Project #2

Larry Bermudez

Due 3/24/2025

Abstract

This project aimed to determine the pressure distribution around a circular cylinder using 32, 64, and 128 linearly varying vortex panels. This was compared with the distributions of the pressure coefficients C_p of potential flow theory. Two cases were simulated: A Lifting and Non-Lifting case. An angle of θ = -150 and 180 for the Lifting and Non-Lifting case, respectively. Separation points along the circular cylinder were also determined using Thwaite's Boundary Layer Separation Theory.

1 Problem Statement

In this project, the goal was to create a model using the vortex panel method to model the flow around a circular cylinder and plot the pressure coefficient for varying vortex panels as well as to determine the boundary separation point. For the two cases, Lifting (asymmetric flow) and Non-Lifting(symmetric flow), the Kutta condition was placed at θ = -150 and 180, respectively. The next step was to use Thwaites Boundary Layer criterion to determine the boundary separation points.

2 Methodology

Using the potential function of the combined field and vortex panels:

$$\phi(x,y) = Ux\cos\alpha + Uy\sin alpha + \Sigma_j \int_j \frac{\gamma(s_j)}{2\pi}\tan^{-1}\left(\frac{y-y_j}{x-x_j}\right) ds_j, \tag{1}$$

where the vortex panel strength varies linearly

$$\gamma(s_j) = \gamma_j + (\gamma_{j+1} - \gamma_j)\frac{s_j}{S_j} \tag{2}$$

The normal velocity components at control points i vanish

$$\left.\frac{\partial\phi}{\partial n}\right|_i = 0 \tag{3}$$

Implementation of this above equation results in the inversion of the following matrix to determine γ_i in equation (2).

$$\sum_{j=1}^{m} (C_{n1ij}\gamma_j + C_{n2ij}\gamma_{j+1}) = 2\pi U \sin(\theta_i - \alpha) \tag{4}$$

Where the pressure is solved using the following equation

$$C_p = 1 - U_{ti}^2 \tag{5}$$

Coefficients can be found in the class notes. Then, to compare with the pressure coefficient using potential flow theory, the following equation was used

$$C_p = 1 - 4\sin^2\theta_i \tag{6}$$

Then to determine the flow separation points along the circular cylinder Thwaites method was implemented

$$\lambda = \frac{0.45}{U_e(x)^6}\frac{dU}{dx}\int_0^x U_e(\xi)d\xi \tag{7}$$

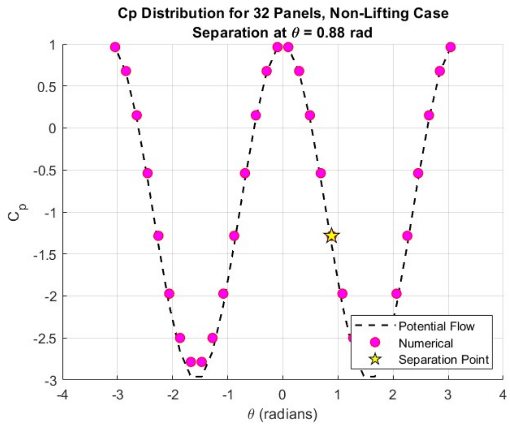
where $\lambda = -0.09$ as this is when boundary separation occurs where U_e is equal to $2\sin\theta$ for Non-Lifting and $2\sin(\theta - \frac{\Gamma}{2\pi R})$ where $\Gamma = -4\pi R\sin\alpha$ for the Lifting case.

3 Model Simulation Results

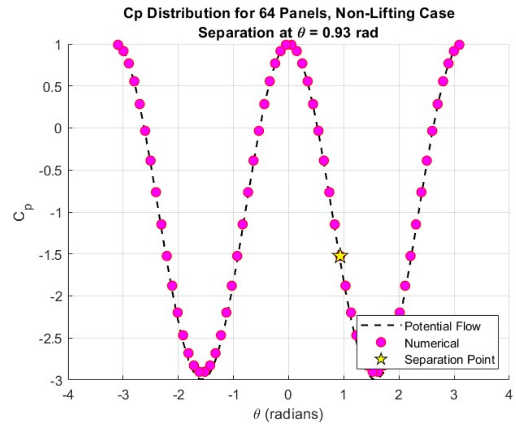
The model was implemented using 32, 64, and 128 linearly varying vortex panels and compared the resulting pressure coefficient to that of potential flow theory. The implementation of Thwaites method was also done to find the separation point for each case.

3.1 Non-Lifting Case

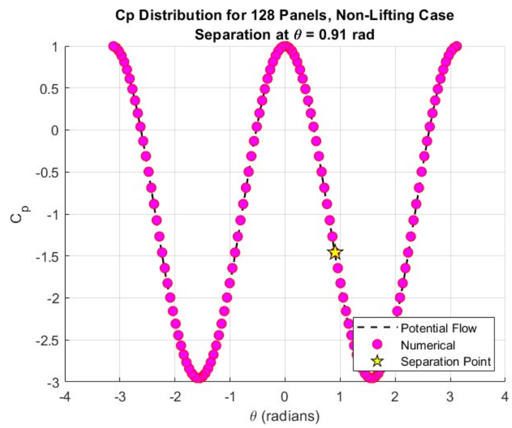
In the non-lifting case, the following plots were attained.



(a) N = 32



(b) N = 64



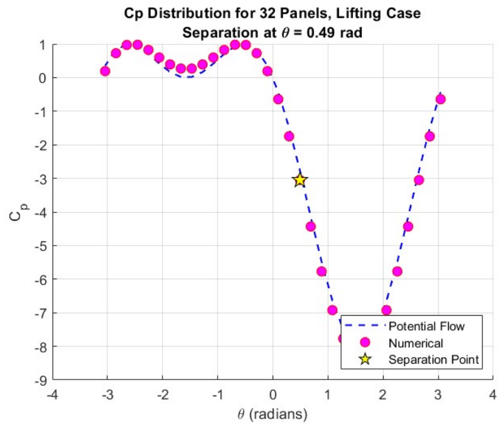
(c) N = 128

Figure 1: C_p for Non-Lifting case with 32, 64, 128 vortex panels

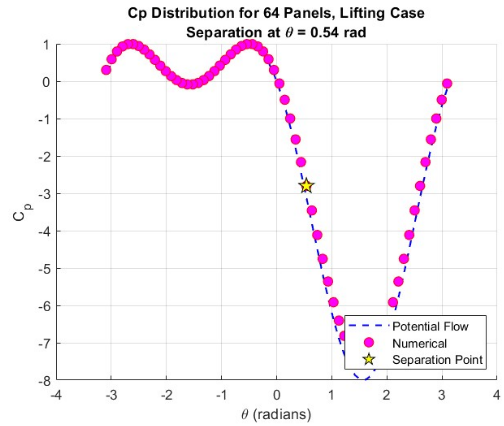
From these plots it is clear that due to the condition of no lift, there is symmetry on both the top and bottom surfaces of the circular cylinder. Also, as the number of panels increases form 32 to 128, it approaches the potential flow theory predictions. The increases number of panels also helps to reduce numerical errors especially when implemented Thwaites method for boundary layer separation calculations. For the non-lift case, the boundary layer separation occured at $\theta = 0.88, 0.93, 0.93$ radians respectively.

3.2 Lifting Case

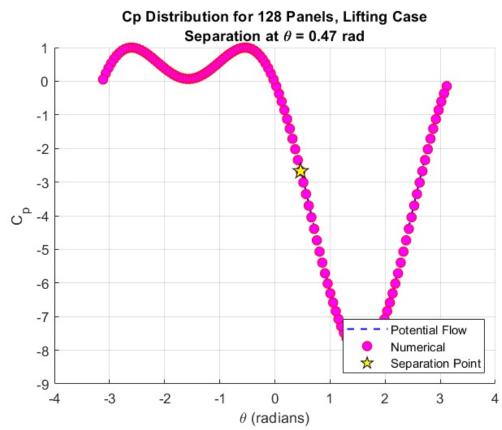
The following plots for the lifting case were attained as well.



(a) N = 32



(b) N = 64



(c) N = 128

Figure 2: C_p for Non-Lifting case with 32, 64, 128 vortex panels

For the lifting case it is evident that there is no symmetry along the cylinder surfaces. This is due to low pressure zone across the top surface and high pressure zone along the bottom surface to induce lift. The separation points were $\theta = 0.49, 0.54, 0.47$ respectively. It is also clear that increasing the number of panels approaches the true C_p value more precisely.

4 Conclusion

This project highlights the use and implementation of the vortex panel method to correctly determine the C_p values for lifting and non-lifting cases. Increasing the number of panels shows that it models each flow case more accurately and precisely. However, implementation of Thwaites method was a challenge and could potentially be incorrect as the θ values attained does not seem entirely correct. In the lifting case the separation point should be closer to around 100-120 degrees. There should also be 2 separation points for the top and bottom surfaces of the cylinder. Although Thwaites Method might have been unsuccessful, correct implementation of the vortex panel method to determine the C_p was acheived.

References

[1] Savas. O *ME 163/Z Engineering Aerodynamics*. Department of Mechanical Engineering at University of California, Berkeley, Spring 2025.

[2] Bermudez. L *Matlab Code*. University of California, Berkeley