Sum of roots and product of roots

Qno.1

Let $p(x) = 2x^5 + x^4 - 26x^3 - 13x^2 + 72x + 36,\ x \in \mathbb{R}$.

a. For the polynomial equation $p(x) = 0$, state

(i) the sum of the roots;

(ii) the product of the roots.

b. A new polynomial is defined by $q(x) = p(x + 4)$.

Find the sum of the roots of the equation $q(x) = 0$.

Qno.2

The quadratic equation $x^2 - 2kx + (k-1) = 0$ has roots α and β such that $\alpha^2 + \beta^2 = 4$. Without solving the equation, find the possible values of the real number k.

Qno.3

Consider the equation $yx^2 + (y-1)x + (y-1) = 0$.

a. Find the set of values of y for which this equation has real roots.

b. Hence determine the range of the function $f : x \to \frac{x+1}{x^2+x+1}$.

c. Explain why f has no inverse.

Qno.4

Consider the polynomial $P(z) = z^5 - 10z^2 + 15z - 6,\ z \in \mathbb{C}$.

The polynomial can be written in the form $P(z) = (z-1)^3(z^2 + bz + c)$.

Consider the function $q(x) = x^5 - 10x^2 + 15x - 6,\ x \in \mathbb{R}$.

a. Write down the sum and the product of the roots of $P(z) = 0$. [2]

b. Show that $(z-1)$ is a factor of $P(z)$. [2]

c. Find the value of b and the value of c. [5]

d. Hence find the complex roots of $P(z) = 0$. [3]

e.i. Show that the graph of $y = q(x)$ is concave up for $x > 1$. [3]

e.ii. Sketch the graph of $y = q(x)$ showing clearly any intercepts with the axes. [3]

Qno.5

The cubic equation $x^3 + px^2 + qx + c = 0$, has roots $\alpha,\ \beta,\ \gamma$. By expanding $(x-\alpha)(x-\beta)(x-\gamma)$ show that

a. (i) $p = -(\alpha + \beta + \gamma)$;

(ii) $q = \alpha\beta + \beta\gamma + \gamma\alpha$;

(iii) $c = -\alpha\beta\gamma$.

b. It is now given that $p = -6$ and $q = 18$ for parts (b) and (c) below.

(i) In the case that the three roots $\alpha,\ \beta,\ \gamma$ form an arithmetic sequence, show that one of the roots is 2.

(ii) Hence determine the value of c.

c. In another case the three roots $\alpha,\ \beta,\ \gamma$ form a geometric sequence. Determine the value of c.

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