Online Science and Maths Tutor

Functions

Qno.1

Markscheme

(a) $\frac{\pi}{4} - \arccos x \geqslant 0$

$\arccos x \leqslant \frac{\pi}{4}$ ***(M1)***

$x \geqslant \frac{\sqrt{2}}{2}$ $\left(\text{accept } x \geqslant \frac{1}{\sqrt{2}}\right)$ ***(A1)***

since $-1 \leqslant x \leqslant 1$ ***(M1)***

$\Rightarrow \frac{\sqrt{2}}{2} \leqslant x \leqslant 1$ $\left(\text{accept } \frac{1}{\sqrt{2}} \leqslant x \leqslant 1\right)$ ***A1***

Note: Penalize the use of $<$ instead of $\leqslant$ only once.

(b) $y = \sqrt{\frac{\pi}{4} - \arccos x} \Rightarrow x = \cos\left(\frac{\pi}{4} - y^2\right)$ ***M1A1***

$f^{-1} : x \to \cos\left(\frac{\pi}{4} - x^2\right)$ ***A1***

$0 \leqslant x \leqslant \sqrt{\frac{\pi}{4}}$ ***A1***

[8 marks]

Qno.2

a. for the equation to have real roots

$(y-1)^2 - 4y(y-1) \geqslant 0$ ***M1***

$\Rightarrow 3y^2 - 2y - 1 \leqslant 0$

(by sign diagram, or algebraic method) ***M1***

$-\frac{1}{3} \leqslant y \leqslant 1$ ***A1A1***

Note: Award first ***A1*** for $-\frac{1}{3}$ and 1, and second ***A1*** for inequalities. These are independent marks.

[4 marks]

b. $f : x \to \frac{x+1}{x^2+x+1} \Rightarrow x + 1 = yx^2 + yx + y$ ***(M1)***

$\Rightarrow 0 = yx^2 + (y-1)x + (y-1)$ ***A1***

hence, from (a) range is $-\frac{1}{3} \leqslant y \leqslant 1$ ***A1***

[3 marks]

c. a value for y would lead to 2 values for x from (a) ***R1***

Note: Do not award ***R1*** if (b) has not been tackled.

[1 mark]

Qno.3

a. **METHOD 1**

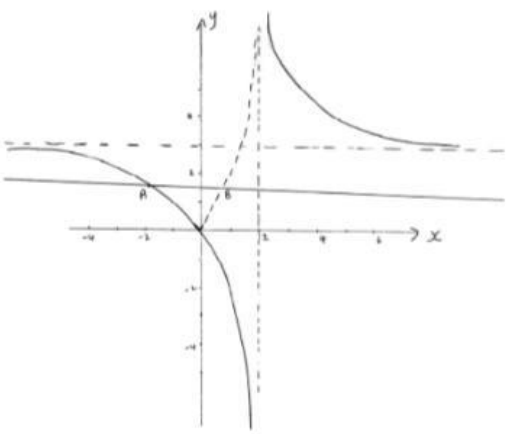
$f(x) = (x+1)(x-1)(x-2)$ ***M1***
$= x^3 - 2x^2 - x + 2$ ***A1A1A1***
$a = -2$, $b = -1$ and $c = 2$
METHOD 2
from the graph or using $f(0) = 2$
$c = 2$ ***A1***
setting up linear equations using $f(1) = 0$ and $f(-1) = 0$ (or $f(2) = 0$) ***M1***
obtain $a = -2$, $b = -1$ ***A1A1***
[4 marks]

b. (i) $(1, 0)$, $(3, 0)$ and $(4, 0)$ ***A1***

(ii) $g(0)$ occurs at $3f(-2)$ ***(M1)***
$= -36$ ***A1***
[3 marks]

Qno.4

a.



Note: In the diagram, points marked A and B refer to part (d) and do not need to be seen in part (a).

shape of curve ***A1***

Note: This mark can only be awarded if there appear to be both horizontal and vertical asymptotes.

intersection at $(0,\ 0)$ ***A1***

horizontal asymptote at $y = 3$ ***A1***

vertical asymptote at $x = 2$ ***A1***

[4 marks]

b. $y = \frac{3x}{x-2}$

$xy - 2y = 3x$ ***M1A1***

$xy - 3x = 2y$

$x = \frac{2y}{y-3}$

$\left(f^{-1}(x)\right) = \frac{2x}{x-3}$ ***M1A1***

Note: Final M1 is for interchanging of x and y, which may be seen at any stage.

[4 marks]

c. **METHOD 1**

attempt to solve $\frac{2x}{x-3} = \frac{3x}{x-2}$ ***(M1)***

$2x(x-2) = 3x(x-3)$

$x\left[2(x-2) - 3(x-3)\right] = 0$

$x(5-x) = 0$

$x = 0$ or $x = 5$ ***A1A1***

METHOD 2

$x = \frac{3x}{x-2}$ or $x = \frac{2x}{x-3}$ ***(M1)***

$x = 0$ or $x = 5$ ***A1A1***

[3 marks]

d. **METHOD 1**

at A : $\frac{3x}{x-2} = \frac{3}{2}$ AND at B : $\frac{3x}{x-2} = -\frac{3}{2}$ ***M1***

$6x = 3x - 6$

$x = -2$ ***A1***

$6x = 6 - 3x$

$x = \frac{2}{3}$ ***A1***

solution is $-2 < x < \frac{2}{3}$ ***A1***

METHOD 2

$\left(\frac{3x}{x-2}\right)^2 < \left(\frac{3}{2}\right)^2$ ***M1***

$9x^2 < \frac{9}{4}(x-2)^2$

$3x^2 + 4x - 4 < 0$

$(3x-2)(x+2) < 0$

$x = -2$ ***(A1)***

$x = \frac{2}{3}$ ***(A1)***

solution is $-2 < x < \frac{2}{3}$ ***A1***

[4 marks]

e. $-2 < x < 2$ ***A1A1***

Note: ***A1*** for correct end points, ***A1*** for correct inequalities.

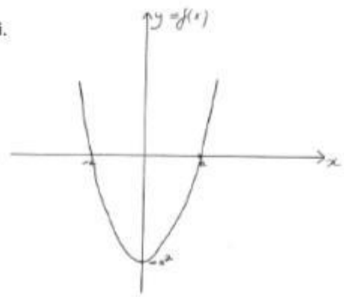
Note: If working is shown, then ***A*** marks may only be awarded following correct working.

[2 marks]

Total [17 marks]

Qno.5

a.i.

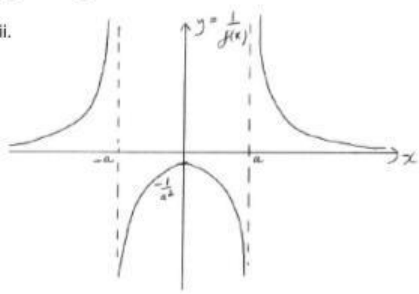


A1 for correct shape

A1 for correct x and y intercepts and minimum point

[2 marks]

a.ii.



A1 for correct shape

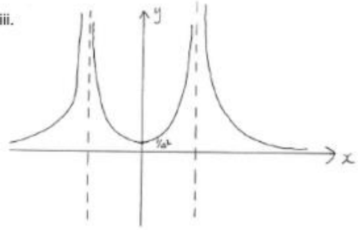
A1 for correct vertical asymptotes

A1 for correct implied horizontal asymptote

A1 for correct maximum point

[??? marks]

a.iii.



A1 for reflecting negative branch from (ii) in the x-axis

A1 for correctly labelled minimum point

[2 marks]

b. **EITHER**

attempt at integration by parts ***(M1)***

$\int (x^2 - a^2) \cos x \mathrm{d}x = (x^2 - a^2) \sin x - \int 2x \sin x \mathrm{d}x$ ***A1A1***

$= (x^2 - a^2) \sin x - 2 \left[-x \cos x + \int \cos x \mathrm{d}x\right]$ ***A1***

$= (x^2 - a^2) \sin x + 2x \cos -2 \sin x + c$ ***A1***

OR

$\int (x^2 - a^2) \cos x \mathrm{d}x = \int x^2 \cos x \mathrm{d}x - \int a^2 \cos x \mathrm{d}x$

attempt at integration by parts ***(M1)***

$\int x^2 \cos x \mathrm{d}x = x^2 \sin x - \int 2x \sin x \mathrm{d}x$ ***A1A1***

$= x^2 \sin x - 2 \left[-x \cos x + \int \cos x \mathrm{d}x\right]$ ***A1***

$= x^2 \sin x + 2x \cos x - 2 \sin x$

$- \int a^2 \cos x \mathrm{d}x = -a^2 \sin x$

$\int (x^2 - a^2) \cos x \mathrm{d}x = (x^2 - a^2) \sin x + 2x \cos x - 2 \sin x + c$ ***A1***

[5 marks]

c. $g(x) = x(x^2 - a^2)^{\frac{1}{2}}$

$g'(x) = (x^2 - a^2)^{\frac{1}{2}} + \frac{1}{2} x (x^2 - a^2)^{-\frac{1}{2}} (2x)$ ***M1A1A1***

Note: Method mark is for differentiating the product. Award ***A1*** for each correct term.

$g'(x) = (x^2 - a^2)^{\frac{1}{2}} + x^2 (x^2 - a^2)^{-\frac{1}{2}}$

both parts of the expression are positive hence $g'(x)$ is positive ***R1***

and therefore g is an increasing function (for $|x| > a$) ***AG***

[4 marks]