Project #3

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Abstract

This project aimed to implement the vortex panel method from Project 2 for flow past two airfoils, the NACA0012 and NACA4412.The vortex panel method was used to calculate the drag and force for flow past the airfoils with an angle of attack from -16° to 16°. A comparison of the lift and drag coefficients was measured against that from literature and thin airfoil theory. Furthermore, using boundary layer separation criterion for Thwaites, the stall angle was determined.

1 Problem Statement

In this project, the goal was to utilize the vortex panel method using 64 or more panels varying in size to model the flow around the NACA0012 and NACA4412 airfoils. This method was applied to linearly varying angle of attacks ranging from -16° to 16°. The lift and pressure coefficients were plotted and compared with the analysis of thin airfoil theory. Afterwards criterion of Thwaites boundary layer separation was applied to determine the stall angles.

2 Methodology

2.1 Obtaining Pressure Coefficient

Using the potential function of the combined field and vortex panels from Project 2:

$$\phi(x, y) = Ux\cos\alpha + Uy\sin\alpha + \Sigma_j \int_j \frac{\gamma(s_j)}{2\pi} \tan^{-1}\left(\frac{y - y_j}{x - x_j}\right) ds_j, \tag{1}$$

where the vortex panel strength varies linearly

$$\gamma(s_j) = \gamma_j + (\gamma_{j+1} - \gamma_j)\frac{s_j}{S_j} \tag{2}$$

The normal velocity components at control points i vanish

$$\left.\frac{\partial\phi}{\partial n}\right|_i = 0 \tag{3}$$

Implementation of this above equation results in the inversion of the following matrix to determine γ_i in equation (2).

$$\sum_{j=1}^{m} (C_{n1ij}\gamma_j + C_{n2ij}\gamma_{j+1}) = 2\pi U \sin(\theta_i - \alpha) \tag{4}$$

Where the pressure coefficient C_p is solved using the following equation

$$C_p = 1 - U_{ti}^2 \tag{5}$$

2.2 Obtaining Lift Coefficient

The lift coefficient C_l was obtained by following the steps from the textbook, Basics of Aerodynamic Design (5th edition) by Kuethe A. M. and Chow C.-Y. Section 5.7, the equation for the lift coefficient becomes:

$$C_l = 2\pi\left[\alpha + \frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}\cos(\theta - 1)d\theta\right] = 2\pi(\alpha_{L0} + \frac{1}{2}\alpha_a) \tag{6}$$

where α_{L0} and α_a are coefficients for the angle of zero lift and absolute angle of attack respectively. In an asymmetric airfoil, $\mathbf{z_m}$ and $\mathbf{p} = 0$, so $\alpha_a = 0$ resulting in the lift coefficient equation to become:

$$C_l = 2\pi\alpha_a \tag{7}$$

2.3 Thwaites Boundary Layer Separation Criterion

To determine the stall angle for both airfoils, the same criterion from Project 2 was used:

$$\lambda = \frac{0.45}{U_e(x)^6}\frac{dU}{dx}\int_0^x U_e(\xi)d\xi = -0.09 \tag{8}$$

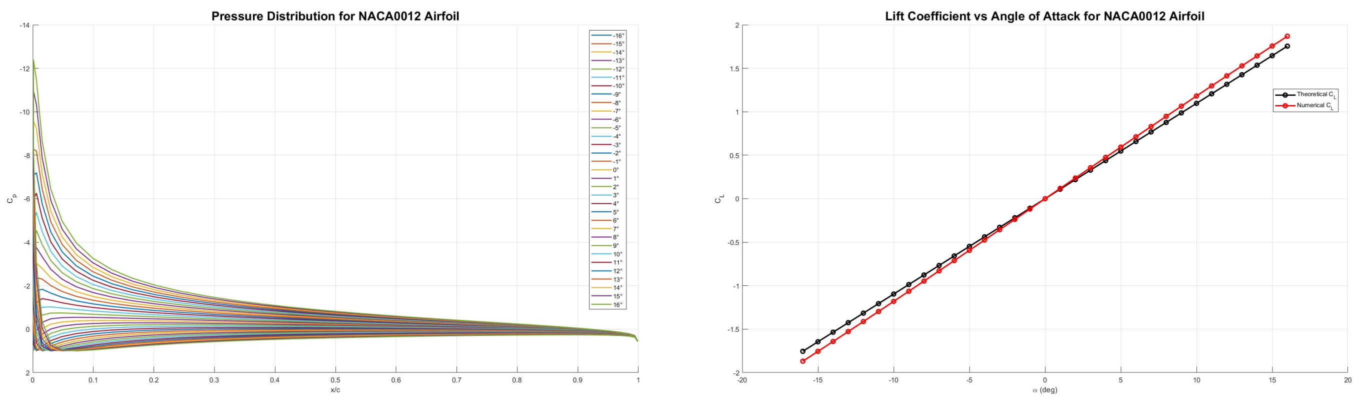
From this equation, the stall angle is determined when x is ≤ 0.2 since the flow separation occurs when it reaches 20% of the chord length from the leading edge. Furthermore, the stall angle is also identified on points where the left side of the equation above is also $\leq$ -0.09.

3 Model Simulation Results

The model was implemented using 64 vortex panels and a range of angles of attack from -16 to 16 with a step of 1. The vortex panel method from Project 2 was used along with Thwaites boundary layer separation criterion. Plots where then made for each airfoil with their C_l vs α and pressure distribution C_p vs x/c.

3.1 NACA0012 Airfoil

For the NACA0012, the following plots were attained for the lift and pressure coefficients:



(a) C_p vs x/c

(b) C_l vs α

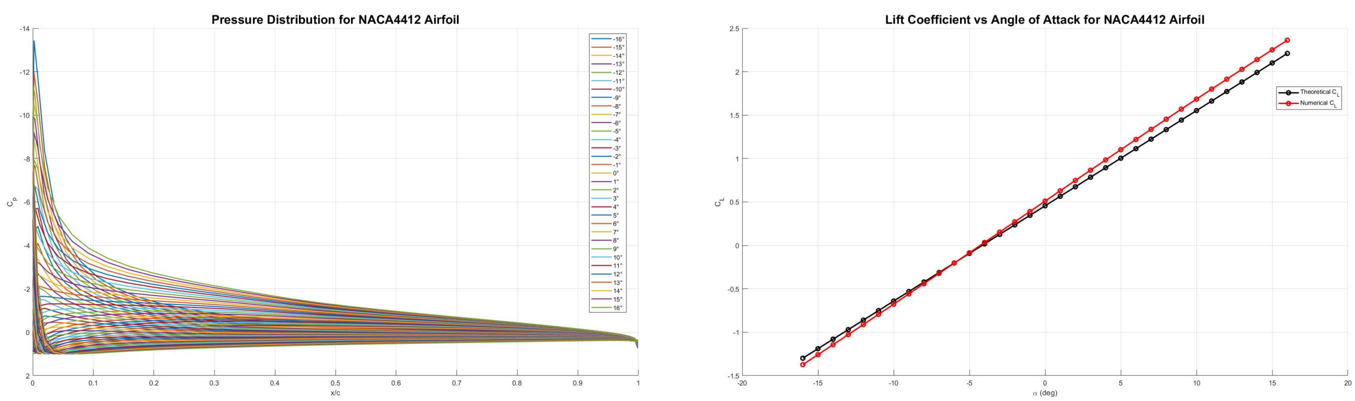
Figure 1: C_p vs x/c and C_l vs α for NACA0012 Airfoil

We can observe that for the C_p vs x/c plot, as α increases, the pressure distribution increases which leads to a higher lift. However, at high angles of attack boundary layer separation is more likely to occur and cause stall. It is also observed that the pressure distribution flips as the angle of attack progresses from negative to positive.

As for the C_l vs α plot, it increases linearly in both numerical and theoretical cases which follows thin airfoil theory closely. However as α increases, there begins to show more deviation from thin airfoil theory which may be attributed to real world conditions and effects such as fluid viscosity and flow separation.

3.2 NACA4412 Airfoil

For the NACA0012, the following plots were attained for the lift and pressure coefficients:



(a) C_p vs x/c

(b) C_l vs α

Figure 2: C_p vs x/c and C_l vs α for NACA4412 Airfoil

For the NACA4412 Airfoil, we can observe that in 2(a) there are key similarities. It is observed that as α increases, the pressure distribution increases which leads to a higher lift. Similarly, at high angles of attack boundary layer separation is more likely to occur and cause stall and C_p flips from negative to positive as α increases.

For the NACA4412 C_l vs α plot, it also increases linearly in both numerical and theoretical cases which follows thin airfoil theory closely. As with the NACA0012 airfoil, it begins to deviate as α increases due to experimental and real world conditions explained above.

4 Stall Angles

The stall angles were obtained as discussed in Section 2.3. For the NACA0012 Airfoil, the stall angle was calculated to be 7° and 10° for the NACA4412. According to literature, from [5], the stall angle was 15° at Re = 170000 for the NACA0012 and 16° for the NACA4412 at the same Re number. The difference in this might have to do with the number of vortex panels used. There was difficulty in using more panels such as 128 as I was limited by the number of pairs of the NACA4412 coordinate points. Experimental factors such as fluid viscosity and turbulence in the air were not taken into account in the code which may explain the low stall angles recorded.

5 Conclusion

This project highlighted the successful implementation of the vortex panel method from Project 2 to be used on two airfoils, the NACA0012 and NACA4412. The pressure and lift coefficient plots were successfully determined. The use of Thwaites Boundary Layer Separation criterion to calculate the stall angles for both airfoils was also successful, although it did not match up with values from literature. The C_l vs α plots for both airfoils successfully followed thin airfoil theory closely but tended to deviate with higher angles of attack. In the future, potentially using 128 vortex panels as opposed to 64 might have provided more accurate results for the stall angle calculations.

References

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[4] S. Martínez-Aranda, A. L. García-González, L. Parras, J. F. Velázquez-Navarro, C. del Pino. *Comparison of the Aerodynamic Characteristics of the NACA0012 Airfoil at Low-to-Moderate Reynolds Numbers for any Aspect Ratio*. International Journal of Aerospace Sciences, Vol. 4 No. 1, 2016, pp. 1-8. doi: 10.5923/j.aerospace.20160401.01.

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