

Units, Dimensions, and Measurements Formulae for NEET & JEE

by AP Sir, Director Sakaar PCMB Classes

Formula /
Topic Name

Formula(e) & Expressions

Conditions & Usage Notes

1. Dimensional Analysis

Fundamental Quantities

Mass [M], Length [L], Time [T], Current [A],
Temp [K], Amount of Substance [mol],
Luminous Intensity [cd]

Base dimensions from which all
others are derived.

Order of Magnitude

Express number as $N = a \times 10^b$

Used to estimate the size of a
quantity.

If $a \leq \sqrt{10}$ (≈ 3.16), Order = b

Example: $4 \times 10^5 \rightarrow$ Since
 $4 > 3.16$, Order is 10^6 .

If $a > \sqrt{10}$, Order = $b + 1$

2. Significant Figures

Counting Rules

1. All non-zero digits are significant.

0.007 (1 SF)

2. Zeros between non-zeros are significant.

2.05 (3 SF)

3. Leading zeros are **never** significant.

2.500 (4 SF)

4. Trailing zeros with a decimal point are
significant.

1200 (Ambiguous, assume 2 unless
specified).

Rounding Off

- Digit > 5 : Round up

Example (= 5):

- Digit < 5 : No change

$2.45 \rightarrow 2.4$ (4 is even)

| | | |
|------------------------------|---|---|
| | - Digit = 5: Round to nearest even number. | 2.35 → 2.4 (round up to even) |
| Arithmetic Operations | Add/Sub: Result has same decimal places as the least precise term. | Add: $12.11 + 18.0 = 30.1$ (1 dec. place) |
| | Mul/Div: Result has same sig figs as the least precise term. | Mul: $2.5 \times 1.25 = 3.1$ (2 SF) |
| 3. Vernier Caliper | | |
| Least Count (L.C.) | $L.C. = 1MSD - 1VSD$ | Where N is total divisions on Vernier scale. |
| | Standard: $L.C. = \frac{1MSD}{N}$ | Common L.C. = 0.1 mm or 0.01 cm. |
| Reading | $Reading = MSR + (VSR \times L.C.)$ | MSR: Main Scale Reading immediately left of zero. |
| | | VSR: Coinciding Vernier division. |
| Zero Error | True Reading = Observed – Zero Error | Negative Error Calculation: |
| | Positive: Zero of VS is right of MS zero. | Error $= -(N - \text{coinciding div}) \times L.C.$ |
| | Negative: Zero of VS is left of MS zero. | |
| 4. Screw Gauge | | |
| Pitch | $Pitch = \frac{\text{Distance moved on Main Scale}}{\text{Number of full rotations}}$ | Usually 1 mm or 0.5 mm. Distance screw moves in 1 rotation. |
| Least Count (L.C.) | $L.C. = \frac{\text{Pitch}}{\text{Total Circular Scale Divisions (CSD)}}$ | Common L.C. = 0.01 mm or 0.001 cm. |
| Reading | $Reading = MSR + (CSR \times L.C.)$ | MSR: Reading visible on linear scale. |

CSR: Circular division coinciding with reference line.

Zero Error

Positive: Zero of CS is below reference line.

Always subtract the error (keeping signs in mind).

Negative: Zero of CS is above reference line.

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Topic: Vectors (Physics)

| Formula Name / Topic | Formula(e) | Conditions / Usage |
|---|---|--|
| 1. Magnitude of a Vector | If $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$: | Used to find the size/length of a vector from its Cartesian components. |
| | $ \vec{A} = \sqrt{A_x^2 + A_y^2 + A_z^2}$ | |
| 2. Unit Vector | $\hat{n} = \frac{\vec{A}}{ \vec{A} }$ | Represents direction only. Magnitude is always 1. |
| 3. Vector Addition (Parallelogram Law) | Resultant (R): | θ is the angle between \vec{A} and \vec{B} (tail-to-tail). |
| | $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$ | $R_{max} = A + B$ (at 0°), $R_{min} = A - B $ (at 180°). |
| | Direction (α with \vec{A}): | |
| | $\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$ | |
| 4. Vector Subtraction | Magnitude: | Used for relative velocity ($\Delta \vec{v}$). θ is the angle between original vectors. |
| | $ \vec{A} - \vec{B} = \sqrt{A^2 + B^2 - 2AB \cos \theta}$ | |
| | Direction: | |
| | $\tan \alpha = \frac{B \sin \theta}{A - B \cos \theta}$ | |
| 5. Resolution of Components | $A_x = A \cos \theta$ | θ is the angle made with the X-axis. |

$$A_y = A \sin \theta$$

6. Direction Cosines $l = \frac{A_x}{A}, \quad m = \frac{A_y}{A}, \quad n = \frac{A_z}{A}$ l, m, n are cosines of angles with X, Y, Z axes.

$$l^2 + m^2 + n^2 = 1$$

7. Dot Product (Scalar) $\vec{A} \cdot \vec{B} = AB \cos \theta$ Result is Scalar.

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Perpendicular if: $\vec{A} \cdot \vec{B} = 0$.

8. Angle Between Vectors $\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$ Vectors must be tail-to-tail.

9. Cross Product (Vector) **Magnitude:** $|\vec{A} \times \vec{B}| = AB \sin \theta$ Result is Vector \perp to \vec{A} and \vec{B} .

Parallel if: $\vec{A} \times \vec{B} = 0$.

Determinant Form:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

10. Lami's Theorem $\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}$ Only for 3 concurrent forces in equilibrium.

11. Relative Velocity $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ Velocity of A w.r.t B.

12. Rain-Man Concept $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$ θ with vertical.

$$\tan \theta = \frac{v_m}{v_r}$$

13. River Boat: Min Time $t_{min} = \frac{d}{v_b}$ Head perpendicular to flow.

$$\text{Drift } x = v_r \times t_{min}$$

14. River Boat: Shortest Path $\sin \theta = \frac{v_r}{v_b}$ Head upstream at angle θ .

$$t = \frac{a}{\sqrt{v_b^2 - v_r^2}}$$
 Cond: $v_b > v_r$.

15. Area of Triangle $\text{Area} = \frac{1}{2} |\vec{A} \times \vec{B}|$ \vec{A}, \vec{B} are adjacent sides.

16. Area of Parallelogram Sides: $|\vec{A} \times \vec{B}|$

Diagonals: $\frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$

17. Vol. of Parallelepiped $V = |\vec{A} \cdot (\vec{B} \times \vec{C})|$ Coplanar if Volume = 0.

$$V = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

Motion in One Dimension - Important Formulae for NEET & JEE

BY AP Sir, Sakaar Classes

| Topic / Formula Name | Formula(e) | Conditions / Notes |
|--|--|--|
| 1. Average Velocity & Speed | $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$ | Valid for any type of motion (uniform or non-uniform). Δx is displacement. |
| | $\text{Speed}_{avg} = \frac{\text{Total Path Length}}{\text{Total Time}}$ | |
| Special Case: Equal Distances | $v_{avg} = \frac{2v_1 v_2}{v_1 + v_2}$ | When a body covers equal distances with different speeds v_1 and v_2 . (Harmonic Mean) |
| Special Case: Equal Time Intervals | $v_{avg} = \frac{v_1 + v_2}{2}$ | When a body travels for equal time intervals with different speeds v_1 and v_2 . (Arithmetic Mean) |
| 2. Instantaneous Velocity & Speed | $\vec{v}_{inst} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$ | $\text{vec}\{v\}_{inst}$ |
| | $\text{Speed}_{inst} =$ | |
| 3. Instantaneous Acceleration | $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ | Used for variable acceleration. The form $v(dv/dx)$ is crucial when v is a function of x . |
| | $a = v \frac{dv}{dx}$ | |

4. Equations of Kinematics (Scalar Form)

1.

$$v = u + at$$

STRICTLY for Constant Acceleration ($a = \text{const.}$).

u : initial velocity, v : final velocity, a : acceleration, s : displacement, t : time.

2.

$$s = ut + \frac{1}{2}at^2$$

3.

$$v^2 = u^2 + 2as$$

4.

$$s = \left(\frac{u + v}{2} \right) t$$

5. Displacement in n^{th} Second

$$S_{n^{\text{th}}} = u + \frac{a}{2}(2n - 1)$$

Displacement covered strictly during the n^{th} second of motion. Only for constant acceleration.

6. Motion Under Gravity (Free Fall)

1.

$$v = u - gt$$

Sign Convention (Upward +ve):

$a = -g$, Displacement h is +ve if up, -ve if down.

2.

$$h = ut - \frac{1}{2}gt^2$$

For dropped object: $u = 0$.

3.

$$v^2 = u^2 - 2gh$$

7. Max Height & Time of Flight

$$H_{\max} = \frac{u^2}{2g}$$

For a particle thrown vertically upward with speed u returning to the same level.

$$T_{flight} = \frac{2u}{g}$$

8. Stopping Distance

$$d_{stop} = \frac{u^2}{2a}$$

Distance traveled before coming to rest ($v = 0$) with retardation a .

9. Relative Velocity in 1D

$$v_{AB} = v_A - v_B$$

Velocity/Acceleration of A with respect to B. Signs are crucial (+ve for one direction, -ve for opposite).

$$a_{AB} = a_A - a_B$$

10. Graphical Interpretations

1. Slope of $x - t$ graph = Velocity (v) Valid for all types of motion.

2. Slope of $v - t$ graph = Acceleration
(a)

3. Area under $v - t$ graph =
Displacement (Δx)

4. Area under Speed-time graph =
Distance

5. Area under $a - t$ graph = Change in
velocity (Δv)

11. Variable Acceleration (Integration)

$$v = \int a \, dt$$

Use definite integrals with limits when acceleration is a function of time ($a = f(t)$).

$$x = \int v \, dt$$

Key Tips for Solving Problems:

- **Sign Convention:** Always choose a positive direction (usually right or up). Any vector (displacement, velocity, acceleration) opposite to this is negative.

- **Vector Form:** For complex problems, use $\vec{v} = \vec{u} + \vec{a}t$.

- **Differentiation vs Integration:**

- $x \xrightarrow{\text{diff}} v \xrightarrow{\text{diff}} a$

- $a \xrightarrow{\text{int}} v \xrightarrow{\text{int}} x$

Motion in 2D and Projectile Motion

BY AP Sir, Sakaar Classes

| Formula Name / Topic | Formula(e) | Conditions / Usage |
|----------------------|------------|--------------------|
|----------------------|------------|--------------------|

1. General Motion in 2D

Position Vector $\vec{r} = x\hat{i} + y\hat{j}$ Cartesian coordinates at time t

Velocity Vector $\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j}$ Instantaneous velocity

Acceleration Vector $\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j}$ Instantaneous acceleration

2. Projectile Motion (Ground to Ground)

Assumptions: Air resistance neglected, g is constant downwards.

Components of Initial Velocity $u_x = u \cos \theta$ θ is angle with horizontal

$$u_y = u \sin \theta$$

Motion Parameters (x & y) $a_x = 0, v_x = u \cos \theta$ (constant) x -motion is uniform; y -motion is under gravity

$$a_y = -g$$

Time of Flight (T) $T = \frac{2u \sin \theta}{g} = \frac{2u_y}{g}$ Projectile lands at same vertical level as launch

Maximum Height (H) $H = \frac{u^2 \sin^2 \theta}{2g} = \frac{u_y^2}{2g}$ Vertical component of velocity becomes zero at max height

Horizontal Range (R) $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x u_y}{g}$ Distance between launch and landing point on same level

Condition for Max Range $\theta = 45^\circ \Rightarrow R_{max} = \frac{u^2}{g}$ For a given initial speed u

Relation between H and R

$$R \tan \theta = 4H$$

Useful for direct relation problems

Equation of Trajectory

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

To find y given x (eliminating t)

OR

$$y = x \tan \theta \left(1 - \frac{x}{R} \right)$$

Instantaneous Velocity

$$\vec{v} = u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j} \quad \text{\textbackslash vec\{v\}}$$

\$

Angle with Horizontal (α)

$$\tan \alpha = \frac{v_y}{v_x} = \frac{u \sin \theta - gt}{u \cos \theta}$$

Direction of motion at time t

3. Horizontal Projectile (From Tower)

Condition: Thrown horizontally ($u_y = 0$) from height h

Time of Flight

$$T = \sqrt{\frac{2h}{g}}$$

Time to reach ground

Horizontal Range

$$R = u \times T = u \sqrt{\frac{2h}{g}}$$

Horizontal distance covered

Velocity at Ground

$$v = \sqrt{u^2 + 2gh}$$

Conservation of energy or vector sum

4. Projectile on Inclined Plane

Incline angle α , Projection angle θ (w.r.t incline)

Time of Flight (Incline)

$$T = \frac{2u \sin \theta}{g \cos \alpha}$$

Component of g perpendicular to plane is $g \cos \alpha$

Range on Incline (R_{inc})

$$R_{inc} = \frac{2u^2 \sin \theta \cos(\theta + \alpha)}{g \cos^2 \alpha}$$

Projecting UP the incline

Range on Incline (R_{inc})

$$R_{inc} = \frac{2u^2 \sin \theta \cos(\theta - \alpha)}{g \cos^2 \alpha}$$

Projecting DOWN the incline

5. Relative Motion in 2D

Relative Velocity

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

Velocity of A with respect to B

Rain-Man Problem

$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$$

\vec{v}_{rm} is how rain appears to the man

River-Boat: Crossing River

$$\vec{v}_{b,g} = \vec{v}_{b,r} + \vec{v}_{r,g}$$

$\vec{v}_{b,r}$ = velocity of boat in still water

Condition: Shortest Path

$$\sin \theta = \frac{v_r}{v_{br}} \text{ (upstream)}$$

Drift = 0 (Requires $v_{br} > v_r$)

Newton's Laws of Motion

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| Formula Name / Topic | Formula(e) | Conditions / Usage |
|---|--|--|
| Newton's Second Law (General) | $\vec{F}_{ext} = \frac{d\vec{p}}{dt}$ | Valid for all systems (even variable mass). |
| Newton's Second Law (Constant Mass) | $\vec{F}_{net} = m\vec{a}$ | Valid only when mass m is constant and in an Inertial Frame. |
| Linear Momentum | $\vec{p} = m\vec{v}$ | Quantity of motion contained in a body. |
| Impulse (J) | $\vec{J} = \int_{t_1}^{t_2} \vec{F}_{ext} dt = \Delta\vec{p}$ | Used when a large force acts for a short time. $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$. |
| | $\vec{J} = \vec{F}_{avg} \cdot \Delta t$ | |
| Impulse-Momentum Theorem | Area under $F - t$ graph = Δp | Used to find change in momentum from a Force-Time graph. |
| Equilibrium of Forces | $\sum \vec{F} = 0 \implies \vec{a} = 0$ | Body is at rest or moving with constant velocity. |
| Lami's Theorem | $\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$ | Valid for 3 coplanar, concurrent forces in equilibrium. |
| Third Law (Action- Reaction) | $\vec{F}_{AB} = -\vec{F}_{BA}$ | Action and reaction act on different bodies simultaneously. |
| Apparent Weight in Lift (Moving Up) | $N = m(g + a)$ | Lift accelerating upwards with acceleration a . |
| Apparent Weight in Lift (Moving Down) | $N = m(g - a)$ | Lift accelerating downwards with acceleration a ($a < g$). |

Apparent Weight (Free Fall)

$$N = 0$$

Lift cable breaks ($a = g$).
Weightlessness.

Conservation of Linear Momentum

$$\vec{p}_{initial} = \vec{p}_{final}$$

Valid if net external force on the system is zero ($\vec{F}_{ext} = 0$).

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Rocket Propulsion (Thrust & Accel)

$$F_{thrust} = u_{rel} \left(-\frac{dm}{dt} \right)$$

u_{rel} is exhaust speed relative to rocket.
 $-\frac{dm}{dt}$ is rate of fuel consumption.

$$a = \frac{u_{rel}}{m} \left(-\frac{dm}{dt} \right) - g$$

Rocket Velocity (at time t)

$$v = u_{rel} \ln \left(\frac{m_0}{m_t} \right) - gt$$

m_0 : initial mass, m_t : mass at time t .
Neglecting initial velocity v_0 .

Force by Liquid Jet (Thrust on Pipe)

$$F = v \frac{dm}{dt} = \rho A v^2$$

Reaction force on a pipe ejecting liquid of density ρ through area A .

Force by Liquid Jet (Striking Wall)

$$F = \rho A v^2$$

Force exerted by a jet striking a vertical wall normally.

(Stops)

$$F = 2\rho A v^2$$

(Reflects)

Connected Bodies (Atwood Machine)

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g$$

Massless, frictionless pulley and string.
 $m_2 > m_1$.

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2} \right) g$$

Block on Smooth Inclined Plane

$$a = g \sin \theta$$

Sliding down a frictionless incline of angle θ .

$$N = mg \cos \theta$$

Static Friction (f_s)

$$f_s \leq \mu_s N$$

Self-adjusting force. Prevents relative motion. N is Normal reaction.

$$f_{s,max} = \mu_s N$$

(Limiting Friction)

Kinetic Friction (f_k)

$$f_k = \mu_k N$$

Opposes relative motion when bodies are actually sliding.

Angle of Friction (λ)

$$\tan \lambda = \mu_s$$

Angle between Normal reaction and Resultant of contact forces.

Angle of Repose (α)

$$\tan \alpha = \mu_s$$

Min angle of incline at which block starts sliding. ($\alpha = \lambda$).

Acceleration on Rough Incline (Down)

$$a = g(\sin \theta - \mu_k \cos \theta)$$

Block sliding down a rough inclined plane.

Acceleration on Rough Incline (Up)

$$a = g(\sin \theta + \mu_k \cos \theta)$$

Block pushed up a rough inclined plane (retardation).

Centripetal Force

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

Net radial force required for circular motion directed towards center.

Safe Turn on Level Road

$$v_{max} = \sqrt{\mu_s r g}$$

Vehicle turning on a flat horizontal road. Friction provides centripetal force.

Banking of Roads (Smooth)

$$\tan \theta = \frac{v^2}{rg}$$

Friction ignored. Ideal banking angle.

Banking of Roads (With Friction)

$$v_{max} = \sqrt{rg \left(\frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta} \right)}$$

Maximum safe speed on a banked rough road.

Bending of Cyclist

$$\tan \theta = \frac{v^2}{rg}$$

Cyclist leans inward to provide necessary centripetal force.

Pseudo Force

Applied to an object when observing

Work, Power, and Energy

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| Formula Name / Topic | Formula | Condition / Notes |
|--|--|---|
| 1. Work Done (Constant Force) | $W = \vec{F} \cdot \vec{S} = FS \cos \theta$ | Force \vec{F} is constant. θ is angle between \vec{F} and displacement \vec{S} . |
| 2. Work Done (Variable Force) | $W = \int_{x_1}^{x_2} F_x dx$ $W = \int \vec{F} \cdot d\vec{r}$ | Force varies with position. |
| 3. Work from Graph | $W = \text{Area under } F-x \text{ graph}$ | Area between force curve and displacement axis (Area above axis is +, below is -). |
| 4. Kinetic Energy (KE) | $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ | $p = mv$ is linear momentum. Relation between KE and Momentum is crucial. |
| 5. Work-Energy Theorem | $W_{\text{net}} = \Delta K = K_f - K_i$ | Valid for all frames (inertial/non-inertial). W_{net} is work by ALL forces (conservative, non-conservative, pseudo). |
| 6. Gravitational Potential Energy (PE) | $U = mgh$ | Near Earth's surface where g is constant. Ref level at ground ($U = 0$). |
| 7. Spring Potential Energy | $U = \frac{1}{2}kx^2$ | x is elongation or compression from natural length . k is spring constant. |
| 8. Conservative Force & PE Relation | $F = -\frac{dU}{dx}$ $\Delta U = -W_{\text{conservative}}$ | Only defined for conservative forces (Gravity, Electrostatic, Spring). |

| | | |
|--|--|--|
| 9. Conservation of Mechanical Energy | $K_i + U_i = K_f + U_f$ | Condition: Only conservative forces do work ($W_{ext} = 0, W_{nc} = 0$). |
| 10. Work by Non-Conservative Forces | $W_{nc} = \Delta E_{mech} = (K_f + U_f) - (K_i + U_i)$ | Used when friction or air resistance is present. |
| 11. Average Power | $P_{avg} = \frac{\Delta W}{\Delta t}$ | Total work done divided by total time taken. |
| 12. Instantaneous Power | $P = \vec{F} \cdot \vec{v} = Fv \cos \theta$ | Rate of work at a specific instant. |
| 13. Vertical Circular Motion (Critical) | $v_{top} \geq \sqrt{gR}, v_{bottom} \geq \sqrt{5gR}$ | Condition to complete a full vertical circle (String/Loop). |
| 14. Vertical Circular Motion (Tension) | $T_{bottom} - T_{top} = 6mg$ | Difference in tension between lowest and highest point. |
| 15. Coefficient of Restitution (e) | $e = \frac{v_{sep}}{v_{app}} = \frac{v_2 - v_1}{u_1 - u_2}$ | $e = 1$ (Elastic), $0 < e < 1$ (Inelastic), $e = 0$ (Perfectly Inelastic). Along Line of Impact. |
| 16. Elastic Collision (1D) | $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left(\frac{2m_2}{m_1 + m_2} \right) u_2$ | Momentum and KE are conserved. |
| 17. Perfectly Inelastic Collision | $V_{common} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$ | Bodies stick together ($e = 0$). Max loss of KE. |
| 18. Loss in KE (Collision) | $\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2 (1 - e^2)$ | General formula for KE loss in any head-on collision. |
| 19. Equilibrium Conditions | Stable: $U'' > 0$, Unstable: $U'' < 0$, Neutral: $U'' = 0$ | $U'' = d^2U/dx^2$. At equilibrium, Net Force is zero. |
| 20. Chain Pulling Problem | $W = \frac{MgL}{2n^2}$ | Work to pull a chain hanging $1/n$ th part off a table back onto the table. |
| 21. Stopping Distance | $d_s = \frac{v^2}{2\mu g}$ | Vehicle stopping distance with friction coefficient μ . |

22. Power of a Pump/Motor $P = \frac{dm}{dt}gh + \frac{1}{2}\frac{dm}{dt}v^2$ Power to lift water rate dm/dt to height h and eject with velocity v .

23. Rebound Height $h_n = e^{2n}h_0$ Height after n^{th} bounce. Total distance $= h_0 \left(\frac{1+e^2}{1-e^2} \right)$.

24. Spring-Block (Sudden Release) $x_{max} = \frac{2mg}{k}$ If a block of mass m attached to a spring is released suddenly from natural length.

25. Bullet Penetration $F_{avg} \cdot d = \frac{1}{2}mv^2$ Work done by resistive force = Change in KE.

26. Oblique Collision $v \sin \alpha = u \sin \theta$ Comp. of velocity \perp to line of impact is unchanged. Along line of impact, use e .

CIRCULAR MOTION

BY AP Sir, Sakaar Classes

Formula / Topic
Name

Formula

Conditions / Usage

1. Kinematics of Circular Motion

Angular Velocity (ω)

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

Rate of change of angular displacement.

$$\omega_{inst} = \frac{d\theta}{dt}$$

Linear Velocity (v)

$$v = r\omega$$

Relation between linear speed (v) and angular speed (ω) for a particle at radius r .

Angular Acceleration
(α)

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Rate of change of angular velocity.

Tangential Acceleration (a_t)

$$a_t = r\alpha = \frac{dv}{dt}$$

Responsible for changing the **magnitude** of velocity (speed). Zero in U.C.M.

Centripetal (Radial) Acceleration (a_c or a_r)

$$a_c = \frac{v^2}{r} = r\omega^2 = v\omega$$

Responsible for changing the **direction** of velocity. Always directs towards center.

Net Acceleration (a_{net})

$$a_{net} = \sqrt{a_c^2 + a_t^2}$$

Vector sum of radial and tangential acceleration.

Angle of Net Acceleration (ϕ)

$$\tan \phi = \frac{a_c}{a_t}$$

ϕ is the angle made by net acceleration with the tangential direction.

Equations of Circular Motion

$$\omega = \omega_0 + \alpha t$$

Condition: Only valid when Angular Acceleration (α) is **constant**.

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

2. Dynamics of Circular Motion

Centripetal Force (F_c)

$$F_c = \frac{mv^2}{r} = mr\omega^2$$

Real force required to keep a body in circular motion (provided by Tension, Friction, Normal, etc.).

Centrifugal Force (Pseudo Force)

$$F_{cf} = \frac{mv^2}{r}$$

Acts radially outward. **Condition:** Only applicable in a **rotating (non-inertial) reference frame**.

3. Banking of Roads & Turning

Bending of Cyclist

$$\tan \theta = \frac{v^2}{rg}$$

θ with vertical. Condition for no skidding while turning.

Car on Level Circular Road (No Banking)

$$v_{max} = \sqrt{\mu_s rg}$$

Maximum safe speed to avoid skidding. μ_s = coefficient of static friction.

Banked Road (Frictionless)

$$\tan \theta = \frac{v^2}{rg}$$

Optimum speed $v_{opt} = \sqrt{rg \tan \theta}$. No wear and tear on tires.

Banked Road (With Friction) - Max Speed

$$v_{max} = \sqrt{rg \left(\frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta} \right)}$$

Speed limit to avoid slipping **outwards** (up the incline).

Banked Road (With Friction) - Min Speed

$$v_{min} = \sqrt{rg \left(\frac{\tan \theta - \mu_s}{1 + \mu_s \tan \theta} \right)}$$

Speed limit to avoid slipping **inwards** (down the incline).

4. Conical Pendulum

Angular Velocity

$$\omega = \sqrt{\frac{g}{L \cos \theta}} = \sqrt{\frac{g}{h}}$$

L = length of string, h = vertical height of point of suspension from circle center.

Time Period (T)

$$T = 2\pi \sqrt{\frac{L \cos \theta}{g}} = 2\pi \sqrt{\frac{h}{g}}$$

Time for one complete revolution.

Tension in String

$$T_{tension} = \frac{mg}{\cos \theta} = mL\omega^2$$

5. Vertical Circular Motion (String)

Velocity at any point

$$v = \sqrt{u^2 - 2gh}$$

u = speed at bottom, h = height from bottom.

Tension at any point

$$T = \frac{mv^2}{r} + mg \cos \theta$$

θ is angle with vertical downward direction.

Critical Velocity (Top)

$$v_{top} = \sqrt{gR}$$

Minimum speed at top to keep string taut ($T_{top} \geq 0$).

Critical Velocity (Bottom)

$$v_{bottom} = \sqrt{5gR}$$

Minimum speed at bottom to complete the full circle.

Critical Velocity (Horizontal Point)

$$v_{mid} = \sqrt{3gR}$$

Speed at the point where string is horizontal.

Tension Difference

$$T_{bottom} - T_{top} = 6mg$$

Valid for any vertical circular motion under gravity.

Condition for Oscillation

$$0 < v_{bottom} \leq \sqrt{2gR}$$

Particle oscillates like a pendulum (doesn't reach horizontal level).

Condition for Leaving Circle

$$\sqrt{2gR} < v_{bottom} < \sqrt{5gR}$$

Particle leaves the circular path in the upper half (T becomes 0 before v).

6. Specific Formulae for Questions

"Death Well" (Rotor)

$$v_{min} = \sqrt{\frac{gR}{\mu}}$$

Min speed to prevent falling. Friction acts upwards balancing weight.

Vehicle on Convex Bridge

$$v_{max} = \sqrt{gR}$$

Max speed to maintain contact with the bridge (Normal reaction $N = 0$).

Radius of Curvature (Projectile)

$$R_{curv} = \frac{v^2}{a_{\perp}}$$

At top of trajectory:

$$R = \frac{u^2 \cos^2 \theta}{g}$$

Toppling of Car on Turn

$$v_{max} = \sqrt{\frac{gra}{h}}$$

$2a$ = distance between wheels (track width),
 h = height of Center of Mass. Condition:
Topples if $v > v_{max}$.

Rotational Motion

BY AP Sir, Sakaar Classes

| Formula / Topic Name | Formula(e) | Conditions / When to use |
|--|--|---|
| 1. KINEMATICS OF ROTATION | | |
| Angular Displacement | $\theta = \frac{l}{r}$ | Angle in radians, l = arc length. |
| Angular Velocity | $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$ | Rate of change of angular position. |
| | $\omega_{inst} = \frac{d\theta}{dt}$ | |
| Angular Acceleration | $\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$ | Rate of change of angular velocity. |
| | $\alpha_{inst} = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta}$ | |
| Equations of Kinematics | <ol style="list-style-type: none"> $\omega = \omega_0 + \alpha t$ $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$ $\theta_{nth} = \omega_0 + \frac{\alpha}{2}(2n - 1)$ | Strictly valid only when angular acceleration (α) is CONSTANT. |
| Linear vs Angular Variables | $v = \omega r$ | v is tangential velocity. |
| | $a_t = \alpha r$ (Tangential acc.) | a_t changes speed. |
| | $a_c = \omega^2 r$ (Centripetal acc.) | a_c changes direction (always exists if $\omega \neq 0$). |

$$a_{net} = \sqrt{a_t^2 + a_c^2}$$

2. MOMENT OF INERTIA (MOI)

Discrete System $I = \sum m_i r_i^2$ r_i is perpendicular distance from the axis of rotation.

Continuous Bodies $I = \int r^2 dm$ Requires integration.

Radius of Gyration (k) $I = Mk^2 \Rightarrow k = \sqrt{\frac{I}{M}}$ Distance where total mass is theoretically concentrated to give same I .

Perpendicular Axis Theorem $I_z = I_x + I_y$ Valid ONLY for planar (2D) bodies (laminar objects). Axes x, y must be in the plane; z perpendicular to plane.

Parallel Axis Theorem $I_{axis} = I_{CM} + Md^2$ Valid for any 3D or 2D body.

d = perpendicular distance between parallel axes.

One axis MUST pass through Center of Mass (CM).

3. MOI OF STANDARD BODIES

(Axis through Center, unless specified)

Ring / Hollow Cylinder $I = MR^2$ Axis perpendicular to plane (Ring) or along geometrical axis (Cylinder).

Disc / Solid Cylinder $I = \frac{MR^2}{2}$ Axis perpendicular to plane (Disc) or along geometrical axis (Cylinder).

| | | |
|---|--|---|
| Thin Rod | $I = \frac{ML^2}{12}$ (Center) | Axis perpendicular to length. |
| | $I = \frac{ML^2}{3}$ (End) | |
| Solid Sphere | $I = \frac{2}{5}MR^2$ | Axis along diameter. |
| Hollow Sphere (Shell) | $I = \frac{2}{3}MR^2$ | Axis along diameter. |
| Rectangular Plate | $I = \frac{M(a^2+b^2)}{12}$ | Axis perpendicular to plate, through center. |
| 4. TORQUE (τ) | | |
| Vector Definition | $\vec{\tau} = \vec{r} \times \vec{F}$ | $\backslash\tau$ |
| | \$ | |
| Newton's 2nd Law (Rotation) | $\vec{\tau}_{net} = I\vec{\alpha}$ | Valid for rigid bodies rotating about a fixed axis or about CM. |
| | $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$ | |
| Couple | $\tau = F \times d$ | Two equal and opposite forces. Torque is independent of the choice of origin. |
| Rotational Equilibrium | $\sum \vec{F}_{ext} = 0$ AND $\sum \vec{\tau}_{ext} = 0$ | Body is neither accelerating translationally nor rotationally. |
| 5. ANGULAR MOMENTUM (L) | | |
| Point Mass | $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$ | About a specific origin. |
| | $L = mvr_{\perp}$ | |

| | | |
|---|---|---|
| Rigid Body (Fixed Axis) | $L = I\omega$ | Axis must be fixed or passing through CM. |
| Combined Motion (Rolling) | $\vec{L} = \vec{L}_{CM} + \vec{L}_{aboutCM} = M(\vec{r}_{cm} \times \vec{v}_{cm}) + I_{cm}\vec{\omega}$ | General formula for a body moving and rotating. |
| Conservation of Angular Momentum | $I_1\omega_1 = I_2\omega_2$ | Condition: Net external torque on the system is ZERO ($\tau_{ext} = 0$). |

6. WORK, POWER, ENERGY

| | | |
|----------------------------------|---|--|
| Rotational Kinetic Energy | $K_{rot} = \frac{1}{2}I\omega^2$ | Pure rotation about an axis. |
| Work Done | $W = \int \tau d\theta = \vec{\tau} \cdot \vec{\Delta\theta}$ | Analogous to $W = \vec{F} \cdot \vec{d}$. |
| Power | $P = \vec{\tau} \cdot \vec{\omega}$ | Instantaneous power delivered by torque. |
| Work-Energy Theorem | $W_{ext} = \Delta K = K_f - K_i$ | Work done by all torques equals change in Rotational KE. |

7. ROLLING MOTION

| | | |
|-----------------------------------|--|---|
| Condition for Pure Rolling | $v_{cm} = \omega R$ | Velocity of the bottom-most point (contact point) is zero relative to ground. |
| Total Kinetic Energy | $K_{total} = K_{trans} + K_{rot}$ | Useful factor $\beta = (1 + \frac{k^2}{R^2})$. |
| | $K_{total} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$ | Ring: $\beta = 2$, Disc: $\beta = 1.5$, Solid Sphere: $\beta = 1.4$. |

| | | |
|-------------------------------------|-------------------------------------|--|
| 8. ROLLING ON INCLINED PLANE | (Specific Question Formulae) | Body rolling down from rest without slipping. |
|-------------------------------------|-------------------------------------|--|

Acceleration
$$a = \frac{g \sin \theta}{1 + \frac{I}{MR^2}} = \frac{g \sin \theta}{\beta}$$

Standard NEET/JEE result.
Solid sphere accelerates
fastest (lowest β).

Velocity at bottom
$$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$$

Depends only on height h
and shape factor $\frac{k^2}{R^2}$, not
Mass or Radius.

Time to reach bottom
$$t = \sqrt{\frac{2L(1 + \frac{k^2}{R^2})}{g \sin \theta}}$$

L = length of incline. Ring
takes max time, Solid
sphere takes min time.

Min Friction Coefficient
$$\mu_{min} = \frac{\tan \theta}{1 + \frac{MR^2}{I}}$$

Condition to prevent
slipping while rolling down.

9. COLLISION & TOPPLING

Angular Impulse (J)
$$J = \int \tau dt = \Delta L$$

Change in Angular
Momentum.

Rod hit by particle
$$L_i = L_f$$
 (about hinge/pivot)

Use Conservation of
Angular Momentum about
the pivot point to find ω
after impact.

$$mvx = \left(\frac{ML^2}{3} + mx^2 \right) \omega$$

Centre of Mass and Collisions

BY AP Sir, Sakaar Classes

| Formula Name / Topic | Formula(e) | Conditions / Usage |
|--|--|---|
| 1. Position of COM (Two Particles) | $X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$ | System of two point masses m_1, m_2 at positions x_1, x_2 . |
| 2. Position Vector of COM (Discrete) | $\vec{R}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n}{M_{total}}$ | General system of n discrete particles. |
| 3. Coordinate Formulae (3D) | $X_{cm} = \frac{\sum m_i x_i}{\sum m_i}$ | To find specific coordinates (x, y, z) of the COM. |
| | $Y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$ | |
| | $Z_{cm} = \frac{\sum m_i z_i}{\sum m_i}$ | |
| 4. COM of Continuous Bodies | $X_{cm} = \frac{1}{M} \int x \, dm$ | Used for rigid bodies (rods, discs, etc.) where mass is distributed continuously. |
| | $Y_{cm} = \frac{1}{M} \int y \, dm$ | |
| 5. Linear Mass Density (λ) | $dm = \lambda \, dx$ | Used for 1D objects like rods/wires. |
| 6. Areal Mass Density (σ) | $dm = \sigma \, dA$ | Used for 2D objects like plates/discs/shells. |

**7. Volumetric
Mass Density (ρ)**

$$dm = \rho dV$$

Used for 3D objects like spheres/cones.

**8. COM: Uniform
Semi-Circular
Ring**

$$Y_{cm} = \frac{2R}{\pi}$$

Center of base is at origin.
Symmetric axis is Y-axis.

**9. COM: Uniform
Semi-Circular
Disc**

$$Y_{cm} = \frac{4R}{3\pi}$$

Center of base is at origin.

**10. COM: Hollow
Hemisphere**

$$Y_{cm} = \frac{R}{2}$$

From the center of the base.

**11. COM: Solid
Hemisphere**

$$Y_{cm} = \frac{3R}{8}$$

From the center of the base.

**12. COM: Hollow
Cone**

$$Y_{cm} = \frac{h}{3}$$

From the center of the base.

**13. COM: Solid
Cone**

$$Y_{cm} = \frac{h}{4}$$

From the center of the base.

**14. COM: Cavity
Problems
(Negative Mass)**

$$\vec{r}_{cm} = \frac{M_{original}\vec{r}_1 - M_{removed}\vec{r}_2}{M_{original} - M_{removed}}$$

When a part is removed from a rigid body. \vec{r}_1 is COM of original, \vec{r}_2 is COM of removed part.

**15. Velocity of
COM**

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \dots}{M_{total}}$$

Velocity of the system's center of mass.

**16. Acceleration
of COM**

$$\vec{a}_{cm} = \frac{\vec{F}_{ext}}{M_{total}}$$

Newton's 2nd Law applied to the whole system.

**17. Momentum
Conservation
(System)**

$$\vec{P}_{initial} = \vec{P}_{final}$$

Valid ONLY if **External Force** (\vec{F}_{ext}) on the system is **Zero**.

**18. Displacement
of COM**

$$\Delta\vec{r}_{cm} = \frac{m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2}{M}$$

If $\vec{F}_{ext} = 0$ initially at rest, then $\Delta\vec{r}_{cm} = 0$. (e.g., Boat & Dog problems).

19. Impulse (\vec{J})

$$\vec{J} = \int \vec{F} dt = \Delta \vec{P} = \vec{P}_f - \vec{P}_i$$

Change in momentum caused by a large force over a short time.

20. Coefficient of Restitution (e)

$$e = \frac{\text{Velocity of Separation}}{\text{Velocity of Approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$

Defined along the **Line of Impact**. $0 \leq e \leq 1$.

21. Head-on Elastic Collision ($e = 1$)

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left(\frac{2m_2}{m_1 + m_2} \right) u_2$$

Final velocity of mass 1.
Conservation of KE & Momentum holds.

22. Head-on Elastic Collision ($e = 1$)

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) u_2 + \left(\frac{2m_1}{m_1 + m_2} \right) u_1$$

Final velocity of mass 2.

23. Elastic Collision: Equal Masses

$$v_1 = u_2, \quad v_2 = u_1$$

If $m_1 = m_2$, velocities are exchanged.

24. Perfectly Inelastic Collision ($e = 0$)

$$V_{common} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

Particles stick together and move with common velocity.
Max KE loss.

25. Inelastic Collision ($0 < e < 1$)

$$v_1 = \frac{m_1 u_1 + m_2 u_2 - m_2 e(u_1 - u_2)}{m_1 + m_2}$$

General formula for 1D collision.

26. Loss in KE (Head-on)

$$\Delta K = \frac{1}{2} \left(\frac{m_1 m_2}{m_1 + m_2} \right) (u_1 - u_2)^2 (1 - e^2)$$

Energy dissipated as heat/sound. Zero if $e = 1$ (Elastic).

27. Oblique Collision (2D)

Along Line of Impact (LOI):

$$e = \frac{v_{2n} - v_{1n}}{u_{1n} - u_{2n}}$$

Momentum is conserved along LOI and Tangential axis (if smooth). Restitution eq applies ONLY along LOI.

28. Rocket Propulsion (Thrust)

$$F_{thrust} = v_{rel} \left(-\frac{dm}{dt} \right)$$

Force on rocket due to ejected gas. v_{rel} is velocity of gas w.r.t rocket.

29. Rocket

Velocity at time t (ignoring

ELASTICITY

BY AP Sir, Sakaar Classes

| Formula Name / Topic | Formula | Condition / Context / Use Case |
|--|--|---|
| Normal Stress (σ) | $\sigma = \frac{F_{\perp}}{A}$ | Restoring force (F_{\perp}) acting per unit area perpendicular to the cross-section. |
| Tangential / Shear Stress (σ_t) | $\sigma_t = \frac{F_{\parallel}}{A}$ | Force (F_{\parallel}) acting parallel to the surface area. Causes shape change without volume change. |
| Longitudinal Strain (ε_l) | $\varepsilon_l = \frac{\Delta L}{L}$ | Change in length per unit original length (Tensile or Compressive). |
| Shearing Strain (ϕ) | $\phi \approx \tan \phi = \frac{x}{L}$ | Relative displacement (x) between parallel layers separated by distance L . |
| Volumetric Strain (ε_v) | $\varepsilon_v = -\frac{\Delta V}{V}$ | Change in volume per unit original volume. Negative sign indicates decrease in volume with pressure increase. |
| Hooke's Law | Stress \propto Strain Stress = $E \times$ Strain | Valid only within the Proportional Limit . E is the Modulus of Elasticity. |
| Young's Modulus (Y) | $Y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$ $Y = \frac{FL}{A\Delta l} = \frac{mgL}{\pi r^2 \Delta l}$ | Used for solids (wires, rods) undergoing length change. Specific for a material. |
| Bulk Modulus (B or K) | $B = \frac{-P}{\Delta V/V} = -V \frac{\Delta P}{\Delta V}$ | Relates volume change to pressure change. Applicable to solids, liquids, and gases. |
| Compressibility (K) | $K = \frac{1}{B}$ | Reciprocal of Bulk Modulus. |

Modulus of Rigidity / Shear Modulus (η or G)

$$\eta = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{F}{A\phi}$$

Resistance to change in shape.
Only for solids.

Poisson's Ratio (σ)

$$\sigma = -\frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

Theoretical limits: -1 to 0.5.

$$\sigma = \frac{\Delta D/D}{\Delta L/L}$$

Practical limits: 0 to 0.5.

Work Done in Stretching (Strain Energy U)

$$U = \frac{1}{2} \times F \times \Delta l$$

Total potential energy stored in a stretched wire.

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{Volume}$$

Energy Density (u)

$$u = \frac{1}{2} \times \text{stress} \times \text{strain}$$

Energy stored per unit volume.

$$u = \frac{1}{2} Y (\text{strain})^2 = \frac{(\text{stress})^2}{2Y}$$

Elongation due to Self Weight

$$\Delta l = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y}$$

Extension of a hanging rod/wire due to its own gravity. M =mass, ρ =density. Note the factor 2 in denominator (acts at Center of Mass).

Thermal Stress

$$\sigma_{\text{thermal}} = Y\alpha\Delta T$$

Rod fixed between rigid supports.

Force $F = Y A \alpha \Delta T$

α = coeff. of linear expansion,
 ΔT = temp change.

Analogy with Spring Constant (k)

$$k = \frac{YA}{L}$$

Treating a wire as a spring ($F = kx$). Useful for series/parallel combination of wires.

Wires in Series

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Composite wire with same Force/Tension acting on both segments.

$$\Delta l_{\text{net}} = \Delta l_1 + \Delta l_2$$

| | | |
|---|--|---|
| Wires in Parallel | $k_{eq} = k_1 + k_2$ | Composite wire where extensions are forced to be equal ($\Delta l_1 = \Delta l_2$). |
| | $F_{net} = F_1 + F_2$ | |
| Interatomic Force Constant (k_a) | $k_a = Y \times r_0$ | r_0 is the equilibrium interatomic distance. |
| Depression of a Beam (Cantilever) | $\delta = \frac{WL^3}{3YI_g}$ | Beam fixed at one end, loaded (W) at the other. I_g is Geometrical Moment of Inertia. |
| Depression of Beam (Supported at ends) | $\delta = \frac{WL^3}{48YI_g}$ | Beam supported at both ends, load W in the center. |
| Torsion of a Cylinder | $C = \frac{\pi\eta r^4}{2L}$ | Restoring couple per unit twist (Torsional rigidity). |
| Breaking Stress | Breaking Force = Breaking Stress $\times A$ | Breaking stress depends on material, not dimensions. Breaking Force depends on area. |
| Relation: Y, B, σ | $Y = 3B(1 - 2\sigma)$ | Relates Young's, Bulk Modulus and Poisson's ratio. |
| Relation: Y, η, σ | $Y = 2\eta(1 + \sigma)$ | Relates Young's, Rigidity Modulus and Poisson's ratio. |
| Relation: Y, B, η | $\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$ | Useful when σ is not given. |
| Relation: σ in terms of B, η | $\sigma = \frac{3B - 2\eta}{6B + 2\eta}$ | Calculation of Poisson's ratio from moduli. |

Quick Tips for Numerical Questions (NEET/JEE)

- Wire Cut into n parts:** If a wire of Young's Modulus Y is cut into n equal parts, the Young's Modulus of each part remains Y (Material property), but the spring constant becomes nk .
- % Change questions:** If strain is small ($< 5\%$), use $\frac{\Delta R}{R} \times 100$. For volume of wire $V = A \times L$ (constant), $\frac{\Delta A}{A} = -\frac{\Delta L}{L}$ (ignoring σ effects for simple resistance type q's) or use conservation of volume $A_1 L_1 = A_2 L_2$.
- Adiabatic vs Isothermal Modulus:**

Fluid Mechanics & Surface Tension

BY AP Sir, Sakaar Classes

Fluid Properties & Hydrostatics

| Formula Name / Topic | Formula | Condition / Note |
|--|---|---|
| Density & Relative Density (RD) | $\rho = \frac{m}{V}$ | $\rho_{\text{water}} = 1000 \text{ kg/m}^3$. RD has no units. |
| | $\text{RD} = \frac{\rho_{\text{substance}}}{\rho_{\text{water at } 4^\circ\text{C}}}$ | |
| Pressure at Depth | $P = P_0 + h\rho g$ | P_0 : Atmospheric Pressure |
| | | h : Depth below free surface. |
| Gauge Pressure | $P_g = P_{\text{absolute}} - P_{\text{atm}} = h\rho g$ | Pressure due to fluid column only. |
| Pascal's Law | $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ | Pressure applied to enclosed fluid is transmitted undiminished. |
| Force on Vertical Dam Wall | $F = \frac{1}{2}\rho g w H^2$ | w : Width, H : Depth. Force acts at $H/3$ from bottom. |
| Archimedes' Principle | $F_B = V_{\text{in}} \cdot \rho_L \cdot g$ | V_{in} : Submerged volume |
| | | ρ_L : Density of Liquid. |
| Condition for Floatation | $mg = F_B$ | Body floats if $\rho_S \leq \rho_L$. Weight of body = Weight of fluid displaced. |
| | $\frac{V_{\text{in}}}{V_{\text{total}}} = \frac{\rho_S}{\rho_L}$ | |
| Accelerated Fluid (Horizontal) | $\tan \theta = \frac{a}{g}$ | θ : Angle of free surface with horizontal. |
| Accelerated Fluid (Vertical) | $P = P_0 + h\rho(g_{\text{eff}})$ | $g_{\text{eff}} = g + a$ (up), $g_{\text{eff}} = g - a$ (down). |

Rotating Fluid (Vortex) $y = \frac{\omega^2 x^2}{2g}$ Parabolic meniscus shape.

Fluid Dynamics

| Formula Name / Topic | Formula | Condition / Note |
|-----------------------------------|--|---|
| Equation of Continuity | $A_1 v_1 = A_2 v_2$ | Conservation of Mass. Incompressible, non-viscous flow. |
| Bernoulli's Principle | $P + \rho gh + \frac{1}{2} \rho v^2 = \text{Constant}$ | Conservation of Energy per unit volume. Ideal fluid. |
| Torricelli's Law | $v = \sqrt{2gh}$ | h : Depth of hole from top. |
| Horizontal Range of Efflux | $R = 2\sqrt{h(H-h)}$ | $R_{\max} = H$ when hole is at $H/2$. |
| Time to empty tank | $t = \frac{A}{a} \sqrt{\frac{2}{g}} (\sqrt{H_1} - \sqrt{H_2})$ | A : Tank area, a : Hole area. |
| Venturimeter | $Q = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$ | h : Height diff in manometer. |

Viscosity

| Formula Name / Topic | Formula | Condition / Note |
|----------------------------------|--|--|
| Newton's Law of Viscosity | $F = -\eta A \frac{dv}{dx}$ | η : Coeff. of viscosity, $\frac{dv}{dx}$: Velocity gradient. |
| Stoke's Law | $F = 6\pi\eta rv$ | Viscous drag on sphere of radius r . |
| Terminal Velocity | $v_T = \frac{2}{9} \frac{r^2(\rho - \sigma)g}{\eta}$ | Constant max velocity. ρ : sphere, σ : fluid. |
| Poiseuille's Equation | $Q = \frac{\pi P r^4}{8\eta l}$ | Volume flow rate in capillary tube. |
| Reynolds Number | $R_e = \frac{\rho v d}{\eta}$ | $R_e < 1000$: Laminar, $R_e > 2000$: Turbulent. |

Surface Tension

| Formula Name / Topic | Formula | Condition / Note |
|----------------------|---------|------------------|
|----------------------|---------|------------------|

| | | |
|---------------------------------|--|---|
| Surface Tension | $T = \frac{F}{L}$ | Force per unit length. |
| Surface Energy | $U = T \times \Delta A$ | ΔA : Change in area. |
| Work done (Liquid Drop) | $W = T \cdot 4\pi(r_2^2 - r_1^2)$ | Single surface. |
| Work done (Soap Bubble) | $W = T \cdot 8\pi(r_2^2 - r_1^2)$ | Two surfaces (inner & outer). |
| Excess Pressure (Drop) | $\Delta P = \frac{2T}{R}$ | Pressure inside > outside. |
| Excess Pressure (Bubble) | $\Delta P = \frac{4T}{R}$ | Two free surfaces. |
| Capillary Rise | $h = \frac{2T \cos \theta}{r \rho g}$ | Jurist's Law. θ : Contact angle. |
| Force to lift wire frame | $F = 2Tl + mg$ | Surface tension acts on both sides. |
| Force to lift Ring | $F \approx 4\pi r T + mg$ | Ring of radius r . |
| Splitting of Drops | $\Delta U = 4\pi R^2 T(n^{1/3} - 1)$ | Energy absorbed (Temp falls). |
| Coalescence of Drops | $E_{\text{released}} = 4\pi T(nr^2 - R^2)$ | Energy released (Temp rises). |

"Success is the sum of small efforts, repeated day in and day out."

Oscillations (Simple Harmonic Motion)

BY AP Sir, Sakaar Classes

| Formula / Topic Name | Formula(e) | Conditions / Notes |
|-----------------------------|--|--|
| 1. Standard Equation of SHM | $\frac{d^2x}{dt^2} + \omega^2 x = 0$ | Differential equation condition for any particle executing SHM. |
| 2. Displacement | $x = A \sin(\omega t + \phi)$ | General displacement from mean position. or ϕ : Initial phase (epoch). |
| | $x = A \cos(\omega t + \phi)$ | Use sin if starts from mean, cos if from extreme. |
| 3. Angular Frequency | $\omega = \frac{2\pi}{T} = 2\pi f = \sqrt{\frac{k}{m}}$ | ω : Angular frequency (rad/s). Depends on system properties (k, m), not amplitude. |
| 4. Velocity (v) | $v = \frac{dx}{dt} = A\omega \cos(\omega t + \phi)$ $v = \pm \omega \sqrt{A^2 - x^2}$ | $v_{max} = A\omega$ (at mean position, $x = 0$). $v_{min} = 0$ (at extreme position, $x = \pm A$). |
| 5. Acceleration (a) | $a = \frac{dv}{dt} = -\omega^2 A \sin(\omega t + \phi)$ $a = -\omega^2 x$ | Direction is always towards mean position. $a_{max} = \omega^2 A$ (at extreme). $a_{min} = 0$ (at mean). |

| | | |
|---|---|--|
| 6. Restoring Force | $F = -kx$ | Linear SHM condition. Force is proportional to displacement and opposite in direction. |
| | $F = -m\omega^2 x$ | |
| 7. Phase Difference | $\Delta\phi = \phi_2 - \phi_1$ | Time diff $\Delta t = \frac{T}{2\pi} \Delta\phi$. |
| | | Path diff $\Delta x = \frac{\lambda}{2\pi} \Delta\phi$ (Wave context). |
| 8. Kinetic Energy (KE) | $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$ | Max at mean position ($K_{max} = \frac{1}{2}kA^2$). |
| | $K = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$ | Zero at extreme position. |
| 9. Potential Energy (PE) | $U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$ | Assuming $U = 0$ at mean position. |
| | $U = \frac{1}{2}kA^2 \sin^2(\omega t + \phi)$ | Max at extreme ($U_{max} = \frac{1}{2}kA^2$). |
| 10. Total Energy (TE) | $E = K + U = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$ | TE is constant (conserved) in undamped SHM. |
| | | $E \propto A^2$ and $E \propto f^2$. |
| 11. Average Energies | $\langle K \rangle_{cycle} = \langle U \rangle_{cycle} = \frac{1}{4}kA^2$ | Over one complete cycle of oscillation. |
| | $\langle E \rangle_{cycle} = \frac{1}{2}kA^2$ | |
| 12. Spring-Mass System (Horizontal/Vertical) | $T = 2\pi \sqrt{\frac{m}{k}}$ | Period is independent of g and amplitude. |
| | | k : Spring constant. |
| 13. Springs in Series | $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$ | End-to-end connection. Force is same, extension adds up. |
| 14. Springs in Parallel | $k_{eq} = k_1 + k_2 + \dots$ | Side-by-side or mass between two fixed springs. Extensions are same. |

15. Cutting a Spring

$$k \cdot l = \text{constant} \implies k \propto \frac{1}{l}$$

If spring of length l is cut into n equal parts, stiffness of each part becomes nk .

**16. Two Block System
(Reduced Mass)**

$$T = 2\pi \sqrt{\frac{\mu}{k}}$$

Where reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$.

Blocks oscillate relative to center of mass.

17. Simple Pendulum

$$T = 2\pi \sqrt{\frac{l}{g_{eff}}}$$

For small angular amplitude ($\theta < 5^\circ$).

l : Length from pivot to CM of bob.

18. Pendulum in Lift

Accelerating Up: $g_{eff} = g + a$

If lift falls freely ($a = g$), $g_{eff} = 0$, $T \rightarrow \infty$ (No oscillation).

$$T = 2\pi \sqrt{\frac{l}{g+a}}$$

Accelerating Down: $g_{eff} = g - a$

$$T = 2\pi \sqrt{\frac{l}{g-a}}$$

**19. Pendulum in
Truck/Car**

$$g_{eff} = \sqrt{g^2 + a^2}$$

Truck moving horizontally with acceleration a . Mean position shifts by $\tan \theta = a/g$.

$$T = 2\pi \sqrt{\frac{l}{(g^2+a^2)^{1/2}}}$$

**20. Pendulum with
Charged Bob**

$$g_{eff} = g + \frac{qE}{m}$$
 (E field down)

Electric field E applied vertically.

$$g_{eff} = g - \frac{qE}{m}$$
 (E field up)

21. Pendulum of Infinite Length

$$T = 2\pi \sqrt{\frac{1}{g(\frac{1}{l} + \frac{1}{R_e})}}$$

If $l \approx R_e$ (Earth's radius).

If $l \rightarrow \infty$,
 $T = 2\pi \sqrt{\frac{R_e}{g}} \approx 84.6 \text{ min.}$

22. Second's Pendulum $T = 2$ seconds Length $l \approx 0.993$ m (on Earth).

23. Physical Pendulum $T = 2\pi\sqrt{\frac{I}{mgd}}$ I : Moment of Inertia about pivot.

24. Torsional Pendulum $T = 2\pi\sqrt{\frac{I}{C}}$ d : Distance between pivot and Center of Mass.

25. Superposition (Same freq) $x_{res} = A \sin(\omega t + \theta)$ I : MOI of disc/body.

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \delta}$$

C : Torsional constant (Nm/rad) of the wire.

26. Liquid in U-Tube $T = 2\pi\sqrt{\frac{h}{g}}$ or $T = 2\pi\sqrt{\frac{L}{2g}}$ h : Height of liquid column in one arm at equilibrium.

L : Total length of liquid column.

27. Body Floating in Liquid $T = 2\pi\sqrt{\frac{m}{A\rho g}} = 2\pi\sqrt{\frac{h_{submerged}}{g}}$ A : Cross-sectional area. ρ : Density of liquid.

Slightly depressed and released.

28. Tunnel through Earth $T = 2\pi\sqrt{\frac{R_e}{g}} \approx 84.6$ min Particle dropped in a tunnel along diameter or chord. (Assuming uniform density).

29. Ball in Concave Dish $T = 2\pi\sqrt{\frac{R-r}{g}}$ R : Radius of curvature of dish. r : Radius of ball.

For small oscillations ($r \ll R$, $T \approx 2\pi\sqrt{R/g}$).

30. Piston in Cylinder $T = 2\pi\sqrt{\frac{mV}{PA^2}}$ V : Volume, P : Pressure, A : Area of piston.

(Adiabatic process usually considered, add factor γ in denominator for adiabatic).

21. Amplitude with

$$A' = A \cdot f^n \text{ where } f > 1$$

If amplitude decays by a constant

Wave Motion (Mechanical Waves)

BY AP Sir, Sakaar Classes

1. Basics of Wave Motion & Progressive Waves

| Formula / Topic Name | Formula | Conditions / Usage Notes |
|----------------------|---------|--------------------------|
|----------------------|---------|--------------------------|

General Plane

Progressive Wave

Equation

$$y = A \sin(\omega t \pm kx + \phi)$$

y

: Displacement,

A

: Amplitude,

ω

: Angular freq,

k

: Propagation constant.

(-) sign: Wave moving in $+x$ direction.

(+) sign: Wave moving in $-x$ direction.

Angular Frequency (ω)

$$\omega = 2\pi f = \frac{2\pi}{T} \quad f$$

: Frequency (Hz),

T

: Time period.

Propagation Constant (k)

$$k = \frac{2\pi}{\lambda}$$

 λ

: Wavelength. Represents phase change per unit length.

Wave Velocity (v)

$$v = f\lambda = \frac{\omega}{k}$$

Speed at which the disturbance travels through the medium.

Particle Velocity (v_p)

$$v_p = \frac{\partial y}{\partial t} = \omega A \cos(\omega t \pm kx)$$

Velocity of a particle oscillating about its mean position.

Relation: Particle vs Wave Velocity

$$v_p = -v \times (\text{slope of y-x graph})$$

Used to find the direction of particle motion if the wave shape is known.

$$v_p = -v \left(\frac{\partial y}{\partial x} \right)$$

Particle Acceleration (a_p)

$$a_p = \frac{\partial^2 y}{\partial t^2} = -\omega^2 y$$

Maximum acceleration

$$a_{max} = \omega^2 A$$

occurs at extreme positions (

$$y = \pm A$$

).

Phase Difference ($\Delta\phi$)

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x$$

Relation between Phase difference and Path difference (Δx) or Time difference (Δt).

$$\Delta\phi = \frac{2\pi}{T} \Delta t$$

2. Speed of Waves in Media

Formula / Topic Name

Formula

Conditions / Usage Notes

Speed of Transverse Wave on String

$$v = \sqrt{\frac{T}{\mu}}$$

: Tension in string,

μ

: Linear mass density (

$$m/L$$

).

Imp: Ensure

μ

is mass per unit length, not volume density.

Speed of Sound (General)

$$v = \sqrt{\frac{E}{\rho}}$$

: Modulus of Elasticity,

ρ

: Density of medium.

Speed in Solids (Rod)

$$v = \sqrt{\frac{Y}{\rho}}$$

: Young's Modulus.

Speed in Fluids (Liquids/Gases)

$$v = \sqrt{\frac{B}{\rho}}$$

: Bulk Modulus.

Newton's Formula (Gases)

$$v = \sqrt{\frac{P}{\rho}}$$

Assumed Isothermal process. (Incorrect historically, gives lower value).

Laplace Correction (Gases)

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

Assumed Adiabatic process.

$$\gamma = C_p/C_v$$

(Adiabatic index).

$$T$$

: Temp in Kelvin,

$$M$$

: Molar mass.

Effect of Temperature on Sound Speed

$$v \propto \sqrt{T}$$

Speed increases with temperature.

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

Effect of Humidity

$$v_{moist} > v_{dry}$$

Moist air is less dense than dry air (

$$\rho_{moist} < \rho_{dry}$$

), so speed increases.

3. Superposition & Interference

Formula / Topic Name

Formula

Conditions / Usage Notes

Resultant Amplitude (

A_{res})

$$A_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$

ϕ

is the phase difference between two interfering waves.

Resultant Intensity (

I_{res})

$$I_{res} = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \phi$$

Since

$$I \propto A^2$$

**Constructive
Interference (Maxima)**

Condition:

Where

$$\phi = 2n\pi$$

$$n = 0, 1, 2, \dots$$

Path diff

$$\Delta x = n\lambda$$

$$A_{max} = A_1 + A_2$$

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

**Destructive
Interference (Minima)**

Condition:

Where

$$\phi = (2n - 1)\pi$$

$$n = 1, 2, 3, \dots$$

Path diff

$$\Delta x = (2n - 1)\frac{\lambda}{2}$$

$$A_{min} = |A_1 - A_2|$$

$$\text{\$\$I_{min} = (\sqrt{I_1} - \sqrt{I_2})}$$

Ratio of Intensities

$$\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2} \right)^2$$

Useful for questions given
amplitude ratio

$$r = A_1/A_2$$

4. Stationary (Standing) Waves

| Formula / Topic Name | Formula | Conditions / Usage Notes |
|----------------------------------|---|---|
| General Equation | $y = 2A \sin(kx) \cos(\omega t)$ | Or $y = 2A \cos(kx) \sin(\omega t)$ |
| | | Amplitude of particle at x is |
| | | $A(x) = 2A \sin(kx)$ |
| Nodes & Antinodes | Nodes: Zero amplitude points. Antinodes: Max amplitude points. | Distance between consecutive Node & Node or Antinode & Antinode = $\lambda/2$ |
| | | Distance between Node & Antinode = $\lambda/4$ |
| String Fixed at Both Ends | $f_n = \frac{nv}{2L} = n \left(\frac{1}{2L} \sqrt{\frac{T}{\mu}} \right)$ | $n = 1, 2, 3\dots$ (Number of loops). |
| | | $n = 1$ |
| | | : Fundamental/1st Harmonic. |

$$n = 2$$

: 2nd Harmonic/1st Overtone.

All harmonics are present.

String Fixed at

One End

$$f_n = \frac{(2n-1)v}{4L} \quad n = 1, 2, 3\dots$$

Only odd harmonics are present (

$$f_1, 3f_1, 5f_1\dots$$

).

Sonometer Law

$$f \propto \frac{1}{L}$$

Used for comparing frequencies when length or tension changes.

$$f \propto \sqrt{T}$$

$$f \propto \frac{1}{\sqrt{\mu}}$$

5. Organ Pipes (Sound Columns)

Formula / Topic
Name

Formula

Conditions / Usage Notes

Open Organ Pipe

$$f_n = \frac{nv}{2L}$$

Open at both ends.

Similar to string fixed at both ends.

All harmonics present (

$$1 : 2 : 3\dots$$

).

Closed Organ Pipe

$$f_n = \frac{(2n-1)v}{4L}$$

Closed at one end.

Similar to string fixed at one end.

Only odd harmonics present (

1 : 3 : 5...

).

End Correction (e)

$$e \approx 0.6r \quad r$$

: Radius of pipe.

Antinode forms slightly outside the open end.

Corrected Lengths

Open Pipe:

Use these lengths in frequency formulas for precise calculation.

$$L_{eff} = L + 2e$$

Closed Pipe:

$$L_{eff} = L + e$$

Resonance Tube

$$v = 2f(L_2 - L_1) \quad L_1$$

: First resonance length ($\lambda/4$).

$$L_2$$

: Second resonance length ($3\lambda/4$).

Eliminates end correction error.

6. Beats & Doppler Effect

| Formula / Topic Name | Formula | Conditions / Usage Notes |
|------------------------------------|---|--|
| Beat Frequency (f_b) | $f_b = f_1 - f_2 $ | Number of beats per second. |
| | | Requires |
| | | $f_1 \approx f_2$ |
| | | . |
| Tuning Fork Loading/Filing | Waxing (Loading): Mass | Used to determine unknown frequency based on change in beat frequency. |
| | ↑ | |
| | , Freq | |
| | | ↓ |
| | | . |
| | Filing: Mass | |
| | | ↓ |
| | , Freq | |
| | | ↑ |
| | | . |
| Doppler Effect (General) | $f' = f_0 \left(\frac{v \pm v_o}{v \mp v_s} \right)$ | f' : Apparent freq, |
| | | f_0 |
| | | : Source freq. |
| | | v |

: Speed of sound.

v_o

: Observer velocity.

v_s

: Source velocity.

Doppler Sign Convention

Numerator (v_o): (+) if Observer moves TOWARDS source.

"Towards" tends to increase frequency.

Denominator (v_s): (-) if Source moves TOWARDS observer.

"Away" tends to decrease frequency.

Effect of Wind (v_w)

$$f' = f_0 \left(\frac{(v \pm v_w) \pm v_o}{(v \pm v_w) \mp v_s} \right)$$

Add

v_w

to

v

if wind blows Source → Observer.

Subtract if wind blows Observer → Source.

7. Intensity & Energy Density

Formula / Topic Name

Formula

Conditions / Usage Notes

Intensity (I)

$$I = 2\pi^2 f^2 A^2 \rho v$$

Power per unit area.

Depends on square of frequency and amplitude.

Intensity vs Distance

Point Source:

Spherical wavefronts vs Cylindrical wavefronts.

Thermal Expansion and Calorimetry

BY AP Sir, Sakaar Classes

| Formula Name / Topic | Formula(e) | Conditions / Notes |
|-------------------------------|--|--|
| Linear Expansion | $\Delta L = L_0\alpha\Delta T$ | Valid for small temperature changes (ΔT). α is the coefficient of linear expansion. |
| | $L_f = L_0(1 + \alpha\Delta T)$ | |
| Superficial (Area) Expansion | $\Delta A = A_0\beta\Delta T$ | β is the coefficient of superficial expansion. |
| | $A_f = A_0(1 + \beta\Delta T)$ | |
| Volume Expansion | $\Delta V = V_0\gamma\Delta T$ | γ is the coefficient of volume expansion. |
| | $V_f = V_0(1 + \gamma\Delta T)$ | |
| Relation between Coefficients | $\alpha : \beta : \gamma = 1 : 2 : 3$ | strictly valid for isotropic solids (properties same in all directions). |
| | $\beta = 2\alpha, \gamma = 3\alpha$ | |
| Anisotropic Expansion | $\gamma = \alpha_x + \alpha_y + \alpha_z$ | For non-isotropic solids where α differs along x, y, z axes. |
| Variation of Density | $\rho' = \frac{\rho_0}{1+\gamma\Delta T} \approx \rho_0(1 - \gamma\Delta T)$ | Approximation valid when $\gamma\Delta T \ll 1$. ρ' decreases as T increases. |
| Thermal Stress | Stress = $Y\alpha\Delta T$ | Rod held between rigid supports preventing expansion. Y is Young's Modulus. |
| | Force = $YA\alpha\Delta T$ | |
| Pendulum Clock (Time Period) | New Period: $T' = T(1 + \frac{1}{2}\alpha\Delta\theta)$ | Due to length change $L' = L(1 + \alpha\Delta\theta)$. $\Delta\theta$ is temp change. |

| | | |
|---|---|--|
| Time Lost/Gained (Pendulum) | $\Delta t = \frac{1}{2}\alpha\Delta\theta \times t$ $\text{Loss/day} = \frac{1}{2}\alpha\Delta\theta \times 86400$ | If temp increases, clock expands, slows down (Loses time). If temp drops, gains time. |
| Bimetallic Strip | $R \approx \frac{d}{(\alpha_1 - \alpha_2)\Delta T}$ | Radius of curvature R when two strips of thickness d bend. $\alpha_1 > \alpha_2$. |
| Expansion of Liquids | $\gamma_{real} = \gamma_{app} + \gamma_{vessel}$ | γ_{real} is actual expansion; γ_{app} is what we see in a container. |
| | $\gamma_{app} = \gamma_{real} - 3\alpha_{vessel}$ | |
| Barometer Correction | $H_0 = H_{obs}[1 - (\gamma_{Hg} - \alpha_{scale})\Delta T]$ | H_0 is true height at $0^\circ C$. Corrects for both Hg expansion and scale expansion. |
| Specific Heat Capacity | $Q = ms\Delta T$ or $Q = mc\Delta T$ | Heat required to change temp without phase change. s or c is specific heat. |
| Molar Heat Capacity | $Q = nC\Delta T$ | n = number of moles. C = Molar heat capacity. |
| Heat Capacity (Thermal Capacity) | $H = ms$ or $H = \frac{Q}{\Delta T}$ | Heat required to raise temp of the <i>whole body</i> by $1^\circ C$. |
| Latent Heat (Phase Change) | $Q = mL$ | Used during melting (L_f) or boiling (L_v). Temp remains constant. |
| Water Equivalent | $W = ms$ | Mass of water that absorbs same heat as the body for same ΔT . (Unit: grams/kg) |
| Principle of Calorimetry | Heat Lost = Heat Gained | System must be isolated (no heat loss to surroundings). |
| | $\sum m_i s_i (T_i - T_{mix}) = \sum m_j s_j (T_{mix} - T_j)$ | |
| Mixture Temperature | $T_{mix} = \frac{m_1 s_1 T_1 + m_2 s_2 T_2}{m_1 s_1 + m_2 s_2}$ | For mixing two substances of same state (e.g., liquid + liquid). |

| | | |
|---|--|---|
| Steam + Ice Mixture | Check heat available vs heat required step-by-step. | 1. Heat released by steam to 100° water (mL_v). 2. Heat to cool water. Compare with ice melting req. |
| Newton's Law of Cooling (Exact) | $\frac{T - T_s}{T_0 - T_s} = e^{-kt}$ | T_s = Surroundings temp. T_0 = Initial temp. T = Temp at time t . |
| Newton's Law (Approx) | $\frac{T_1 - T_2}{t} = K \left[\frac{T_1 + T_2}{2} - T_s \right]$ | Valid only when temp difference $(T - T_s)$ is small (approx $< 30^\circ C$). |
| Stefan's Law | $E = \sigma A e T^4$ | Rate of energy radiated. e = emissivity ($0 \leq e \leq 1$). |
| Net Rate of Heat Loss (Radiation) | $\frac{dQ}{dt} = \sigma A e (T^4 - T_s^4)$ | Body at temp T placed in surroundings at temp T_s . If $T > T_s$, heat is lost. |
| Growth of Ice (Ice Formation Time) | $t = \frac{\rho L}{2K\theta} (x_2^2 - x_1^2)$ | Time taken to increase thickness from x_1 to x_2 . θ = temp difference (Air - $0^\circ C$). K = Thermal conductivity. |

Kinetic Theory of Gases and Thermodynamics

BY AP Sir, Sakaar Classes

1. Kinetic Theory of Gases (KTG) & Gas Laws

| Formula Name / Topic | Formula(e) | Conditions / specific usage |
|---------------------------|--|--|
| Ideal Gas Equation | $PV = nRT$ | General Ideal Gas Law. |
| | $PV = \frac{m}{M}RT$ | n : moles, m : mass, M : Molar mass, ρ : density, N : number of molecules, k_B : Boltzmann constant ($1.38 \times 10^{-23} J/K$). |
| | $P = \rho \frac{RT}{M}$ | |
| | $PV = Nk_B T$ | |
| Boyle's Law | $P \propto \frac{1}{V} \implies P_1 V_1 = P_2 V_2$ | Constant Temperature (T) (Isothermal). |
| | | Graph P vs $1/V$ is a straight line through origin. |
| Charles's Law | $V \propto T \implies \frac{V_1}{T_1} = \frac{V_2}{T_2}$ | Constant Pressure (P) (Isobaric). |
| | | T must be in Kelvin. |
| Gay-Lussac's Law | $P \propto T \implies \frac{P_1}{T_1} = \frac{P_2}{T_2}$ | Constant Volume (V) (Isochoric). |
| | | Pressure Law. T must be in Kelvin. |
| Avogadro's Law | $V \propto n \implies \frac{V_1}{n_1} = \frac{V_2}{n_2}$ | Constant P and T. |
| | | Equal volumes of gases contain equal number of molecules. |

Dalton's Law of Partial Pressure

$$P_{total} = P_1 + P_2 + \dots + P_n$$

For non-reacting gas mixture.

$$P_i = x_i P_{total}$$

$$x_i = \frac{n_i}{n_{total}} \text{ (Mole fraction).}$$

Graham's Law of Diffusion

$$r \propto \frac{1}{\sqrt{M}} \implies \frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{\rho_2}{\rho_1}}$$

r : Rate of diffusion/effusion.

Lighter gases diffuse faster.

At const P and T .

Pressure of an Ideal Gas

$$P = \frac{1}{3} \rho v_{rms}^2$$

v_{rms} : Root Mean Square speed.

$$P = \frac{1}{3} \frac{mN}{V} v_{rms}^2$$

Pressure depends on density and square of RMS speed.

$$P = \frac{2}{3} E \text{ (where } E \text{ is K.E. per unit volume)}$$

E : Total translational K.E. per unit volume.

Root Mean Square Speed (v_{rms})

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_B T}{m_{molecule}}} = \sqrt{\frac{3P}{\rho}}$$

Speed effective in calculating kinetic energy.

Note: T must be in Kelvin, M in kg/mol.

For mixture:

$$v_{rms(mix)} = \sqrt{\frac{n_1 M_1 v_1^2 + n_2 M_2 v_2^2}{n_1 M_1 + n_2 M_2}}$$

Average Speed (v_{avg})

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8k_B T}{\pi m}}$$

Arithmetic mean of speeds.

$$v_{avg} \approx 0.92 v_{rms}$$

Most Probable Speed (v_{mp})

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2k_B T}{m}}$$

Speed possessed by maximum fraction of molecules.

Ratio

$$v_{mp} : v_{avg} : v_{rms} = \sqrt{2} : \sqrt{8/\pi} : \sqrt{3}$$

| | | |
|--|---|--|
| Kinetic Energy (Translational) | Per molecule: $K.E. = \frac{3}{2}k_B T$ | depends ONLY on Temperature (T). |
| | Per mole: $K.E. = \frac{3}{2}RT$ | Independent of nature of gas. |
| Degrees of Freedom (f) | Monoatomic: $f = 3$ (3 trans) | Used to calculate internal energy and C_v . |
| | Diatomeric (rigid): $f = 5$ (3 trans + 2 rot) | Note: For NEET/JEE, unless specified "high temp", take Diatomic $f = 5$. |
| | Diatomeric (vib at high T): $f = 7$ | |
| | Triatomic (linear): $f = 5$ (rigid) / 7 (vib) | |
| | Triatomic (non-linear): $f = 6$ | |
| Law of Equipartition of Energy | Energy associated with each D.O.F = $\frac{1}{2}k_B T$ (per molecule) | Total Internal Energy (U) depends on f . |
| Internal Energy (U) | $U = \frac{f}{2}nRT$ | For an ideal gas, U is a function of T only. |
| | | Change: $\Delta U = \frac{f}{2}nR\Delta T = nC_v\Delta T$ |
| Mean Free Path (λ) | $\lambda = \frac{1}{\sqrt{2\pi d^2 n_v}}$ | d : diameter of molecule, n_v : number density (N/V). |
| | | $\lambda \propto \frac{T}{P}$ (since $n_v = P/k_B T$). |
| Mixture of Gases | $M_{mix} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$ | Used when non-reacting gases are mixed. |
| | $C_{v(mix)} = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}$ | |
| | $\gamma_{mix} = \frac{C_{p(mix)}}{C_{v(mix)}}$ | |

2. Thermodynamics

| Formula Name / Topic | Formula(e) | Conditions / specific usage |
|--|---|--|
| First Law of Thermodynamics (FLOT) | $dQ = dU + dW$ $Q = \Delta U + W$ | Conservation of Energy. Sign Convention (Chemistry opposite for W): Heat added to system: $Q > 0$ |
| | | Work done BY gas (expansion): $W > 0$ |
| | | Internal Energy increases: $\Delta U > 0$ |
| Work Done (General) | $W = \int_{V_1}^{V_2} P \, dV$ | Area under P-V curve on Volume axis gives Work Done. |
| Mayer's Relation | $C_P - C_V = R$ | Valid for Ideal Gas (per mole). |
| | | $C_P > C_V$ because work is done in isobaric expansion. |
| Specific Heat Ratio (γ) | $\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$ | Monoatomic $\gamma = 5/3 = 1.67$ Diatomeric $\gamma = 7/5 = 1.4$ |
| | | Triatomic $\gamma = 4/3 = 1.33$ |
| Bulk Modulus of Gas (B) | General: $B = -V \frac{dP}{dV}$ | Resistance to compression. |
| | Isothermal (B_T): $B_T = P$ | Adiabatic elasticity is γ times Isothermal elasticity ($B_S = \gamma B_T$). |
| | Adiabatic (B_S): $B_S = \gamma P$ | Isobaric ($P=\text{const}$): $B = 0$. |

| | | |
|---|---|---|
| | | Isochoric ($V=\text{const}$): $B = \infty$. |
| Isochoric Process ($V = \text{const}$) | $W = 0$ | Volume constant ($\Delta V = 0$). |
| | $Q = \Delta U = nC_V\Delta T$ | Gay-Lussac's Law holds. |
| | $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ | |
| Isobaric Process ($P = \text{const}$) | $W = P(V_2 - V_1) = nR(T_2 - T_1)$ | Pressure constant. |
| | $Q = nC_P\Delta T$ | Charles's Law holds. |
| | $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ | Fraction of heat into internal energy: $1/\gamma$. |
| | | Fraction of heat into work: $1 - 1/\gamma$. |
| Isothermal Process ($T = \text{const}$) | $W = nRT \ln\left(\frac{V_2}{V_1}\right) = 2.303nRT \log\left(\frac{V_2}{V_1}\right)$ | Temperature constant ($\Delta T = 0$). |
| | $W = nRT \ln\left(\frac{P_1}{P_2}\right)$ | Internal energy change is zero for ideal gas. |
| | $\Delta U = 0 \implies Q = W$ | Boyle's Law ($P_1V_1 = P_2V_2$). |
| Adiabatic Process ($Q = 0$) | Equation: $PV^\gamma = \text{const}$ | No heat exchange ($dQ = 0$). |
| | $TV^{\gamma-1} = \text{const}$ | Occurs suddenly/quickly or in insulated containers. |
| | $P^{1-\gamma}T^\gamma = \text{const}$ | |
| Work in Adiabatic Process | $W = \frac{P_1V_1 - P_2V_2}{\gamma-1} = \frac{nR(T_1 - T_2)}{\gamma-1}$ | Since $Q = 0$, $W = -\Delta U$. |

| | | |
|--|--|--|
| | | Expansion causes cooling ($T_2 < T_1$). |
| Slope of P-V Graph | Isothermal Slope: $\frac{dP}{dV} = -\frac{P}{V}$ | Adiabatic curve is steeper than Isothermal by a factor of γ . |
| | Adiabatic Slope: $\frac{dP}{dV} = -\gamma \frac{P}{V}$ | |
| Polytropic Process | $PV^x = \text{const}$ | General process. |
| | | Molar Heat Capacity: $C = C_V + \frac{R}{1-x}$ |
| | | Work: $W = \frac{nR(T_1 - T_2)}{x-1}$ |
| Cyclic Process | $\Delta U_{net} = 0$ | Work = Area enclosed by the loop. |
| | $Q_{net} = W_{net}$ | Clockwise = +ve Work (Engine). |
| | | Anticlockwise = -ve Work (Refrigerator). |
| Efficiency of Heat Engine (η) | $\eta = \frac{\text{Work Output}}{\text{Heat Input}} = \frac{W}{Q_{in}}$ | Q_{in} : Heat absorbed from source. |
| | $\eta = 1 - \frac{Q_{out}}{Q_{in}}$ | Q_{out} : Heat rejected to sink. |
| Carnot Engine | $\eta = 1 - \frac{T_{sink}}{T_{source}}$ | Maximum theoretical efficiency. |
| | | T must be in Kelvin. |
| | | Valid for reversible cycle only. |

Refrigerator (COP - β) $\beta = \frac{\text{Heat Extracted}}{\text{Work Input}} = \frac{Q_{cold}}{W}$ Coefficient of Performance.

$$\beta = \frac{Q_{cold}}{Q_{hot} - Q_{cold}} = \frac{T_{cold}}{T_{hot} - T_{cold}}$$

Relationship with efficiency:
 $\beta = \frac{1-\eta}{\eta}$

Entropy (ΔS) $\Delta S = \int \frac{dQ_{rev}}{T}$ Measure of disorder.

Ideal Gas:
 $\Delta S = nC_v \ln\left(\frac{T_2}{T_1}\right) + nR \ln\left(\frac{V_2}{V_1}\right)$

Adiabatic Reversible: $\Delta S = 0$
(Isentropic).

Phase Change: $\Delta S = \frac{mL}{T}$

Free Expansion $W = 0, Q = 0, \Delta U = 0, \Delta T = 0$ Expansion against vacuum ($P_{ext} = 0$).

Neither isothermal nor
adiabatic in strict sense, but
 $\Delta T = 0$ for ideal gas.

Note from AP Sir:

- Always check units: Pressure in Pa (N/m^2), Volume in m^3 , Temperature in Kelvin (K).
- $R = 8.314 \text{ J}/(\text{mol} \cdot \text{K})$ when using SI units.
- $R = 0.0821 \text{ (L} \cdot \text{atm})/(\text{mol} \cdot \text{K})$ when Pressure is atm and Volume is Liters.