Functions

Qno.1

Consider the function $f : x \to \sqrt{\frac{\pi}{4} - \arccos x}$.

(a) Find the largest possible domain of f.

(b) Determine an expression for the inverse function, f^{-1}, and write down its domain.

Qno.2

Consider the equation $yx^2 + (y-1)x + (y-1) = 0$.

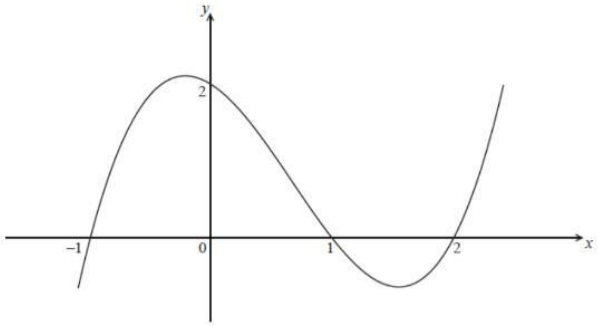
a. Find the set of values of y for which this equation has real roots. [4]

b. Hence determine the range of the function $f : x \to \frac{x+1}{x^2+x+1}$. [3]

c. Explain why f has no inverse. [1]

Qno.3

Let $f(x) = x^3 + ax^2 + bx + c$, where $a, b, c \in \mathbb{Z}$. The diagram shows the graph of $y = f(x)$.



a. Using the information shown in the diagram, find the values of a, b and c. [4]

b. If $g(x) = 3f(x-2)$, [3]

(i) state the coordinates of the points where the graph of g intercepts the x-axis.
(ii) Find the y-intercept of the graph of g.

Qno.4

The function f is defined by $f(x) = \frac{3x}{x-2}$, $x \in \mathbb{R}$, $x \neq 2$.

a. Sketch the graph of $y = f(x)$, indicating clearly any asymptotes and points of intersection with the x and y axes. [4]

b. Find an expression for $f^{-1}(x)$. [4]

c. Find all values of x for which $f(x) = f^{-1}(x)$. [3]

d. Solve the inequality $|f(x)| < \frac{3}{2}$. [4]

e. Solve the inequality $f(|x|) < \frac{3}{2}$. [2]

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Qno.5

Consider the function f defined by $f(x) = x^2 - a^2,\ x \in \mathbb{R}$ where a is a positive constant.

The function g is defined by $g(x) = x\sqrt{f(x)}$ for $|x| > a$.

a.i. Showing any x and y intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes. [2]

$y = f(x)$;

a.ii Showing any x and y intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes. [4]

$y = \frac{1}{f(x)}$;

a.iii Showing any x and y intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes. [2]

$y = \left|\frac{1}{f(x)}\right|$.

b. Find $\int f(x)\cos x \mathrm{d}x$. [5]

c. By finding $g'(x)$ explain why g is an increasing function. [4]

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