysis of stall and surge in a high-speed centrifugal compressor. AIAA Paper 2000–0226.
- STENNING, A. H. (1980). Rotating stall and surge. *ASME J. Fluids Engineering*, **102**, 14–20.
- SYCHEV, V. V., VASSERMAN, A. A., and KOZLOV, A. D. (1987). *Tables of Thermodynamic Properties of Nitrogen*. Springer, Berlin.
- VENTURINI, M. (2005). Development and experimental validation of a compressor dynamic model. *ASME J. Turbomachinery*, **127**, 599–608.
- WANG, H.-H., KRSTIĆ, M., and LARSEN, M. (2000). Control of deep-hysteresis aeroengine compressors. *ASME J. Dynamic Systems, Measurements, and Control*, **122**, 140–152.
- WEIGL, H. J., PADUANO, J. D., FRÉCHETTE, L. G., EPSTEIN, A. H., GREITZER, E. M., BRIGHT, M. M., and STRAZISAR, A. J. (1998). Active stabilization of rotating stall and surge in a transonic single-stage axial compressor. *ASME J. Turbomachinery*, **120**, 625–636.
- WHITFIELD, A. and BAINES, N. C. (1990). *Design of Radial Turbomachines*. Longman Scientific & Technical, Harlow.
- WHITMORE, S. A. (1988). Formulation of a general technique for predicting pneumatic attenuation errors in airborne pressure sensing devices. Technical Report TM-100430, NASA.
- WHITMORE, S. A. and MOES, T. R. (1991). The effects of pressure sensor acoustics on airdata derived from a high-angle-of-attack flush airdata sensing (HI-FADS) system. Technical Report TM-101736, NASA.
- WILLEMS, F. and DE JAGER, B. (1999). Modeling and control of compressor flow instabilities. *IEEE Control Systems Magazine*, **19**, 8–18.
- WILLEMS, F., HEEMELS, W. P. M. H., DE JAGER, B., and STOORVOGEL, A. A. (2002). Positive feedback stabilization of centrifugal compressor surge. *Automatica*, **38**, 311–318.
- WILLEMS, F. P. T. (2000). *Modeling and Bounded Feedback Stabilization of Centrifugal Compressor Surge*. Ph.D. thesis, Technische Universiteit Eindhoven, Eindhoven, The Netherlands.
- YANG, W. C. and TOBLER, W. E. (1991). Dissipative model approximation of fluid transmission lines using linear friction model. *ASME J. Dynamic Systems, Measurements, and Control*, **113**, 152–162.
- VAN DE WAL, M., WILLEMS, F., and DE JAGER, B. (2002). Selection of actuators and sensors for active surge control. *J. Propulsion and Power*, **18**, 84–92.



# Samenvatting

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Surge is een instabiele modus van een compressorsysteem welke optreedt bij lage massa stromen beneden de zogenaamde surge lijn. De instabiliteit wordt gekarakteriseerd door een periodieke oscillatie van de massa stroom door en de drukverhoging over de compressor. Het optreden van deze limiet cycli vermindert de prestaties van de compressor en de resulterende thermische en mechanische belastingen kunnen mogelijk schade aan het compressorsysteem veroorzaken.

Actieve terugkoppeling is een veelbelovend concept voor het onderdrukken van surge in compressorsysteem. Echter, een doorbraak in de praktische toepassing van dit concept is nog niet gerealiseerd. De doelen van het hier gepresenteerde onderzoek zijn het bepalen van de kritische barrières voor de industriële toepassing van actieve surge onderdrukking en het onderzoeken van mogelijkheden om deze barrières weg te nemen. Om deze onderzoeksvragen te beantwoorden, richt het onderzoek zich in de eerste plaats op het modelleren en identificeren van de relevante compressor dynamica van een centrifugaal compressor en vervolgens op het ontwerp, realisatie en testen van een actief regelsysteem voor het onderdrukken van surge.

Voor het beschrijven van het dynamische gedrag van een tweetal centrifugaal compressoren wordt een lage orde model gebruikt dat is afgeleid van het zogenaamde Greitzer model. Om de aeroakoestische effecten te verdisconteren die tijdens surge in één van de testopstellingen optreden, wordt het Greitzer model uitgebreid met een transmissielijn model. Op basis van experimentele validatie kan worden geconcludeerd dat de ontwikkelde modellen in staat zijn om het gedrag van de industriële centrifugaal compressoren tijdens surge met acceptabele nauwkeurigheid en onder verschillende condities te beschrijven.

De ontwikkelde dynamische modellen zijn vervolgens gebruikt om te onderzoeken welke dynamica van industriële compressoren relevant is voor het ontwerp van een regelsysteem. De analyse bevestigt het belang van de zogenoemde stabiliteitsparameter en motiveert het ontwikkelen van een tweetal methoden om deze parameter te identificeren in de onderzochte compressor systemen. De eerste methode is gebaseerd op een geometrische benadering van de hydraulische inductie van een centrifugaal compressor. De

tweede methode gebruikt benaderde realisatie theorie voor het identificeren van de dominante dynamica in een compressor uit stap responsie metingen. Door de uitkomsten van beide methoden te combineren kan worden geconcludeerd dat identificatie een meer betrouwbare schatting voor de stabiliteitsparameter kan opleveren dan het afstemmen van model parameters.

Vervolgens wordt het model van het compressorsysteem gebruikt om een actief regelsysteem te ontwerpen dat bestaat uit een druksensor en regelklep in de compressor uitlaat en een lineair kwadratische Gaussische regelaar. Simulaties met het niet-lineaire compressorsysteem hebben laten zien dat stabilisatie op 95% van de surge massa stroom mogelijk is bij lage compressor snelheden. Door middel van simulaties van het geregelde systeem zijn vervolgens de benodigde capaciteit, bandbreedte en de toelaatbare tijdvertraging van de regelklep bepaalt.

Om aan de actuator specificaties te voldoen is een nieuwe regelklep ontwikkeld. De excellente prestaties met betrekking tot de snelheid van de klep zijn door middel van experimenten aangetoond. Gezien het potentieel van de elektromechanische regelklep hoeven de beperkingen van een actuator niet langer een obstakel te vormen voor de toepassing van actieve surge onderdrukking.

Uiteindelijk is het ontworpen regelsysteem geïmplementeerd in een van de bestudeerde compressorsystemen. Uit experimenten met het geregelde compressorsysteem is gebleken dat de regelaar niet in staat is om surge in de testopstelling te onderdrukken. De meest waarschijnlijke oorzaak hiervoor is het gebrek aan robuustheid met betrekking tot modelonzekerheden, ruis en variërende proces condities en het kleine attractiegebied van de regelaar. Op basis van de inzichten die gedurende het onderzoek zijn verkregen, worden verschillende aanpakken voorgesteld om de barrières op te heffen die een succesvolle demonstratie van actieve surge onderdrukking op industriële schaal in de weg staan. Verder onderzoek dient zich in de eerste plaats te richten op het verder verminderen van onzekerheden in de schattingen voor de stabiliteitsparameter en het modelgebaseerde ontwerpen van een robuuste regelstrategie.

# Acknowledgements

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The research as presented in this thesis has been made possible by the contributions of numerous people. Maarten Steinbuch and Bram de Jager from the Technische Universiteit Eindhoven, Jan Smeulders from TNO Science and Industry, Patrick van der Span and Theodor Wallmann from Siemens PGI are acknowledged for their supervision, support and valuable feedback during the project. In particular I thank Eric Borg, Erik van Zalk and Ulrich Knörr from Siemens PGI for their valuable assistance during the extensive experimental work and Nick Rosielle and Jan de Vries for their role in the design and realization of the new control valve actuator. Furthermore, I thank Jan Tommy Gravdahl and especially Bjørnar Bøhagen from the NTNU for the pleasant and fruitful cooperation in Eindhoven and Trondheim. I also thank the following students from the Technische Universiteit Eindhoven for their contributions: Peter Tijl, Eddy Peters, Mustafa Uyanik, Dennis Leermakers, William van den Bremer, Tom Marechal, René Vugts, Ruud Mickers, Philippe Zilligen-Molenaar en Marjan Nieuwenhuizen. All other people who were involved are equally acknowledged.

However, working on and finishing a PhD thesis requires more than only research. I thank my roommates Rob Mestrom and Jan-Kees de Bruin, all my other (former) colleagues from the Technische Universiteit Eindhoven and in particular Rogier Hesselink and Lia Neervoort for providing some of the other essential ingredients. However, the most important ingredients have been provided by my parents Hans and Ricky van Helvoirt in the form of 30 years of unconditional love, support, criticism and guidance. Finally, I thank my sister Marieke, her husband Teun Kerckhoffs and also my grandmother 'oma' Van den Berk for their continuing support and genuine interest.

Jan van Helvoirt  
August, 2007



# Curriculum Vitae

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Jan van Helvoirt was born on April 23th, 1977 in Utrecht, The Netherlands. He graduated from secondary school at the Paulus Lyceum, Tilburg, in 1995. He received the M.Sc. degree (cum laude) in Mechanical Engineering from the Technische Universiteit Eindhoven (TU/e), The Netherlands, in 2002. During his traineeships at the TU/e and at Mechanical Development, Philips Optical Storage, Singapore, he worked on playability improvements of optical disc drives. During his M.Sc. project 'Disc defect handling in optical disc drives' he continued this work at the Emerging Technologies and Systems Laboratory of Philips Optical Storage, Eindhoven.

In September 2002, he started as a Ph.D. student in the Control Systems Technology group at the TU/e. During his research he was partially employed by the Netherlands Organisation for Applied Scientific Research (TNO) where he worked in the Flow and Structural Dynamics group of the Industrial Modeling & Control division of TNO Science and Industry. The Ph.D. project focused on the research and development of active surge control for industrial scale centrifugal compression systems and the results of the project are presented in this thesis.

In 2006 he initiated the start-up of a new company in the field of measurement and control technology. The company, named CST Innovations, will be a full subsidiary of the TU/e Holding and it will start in 2007.



# Stellingen

Behorende bij het proefschrift

## Centrifugal Compressor Surge

Modeling and Identification for Control

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1. De noodzaak om parameters in het Greitzer model te tunen duidt niet op een theoretische tekortkoming maar op een gebrek aan data.  
*Hoofdstukken 3 en 6 van dit proefschrift*
2. De bandbreedte van een regelklep hoeft geen beperking te vormen voor het onderdrukken van surge in industriële compressoren.  
*Hoofdstuk 7 van dit proefschrift*
3. Succesvol onderdrukken van surge in een industriële centrifugaal compressor vereist een robuuste regelaar.  
*Hoofdstuk 8 van dit proefschrift*
4. Een model is nooit geheel objectief.
5. Het gebruik van het woord methodologie in de technisch-wetenschappelijke literatuur is meestal misplaatst.
6. Proefschriften binnen de technische wetenschappen dienen een volledige symbolenlijst te bevatten.
7. Iets bestaat dan en slechts dan indien het interacties met het bestaande toelaat. Iets wordt bepaald door alle stoffelijke en onstoffelijke interacties die plaatsvinden.
8. Om het bestaande te begrijpen zal men zich eerder moeten verbreden dan verdiepen.
9. Statistiek is een intelligent zwaktebod.  
*Pierre-Simon Laplace, Essai philosophique sur les probabilités, 1814*

10. Het verkleinen van de kloof tussen universiteit en industrie vermindert het vermogen tot innovatie. Het is beter om een brug te slaan.
11. Het is in strijd met de democratie om te stellen dat de vrijheid van meningsuiting absoluut en onveranderlijk is.
12. Gezien de hoge ethische waarde is het woord belangeloos misleidend.
13. Een oprechte vriend is een gelukkig mens.
14. Geslepen staal ruikt het lekkerst.

Jan van Helvoirt  
Augustus, 2007