Series

Qno.1

$u_4 = u_1 + 3d = 7$, $u_9 = u_1 + 8d = 22$ ***A1A1***

Note: $5d = 15$ gains both above marks

$u_1 = -2$, $d = 3$ ***A1***

$S_n = \frac{n}{2}(-4 + (n-1)3) > 10\,000$ ***M1***

$n = 83$ ***A1***

[5 marks]

Qno.2

we are given that $ar^2 = 9$ and $\frac{a}{1-r} = 64$ ***A1***

dividing, $r^2(1-r) = \frac{9}{64}$ ***M1***

$64r^3 - 64r^2 + 9 = 0$ ***A1***

$r = 0.75,\ a = 16$ ***A1A1***

[5 marks]

Qno.3

a. $S_n = \frac{n}{2}[2a + (n-1)d]$

$212 = \frac{16}{2}(2a + 15d) \quad (= 16a + 120d)$ ***A1***

n^{th} term is $a + (n-1)d$

$8 = a + 4d$ ***A1***

solving simultaneously: ***(M1)***

$d = 1.5, \ a = 2$ ***A1***

[4 marks]

b. $\frac{n}{2}[4 + 1.5(n-1)] > 600$ ***(M1)***

$\Rightarrow 3n^2 + 5n - 2400 > 0$ ***(A1)***

$\Rightarrow n > 27.4..., \ (n < -29.1...)$

[redacted]

Note: Do not penalize improper use of inequalities.

$\Rightarrow n = 28$ ***A1***

[3 marks]

Qno.4

a. $S_{2n} = \frac{2n}{2}\left(2(8) + (2n-1)\frac{1}{4}\right)$ ***(M1)***

$= n\left(16 + \frac{2n-1}{4}\right)$ ***A1***

$S_{3n} = \frac{3n}{2}\left(2 \times 8 + (3n-1)\frac{1}{4}\right)$ ***(M1)***

$= \frac{3n}{2}\left(16 + \frac{3n-1}{4}\right)$ ***A1***

$S_{2n} = S_{3n} - S_{2n} \Rightarrow 2S_{2n} = S_{3n}$ ***M1***

solve $2S_{2n} = S_{3n}$

$\Rightarrow 2n\left(16 + \frac{2n-1}{4}\right) = \frac{3n}{2}\left(16 + \frac{3n-1}{4}\right)$ ***A1***

$\left(\Rightarrow 2\left(16 + \frac{2n-1}{4}\right) = \frac{3}{2}\left(16 + \frac{3n-1}{4}\right)\right)$

(gdc or algebraic solution) ***(M1)***

[redacted]

$n = 63$ ***A2***

[9 marks]

b. $(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_4)^2 + \ldots$

$= (a_1 - a_1 r)^2 + (a_1 r - a_1 r^2)^2 + (a_1 r^2 - a_1 r^3) + \ldots$ ***M1A1***

$= [a_1(1-r)]^2 + [a_1 r(1-r)]^2 + \left[a_1 r^2(1-r)\right]^2 + \ldots + \left[a_1 r^{n-1}(1-r)\right]^2$ ***(A1)***

Note: This ***A1*** is for the expression for the last term.

$= a_1^2(1-r)^2 + a_1^2 r^2(1-r)^2 + a_1^2 r^4(1-r)^2 + \ldots + a_1^2 r^{2n-2}(1-r)^2$ ***A1***

$= a_1^2(1-r)^2(1 + r^2 + r^4 + \ldots + r^{2n-2})$ ***A1***

$= a_1^2(1-r)^2\left(\frac{1-r^{2n}}{1-r^2}\right)$ ***M1A1***

$= \frac{a_1^2(1-r)(1-r^{2n})}{1+r}$ ***AG***

[7 marks]

Qno.5

$2 \times 1.05^{n-1} > 500$ ***M1***

$n - 1 > \frac{\log 250}{\log 1.05}$ ***M1***

$n - 1 > 113.1675\ldots$ ***A1***

$n = 115$ ***(A1)***

$u_{115} = 521$ ***A1*** ***N5***

Note: Accept graphical solution with appropriate sketch.

[5 marks]