MasterLeap Tuition Grade 9 formula sheet

Topics Covered:

- Number Systems
- Polynomials
- Coordinate Geometry
- Linear Equations in Two Variables
- Introduction to Euclid's Geometry
- Lines and Angles
- Triangles
- Quadrilaterals
- Circles
- Heron's Formula
- Surface Areas and Volumes
- Statistics
- Probability

SECTION 1: NUMBER SYSTEMS

Types of Numbers

Number Type	Definition	Example
Natural Numbers (N)	Counting numbers	1,2,3,4,...

Whole Numbers (W)	Natural numbers + 0	0,1,2,3,4,...
Integers (Z)	Whole numbers + negatives	...,-3,-2,-1,0,1,2,3,...
Rational Numbers (Q)	Numbers in form p/q, q≠0	2/3, -4/5, 7, 0.25
Irrational Numbers	Cannot be written as p/q	√2, √3, π, e
Real Numbers (R)	All rational + irrational numbers	

Properties of Real Numbers

Property	Addition	Multiplication
Closure	a + b is real	a × b is real

Commutative	$a + b = b + a$	$a \times b = b \times a$
Associative	$(a+b)+c = a+(b+c)$	$(a \times b) \times c = a \times (b \times c)$
Identity	$a + 0 = a$	$a \times 1 = a$
Inverse	$a + (-a) = 0$	$a \times (1/a) = 1$ $(a \neq 0)$
Distributive	$a \times (b + c) = a \times b + a \times c$	

Representing Irrational Numbers on Number Line

- √2: Construct right triangle with legs 1,1 → hypotenuse √2
- √3: Construct right triangle with legs √2 and 1

Laws of Exponents for Real Numbers

Law	Formula
Product	$a^m \times a^n = a^{m+n}$

Quotient	$a^m \div a^n = a^{m-n}$
Power of Power	$(a^m)^n = a^{mn}$
Power of Product	$(ab)^m = a^m b^m$
Power of Quotient	$(a/b)^m = a^m/b^m$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-n} = 1/a^n$
Fractional Exponent	$a^(m/n) = \sqrt[n]{(a^m)}$

Rationalization

- Rationalizing factor: Multiply numerator and denominator by conjugate
- For $1/(\sqrt{a} + \sqrt{b})$: Multiply by $(\sqrt{a} - \sqrt{b})/(\sqrt{a} - \sqrt{b})$
- For $1/(\sqrt{a} - \sqrt{b})$: Multiply by $(\sqrt{a} + \sqrt{b})/(\sqrt{a} + \sqrt{b})$

Laws of Radicals

Law	Formula
√(ab)	√a × √b
√(a/b)	√a ÷ √b
$\sqrt[n]{(a^m)}$	a^(m/n)
$(\sqrt[n]{a})^m$	a^(m/n)

SECTION 2: POLYNOMIALS

Basic Terms

Term	Definition	Example

Polynomial	Expression with variables, constants, and non-negative integer exponents	$3x^2 + 2x - 5$
Degree	Highest power of variable	In $3x^2 + 2x - 5$, degree = 2
Coefficient	Number multiplied by variable	In $3x^2$, coefficient = 3
Constant Term	Term with no variable	In above, -5
Monomial	Polynomial with one term	$5x$, $-3y^2$, 7
Binomial	Polynomial with two terms	$2x + 3$, $4a - 5b$
Trinomial	Polynomial with three terms	$x^2 + 2x + 1$

Types of Polynomials (Based on Degree)

Degree	Name	Example
0	Constant	5
1	Linear	2x + 3
2	Quadratic	$x^2 + 5x + 6$
3	Cubic	$x^3 - 3x^2 + 2x$
4	Biquadratic	$x^4 + 2x^2 - 3$

Zeroes of Polynomial

- Zero of polynomial p(x) is value of x for which p(x) = 0
- Number of zeroes ≤ Degree of polynomial
- Geometrically: x-coordinate where graph intersects x-axis

Remainder Theorem

- Statement: If p(x) is divided by (x - a), remainder = p(a)

- Example: $p(x) = x^2 + 2x - 3$ divided by $(x - 2)$, remainder = $p(2) = 4 + 4 - 3 = 5$

Factor Theorem

- Statement: $(x - a)$ is factor of $p(x)$ if and only if $p(a) = 0$
- Example: For $p(x) = x^2 - 5x + 6$, $p(2) = 0 \rightarrow (x-2)$ is factor

Algebraic Identities

Identity	Formula
I	$(x + y)^2 = x^2 + 2xy + y^2$
II	$(x - y)^2 = x^2 - 2xy + y^2$
III	$x^2 - y^2 = (x + y)(x - y)$
IV	$(x + a)(x + b) = x^2 + (a+b)x + ab$
V	$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

VI	$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
VII	$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
VIII	$x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2+y^2+z^2 - xy - yz - zx)$

Factorization of Cubic Polynomials

- Step 1: Find one factor by trial (using factor theorem)
- Step 2: Divide polynomial by that factor
- Step 3: Factorize the quotient (quadratic)

SECTION 3: COORDINATE GEOMETRY

Cartesian System

Component	Description
x-axis	Horizontal line

y-axis	Vertical line
Origin	(0,0) - intersection of axes
Quadrants	Four regions divided by axes

Quadrant Signs

Quadrant	x-coordinate	y-coordinate	Example
I	Positive	Positive	(3,4)
II	Negative	Positive	(-3,4)
III	Negative	Negative	(-3,-4)

IV	Positive	Negative	(3,-4)

Coordinates of a Point

- Ordered pair (x,y)
- x-coordinate (abscissa): Distance from y-axis
- y-coordinate (ordinate): Distance from x-axis

Distance Formula

- Distance between $P(x_1,y_1)$ and $Q(x_2,y_2)$:
  - $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Section Formula

- Coordinates of point dividing line joining (x_1,y_1) and (x_2,y_2) in ratio m:n:
  - Internal division: $[(mx_2 + nx_1)/(m+n), (my_2 + ny_1)/(m+n)]$
  - External division: $[(mx_2 - nx_1)/(m-n), (my_2 - ny_1)/(m-n)]$

Midpoint Formula

- Midpoint of line joining (x_1,y_1) and (x_2,y_2):
  - $[(x_1 + x_2)/2, (y_1 + y_2)/2]$

Area of Triangle

- Area = $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$
- Collinearity: Points are collinear if area = 0

SECTION 4: LINEAR EQUATIONS IN TWO VARIABLES

Standard Form

- ax + by + c = 0, where a and b are not both zero

Solution of Linear Equation

- Every linear equation in two variables has infinitely many solutions
- Each solution is an ordered pair (x,y) that satisfies the equation

Graph of Linear Equation

- Always a straight line
- To draw: Find at least two points
- x-intercept: Put y = 0, solve for x
- y-intercept: Put x = 0, solve for y

Equations of Lines Parallel to Axes

Line	Equation	Description
Parallel to x-axis	y = k	Horizontal line at distance k from x-axis

Parallel to y-axis	x = k	Vertical line at distance k from y-axis

Slope of a Line

- Slope (m) = $(y_2 - y_1)/(x_2 - x_1)$
- Positive slope: Line goes upward from left to right
- Negative slope: Line goes downward from left to right
- Zero slope: Horizontal line (y = constant)
- Undefined slope: Vertical line (x = constant)

SECTION 5: LINES AND ANGLES

Basic Terms

Term	Definition
Line	Straight path extending infinitely in both directions
Ray	Part of line with one endpoint

Line Segment	Part of line with two endpoints
Angle	Formed by two rays with common endpoint
Intersecting Lines	Lines that meet at a point
Parallel Lines	Lines that never meet
Transversal	Line intersecting two or more lines

Angle Pairs

Pair	Definition	Property
Complementary	Sum = 90°	
Supplementary	Sum = 180°	

Adjacent	Share common arm and vertex	
Linear Pair	Adjacent angles on a straight line	Sum = 180°
Vertically Opposite	Formed when two lines intersect	Equal

Angles Formed by Transversal

Angle Type	Definition	When lines are parallel
Corresponding	Same position relative to intersection	Equal
Alternate Interior	Inside parallel lines, opposite sides of transversal	Equal

Alternate Exterior	Outside parallel lines, opposite sides of transversal	Equal
Co-interior	Inside parallel lines, same side of transversal	Sum = 180°

Angle Sum Properties

Figure	Property
Triangle	Sum of angles = 180°
Quadrilateral	Sum of angles = 360°
Polygon with n sides	Sum of interior angles = (n-2) × 180°
Exterior angle of polygon	Sum of exterior angles = 360°

SECTION 6: TRIANGLES

Congruence of Triangles

Criterion	Condition
SSS	Three sides of one triangle equal to three sides of another
SAS	Two sides and included angle equal
ASA	Two angles and included side equal
AAS	Two angles and a non-included side equal
RHS	Right angle, hypotenuse, and one side equal

Properties of Triangles

Property	Statement
Angles opposite equal sides	Equal
Sides opposite equal angles	Equal
In isosceles triangle	Altitude from vertex bisects base
In equilateral triangle	All angles = 60°, all sides equal

Inequalities in Triangles

Inequality	Statement
Angle-Side Relationship	Larger side opposite larger angle
Triangle Inequality	Sum of any two sides > Third side

Difference of two sides	Less than third side

Pythagoras Theorem

- In right triangle: (Hypotenuse)² = (Base)² + (Height)²
- Converse: If $a^2 + b^2 = c^2$, then triangle is right-angled

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SECTION 7: QUADRILATERALS

Types of Quadrilaterals

Quadrilateral	Properties
Parallelogram	Opposite sides parallel and equal, opposite angles equal, diagonals bisect each other
Rectangle	Parallelogram with all angles 90°, diagonals equal

Rhombus	Parallelogram with all sides equal, diagonals perpendicular bisectors
Square	Rectangle with all sides equal, diagonals equal and perpendicular
Trapezium	One pair of opposite sides parallel
Isosceles Trapezium	Non-parallel sides equal, base angles equal
Kite	Two pairs of adjacent sides equal, diagonals perpendicular

Mid-Point Theorem

- Statement: Line joining mid-points of two sides of triangle is parallel to third side and half of it
- Converse: Line through mid-point of one side parallel to another side bisects third side

Important Theorems

Theorem	Statement
Diagonal of parallelogram	Divides it into two congruent triangles
In parallelogram	Opposite sides are equal
In parallelogram	Opposite angles are equal
In parallelogram	Diagonals bisect each other
In rectangle	Diagonals are equal
In rhombus	Diagonals are perpendicular bisectors

SECTION 8: CIRCLES

Basic Terms

Term	Definition
Circle	Set of points equidistant from fixed point (center)
Radius	Distance from center to circle
Diameter	Twice radius, longest chord
Chord	Line segment joining two points on circle
Arc	Part of circumference
Sector	Region between two radii and arc
Segment	Region between chord and arc

Secant	Line intersecting circle at two points
Tangent	Line touching circle at one point

Chord Properties

Property	Statement
Perpendicular from center to chord	Bisects the chord
Line from center to midpoint of chord	Perpendicular to chord
Equal chords	Equidistant from center
Chords equidistant from center	Equal

Angle Properties

Property	Statement
Angle in semicircle	90° (right angle)
Angle subtended by chord at center	Twice angle at circumference
Angles in same segment	Equal
Opposite angles of cyclic quadrilateral	Sum = 180°
Exterior angle of cyclic quadrilateral	Equal to interior opposite angle

Cyclic Quadrilateral

- Definition: All four vertices lie on circle
- Property: Sum of opposite angles = 180°
- Converse: If sum of opposite angles = 180°, quadrilateral is cyclic

Tangent Properties

Property	Statement
Radius to point of tangency	Perpendicular to tangent
Tangents from external point	Equal in length
Angle between tangent and chord	Equal to angle in alternate segment

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SECTION 9: HERON'S FORMULA

Heron's Formula for Area of Triangle

- Area = √[s(s-a)(s-b)(s-c)]
- Where s = (a + b + c)/2 (semi-perimeter)
- a, b, c are sides of triangle

Area of Quadrilateral Using Heron's Formula

- Step 1: Divide quadrilateral into two triangles by diagonal
- Step 2: Find area of each triangle using Heron's formula
- Step 3: Add the areas

SECTION 10: SURFACE AREAS AND VOLUMES

Cuboid and Cube

Shape	Lateral Surface Area	Total Surface Area	Volume
Cuboid	$2h(l + b)$	$2(lb + bh + hl)$	$l \times b \times h$
Cube	$4a^2$	$6a^2$	a^3

Right Circular Cylinder

Measurement	Formula
Curved Surface Area	$2\pi rh$
Total Surface Area	$2\pi r(h + r)$

Volume	$\pi r^2 h$

Right Circular Cone

Measurement	Formula
Slant Height (l)	$l = \sqrt{(r^2 + h^2)}$
Curved Surface Area	$\pi r l$
Total Surface Area	$\pi r(l + r)$
Volume	$\frac{1}{3}\pi r^2 h$

Sphere and Hemisphere

Shape	Surface Area	Volume

Sphere	$4\pi r^2$	$^4/_3\pi r^3$
Hemisphere (curved)	$2\pi r^2$	$^2/_3\pi r^3$
Hemisphere (total)	$3\pi r^2$	$^2/_3\pi r^3$

SECTION 11: STATISTICS

Measures of Central Tendency

Measure	Definition	Formula
Mean	Average	Sum of observations / Number of observations
Median	Middle value	Arrange in order: if n odd → (n+1)/2th term; if n even →

		average of n/2 and (n/2+1)th terms
Mode	Most frequent value	Value with highest frequency

Grouped Data

Term	Definition
Class Interval	Range of values (e.g., 0-10, 10-20)
Class Mark	Mid-point of class = (Upper limit + Lower limit)/2
Frequency	Number of observations in each class
Cumulative Frequency	Sum of frequencies up to that class

Mean for Grouped Data

Method	Formula
Direct Method	Mean = $\Sigma(f_i x_i)/\Sigma f_i$
Assumed Mean Method	Mean = $a + [\Sigma(f_i d_i)/\Sigma f_i]$ where $d_i = x_i - a$
Step Deviation Method	Mean = $a + [\Sigma(f_i u_i)/\Sigma f_i] \times h$ where $u_i = (x_i - a)/h$

Graphical Representation

Graph	Use
Bar Graph	Compare categories
Histogram	Continuous data (no gaps between bars)
Frequency Polygon	Join mid-points of tops of bars

Ogive	Cumulative frequency curve

SECTION 12: PROBABILITY

Basic Probability

Term	Definition
Experiment	An action with uncertain outcome
Trial	Single performance of experiment
Outcome	Result of a trial
Event	Set of desired outcomes
Sample Space	Set of all possible outcomes

Probability Formula

- P(E) = (Number of favorable outcomes)/(Total number of outcomes)
- Range: $0 \leq P(E) \leq 1$
- P(not E) = 1 - P(E)

Experimental (Empirical) Probability

- P(E) = (Number of times event occurred)/(Total number of trials)
- As number of trials increases, experimental probability approaches theoretical probability

Important Facts

- Probability of sure event = 1
- Probability of impossible event = 0
- Sum of probabilities of all elementary events = 1