Mathematical induction, Geometric and Arithmetic sequence

Qno.1

a. Find the sum of the infinite geometric sequence $27, -9, 3, -1, \ldots$. [3]

b. Use mathematical induction to prove that for $n \in \mathbb{Z}^+$, [7]

$$a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}.$$

Qno.2

Use mathematical induction to prove that $n(n^2+5)$ is divisible by 6 for $n \in \mathbb{Z}^+$.

(8 marks)

Qno.3

An 81 metre rope is cut into n pieces of increasing lengths that form an arithmetic sequence with a common difference of d metres. Given that the lengths of the shortest and longest pieces are 1.5 metres and 7.5 metres respectively, find the values of n and d.

(marks 4)

Qno.4

(a) The sum of the first six terms of an arithmetic series is 81. The sum of its first eleven terms is 231. Find the first term and the common difference. [14]

(b) The sum of the first two terms of a geometric series is 1 and the sum of its first four terms is 5. If all of its terms are positive, find the first term and the common ratio.

(c) The r^{th} term of a new series is defined as the product of the r^{th} term of the arithmetic series and the r^{th} term of the geometric series above. Show that the r^{th} term of this new series is $(r+1)2^{r-1}$.

d. Using mathematical induction, prove that [7]

$$\sum_{r=1}^{n}(r+1)2^{r-1} = n2^n,\ n \in \mathbb{Z}^+.$$

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Qno.5

The 1st, 4th and 8th terms of an arithmetic sequence, with common difference d, $d \neq 0$, are the first three terms of a geometric sequence, with common ratio r. Given that the 1st term of both sequences is 9 find

a. the value of d; [4]

b. the value of r; [1]

Qno.6

A geometric sequence has first term a, common ratio r and sum to infinity 76. A second geometric sequence has first term a, common ratio r^3 and sum to infinity 36.

Find r.

(7 marks)