Bionomial Expansion

Qno.1

Markscheme

a. $(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$ ***(M1)A1***

[2 marks]

b. $f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$ ***(M1)***

$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$

$= \lim_{h \to 0} (3x^2 + 3xh + h^2)$ ***A1***

$= 3x^2$ ***A1***

Note: Do not award final A1 on FT if $= 3x^2$ is not obtained

Note: Final A1 can only be obtained if previous A1 is given

[3 marks]

Total [5 marks]

Qno.2

Markscheme

a. using the factor theorem $z+1$ is a factor ***(M1)***

$z^3+1=(z+1)(z^2-z+1)$ ***A1***

[2 marks]

b. (i) **METHOD 1**

$z^3=-1 \Rightarrow z^3+1=(z+1)(z^2-z+1)=0$ ***(M1)***

solving $z^2-z+1=0$ ***M1***

$z=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm i\sqrt{3}}{2}$ ***A1***

therefore one cube root of -1 is γ ***AG***

METHOD 2

$\gamma^2=\left(\frac{1+i\sqrt{3}}{2}^2\right)=\frac{-1+i\sqrt{3}}{2}$ ***M1A1***

$\gamma^2=\frac{-1+i\sqrt{3}}{2}\times\frac{1+i\sqrt{3}}{2}=\frac{-1-3}{4}$ ***A1***

$=-1$ AG

METHOD 3

$\gamma=\frac{1+i\sqrt{3}}{2}=e^{i\frac{\pi}{3}}$ ***M1A1***

$\gamma^3=e^{i\pi}=-1$ ***A1***

(ii) **METHOD 1**

as γ is a root of $z^2-z+1=0$ then $\gamma^2-\gamma+1=0$ ***M1R1***

$\therefore \gamma^2=\gamma-1$ ***AG***

[redacted]

Note: Award ***M1*** for the use of $z^2-z+1=0$ in any way.

Award ***R1*** for a correct reasoned approach.

METHOD 2

$\gamma^2=\frac{-1+i\sqrt{3}}{2}$ ***M1***

$\gamma-1=\frac{1+i\sqrt{3}}{2}-1=\frac{-1+i\sqrt{3}}{2}$ ***A1***

(iii) **METHOD 1**

$(1-\gamma)^6=(-\gamma^2)^6$ ***(M1)***

$=(\gamma)^{12}$ ***A1***

$=(\gamma^3)^4$ ***(M1)***

$=(-1)^4$

$=1$ ***A1***

METHOD 2

$(1-\gamma)^6$

$=1-6\gamma+15\gamma^2-20\gamma^3+15\gamma^4-6\gamma^5+\gamma^6$ ***M1A1***

Note: Award ***M1*** for attempt at binomial expansion.

use of any previous result *e.g.* $=1-6\gamma+15\gamma^2+20-15\gamma+6\gamma^2+1$ ***M1***

$=1$ ***A1***

Note: As the question uses the word 'hence', other methods that do not use previous results are awarded no marks.

[9 marks]

Qno.3

clear attempt at binomial expansion for exponent 5 ***M1***

$2^5 + 5 \times 2^4 \times (-3x) + \frac{5\times4}{2} \times 2^3 \times (-3x)^2 + \frac{5\times4\times3}{6} \times 2^2 \times (-3x)^3$

$+\frac{5\times4\times3\times2}{24} \times 2 \times (-3x)^4 + (-3x)^5$ ***(A1)***

Note: Only award ***M1*** if binomial coefficients are seen.

$= 32 - 240x + 720x^2 - 1080x^3 + 810x^4 - 243x^5$ ***A2***

Note: Award ***A1*** for correct moduli of coefficients and powers. ***A1*** for correct signs.

Total [4 marks]

Qno.4

each term is of the form $\binom{7}{r}(x^2)^{7-r}\left(\frac{-2}{x}\right)^r$ ***(M1)***

$= \binom{7}{r} x^{14-2r}(-2)^r x^{-r}$

[redacted]

so $14 - 3r = 8$ ***(A1)***

$r = 2$

so require $\binom{7}{2}(x^2)^5\left(\frac{-2}{x}\right)^2$ (or simply $\binom{7}{2}(-2)^2$) ***A1***

$= 21 \times 4$

$= 84$ ***A1***

Note: Candidates who attempt a full expansion, including the correct term, may only be awarded ***M1A0A0A0***.

[4 marks]

Qno.5

$$\left(\frac{x}{y}-\frac{y}{x}\right)^4=\left(\frac{x}{y}\right)^4+4\left(\frac{x}{y}\right)^3\left(-\frac{y}{x}\right)+6\left(\frac{x}{y}\right)^2\left(-\frac{y}{x}\right)^2+4\left(\frac{x}{y}\right)\left(-\frac{y}{x}\right)^3+\left(-\frac{y}{x}\right)^4$$ ***(M1)(A1)***

Note: Award ***M1*** for attempt to expand and ***A1*** for correct unsimplified expansion.

$$=\frac{x^4}{y^4}-4\frac{x^2}{y^2}+6-4\frac{y^2}{x^2}+\frac{y^4}{x^4}\quad\left(=\frac{x^8-4x^6y^2+6x^4y^4-4x^2y^6+y^8}{x^4y^4}\right)$$ ***A1A1***

Note: Award ***A1*** for powers, ***A1*** for coefficients and signs.

Note: Final two ***A*** marks are independent of first ***A*** mark.

[4 marks]

Qno.6

METHOD 1

constant term: $\binom{5}{0}(-2x)^0\binom{7}{0}x^0 = 1$ *A1*

term in x: $\binom{7}{1}x + \binom{5}{1}(-2x) = -3x$ *(M1)A1*

term in x^2 : $\binom{7}{2}x^2 + \binom{5}{2}(-2x)^2 + \binom{7}{1}x\binom{5}{1}(-2x) = -9x^2$ *M1A1* *N3*

[5 marks]

METHOD 2

$(1-2x)^5(1+x)^7 = \left(1+5(-2x)+\frac{5\times4(-2x)^2}{2!}+\ldots\right)\left(1+7x+\frac{7\times6}{2}x^2+\ldots\right)$ *M1M1*

$= (1-10x+40x^2+\ldots)(1+7x+21x^2+\ldots)$

$= 1+7x+21x^2-10x-70x^2+40x^2+\ldots$

$= 1-3x-9x^2+\ldots$ *A1A1A1* *N3*

[5 marks]