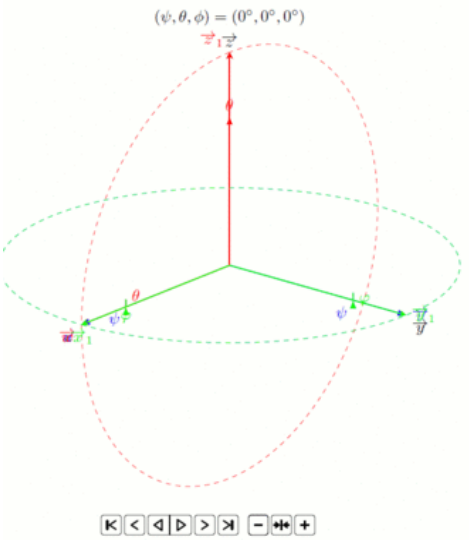
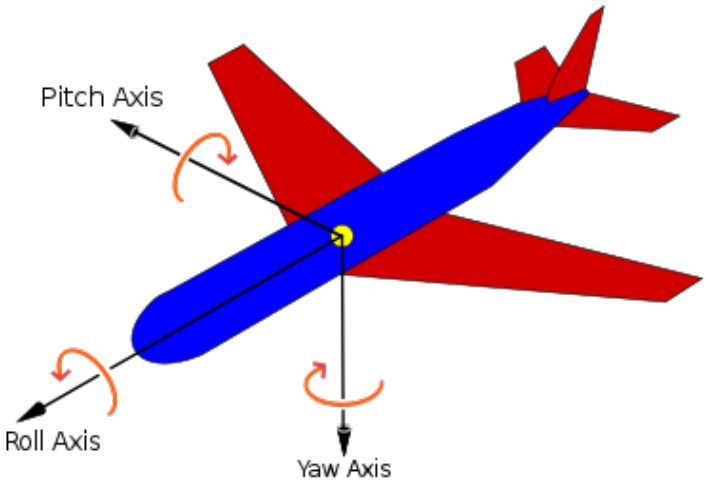
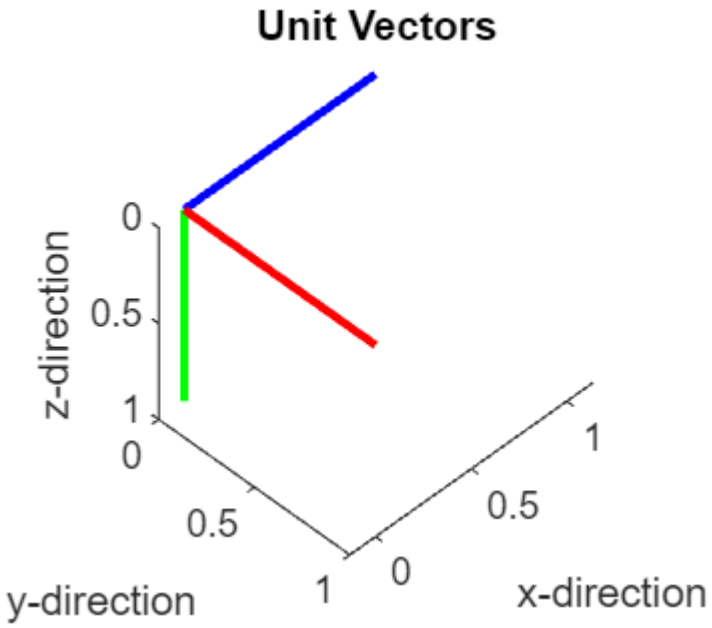
Euler Angles





This live script walks through Euler Angles and how they are used to transform one vector (or in this case a basis) to another vector in $\Re^3$

```
% Define a set of unit vectors
x = [0,1; 0,0; 0,0];
y = [0,0; 0,1; 0,0];
z = [0,0; 0,0; 0,1];

figure(1)
clf
hold on
axis equal

set(gca,'ydir', 'reverse')
set(gca,'zdir', 'reverse')

plot3(x(1,:), x(2,:), x(3,:),'LineWidth',2, 'Color','blue')
plot3(y(1,:), y(2,:), y(3,:),'LineWidth',2, 'Color','red')
plot3(z(1,:), z(2,:), z(3,:),'LineWidth',2, 'Color','green')

xlabel('x-direction')
ylabel('y-direction')
zlabel('z-direction')
title('Unit Vectors')

view([-45 45])
```



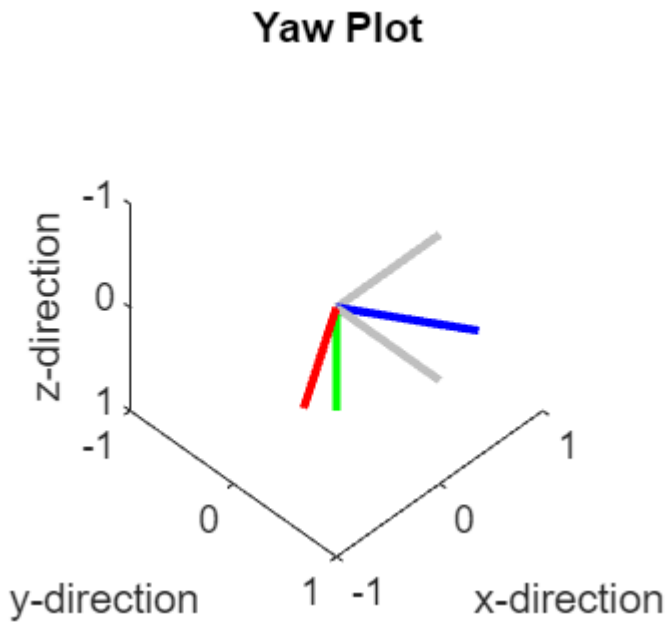
Yaw

- Rotation about the z-axis ($z_1 = z_0$)
- Anticlockwise rotation

Yaw: $\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$

```
psi =58;

yaw = [cosd(psi), -sind(psi), 0; sind(psi), cosd(psi), 0; 0,0,1];
x1 = yaw*x;
y1 = yaw*y;
z1 = yaw*z;

figure(2)
clf
hold on
axis equal

xlim([-1 1])
ylim([-1 1])
zlim([-1 1])

set(gca,'ydir', 'reverse')
set(gca,'zdir', 'reverse')

plot3(x(1,:), x(2,:), x(3,:),'LineWidth',2, 'Color','#BFBFBF')
plot3(y(1,:), y(2,:), y(3,:),'LineWidth',2, 'Color','#BFBFBF')
```

```
plot3(z(1,:), z(2,:), z(3,:),'LineWidth',2, 'Color','#BFBFBF')

plot3(x1(1,:), x1(2,:), x1(3,:),'LineWidth',2, 'Color','blue')
plot3(y1(1,:), y1(2,:), y1(3,:),'LineWidth',2, 'Color','red')
plot3(z1(1,:), z1(2,:), z1(3,:),'LineWidth',2, 'Color','green')

xlabel('x-direction')
ylabel('y-direction')
zlabel('z-direction')
title('Yaw Plot')

view([-45 45])
```



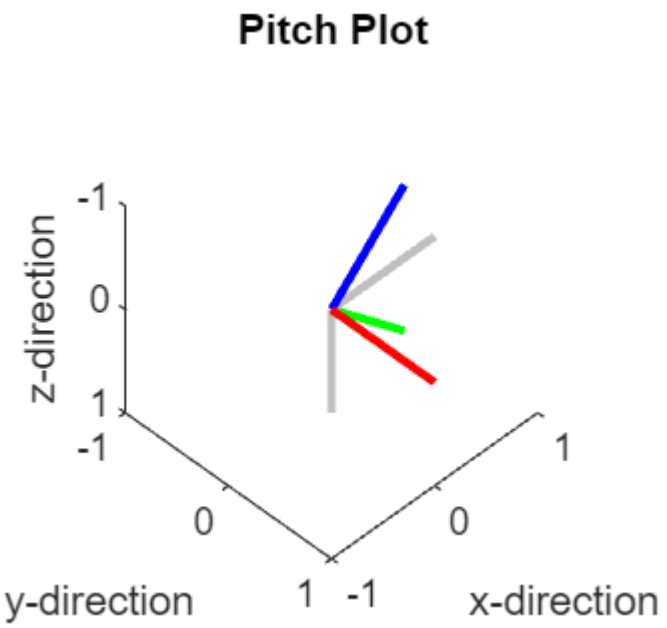
Pitch

- Rotation about the y-axis ($y_1 = y_0$)
- Anticlockwise rotation

Pitch: $\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$

```
theta =45;

pitch = [cosd(theta), 0, sind(theta); 0,1,0; -sind(theta), 0, cosd(theta)];
x1 = pitch*x;
y1 = pitch*y;
z1 = pitch*z;

figure(3)
clf
```

```
hold on
axis equal

xlim([-1 1])
ylim([-1 1])
zlim([-1 1])

set(gca,'ydir', 'reverse')
set(gca,'zdir', 'reverse')

plot3(x(1,:), x(2,:), x(3,:),'LineWidth',2, 'Color','#BFBFBF')
plot3(y(1,:), y(2,:), y(3,:),'LineWidth',2, 'Color','#BFBFBF')
plot3(z(1,:), z(2,:), z(3,:),'LineWidth',2, 'Color','#BFBFBF')

plot3(x1(1,:), x1(2,:), x1(3,:),'LineWidth',2, 'Color','blue')
plot3(y1(1,:), y1(2,:), y1(3,:),'LineWidth',2, 'Color','red')
plot3(z1(1,:), z1(2,:), z1(3,:),'LineWidth',2, 'Color','green')

xlabel('x-direction')
ylabel('y-direction')
zlabel('z-direction')
title('Pitch Plot')

view([-45 45])
```

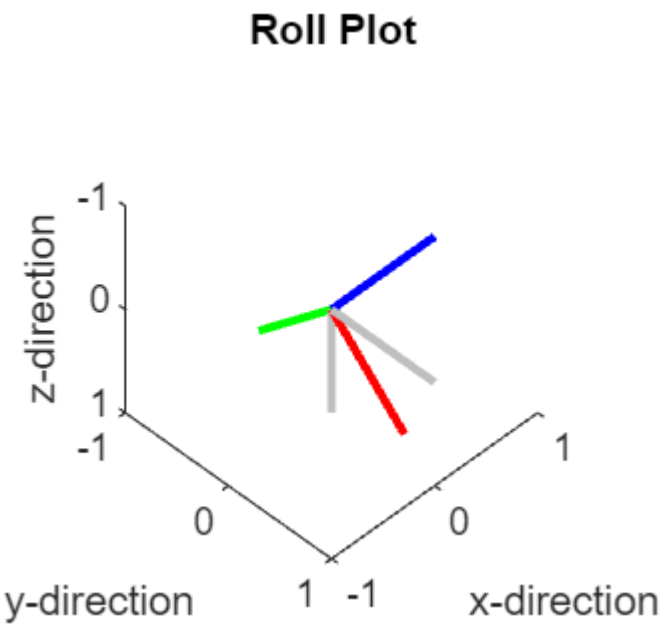


Roll

- Rotation about the x-axis ($x_1 = x_0$)
- Anticlockwise rotation

Roll: $\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$

```
phi =45;
roll = [1,0,0; 0, cosd(phi), -sind(phi); 0, sind(phi), cosd(phi)];
x1 = roll*x;
y1 = roll*y;
z1 = roll*z;

figure(4)
clf
hold on
axis equal

xlim([-1 1])
ylim([-1 1])
zlim([-1 1])

set(gca,'ydir', 'reverse')
set(gca,'zdir', 'reverse')

plot3(x(1,:), x(2,:), x(3,:),'LineWidth',2, 'Color','#BFBFBF')
plot3(y(1,:), y(2,:), y(3,:),'LineWidth',2, 'Color','#BFBFBF')
plot3(z(1,:), z(2,:), z(3,:),'LineWidth',2, 'Color','#BFBFBF')

plot3(x1(1,:), x1(2,:), x1(3,:),'LineWidth',2, 'Color','blue')
plot3(y1(1,:), y1(2,:), y1(3,:),'LineWidth',2, 'Color','red')
plot3(z1(1,:), z1(2,:), z1(3,:),'LineWidth',2, 'Color','green')

xlabel('x-direction')
ylabel('y-direction')
zlabel('z-direction')
title('Roll Plot')

view([-45 45])
```



Compiled Rotations

$$\begin{bmatrix} x_f \\ y_f \\ z_f \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

```
psi = 10;
theta = 10;
phi =10;
yaw = [cosd(psi), -sind(psi), 0; sind(psi), cosd(psi), 0; 0,0,1];
pitch = [cosd(theta), 0, -sind(theta); 0,1,0; sind(theta), 0, cosd(theta)];
roll = [1,0,0; 0, cosd(phi), -sind(phi); 0, sind(phi), cosd(phi)];

cosine_matrix = yaw*pitch*roll;

x1 = cosine_matrix*x;
y1 = cosine_matrix*y;
z1 = cosine_matrix*z;


figure(5)
clf
hold on
axis equal

a = 1;
xlim([-a a])
ylim([-a a])
zlim([-a a])
```

```
set(gca,'ydir', 'reverse')
set(gca,'zdir', 'reverse')

plot3(x(1,:), x(2,:), x(3,:),'LineWidth',2, 'Color','#BFBFBF')
plot3(y(1,:), y(2,:), y(3,:),'LineWidth',2, 'Color','#BFBFBF')
plot3(z(1,:), z(2,:), z(3,:),'LineWidth',2, 'Color','#BFBFBF')

plot3(x1(1,:), x1(2,:), x1(3,:),'LineWidth',2, 'Color','blue')
plot3(y1(1,:), y1(2,:), y1(3,:),'LineWidth',2, 'Color','red')
plot3(z1(1,:), z1(2,:), z1(3,:),'LineWidth',2, 'Color','green')

xlabel('x-direction')
ylabel('y-direction')
zlabel('z-direction')
title('Compiled Rotations Plot')

view([-45 45])
```