

# Units, Dimensions, and Measurements Formulae for NEET & JEE

by AP Sir, Director Sakaar PCMB Classes

Formula / Topic Name	Formula(e) & Expressions	Conditions & Usage Notes
<b>1. Dimensional Analysis</b>		
<b>Fundamental Quantities</b>	Mass [ $M$ ], Length [ $L$ ], Time [ $T$ ], Current [ $A$ ], Temp [ $K$ ], Amount of Substance [ $mol$ ], Luminous Intensity [ $cd$ ]	Base dimensions from which all others are derived.
<b>Order of Magnitude</b>	Express number as $N = a \times 10^b$  If $a \leq \sqrt{10} (\approx 3.16)$ , Order = $b$  If $a > \sqrt{10}$ , Order = $b + 1$	Used to estimate the size of a quantity.  Example: $4 \times 10^5 \rightarrow$ Since $4 > 3.16$ , Order is $10^6$ .
<b>2. Significant Figures</b>		
<b>Counting Rules</b>	1. All non-zero digits are significant.  2. Zeros between non-zeros are significant.  3. Leading zeros are <b>never</b> significant.  4. Trailing zeros with a decimal point are significant.	0.007 (1 SF)  2.05 (3 SF)  2.500 (4 SF)  1200 (Ambiguous, assume 2 unless specified).
<b>Rounding Off</b>	- Digit $> 5$ : Round up  - Digit $< 5$ : No change	Example ( $= 5$ ):  2.45 $\rightarrow$ 2.4 (4 is even)

- Digit = 5: Round to nearest **even** number.

2.35 → 2.4 (round up to even)

### Arithmetic Operations

**Add/Sub:** Result has same **decimal places** as the least precise term.

Add:  $12.11 + 18.0 = 30.1$  (1 dec. place)

**Mul/Div:** Result has same **sig figs** as the least precise term.

Mul:  $2.5 \times 1.25 = 3.1$  (2 SF)

### 3. Vernier Caliper

#### Least Count (L.C.)

$$L.C. = 1MSD - 1VSD$$

Where  $N$  is total divisions on Vernier scale.

$$\text{Standard: } L.C. = \frac{1MSD}{N}$$

Common L.C. = 0.1 mm or 0.01 cm.

#### Reading

$$\text{Reading} = MSR + (VSR \times L.C.)$$

$MSR$ : Main Scale Reading immediately left of zero.

$VSR$ : Coinciding Vernier division.

#### Zero Error

$$\text{True Reading} = \text{Observed} - \text{Zero Error}$$

#### Negative Error Calculation:

**Positive:** Zero of VS is right of MS zero.

$$\text{Error} = -(N - \text{coinciding div}) \times L.C.$$

**Negative:** Zero of VS is left of MS zero.

### 4. Screw Gauge

#### Pitch

$$\text{Pitch} = \frac{\text{Distance moved on Main Scale}}{\text{Number of full rotations}}$$

Usually 1 mm or 0.5 mm. Distance screw moves in 1 rotation.

#### Least Count (L.C.)

$$L.C. = \frac{\text{Pitch}}{\text{Total Circular Scale Divisions (CSD)}}$$

Common L.C. = 0.01 mm or 0.001 cm.

#### Reading

$$\text{Reading} = MSR + (CSR \times L.C.)$$

$MSR$ : Reading visible on linear scale.

*CSR*: Circular division coinciding with reference line.

**Zero Error**

**Positive:** Zero of CS is below reference line.

Always subtract the error (keeping signs in mind).

**Negative:** Zero of CS is above reference line.

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## Topic: Vectors (Physics)

Formula Name / Topic	Formula(e)	Conditions / Usage
1. Magnitude of a Vector	If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ :  $ \vec{A}  = \sqrt{A_x^2 + A_y^2 + A_z^2}$	Used to find the size/length of a vector from its Cartesian components.
2. Unit Vector	$\hat{n} = \frac{\vec{A}}{ \vec{A} }$	Represents direction only. Magnitude is always 1.
3. Vector Addition (Parallelogram Law)	<b>Resultant (<math>R</math>):</b>  $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$  <b>Direction (<math>\alpha</math> with <math>\vec{A}</math>):</b>  $\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$	$\theta$ is the angle between $\vec{A}$ and $\vec{B}$ (tail-to-tail).  $R_{max} = A + B$ (at $0^\circ$ ), $R_{min} =  A - B $ (at $180^\circ$ ).
4. Vector Subtraction	<b>Magnitude:</b>  $ \vec{A} - \vec{B}  = \sqrt{A^2 + B^2 - 2AB \cos \theta}$  <b>Direction:</b>  $\tan \alpha = \frac{B \sin \theta}{A - B \cos \theta}$	Used for relative velocity ( $\Delta \vec{v}$ ). $\theta$ is the angle between original vectors.
5. Resolution of Components	$A_x = A \cos \theta$	$\theta$ is the angle made with the X-axis.

$$A_y = A \sin \theta$$

#### 6. Direction Cosines

$$l = \frac{A_x}{A}, \quad m = \frac{A_y}{A}, \quad n = \frac{A_z}{A}$$

$l, m, n$  are cosines of angles with X, Y, Z axes.

$$l^2 + m^2 + n^2 = 1$$

#### 7. Dot Product (Scalar)

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

Result is Scalar.

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

**Perpendicular if:**  $\vec{A} \cdot \vec{B} = 0$ .

#### 8. Angle Between Vectors

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

Vectors must be tail-to-tail.

#### 9. Cross Product (Vector)

$$\text{Magnitude: } |\vec{A} \times \vec{B}| = AB \sin \theta$$

Result is Vector  $\perp$  to  $\vec{A}$  and  $\vec{B}$ .

**Parallel if:**  $\vec{A} \times \vec{B} = 0$ .

**Determinant Form:**

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

#### 10. Lami's Theorem

$$\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}$$

Only for 3 concurrent forces in equilibrium.

#### 11. Relative Velocity

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

Velocity of A w.r.t B.

#### 12. Rain-Man Concept

$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$$

$\theta$  with vertical.

$$\tan \theta = \frac{v_m}{v_r}$$

#### 13. River Boat: Min Time

$$t_{min} = \frac{d}{v_b}$$

Head perpendicular to flow.

$$\text{Drift } x = v_r \times t_{min}$$

#### 14. River Boat: Shortest Path

$$\sin \theta = \frac{v_r}{v_b}$$

Head upstream at angle  $\theta$ .

$$t = \frac{a}{\sqrt{v_b^2 - v_r^2}}$$

Cond:  $v_b > v_r$ .

### 15. Area of Triangle

$$\text{Area} = \frac{1}{2} |\vec{A} \times \vec{B}|$$

$\vec{A}, \vec{B}$  are adjacent sides.

### 16. Area of Parallelogram

$$\text{Sides: } |\vec{A} \times \vec{B}|$$

$$\text{Diagonals: } \frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$$

### 17. Vol. of Parallelepiped

$$V = |\vec{A} \cdot (\vec{B} \times \vec{C})|$$

Coplanar if Volume = 0.

$$V = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

# Motion in One Dimension - Important Formulae for NEET & JEE

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Topic / Formula Name	Formula(e)	Conditions / Notes
1. Average Velocity & Speed	$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	Valid for any type of motion (uniform or non-uniform). $\Delta x$ is displacement.
	$\text{Speed}_{avg} = \frac{\text{Total Path Length}}{\text{Total Time}}$	
Special Case: Equal Distances	$v_{avg} = \frac{2v_1 v_2}{v_1 + v_2}$	When a body covers equal distances with different speeds $v_1$ and $v_2$ . (Harmonic Mean)
Special Case: Equal Time Intervals	$v_{avg} = \frac{v_1 + v_2}{2}$	When a body travels for equal time intervals with different speeds $v_1$ and $v_2$ . (Arithmetic Mean)
2. Instantaneous Velocity & Speed	$\vec{v}_{inst} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$	$\vec{v}_{inst}$
	$\text{Speed}_{inst} =$	
3. Instantaneous Acceleration	$a = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$	Used for variable acceleration. The form $v(dv/dx)$ is crucial when $v$ is a function of $x$ .
	$a = v \frac{dv}{dx}$	

**4. Equations of Kinematics (Scalar Form)**

1.

$$v = u + at$$

2.

$$s = ut + \frac{1}{2}at^2$$

3.

$$v^2 = u^2 + 2as$$

4.

$$s = \left( \frac{u + v}{2} \right) t$$

**5. Displacement in  $n^{th}$  Second**

$$S_{n^{th}} = u + \frac{a}{2}(2n - 1)$$

**STRICTLY for Constant Acceleration ( $a = \text{const}$ ).**

$u$ : initial velocity,  $v$ : final velocity,  $a$ : acceleration,  $s$ : displacement,  $t$ : time.

Displacement covered strictly during the  $n^{th}$  second of motion. Only for constant acceleration.

**6. Motion Under Gravity (Free Fall)**

1.

$$v = u - gt$$

2.

$$h = ut - \frac{1}{2}gt^2$$

3.

$$v^2 = u^2 - 2gh$$

Sign Convention (Upward +ve):

$a = -g$ , Displacement  $h$  is +ve if up, -ve if down.

For dropped object:  $u = 0$ .

**7. Max Height & Time of Flight**

$$H_{max} = \frac{u^2}{2g}$$

For a particle thrown vertically upward with speed  $u$  returning to the same level.



$$T_{flight} = \frac{2u}{g}$$

### 8. Stopping Distance

$$d_{stop} = \frac{u^2}{2a}$$

Distance traveled before coming to rest ( $v = 0$ ) with retardation  $a$ .

### 9. Relative Velocity in 1D

$$v_{AB} = v_A - v_B$$

Velocity/Acceleration of A with respect to B. Signs are crucial (+ve for one direction, -ve for opposite).

$$a_{AB} = a_A - a_B$$

### 10. Graphical Interpretations

1. Slope of  $x - t$  graph = Velocity ( $v$ )

Valid for all types of motion.

2. Slope of  $v - t$  graph = Acceleration ( $a$ )

3. Area under  $v - t$  graph = Displacement ( $\Delta x$ )

4. Area under Speed-time graph = Distance

5. Area under  $a - t$  graph = Change in velocity ( $\Delta v$ )

### 11. Variable Acceleration (Integration)

$$v = \int a \, dt$$

Use definite integrals with limits when acceleration is a function of time ( $a = f(t)$ ).

$$x = \int v \, dt$$

### Key Tips for Solving Problems:

- **Sign Convention:** Always choose a positive direction (usually right or up). Any vector (displacement, velocity, acceleration) opposite to this is negative.

- **Vector Form:** For complex problems, use  $\vec{v} = \vec{u} + \vec{a}t$ .

- **Differentiation vs Integration:**

- $x \xrightarrow{\text{diff}} v \xrightarrow{\text{diff}} a$

- $a \xrightarrow{\text{int}} v \xrightarrow{\text{int}} x$

# Motion in 2D and Projectile Motion

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Formula Name / Topic	Formula(e)	Conditions / Usage
<b>1. General Motion in 2D</b>		
Position Vector	$\vec{r} = x\hat{i} + y\hat{j}$	Cartesian coordinates at time $t$
Velocity Vector	$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j}$	Instantaneous velocity
Acceleration Vector	$\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j}$	Instantaneous acceleration
<b>2. Projectile Motion (Ground to Ground)</b>		<b>Assumptions:</b> Air resistance neglected, $g$ is constant downwards.
Components of Initial Velocity	$u_x = u \cos \theta$	$\theta$ is angle with horizontal
	$u_y = u \sin \theta$	
Motion Parameters (x & y)	$a_x = 0, v_x = u \cos \theta$ (constant)	$x$ -motion is uniform; $y$ -motion is under gravity
	$a_y = -g$	
Time of Flight ( $T$ )	$T = \frac{2u \sin \theta}{g} = \frac{2u_y}{g}$	Projectile lands at same vertical level as launch
Maximum Height ( $H$ )	$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{u_y^2}{2g}$	Vertical component of velocity becomes zero at max height
Horizontal Range ( $R$ )	$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x u_y}{g}$	Distance between launch and landing point on same level
Condition for Max Range	$\theta = 45^\circ \Rightarrow R_{max} = \frac{u^2}{g}$	For a given initial speed $u$

Relation between  $H$  and  $R$

$$R \tan \theta = 4H$$

Useful for direct relation problems

Equation of Trajectory

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

To find  $y$  given  $x$  (eliminating  $t$ )

OR

$$y = x \tan \theta \left( 1 - \frac{x}{R} \right)$$

Instantaneous Velocity

$$\vec{v} = u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}$$

$\vec{v}$

\$

Angle with Horizontal ( $\alpha$ )

$$\tan \alpha = \frac{v_y}{v_x} = \frac{u \sin \theta - gt}{u \cos \theta}$$

Direction of motion at time  $t$

### 3. Horizontal Projectile (From Tower)

**Condition:** Thrown horizontally ( $u_y = 0$ ) from height  $h$

Time of Flight

$$T = \sqrt{\frac{2h}{g}}$$

Time to reach ground

Horizontal Range

$$R = u \times T = u \sqrt{\frac{2h}{g}}$$

Horizontal distance covered

Velocity at Ground

$$v = \sqrt{u^2 + 2gh}$$

Conservation of energy or vector sum

### 4. Projectile on Inclined Plane

**Incline angle  $\alpha$ , Projection angle  $\theta$   
(w.r.t incline)**

Time of Flight (Incline)

$$T = \frac{2u \sin \theta}{g \cos \alpha}$$

Component of  $g$  perpendicular to  
plane is  $g \cos \alpha$

Range on Incline ( $R_{inc}$ )

$$R_{inc} = \frac{2u^2 \sin \theta \cos(\theta + \alpha)}{g \cos^2 \alpha}$$

Projecting UP the incline

Range on Incline ( $R_{inc}$ )

$$R_{inc} = \frac{2u^2 \sin \theta \cos(\theta - \alpha)}{g \cos^2 \alpha}$$

Projecting DOWN the incline

## 5. Relative Motion in 2D

Relative Velocity

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

Velocity of A with respect to B

Rain-Man Problem

$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$$

$\vec{v}_{rm}$  is how rain appears to the man

River-Boat: Crossing River

$$\vec{v}_{b,g} = \vec{v}_{b,r} + \vec{v}_{r,g}$$

$\vec{v}_{b,r}$  = velocity of boat in still water

Condition: Shortest Path

$$\sin \theta = \frac{v_r}{v_{br}} \text{ (upstream)}$$

Drift = 0 (Requires  $v_{br} > v_r$ )

# Newton's Laws of Motion

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Formula Name / Topic	Formula(e)	Conditions / Usage
<b>Newton's Second Law</b> (General)	$\vec{F}_{ext} = \frac{d\vec{p}}{dt}$	Valid for all systems (even variable mass).
<b>Newton's Second Law</b> (Constant Mass)	$\vec{F}_{net} = m\vec{a}$	Valid only when mass $m$ is constant and in an Inertial Frame.
<b>Linear Momentum</b>	$\vec{p} = m\vec{v}$	Quantity of motion contained in a body.
<b>Impulse (<math>J</math>)</b>	$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{ext} dt = \Delta\vec{p}$ $\vec{J} = \vec{F}_{avg} \cdot \Delta t$	Used when a large force acts for a short time. $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$ .
<b>Impulse-Momentum Theorem</b>	Area under $F - t$ graph = $\Delta p$	Used to find change in momentum from a Force-Time graph.
<b>Equilibrium of Forces</b>	$\sum \vec{F} = 0 \implies \vec{a} = 0$	Body is at rest or moving with constant velocity.
<b>Lami's Theorem</b>	$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$	Valid for 3 coplanar, concurrent forces in equilibrium.
<b>Third Law (Action-Reaction)</b>	$\vec{F}_{AB} = -\vec{F}_{BA}$	Action and reaction act on different bodies simultaneously.
<b>Apparent Weight in Lift</b> (Moving Up)	$N = m(g + a)$	Lift accelerating upwards with acceleration $a$ .
<b>Apparent Weight in Lift</b> (Moving Down)	$N = m(g - a)$	Lift accelerating downwards with acceleration $a$ ( $a < g$ ).

**Apparent Weight** (Free Fall)

$$N = 0$$

Lift cable breaks ( $a = g$ ).  
Weightlessness.

**Conservation of Linear Momentum**

$$\vec{p}_{initial} = \vec{p}_{final}$$

Valid if net external force on the system is zero ( $\vec{F}_{ext} = 0$ ).

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

**Rocket Propulsion**  
(Thrust & Accel)

$$F_{thrust} = u_{rel} \left( -\frac{dm}{dt} \right)$$

$u_{rel}$  is exhaust speed relative to rocket.  
 $-\frac{dm}{dt}$  is rate of fuel consumption.

$$a = \frac{u_{rel}}{m} \left( -\frac{dm}{dt} \right) - g$$

**Rocket Velocity** (at time  $t$ )

$$v = u_{rel} \ln \left( \frac{m_0}{m_t} \right) - gt$$

$m_0$ : initial mass,  $m_t$ : mass at time  $t$ .  
Neglecting initial velocity  $v_0$ .

**Force by Liquid Jet**  
(Thrust on Pipe)

$$F = v \frac{dm}{dt} = \rho A v^2$$

Reaction force on a pipe ejecting liquid of density  $\rho$  through area  $A$ .

**Force by Liquid Jet**  
(Striking Wall)

$$F = \rho A v^2$$

Force exerted by a jet striking a vertical wall normally.

(Stops)

$$F = 2\rho A v^2$$

(Reflects)

**Connected Bodies**  
(Atwood Machine)

$$a = \left( \frac{m_2 - m_1}{m_1 + m_2} \right) g$$

Massless, frictionless pulley and string.  
 $m_2 > m_1$ .

$$T = \left( \frac{2m_1 m_2}{m_1 + m_2} \right) g$$

**Block on Smooth  
Inclined Plane**

$$a = g \sin \theta$$

Sliding down a frictionless incline of angle  $\theta$ .

$$N = mg \cos \theta$$

**Static Friction ( $f_s$ )**

$$f_s \leq \mu_s N$$

Self-adjusting force. Prevents relative motion.  $N$  is Normal reaction.

$$f_{s,max} = \mu_s N$$

(Limiting Friction)

**Kinetic Friction ( $f_k$ )**

$$f_k = \mu_k N$$

Opposes relative motion when bodies are actually sliding.

**Angle of Friction ( $\lambda$ )**

$$\tan \lambda = \mu_s$$

Angle between Normal reaction and Resultant of contact forces.

**Angle of Repose ( $\alpha$ )**

$$\tan \alpha = \mu_s$$

Min angle of incline at which block starts sliding. ( $\alpha = \lambda$ ).

**Acceleration on Rough  
Incline (Down)**

$$a = g(\sin \theta - \mu_k \cos \theta)$$

Block sliding down a rough inclined plane.

**Acceleration on Rough  
Incline (Up)**

$$a = g(\sin \theta + \mu_k \cos \theta)$$

Block pushed up a rough inclined plane (retardation).

**Centripetal Force**

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

Net radial force required for circular motion directed towards center.

**Safe Turn on Level  
Road**

$$v_{max} = \sqrt{\mu_s r g}$$

Vehicle turning on a flat horizontal road. Friction provides centripetal force.

**Banking of Roads  
(Smooth)**

$$\tan \theta = \frac{v^2}{rg}$$

Friction ignored. Ideal banking angle.

**Banking of Roads  
(With Friction)**

$$v_{max} = \sqrt{rg \left( \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta} \right)}$$

Maximum safe speed on a banked rough road.



### Bending of Cyclist

$$\tan \theta = \frac{v^2}{rg}$$

Cyclist leans inward to provide necessary centripetal force.

### Pseudo Force

→

Applied to an object when observing

# Work, Power, and Energy

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Formula Name / Topic	Formula	Condition / Notes
1. Work Done (Constant Force)	$W = \vec{F} \cdot \vec{S} = FS \cos \theta$	Force $\vec{F}$ is constant. $\theta$ is angle between $\vec{F}$ and displacement $\vec{S}$ .
2. Work Done (Variable Force)	$W = \int_{x_1}^{x_2} F_x dx$ $W = \int \vec{F} \cdot d\vec{r}$	Force varies with position.
3. Work from Graph	$W = \text{Area under } F\text{-}x \text{ graph}$	Area between force curve and displacement axis (Area above axis is +, below is -).
4. Kinetic Energy (KE)	$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$	$p = mv$ is linear momentum. Relation between KE and Momentum is crucial.
5. Work-Energy Theorem	$W_{\text{net}} = \Delta K = K_f - K_i$	Valid for <b>all</b> frames (inertial/non-inertial). $W_{\text{net}}$ is work by ALL forces (conservative, non-conservative, pseudo).
6. Gravitational Potential Energy (PE)	$U = mgh$	Near Earth's surface where $g$ is constant. Ref level at ground ( $U = 0$ ).
7. Spring Potential Energy	$U = \frac{1}{2}kx^2$	$x$ is elongation or compression from <b>natural length</b> . $k$ is spring constant.
8. Conservative Force & PE Relation	$F = -\frac{dU}{dx}$ $\Delta U = -W_{\text{conservative}}$	Only defined for conservative forces (Gravity, Electrostatic, Spring).

<b>9. Conservation of Mechanical Energy</b>	$K_i + U_i = K_f + U_f$	Condition: Only conservative forces do work ( $W_{ext} = 0, W_{nc} = 0$ ).
<b>10. Work by Non-Conservative Forces</b>	$W_{nc} = \Delta E_{mech} = (K_f + U_f) - (K_i + U_i)$	Used when friction or air resistance is present.
<b>11. Average Power</b>	$P_{avg} = \frac{\Delta W}{\Delta t}$	Total work done divided by total time taken.
<b>12. Instantaneous Power</b>	$P = \vec{F} \cdot \vec{v} = Fv \cos \theta$	Rate of work at a specific instant.
<b>13. Vertical Circular Motion (Critical)</b>	$v_{top} \geq \sqrt{gR}, v_{bottom} \geq \sqrt{5gR}$	Condition to complete a full vertical circle (String/Loop).
<b>14. Vertical Circular Motion (Tension)</b>	$T_{bottom} - T_{top} = 6mg$	Difference in tension between lowest and highest point.
<b>15. Coefficient of Restitution (<math>e</math>)</b>	$e = \frac{v_{sep}}{v_{app}} = \frac{v_2 - v_1}{u_1 - u_2}$	$e = 1$ (Elastic), $0 < e < 1$ (Inelastic), $e = 0$ (Perfectly Inelastic). Along Line of Impact.
<b>16. Elastic Collision (1D)</b>	$v_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left( \frac{2m_2}{m_1 + m_2} \right) u_2$	Momentum and KE are conserved.
<b>17. Perfectly Inelastic Collision</b>	$V_{common} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$	Bodies stick together ( $e = 0$ ). Max loss of KE.
<b>18. Loss in KE (Collision)</b>	$\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2 (1 - e^2)$	General formula for KE loss in any head-on collision.
<b>19. Equilibrium Conditions</b>	Stable: $U'' > 0$ , Unstable: $U'' < 0$ , Neutral: $U'' = 0$	$U'' = d^2U/dx^2$ . At equilibrium, Net Force is zero.
<b>20. Chain Pulling Problem</b>	$W = \frac{MgL}{2n^2}$	Work to pull a chain hanging $1/n$ th part off a table back onto the table.
<b>21. Stopping Distance</b>	$d_s = \frac{v^2}{2\mu g}$	Vehicle stopping distance with friction coefficient $\mu$ .

**22. Power of a Pump/Motor**

$$P = \frac{dm}{dt} gh + \frac{1}{2} \frac{dm}{dt} v^2$$

Power to lift water rate  $dm/dt$  to height  $h$  and eject with velocity  $v$ .

**23. Rebound Height**

$$h_n = e^{2n} h_0$$

Height after  $n^{th}$  bounce. Total distance =  $h_0 \left( \frac{1+e^2}{1-e^2} \right)$ .

**24. Spring-Block (Sudden Release)**

$$x_{max} = \frac{2mg}{k}$$

If a block of mass  $m$  attached to a spring is released suddenly from natural length.

**25. Bullet Penetration**

$$F_{avg} \cdot d = \frac{1}{2} mv^2$$

Work done by resistive force = Change in KE.

**26. Oblique Collision**

$$v \sin \alpha = u \sin \theta$$

Comp. of velocity  $\perp$  to line of impact is unchanged. Along line of impact, use  $e$ .

# CIRCULAR MOTION

BY AP Sir, Sakaar Classes

Formula / Topic Name	Formula	Conditions / Usage
<b>1. Kinematics of Circular Motion</b>		
Angular Velocity ( $\omega$ )	$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$ $\omega_{inst} = \frac{d\theta}{dt}$	Rate of change of angular displacement.
Linear Velocity ( $v$ )	$v = r\omega$	Relation between linear speed ( $v$ ) and angular speed ( $\omega$ ) for a particle at radius $r$ .
Angular Acceleration ( $\alpha$ )	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$	Rate of change of angular velocity.
Tangential Acceleration ( $a_t$ )	$a_t = r\alpha = \frac{dv}{dt}$	Responsible for changing the <b>magnitude</b> of velocity (speed). Zero in U.C.M.
Centripetal (Radial) Acceleration ( $a_c$ or $a_r$ )	$a_c = \frac{v^2}{r} = r\omega^2 = v\omega$	Responsible for changing the <b>direction</b> of velocity. Always directs towards center.
Net Acceleration ( $a_{net}$ )	$a_{net} = \sqrt{a_c^2 + a_t^2}$	Vector sum of radial and tangential acceleration.
Angle of Net Acceleration ( $\phi$ )	$\tan \phi = \frac{a_c}{a_t}$	$\phi$ is the angle made by net acceleration with the tangential direction.
Equations of Circular Motion	$\omega = \omega_0 + \alpha t$	<b>Condition:</b> Only valid when Angular Acceleration ( $\alpha$ ) is <b>constant</b> .

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

## 2. Dynamics of Circular Motion

Centripetal Force ( $F_c$ )

$$F_c = \frac{mv^2}{r} = mr\omega^2$$

Real force required to keep a body in circular motion (provided by Tension, Friction, Normal, etc.).

Centrifugal Force (Pseudo Force)

$$F_{cf} = \frac{mv^2}{r}$$

Acts radially outward. **Condition:** Only applicable in a **rotating (non-inertial) reference frame**.

## 3. Banking of Roads & Turning

Bending of Cyclist

$$\tan \theta = \frac{v^2}{rg}$$

$\theta$  with vertical. Condition for no skidding while turning.

Car on Level Circular Road (No Banking)

$$v_{max} = \sqrt{\mu_s rg}$$

Maximum safe speed to avoid skidding.  $\mu_s$  = coefficient of static friction.

Banked Road (Frictionless)

$$\tan \theta = \frac{v^2}{rg}$$

Optimum speed  $v_{opt} = \sqrt{rg \tan \theta}$ . No wear and tear on tires.

Banked Road (With Friction) - Max Speed

$$v_{max} = \sqrt{rg \left( \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta} \right)}$$

Speed limit to avoid slipping **outwards** (up the incline).

Banked Road (With Friction) - Min Speed

$$v_{min} = \sqrt{rg \left( \frac{\tan \theta - \mu_s}{1 + \mu_s \tan \theta} \right)}$$

Speed limit to avoid slipping **inwards** (down the incline).

## 4. Conical Pendulum

Angular Velocity

$$\omega = \sqrt{\frac{g}{L \cos \theta}} = \sqrt{\frac{g}{h}}$$

$L$  = length of string,  $h$  = vertical height of point of suspension from circle center.

Time Period ( $T$ )

$$T = 2\pi \sqrt{\frac{L \cos \theta}{g}} = 2\pi \sqrt{\frac{h}{g}}$$

Time for one complete revolution.

Tension in String

$$T_{tension} = \frac{mg}{\cos \theta} = mL\omega^2$$

## 5. Vertical Circular Motion (String)

Velocity at any point

$$v = \sqrt{u^2 - 2gh}$$

$u$  = speed at bottom,  $h$  = height from bottom.

Tension at any point

$$T = \frac{mv^2}{r} + mg \cos \theta$$

$\theta$  is angle with vertical downward direction.

Critical Velocity (Top)

$$v_{top} = \sqrt{gR}$$

Minimum speed at top to keep string taut ( $T_{top} \geq 0$ ).

Critical Velocity (Bottom)

$$v_{bottom} = \sqrt{5gR}$$

Minimum speed at bottom to complete the full circle.

Critical Velocity (Horizontal Point)

$$v_{mid} = \sqrt{3gR}$$

Speed at the point where string is horizontal.

Tension Difference

$$T_{bottom} - T_{top} = 6mg$$

Valid for any vertical circular motion under gravity.

Condition for Oscillation

$$0 < v_{bottom} \leq \sqrt{2gR}$$

Particle oscillates like a pendulum (doesn't reach horizontal level).

Condition for Leaving Circle

$$\sqrt{2gR} < v_{bottom} < \sqrt{5gR}$$

Particle leaves the circular path in the upper half ( $T$  becomes 0 before  $v$ ).

## 6. Specific Formulae for Questions

"Death Well" (Rotor)

$$v_{min} = \sqrt{\frac{gR}{\mu}}$$

Min speed to prevent falling. Friction acts upwards balancing weight.

Vehicle on Convex Bridge

$$v_{max} = \sqrt{gR}$$

Max speed to maintain contact with the bridge (Normal reaction  $N = 0$ ).

Radius of Curvature (Projectile)

$$R_{curv} = \frac{v^2}{a_{\perp}}$$

At top of trajectory:

$$R = \frac{u^2 \cos^2 \theta}{g}$$

Toppling of Car on Turn

$$v_{max} = \sqrt{\frac{gra}{h}}$$

$2a$  = distance between wheels (track width),  
 $h$  = height of Center of Mass. Condition:  
Topples if  $v > v_{max}$ .



# Rotational Motion

BY AP Sir, Sakaar Classes

Formula / Topic Name	Formula(e)	Conditions / When to use
<b>1. KINEMATICS OF ROTATION</b>		
<b>Angular Displacement</b>	$\theta = \frac{l}{r}$	Angle in radians, $l$ = arc length.
<b>Angular Velocity</b>	$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$	Rate of change of angular position.
	$\omega_{inst} = \frac{d\theta}{dt}$	
<b>Angular Acceleration</b>	$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$	Rate of change of angular velocity.
	$\alpha_{inst} = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta}$	
<b>Equations of Kinematics</b>	1. $\omega = \omega_0 + \alpha t$ 2. $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ 3. $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$ 4. $\theta_{nth} = \omega_0 + \frac{\alpha}{2}(2n - 1)$	<b>Strictly valid only when angular acceleration (<math>\alpha</math>) is CONSTANT.</b>
<b>Linear vs Angular Variables</b>	$v = \omega r$	$v$ is tangential velocity.
	$a_t = \alpha r$ (Tangential acc.)	$a_t$ changes speed.
	$a_c = \omega^2 r$ (Centripetal acc.)	$a_c$ changes direction (always exists if $\omega \neq 0$ ).

$$a_{net} = \sqrt{a_t^2 + a_c^2}$$

## 2. MOMENT OF INERTIA (MOI)

Discrete System

$$I = \sum m_i r_i^2$$

$r_i$  is perpendicular distance from the axis of rotation.

Continuous Bodies

$$I = \int r^2 dm$$

Requires integration.

Radius of Gyration ( $k$ )

$$I = Mk^2 \Rightarrow k = \sqrt{\frac{I}{M}}$$

Distance where total mass is theoretically concentrated to give same  $I$ .

Perpendicular Axis Theorem

$$I_z = I_x + I_y$$

**Valid ONLY for planar (2D) bodies** (laminar objects). Axes  $x, y$  must be in the plane;  $z$  perpendicular to plane.

Parallel Axis Theorem

$$I_{axis} = I_{CM} + Md^2$$

Valid for **any** 3D or 2D body.

$d$  = perpendicular distance between parallel axes.

One axis **MUST** pass through Center of Mass (CM).

## 3. MOI OF STANDARD BODIES

(Axis through Center, unless specified)

Ring / Hollow Cylinder

$$I = MR^2$$

Axis perpendicular to plane (Ring) or along geometrical axis (Cylinder).

Disc / Solid Cylinder

$$I = \frac{MR^2}{2}$$

Axis perpendicular to plane (Disc) or along geometrical axis (Cylinder).

Thin Rod	$I = \frac{ML^2}{12}$ (Center)	Axis perpendicular to length.
	$I = \frac{ML^2}{3}$ (End)	
Solid Sphere	$I = \frac{2}{5}MR^2$	Axis along diameter.
Hollow Sphere (Shell)	$I = \frac{2}{3}MR^2$	Axis along diameter.
Rectangular Plate	$I = \frac{M(a^2+b^2)}{12}$	Axis perpendicular to plate, through center.

#### 4. TORQUE ( $\tau$ )

Vector Definition	$\vec{\tau} = \vec{r} \times \vec{F}$	$\tau$
	$\tau = rF \sin \theta$	
Newton's 2nd Law (Rotation)	$\vec{\tau}_{net} = I\vec{\alpha}$	Valid for rigid bodies rotating about a fixed axis or about CM.
	$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$	
Couple	$\tau = F \times d$	Two equal and opposite forces. Torque is independent of the choice of origin.
Rotational Equilibrium	$\sum \vec{F}_{ext} = 0$ AND $\sum \vec{\tau}_{ext} = 0$	Body is neither accelerating translationally nor rotationally.

#### 5. ANGULAR MOMENTUM ( $L$ )

Point Mass	$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$	About a specific origin.
	$L = mvr_{\perp}$	

<b>Rigid Body (Fixed Axis)</b>	$L = I\omega$	Axis must be fixed or passing through CM.
<b>Combined Motion (Rolling)</b>	$\vec{L} = \vec{L}_{CM} + \vec{L}_{aboutCM} = M(\vec{r}_{cm} \times \vec{v}_{cm}) + I_{cm}\vec{\omega}$	General formula for a body moving and rotating.
<b>Conservation of Angular Momentum</b>	$I_1\omega_1 = I_2\omega_2$	<b>Condition:</b> Net external torque on the system is ZERO ( $\tau_{ext} = 0$ ).
<b>6. WORK, POWER, ENERGY</b>		
<b>Rotational Kinetic Energy</b>	$K_{rot} = \frac{1}{2}I\omega^2$	Pure rotation about an axis.
<b>Work Done</b>	$W = \int \tau d\theta = \vec{\tau} \cdot \Delta\vec{\theta}$	Analogous to $W = \vec{F} \cdot \vec{d}$ .
<b>Power</b>	$P = \vec{\tau} \cdot \vec{\omega}$	Instantaneous power delivered by torque.
<b>Work-Energy Theorem</b>	$W_{ext} = \Delta K = K_f - K_i$	Work done by all torques equals change in Rotational KE.
<b>7. ROLLING MOTION</b>		
<b>Condition for Pure Rolling</b>	$v_{cm} = \omega R$	Velocity of the bottom-most point (contact point) is zero relative to ground.
	$a_{cm} = \alpha R$	
<b>Total Kinetic Energy</b>	$K_{total} = K_{trans} + K_{rot}$	Useful factor $\beta = (1 + \frac{k^2}{R^2})$ .
	$K_{total} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$	Ring: $\beta = 2$ , Disc: $\beta = 1.5$ , Solid Sphere: $\beta = 1.4$ .
	$K_{total} = \frac{1}{2}Mv_{cm}^2(1 + \frac{k^2}{R^2})$	
<b>8. ROLLING ON INCLINED PLANE</b>	<b>(Specific Question Formulae)</b>	<b>Body rolling down from rest without slipping.</b>

**Acceleration**

$$a = \frac{g \sin \theta}{1 + \frac{I}{MR^2}} = \frac{g \sin \theta}{\beta}$$

Standard NEET/JEE result.  
Solid sphere accelerates fastest (lowest  $\beta$ ).

**Velocity at bottom**

$$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$$

Depends only on height  $h$  and shape factor  $\frac{k^2}{R^2}$ , not Mass or Radius.

**Time to reach bottom**

$$t = \sqrt{\frac{2L(1 + \frac{k^2}{R^2})}{g \sin \theta}}$$

$L$  = length of incline. Ring takes max time, Solid sphere takes min time.

**Min Friction Coefficient**

$$\mu_{min} = \frac{\tan \theta}{1 + \frac{MR^2}{I}}$$

Condition to prevent slipping while rolling down.

## 9. COLLISION & TOPPLING

**Angular Impulse ( $J$ )**

$$J = \int \tau dt = \Delta L$$

Change in Angular Momentum.

**Rod hit by particle**

$$L_i = L_f \text{ (about hinge/pivot)}$$

$$mvx = \left(\frac{ML^2}{3} + mx^2\right)\omega$$

Use Conservation of Angular Momentum about the pivot point to find  $\omega$  after impact.

# Centre of Mass and Collisions

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Formula Name / Topic	Formula(e)	Conditions / Usage
1. Position of COM (Two Particles)	$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$	System of two point masses $m_1, m_2$ at positions $x_1, x_2$ .
2. Position Vector of COM (Discrete)	$\vec{R}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n}{M_{total}}$	General system of $n$ discrete particles.
3. Coordinate Formulae (3D)	$X_{cm} = \frac{\sum m_i x_i}{\sum m_i}$ $Y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$ $Z_{cm} = \frac{\sum m_i z_i}{\sum m_i}$	To find specific coordinates ( $x, y, z$ ) of the COM.
4. COM of Continuous Bodies	$X_{cm} = \frac{1}{M} \int x \, dm$ $Y_{cm} = \frac{1}{M} \int y \, dm$	Used for rigid bodies (rods, discs, etc.) where mass is distributed continuously.
5. Linear Mass Density ( $\lambda$ )	$dm = \lambda \, dx$	Used for 1D objects like rods/wires.
6. Areal Mass Density ( $\sigma$ )	$dm = \sigma \, dA$	Used for 2D objects like plates/discs/shells.

7. Volumetric  
Mass Density ( $\rho$ )

$$dm = \rho dV$$

Used for 3D objects like  
spheres/cones.

8. COM: Uniform  
Semi-Circular  
Ring

$$Y_{cm} = \frac{2R}{\pi}$$

Center of base is at origin.  
Symmetric axis is Y-axis.

9. COM: Uniform  
Semi-Circular  
Disc

$$Y_{cm} = \frac{4R}{3\pi}$$

Center of base is at origin.

10. COM: Hollow  
Hemisphere

$$Y_{cm} = \frac{R}{2}$$

From the center of the base.

11. COM: Solid  
Hemisphere

$$Y_{cm} = \frac{3R}{8}$$

From the center of the base.

12. COM: Hollow  
Cone

$$Y_{cm} = \frac{h}{3}$$

From the center of the base.

13. COM: Solid  
Cone

$$Y_{cm} = \frac{h}{4}$$

From the center of the base.

14. COM: Cavity  
Problems  
(Negative Mass)

$$\vec{r}_{cm} = \frac{M_{original}\vec{r}_1 - M_{removed}\vec{r}_2}{M_{original} - M_{removed}}$$

When a part is removed from a  
rigid body.  $\vec{r}_1$  is COM of original,  
 $\vec{r}_2$  is COM of removed part.

15. Velocity of  
COM

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \dots}{M_{total}}$$

Velocity of the system's center  
of mass.

16. Acceleration  
of COM

$$\vec{a}_{cm} = \frac{\vec{F}_{ext}}{M_{total}}$$

Newton's 2nd Law applied to  
the whole system.

17. Momentum  
Conservation  
(System)

$$\vec{P}_{initial} = \vec{P}_{final}$$

Valid ONLY if **External Force** ( $\vec{F}_{ext}$ ) on the system is **Zero**.

18. Displacement  
of COM

$$\Delta\vec{r}_{cm} = \frac{m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2}{M}$$

If  $\vec{F}_{ext} = 0$  initially at rest, then  
 $\Delta\vec{r}_{cm} = 0$ . (e.g., Boat & Dog  
problems).

**19. Impulse ( $\vec{J}$ )**

$$\vec{J} = \int \vec{F} dt = \Delta \vec{P} = \vec{P}_f - \vec{P}_i$$

Change in momentum caused by a large force over a short time.

**20. Coefficient of Restitution ( $e$ )**

$$e = \frac{\text{Velocity of Separation}}{\text{Velocity of Approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$

Defined along the **Line of Impact**.  $0 \leq e \leq 1$ .

**21. Head-on Elastic Collision ( $e = 1$ )**

$$v_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left( \frac{2m_2}{m_1 + m_2} \right) u_2$$

Final velocity of mass 1.  
Conservation of KE & Momentum holds.

**22. Head-on Elastic Collision ( $e = 1$ )**

$$v_2 = \left( \frac{m_2 - m_1}{m_1 + m_2} \right) u_2 + \left( \frac{2m_1}{m_1 + m_2} \right) u_1$$

Final velocity of mass 2.

**23. Elastic Collision: Equal Masses**

$$v_1 = u_2, \quad v_2 = u_1$$

If  $m_1 = m_2$ , velocities are **exchanged**.

**24. Perfectly Inelastic Collision ( $e = 0$ )**

$$V_{\text{common}} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

Particles stick together and move with common velocity.  
Max KE loss.

**25. Inelastic Collision ( $0 < e < 1$ )**

$$v_1 = \frac{m_1 u_1 + m_2 u_2 - m_2 e (u_1 - u_2)}{m_1 + m_2}$$

General formula for 1D collision.

**26. Loss in KE (Head-on)**

$$\Delta K = \frac{1}{2} \left( \frac{m_1 m_2}{m_1 + m_2} \right) (u_1 - u_2)^2 (1 - e^2)$$

Energy dissipated as heat/sound. Zero if  $e = 1$  (Elastic).

**27. Oblique Collision (2D)**

Along Line of Impact (LOI):

$$e = \frac{v_{2n} - v_{1n}}{u_{1n} - u_{2n}}$$

Momentum is conserved along LOI and Tangential axis (if smooth). Restitution eq applies **ONLY** along LOI.

**28. Rocket Propulsion (Thrust)**

$$F_{\text{thrust}} = v_{\text{rel}} \left( -\frac{dm}{dt} \right)$$

Force on rocket due to ejected gas.  $v_{\text{rel}}$  is velocity of gas w.r.t rocket.

**29. Rocket**

Velocity at time  $t$  (ignoring



# ELASTICITY

BY AP Sir, Sakaar Classes

Formula Name / Topic	Formula	Condition / Context / Use Case
Normal Stress ( $\sigma$ )	$\sigma = \frac{F_{\perp}}{A}$	Restoring force ( $F_{\perp}$ ) acting per unit area perpendicular to the cross-section.
Tangential / Shear Stress ( $\sigma_t$ )	$\sigma_t = \frac{F_{\parallel}}{A}$	Force ( $F_{\parallel}$ ) acting parallel to the surface area. Causes shape change without volume change.
Longitudinal Strain ( $\varepsilon_l$ )	$\varepsilon_l = \frac{\Delta L}{L}$	Change in length per unit original length (Tensile or Compressive).
Shearing Strain ( $\phi$ )	$\phi \approx \tan \phi = \frac{x}{L}$	Relative displacement ( $x$ ) between parallel layers separated by distance $L$ .
Volumetric Strain ( $\varepsilon_v$ )	$\varepsilon_v = -\frac{\Delta V}{V}$	Change in volume per unit original volume. Negative sign indicates decrease in volume with pressure increase.
Hooke's Law	Stress $\propto$ Strain  Stress = $E \times$ Strain	Valid only within the <b>Proportional Limit</b> . $E$ is the Modulus of Elasticity.
Young's Modulus ( $Y$ )	$Y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$  $Y = \frac{FL}{A\Delta l} = \frac{mgL}{\pi r^2 \Delta l}$	Used for solids (wires, rods) undergoing length change. Specific for a material.
Bulk Modulus ( $B$ or $K$ )	$B = \frac{-P}{\Delta V/V} = -V \frac{\Delta P}{\Delta V}$	Relates volume change to pressure change. Applicable to solids, liquids, and gases.
Compressibility ( $K$ )	$K = \frac{1}{B}$	Reciprocal of Bulk Modulus.

**Modulus of Rigidity / Shear Modulus ( $\eta$  or  $G$ )**

$$\eta = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{F}{A\phi}$$

Resistance to change in shape.  
Only for solids.

**Poisson's Ratio ( $\sigma$ )**

$$\sigma = -\frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

Theoretical limits:  $-1$  to  $0.5$ .

$$\sigma = \frac{\Delta D/D}{\Delta L/L}$$

Practical limits:  $0$  to  $0.5$ .

**Work Done in Stretching (Strain Energy  $U$ )**

$$U = \frac{1}{2} \times F \times \Delta l$$

Total potential energy stored in a stretched wire.

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{Volume}$$

**Energy Density ( $u$ )**

$$u = \frac{1}{2} \times \text{stress} \times \text{strain}$$

Energy stored per unit volume.

$$u = \frac{1}{2} Y (\text{strain})^2 = \frac{(\text{stress})^2}{2Y}$$

**Elongation due to Self Weight**

$$\Delta l = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y}$$

Extension of a hanging rod/wire due to its own gravity.  $M$ =mass,  $\rho$ =density. Note the factor **2** in denominator (acts at Center of Mass).

**Thermal Stress**

$$\sigma_{\text{thermal}} = Y\alpha\Delta T$$

Rod fixed between rigid supports.

$$\text{Force } F = Y A \alpha \Delta T$$

$\alpha$  = coeff. of linear expansion,  
 $\Delta T$  = temp change.

**Analogy with Spring Constant ( $k$ )**

$$k = \frac{YA}{L}$$

Treating a wire as a spring ( $F = kx$ ). Useful for series/parallel combination of wires.

**Wires in Series**

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Composite wire with same Force/Tension acting on both segments.

$$\Delta l_{net} = \Delta l_1 + \Delta l_2$$

<b>Wires in Parallel</b>	$k_{eq} = k_1 + k_2$	Composite wire where extensions are forced to be equal ( $\Delta l_1 = \Delta l_2$ ).
	$F_{net} = F_1 + F_2$	
<b>Interatomic Force Constant (<math>k_a</math>)</b>	$k_a = Y \times r_0$	$r_0$ is the equilibrium interatomic distance.
<b>Depression of a Beam (Cantilever)</b>	$\delta = \frac{WL^3}{3YI_g}$	Beam fixed at one end, loaded ( $W$ ) at the other. $I_g$ is Geometrical Moment of Inertia.
<b>Depression of Beam (Supported at ends)</b>	$\delta = \frac{WL^3}{48YI_g}$	Beam supported at both ends, load $W$ in the center.
<b>Torsion of a Cylinder</b>	$C = \frac{\pi\eta r^4}{2L}$	Restoring couple per unit twist (Torsional rigidity).
<b>Breaking Stress</b>	Breaking Force = Breaking Stress $\times A$	Breaking stress depends on material, not dimensions. Breaking Force depends on area.
<b>Relation: <math>Y, B, \sigma</math></b>	$Y = 3B(1 - 2\sigma)$	Relates Young's, Bulk Modulus and Poisson's ratio.
<b>Relation: <math>Y, \eta, \sigma</math></b>	$Y = 2\eta(1 + \sigma)$	Relates Young's, Rigidity Modulus and Poisson's ratio.
<b>Relation: <math>Y, B, \eta</math></b>	$\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$	Useful when $\sigma$ is not given.
<b>Relation: <math>\sigma</math> in terms of <math>B, \eta</math></b>	$\sigma = \frac{3B - 2\eta}{6B + 2\eta}$	Calculation of Poisson's ratio from moduli.

### Quick Tips for Numerical Questions (NEET/JEE)

- Wire Cut into n parts:** If a wire of Young's Modulus  $Y$  is cut into  $n$  equal parts, the Young's Modulus of each part remains  $Y$  (Material property), but the spring constant becomes  $nk$ .
- % Change questions:** If strain is small ( $< 5\%$ ), use  $\frac{\Delta R}{R} \times 100$ . For volume of wire  $V = A \times L$  (constant),  $\frac{\Delta A}{A} = -\frac{\Delta L}{L}$  (ignoring  $\sigma$  effects for simple resistance type q's) or use conservation of volume  $A_1 L_1 = A_2 L_2$ .
- Adiabatic vs Isothermal Modulus:**

# Fluid Mechanics & Surface Tension

BY AP Sir, Sakaar Classes

## Fluid Properties & Hydrostatics

Formula Name / Topic	Formula	Condition / Note
Density & Relative Density (RD)	$\rho = \frac{m}{V}$	$\rho_{\text{water}} = 1000 \text{ kg/m}^3$ . RD has no units.
	$\text{RD} = \frac{\rho_{\text{substance}}}{\rho_{\text{water at } 4^\circ \text{C}}}$	
Pressure at Depth	$P = P_0 + h\rho g$	$P_0$ : Atmospheric Pressure
		$h$ : Depth below free surface.
Gauge Pressure	$P_g = P_{\text{absolute}} - P_{\text{atm}} = h\rho g$	Pressure due to fluid column only.
Pascal's Law	$\frac{F_1}{A_1} = \frac{F_2}{A_2}$	Pressure applied to enclosed fluid is transmitted undiminished.
Force on Vertical Dam Wall	$F = \frac{1}{2}\rho g w H^2$	$w$ : Width, $H$ : Depth. Force acts at $H/3$ from bottom.
Archimedes' Principle	$F_B = V_{\text{in}} \cdot \rho_L \cdot g$	$V_{\text{in}}$ : Submerged volume
		$\rho_L$ : Density of Liquid.
Condition for Floatation	$mg = F_B$	Body floats if $\rho_S \leq \rho_L$ . Weight of body = Weight of fluid displaced.
	$\frac{V_{\text{in}}}{V_{\text{total}}} = \frac{\rho_S}{\rho_L}$	
Accelerated Fluid (Horizontal)	$\tan \theta = \frac{a}{g}$	$\theta$ : Angle of free surface with horizontal.
Accelerated Fluid (Vertical)	$P = P_0 + h\rho(g_{\text{eff}})$	$g_{\text{eff}} = g + a$ (up), $g_{\text{eff}} = g - a$ (down).

**Rotating Fluid  
(Vortex)**

$$y = \frac{\omega^2 x^2}{2g}$$

Parabolic meniscus shape.

**Fluid Dynamics**

Formula Name / Topic

Formula

Condition / Note

**Equation of  
Continuity**

$$A_1 v_1 = A_2 v_2$$

Conservation of Mass. Incompressible, non-viscous flow.

**Bernoulli's Principle**

$$P + \rho gh + \frac{1}{2} \rho v^2 = \text{Constant}$$

Conservation of Energy per unit volume. Ideal fluid.

**Torricelli's Law**

$$v = \sqrt{2gh}$$

$h$ : Depth of hole from top.

**Horizontal Range of  
Efflux**

$$R = 2\sqrt{h(H-h)}$$

$R_{\max} = H$  when hole is at  $H/2$ .

**Time to empty tank**

$$t = \frac{A}{a} \sqrt{\frac{2}{g}} (\sqrt{H_1} - \sqrt{H_2})$$

$A$ : Tank area,  $a$ : Hole area.

**Venturimeter**

$$Q = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

$h$ : Height diff in manometer.

**Viscosity**

Formula Name / Topic

Formula

Condition / Note

**Newton's Law of Viscosity**

$$F = -\eta A \frac{dv}{dx}$$

$\eta$ : Coeff. of viscosity,  $\frac{dv}{dx}$ : Velocity gradient.

**Stoke's Law**

$$F = 6\pi\eta r v$$

Viscous drag on sphere of radius  $r$ .

**Terminal Velocity**

$$v_T = \frac{2}{9} \frac{r^2(\rho - \sigma)g}{\eta}$$

Constant max velocity.  $\rho$ : sphere,  $\sigma$ : fluid.

**Poiseuille's Equation**

$$Q = \frac{\pi P r^4}{8\eta l}$$

Volume flow rate in capillary tube.

**Reynolds Number**

$$R_e = \frac{\rho v d}{\eta}$$

$R_e < 1000$ : Laminar,  $R_e > 2000$ : Turbulent.

**Surface Tension**

Formula Name / Topic

Formula

Condition / Note

<b>Surface Tension</b>	$T = \frac{F}{L}$	Force per unit length.
<b>Surface Energy</b>	$U = T \times \Delta A$	$\Delta A$ : Change in area.
<b>Work done (Liquid Drop)</b>	$W = T \cdot 4\pi(r_2^2 - r_1^2)$	Single surface.
<b>Work done (Soap Bubble)</b>	$W = T \cdot 8\pi(r_2^2 - r_1^2)$	Two surfaces (inner & outer).
<b>Excess Pressure (Drop)</b>	$\Delta P = \frac{2T}{R}$	Pressure inside > outside.
<b>Excess Pressure (Bubble)</b>	$\Delta P = \frac{4T}{R}$	Two free surfaces.
<b>Capillary Rise</b>	$h = \frac{2T \cos \theta}{r \rho g}$	Jurist's Law. $\theta$ : Contact angle.
<b>Force to lift wire frame</b>	$F = 2Tl + mg$	Surface tension acts on both sides.
<b>Force to lift Ring</b>	$F \approx 4\pi rT + mg$	Ring of radius $r$ .
<b>Splitting of Drops</b>	$\Delta U = 4\pi R^2 T (n^{1/3} - 1)$	Energy absorbed (Temp falls).
<b>Coalescence of Drops</b>	$E_{\text{released}} = 4\pi T (nr^2 - R^2)$	Energy released (Temp rises).

*“Success is the sum of small efforts repeated day in and day out”*

# Oscillations (Simple Harmonic Motion)

BY AP Sir, Sakaar Classes

Formula / Topic Name	Formula(e)	Conditions / Notes
1. Standard Equation of SHM	$\frac{d^2x}{dt^2} + \omega^2x = 0$	Differential equation condition for any particle executing SHM.
2. Displacement	$x = A \sin(\omega t + \phi)$  or  $x = A \cos(\omega t + \phi)$	General displacement from mean position.  $\phi$ : Initial phase (epoch).  Use $\sin$ if starts from mean, $\cos$ if from extreme.
3. Angular Frequency	$\omega = \frac{2\pi}{T} = 2\pi f = \sqrt{\frac{k}{m}}$	$\omega$ : Angular frequency (rad/s).  Depends on system properties ( $k, m$ ), not amplitude.
4. Velocity ( $v$ )	$v = \frac{dx}{dt} = A\omega \cos(\omega t + \phi)$  $v = \pm\omega\sqrt{A^2 - x^2}$	$v_{max} = A\omega$ (at mean position, $x = 0$ ).  $v_{min} = 0$ (at extreme position, $x = \pm A$ ).
5. Acceleration ( $a$ )	$a = \frac{dv}{dt} = -\omega^2 A \sin(\omega t + \phi)$  $a = -\omega^2 x$	Direction is always towards mean position.  $a_{max} = \omega^2 A$ (at extreme).  $a_{min} = 0$ (at mean).

**6. Restoring Force**

$$F = -kx$$

Linear SHM condition. Force is proportional to displacement and opposite in direction.

$$F = -m\omega^2 x$$

**7. Phase Difference**

$$\Delta\phi = \phi_2 - \phi_1$$

$$\text{Time diff } \Delta t = \frac{T}{2\pi} \Delta\phi.$$

$$\text{Path diff } \Delta x = \frac{\lambda}{2\pi} \Delta\phi \text{ (Wave context).}$$

**8. Kinetic Energy (KE)**

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$

Max at mean position ( $K_{max} = \frac{1}{2}kA^2$ ).

$$K = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

Zero at extreme position.

**9. Potential Energy (PE)**

$$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$$

Assuming  $U = 0$  at mean position.

$$U = \frac{1}{2}kA^2 \sin^2(\omega t + \phi)$$

Max at extreme ( $U_{max} = \frac{1}{2}kA^2$ ).

**10. Total Energy (TE)**

$$E = K + U = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$$

TE is constant (conserved) in undamped SHM.

$$E \propto A^2 \text{ and } E \propto f^2.$$

**11. Average Energies**

$$\langle K \rangle_{cycle} = \langle U \rangle_{cycle} = \frac{1}{4}kA^2$$

Over one complete cycle of oscillation.

$$\langle E \rangle_{cycle} = \frac{1}{2}kA^2$$

**12. Spring-Mass System (Horizontal/Vertical)**

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Period is independent of  $g$  and amplitude.

$k$ : Spring constant.

**13. Springs in Series**

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$$

End-to-end connection. Force is same, extension adds up.

**14. Springs in Parallel**

$$k_{eq} = k_1 + k_2 + \dots$$

Side-by-side or mass between two fixed springs. Extensions are same.



**15. Cutting a Spring**

$$k \cdot l = \text{constant} \implies k \propto \frac{1}{l}$$

If spring of length  $l$  is cut into  $n$  equal parts, stiffness of each part becomes  $nk$ .

**16. Two Block System (Reduced Mass)**

$$T = 2\pi \sqrt{\frac{\mu}{k}}$$

Where reduced mass  $\mu = \frac{m_1 m_2}{m_1 + m_2}$ .

Blocks oscillate relative to center of mass.

**17. Simple Pendulum**

$$T = 2\pi \sqrt{\frac{l}{g_{eff}}}$$

For small angular amplitude ( $\theta < 5^\circ$ ).

$l$ : Length from pivot to CM of bob.

**18. Pendulum in Lift**

**Accelerating Up:**  $g_{eff} = g + a$

If lift falls freely ( $a = g$ ),  $g_{eff} = 0$ ,  $T \rightarrow \infty$  (No oscillation).

$$T = 2\pi \sqrt{\frac{l}{g+a}}$$

**Accelerating Down:**  $g_{eff} = g - a$

$$T = 2\pi \sqrt{\frac{l}{g-a}}$$

**19. Pendulum in Truck/Car**

$$g_{eff} = \sqrt{g^2 + a^2}$$

Truck moving horizontally with acceleration  $a$ . Mean position shifts by  $\tan \theta = a/g$ .

$$T = 2\pi \sqrt{\frac{l}{(g^2 + a^2)^{1/2}}}$$

**20. Pendulum with Charged Bob**

$$g_{eff} = g + \frac{qE}{m} \text{ (E field down)}$$

Electric field  $E$  applied vertically.

$$g_{eff} = g - \frac{qE}{m} \text{ (E field up)}$$

**21. Pendulum of Infinite Length**

$$T = 2\pi \sqrt{\frac{1}{g(\frac{1}{l} + \frac{1}{R_e})}}$$

If  $l \approx R_e$  (Earth's radius).

If  $l \rightarrow \infty$ ,

$$T = 2\pi \sqrt{\frac{R_e}{g}} \approx 84.6 \text{ min.}$$

22. Second's Pendulum	$T = 2 \text{ seconds}$	Length $l \approx 0.993 \text{ m}$ (on Earth).
23. Physical Pendulum	$T = 2\pi\sqrt{\frac{I}{mgd}}$	$I$ : Moment of Inertia about pivot.  $d$ : Distance between pivot and Center of Mass.
24. Torsional Pendulum	$T = 2\pi\sqrt{\frac{I}{C}}$	$I$ : MOI of disc/body.  $C$ : Torsional constant ( $Nm/rad$ ) of the wire.
25. Superposition (Same freq)	$x_{res} = A \sin(\omega t + \theta)$  $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \delta}$	$\delta$ : Phase difference between two waves.  $\tan \theta = \frac{A_1 \sin \phi_1 + A_2 \sin \phi_2}{A_1 \cos \phi_1 + A_2 \cos \phi_2}$
26. Liquid in U-Tube	$T = 2\pi\sqrt{\frac{h}{g}}$ or $T = 2\pi\sqrt{\frac{L}{2g}}$	$h$ : Height of liquid column in one arm at equilibrium.  $L$ : Total length of liquid column.
27. Body Floating in Liquid	$T = 2\pi\sqrt{\frac{m}{A\rho g}} = 2\pi\sqrt{\frac{h_{submerged}}{g}}$	$A$ : Cross-sectional area. $\rho$ : Density of liquid.  Slightly depressed and released.
28. Tunnel through Earth	$T = 2\pi\sqrt{\frac{R_e}{g}} \approx 84.6 \text{ min}$	Particle dropped in a tunnel along diameter or chord. (Assuming uniform density).
29. Ball in Concave Dish	$T = 2\pi\sqrt{\frac{R-r}{g}}$	$R$ : Radius of curvature of dish. $r$ : Radius of ball.  For small oscillations ( $r \ll R$ ), $T \approx 2\pi\sqrt{R/g}$ .
30. Piston in Cylinder	$T = 2\pi\sqrt{\frac{mV}{PA^2}}$	$V$ : Volume, $P$ : Pressure, $A$ : Area of piston.

(Adiabatic process usually considered, add factor  $\gamma$  in denominator for adiabatic).

21. Amplitude with

$$A = A_0 f^n \text{ where } f < 1$$

If amplitude decays by a constant

# Wave Motion (Mechanical Waves)

BY AP Sir, Sakaar Classes

## 1. Basics of Wave Motion & Progressive Waves

Formula / Topic Name	Formula	Conditions / Usage Notes
<b>General Plane Progressive Wave Equation</b>	$y = A \sin(\omega t \pm kx + \phi)$	$y$ : Displacement,
		$A$ : Amplitude,
		$\omega$ : Angular freq,
		$k$ : Propagation constant.
		<b>(-) sign:</b> Wave moving in +x direction.  <b>(+) sign:</b> Wave moving in -x direction.
<b>Angular Frequency (<math>\omega</math>)</b>	$\omega = 2\pi f = \frac{2\pi}{T}$	$f$ : Frequency (Hz),
		$T$ : Time period.

**Propagation Constant  
( $k$ )**

$$k = \frac{2\pi}{\lambda}$$

$\lambda$

: Wavelength. Represents phase change per unit length.

**Wave Velocity ( $v$ )**

$$v = f\lambda = \frac{\omega}{k}$$

Speed at which the disturbance travels through the medium.

**Particle Velocity ( $v_p$ )**

$$v_p = \frac{\partial y}{\partial t} = \omega A \cos(\omega t \pm kx)$$

Velocity of a particle oscillating about its mean position.

**Relation: Particle vs  
Wave Velocity**

$$v_p = -v \times (\text{slope of y-x graph})$$

Used to find the direction of particle motion if the wave shape is known.

$$v_p = -v \left( \frac{\partial y}{\partial x} \right)$$

**Particle Acceleration ( $a_p$ )**

$$a_p = \frac{\partial^2 y}{\partial t^2} = -\omega^2 y$$

Maximum acceleration

$$a_{max} = \omega^2 A$$

occurs at extreme positions (

$$y = \pm A$$

).

**Phase Difference ( $\Delta\phi$ )**

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x$$

Relation between Phase difference and Path difference ( $\Delta x$ ) or Time difference ( $\Delta t$ ).

$$\Delta\phi = \frac{2\pi}{T} \Delta t$$

## 2. Speed of Waves in Media

Formula / Topic Name

Formula

Conditions / Usage Notes

### Speed of Transverse Wave on String

$$v = \sqrt{\frac{T}{\mu}}$$

$T$

: Tension in string,

$\mu$

: Linear mass density (

$m/L$

).

**Imp:** Ensure

$\mu$

is mass per unit length, not volume density.

### Speed of Sound (General)

$$v = \sqrt{\frac{E}{\rho}}$$

$E$

: Modulus of Elasticity,

$\rho$

: Density of medium.

### Speed in Solids (Rod)

$$v = \sqrt{\frac{Y}{\rho}}$$

$Y$

: Young's Modulus.

### Speed in Fluids (Liquids/Gases)

$$v = \sqrt{\frac{B}{\rho}}$$

$B$

: Bulk Modulus.

### Newton's Formula (Gases)

$$v = \sqrt{\frac{P}{\rho}}$$

Assumed Isothermal process. (Incorrect historically, gives lower value).

### Laplace Correction (Gases)

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

Assumed Adiabatic process.

$$\gamma = C_p/C_v$$

(Adiabatic index).

$$T$$

: Temp in Kelvin,

$$M$$

: Molar mass.

Effect of Temperature on  
Sound Speed

$$v \propto \sqrt{T}$$

Speed increases with temperature.

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

Effect of Humidity

$$v_{moist} > v_{dry}$$

Moist air is less dense than dry air (

$$\rho_{moist} < \rho_{dry}$$

), so speed increases.

### 3. Superposition & Interference

Formula / Topic Name

Formula

Conditions / Usage Notes

Resultant Amplitude (  
 $A_{res}$ )

$$A_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \phi}$$

$$\phi$$

is the phase difference between  
two interfering waves.

Resultant Intensity (  
 $I_{res}$ )

$$I_{res} = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \phi$$

Since

$$I \propto A^2$$

.

**Constructive  
Interference (Maxima)**

Condition:

$$\phi = 2n\pi$$

Where

$$n = 0, 1, 2, \dots$$

Path diff

$$\Delta x = n\lambda$$

$$A_{max} = A_1 + A_2$$

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

**Destructive  
Interference (Minima)**

Condition:

$$\phi = (2n - 1)\pi$$

Where

$$n = 1, 2, 3, \dots$$

Path diff

$$\Delta x = (2n - 1)\frac{\lambda}{2}$$

$$A_{min} = |A_1 - A_2|$$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

**Ratio of Intensities**

$$\frac{I_{max}}{I_{min}} = \left( \frac{A_1 + A_2}{A_1 - A_2} \right)^2$$

Useful for questions given  
amplitude ratio

$$r = A_1/A_2$$

## 4. Stationary (Standing) Waves



Formula / Topic  
Name

Formula

Conditions / Usage Notes

General Equation

$$y = 2A \sin(kx) \cos(\omega t)$$

Or

$$y = 2A \cos(kx) \sin(\omega t)$$

.

Amplitude of particle at

$$x$$

is

$$A(x) = 2A \sin(kx)$$

.

Nodes &  
Antinodes

**Nodes:** Zero amplitude points.

Distance between consecutive Node & Node or  
Antinode & Antinode =

**Antinodes:** Max amplitude  
points.

$$\lambda/2$$

.

Distance between Node & Antinode =

$$\lambda/4$$

.

String Fixed at  
Both Ends

$$f_n = \frac{nv}{2L} = n \left( \frac{1}{2L} \sqrt{\frac{T}{\mu}} \right)$$

$$n = 1, 2, 3 \dots$$

(Number of loops).

$$n = 1$$

: Fundamental/1st Harmonic.

$$n = 2$$

: 2nd Harmonic/1st Overtone.

All harmonics are present.

**String Fixed at  
One End**

$$f_n = \frac{(2n - 1)v}{4L}$$

$$n = 1, 2, 3...$$

Only odd harmonics are present (

$$f_1, 3f_1, 5f_1...$$

).

**Sonometer Law**

$$f \propto \frac{1}{L}$$

Used for comparing frequencies when length or tension changes.

$$f \propto \sqrt{T}$$

$$f \propto \frac{1}{\sqrt{\mu}}$$

## 5. Organ Pipes (Sound Columns)

Formula / Topic  
Name

Formula

Conditions / Usage Notes

**Open Organ Pipe**

$$f_n = \frac{nv}{2L}$$

Open at both ends.

Similar to string fixed at both ends.

All harmonics present (

$$1 : 2 : 3...$$

).

**Closed Organ Pipe**

$$f_n = \frac{(2n - 1)v}{4L}$$

Closed at one end.

Similar to string fixed at one end.

Only odd harmonics present (

$$1 : 3 : 5 \dots$$

).

**End Correction ( $e$ )**

$$e \approx 0.6r$$

$$r$$

: Radius of pipe.

Antinode forms slightly outside the open end.

**Corrected Lengths**

Open Pipe:

Use these lengths in frequency formulas for precise calculation.

$$L_{eff} = L + 2e$$

Closed Pipe:

$$L_{eff} = L + e$$

**Resonance Tube**

$$v = 2f(L_2 - L_1)$$

$$L_1$$

: First resonance length ( $\lambda/4$ ).

$$L_2$$

: Second resonance length ( $3\lambda/4$ ).

Eliminates end correction error.

6. Beats & Doppler Effect

Formula / Topic Name	Formula	Conditions / Usage Notes
Beat Frequency ( $f_b$ )	$f_b =   f_1 - f_2$	Number of beats per second.  Requires  $f_1 \approx f_2$  .
Tuning Fork Loading/Filing	<div>Waxing (Loading): Mass  ↑  , Freq  ↓  .  Filing: Mass  ↓  , Freq  ↑  .</div>	Used to determine unknown frequency based on change in beat frequency.
Doppler Effect (General)	$f' = f_0 \left( \frac{v \pm v_o}{v \mp v_s} \right)$	$f'$ : Apparent freq,  $f_0$ : Source freq.  $v$

		: Speed of sound.
		$v_o$
		: Observer velocity.
		$v_s$
		: Source velocity.
<b>Doppler Sign Convention</b>	<b>Numerator (<math>v_o</math>):</b> (+) if Observer moves TOWARDS source.	"Towards" tends to increase frequency.
	<b>Denominator (<math>v_s</math>):</b> (-) if Source moves TOWARDS observer.	"Away" tends to decrease frequency.
<b>Effect of Wind (<math>v_w</math>)</b>	$f' = f_0 \left( \frac{(v \pm v_w) \pm v_o}{(v \pm v_w) \mp v_s} \right)$	Add $v_w$ to $v$ if wind blows Source $\rightarrow$ Observer.  Subtract if wind blows Observer $\rightarrow$ Source.

## 7. Intensity & Energy Density

Formula / Topic Name	Formula	Conditions / Usage Notes
<b>Intensity (<math>I</math>)</b>	$I = 2\pi^2 f^2 A^2 \rho v$	Power per unit area.  Depends on square of frequency and amplitude.
<b>Intensity vs Distance</b>	<b>Point Source:</b>	Spherical wavefronts vs Cylindrical wavefronts.

# Thermal Expansion and Calorimetry

BY AP Sir, Sakaar Classes

Formula Name / Topic	Formula(e)	Conditions / Notes
<b>Linear Expansion</b>	$\Delta L = L_0 \alpha \Delta T$ $L_f = L_0 (1 + \alpha \Delta T)$	Valid for small temperature changes ( $\Delta T$ ). $\alpha$ is the coefficient of linear expansion.
<b>Superficial (Area) Expansion</b>	$\Delta A = A_0 \beta \Delta T$ $A_f = A_0 (1 + \beta \Delta T)$	$\beta$ is the coefficient of superficial expansion.
<b>Volume Expansion</b>	$\Delta V = V_0 \gamma \Delta T$ $V_f = V_0 (1 + \gamma \Delta T)$	$\gamma$ is the coefficient of volume expansion.
<b>Relation between Coefficients</b>	$\alpha : \beta : \gamma = 1 : 2 : 3$ $\beta = 2\alpha, \quad \gamma = 3\alpha$	strictly valid for isotropic solids (properties same in all directions).
<b>Anisotropic Expansion</b>	$\gamma = \alpha_x + \alpha_y + \alpha_z$	For non-isotropic solids where $\alpha$ differs along x, y, z axes.
<b>Variation of Density</b>	$\rho' = \frac{\rho_0}{1 + \gamma \Delta T} \approx \rho_0 (1 - \gamma \Delta T)$	Approximation valid when $\gamma \Delta T \ll 1$ . $\rho'$ decreases as T increases.
<b>Thermal Stress</b>	$\text{Stress} = Y \alpha \Delta T$ $\text{Force} = Y A \alpha \Delta T$	Rod held between rigid supports preventing expansion. $Y$ is Young's Modulus.
<b>Pendulum Clock (Time Period)</b>	New Period: $T' = T(1 + \frac{1}{2} \alpha \Delta \theta)$	Due to length change $L' = L(1 + \alpha \Delta \theta)$ . $\Delta \theta$ is temp change.

<b>Time Lost/Gained (Pendulum)</b>	$\Delta t = \frac{1}{2}\alpha\Delta\theta \times t$ $\text{Loss/day} = \frac{1}{2}\alpha\Delta\theta \times 86400$	If temp increases, clock expands, slows down (Loses time). If temp drops, gains time.
<b>Bimetallic Strip</b>	$R \approx \frac{d}{(\alpha_1 - \alpha_2)\Delta T}$	Radius of curvature $R$ when two strips of thickness $d$ bend. $\alpha_1 > \alpha_2$ .
<b>Expansion of Liquids</b>	$\gamma_{real} = \gamma_{app} + \gamma_{vessel}$ $\gamma_{app} = \gamma_{real} - 3\alpha_{vessel}$	$\gamma_{real}$ is actual expansion; $\gamma_{app}$ is what we see in a container.
<b>Barometer Correction</b>	$H_0 = H_{obs}[1 - (\gamma_{Hg} - \alpha_{scale})\Delta T]$	$H_0$ is true height at $0^\circ C$ . Corrects for both Hg expansion and scale expansion.
<b>Specific Heat Capacity</b>	$Q = ms\Delta T \text{ or } Q = mc\Delta T$	Heat required to change temp without phase change. $s$ or $c$ is specific heat.
<b>Molar Heat Capacity</b>	$Q = nC\Delta T$	$n$ = number of moles. $C$ = Molar heat capacity.
<b>Heat Capacity (Thermal Capacity)</b>	$H = ms \text{ or } H = \frac{Q}{\Delta T}$	Heat required to raise temp of the <i>whole body</i> by $1^\circ C$ .
<b>Latent Heat (Phase Change)</b>	$Q = mL$	Used during melting ( $L_f$ ) or boiling ( $L_v$ ). Temp remains constant.
<b>Water Equivalent</b>	$W = ms$	Mass of water that absorbs same heat as the body for same $\Delta T$ . (Unit: grams/kg)
<b>Principle of Calorimetry</b>	<p>Heat Lost = Heat Gained</p> $\sum m_i s_i (T_i - T_{mix}) = \sum m_j s_j (T_{mix} - T_j)$	System must be isolated (no heat loss to surroundings).
<b>Mixture Temperature</b>	$T_{mix} = \frac{m_1 s_1 T_1 + m_2 s_2 T_2}{m_1 s_1 + m_2 s_2}$	For mixing two substances of same state (e.g., liquid + liquid).

**Steam + Ice Mixture**

Check heat available vs heat required step-by-step.

1. Heat released by steam to  $100^\circ$  water ( $mL_v$ ).

2. Heat to cool water. Compare with ice melting req.

**Newton's Law of Cooling (Exact)**

$$\frac{T - T_s}{T_0 - T_s} = e^{-kt}$$

$T_s$  = Surroundings temp.  $T_0$  = Initial temp.  $T$  = Temp at time  $t$ .

**Newton's Law (Approx)**

$$\frac{T_1 - T_2}{t} = K \left[ \frac{T_1 + T_2}{2} - T_s \right]$$

Valid only when temp difference ( $T - T_s$ ) is small (approx  $< 30^\circ C$ ).

**Stefan's Law**

$$E = \sigma A e T^4$$

Rate of energy radiated.  $e$  = emissivity ( $0 \leq e \leq 1$ ).

**Net Rate of Heat Loss (Radiation)**

$$\frac{dQ}{dt} = \sigma A e (T^4 - T_s^4)$$

Body at temp  $T$  placed in surroundings at temp  $T_s$ . If  $T > T_s$ , heat is lost.

**Growth of Ice (Ice Formation Time)**

$$t = \frac{\rho L}{2K\theta} (x_2^2 - x_1^2)$$

Time taken to increase thickness from  $x_1$  to  $x_2$ .  $\theta$  = temp difference (Air -  $0^\circ C$ ).  $K$  = Thermal conductivity.



# Kinetic Theory of Gases and Thermodynamics

BY AP Sir, Sakaar Classes

## 1. Kinetic Theory of Gases (KTG) & Gas Laws

Formula Name / Topic	Formula(e)	Conditions / specific usage
<b>Ideal Gas Equation</b>	$PV = nRT$	<b>General Ideal Gas Law.</b>
	$PV = \frac{m}{M}RT$	$n$ : moles, $m$ : mass, $M$ : Molar mass, $\rho$ : density, $N$ : number of molecules, $k_B$ : Boltzmann constant ( $1.38 \times 10^{-23} J/K$ ).
	$P = \rho \frac{RT}{M}$	
	$PV = Nk_B T$	
<b>Boyle's Law</b>	$P \propto \frac{1}{V} \implies P_1 V_1 = P_2 V_2$	<b>Constant Temperature (<math>T</math>)</b> (Isothermal).  Graph $P$ vs $1/V$ is a straight line through origin.
<b>Charles's Law</b>	$V \propto T \implies \frac{V_1}{T_1} = \frac{V_2}{T_2}$	<b>Constant Pressure (<math>P</math>)</b> (Isobaric).  $T$ must be in Kelvin.
<b>Gay-Lussac's Law</b>	$P \propto T \implies \frac{P_1}{T_1} = \frac{P_2}{T_2}$	<b>Constant Volume (<math>V</math>)</b> (Isochoric).  Pressure Law. $T$ must be in Kelvin.
<b>Avogadro's Law</b>	$V \propto n \implies \frac{V_1}{n_1} = \frac{V_2}{n_2}$	<b>Constant <math>P</math> and <math>T</math>.</b>  Equal volumes of gases contain equal number of molecules.

**Dalton's Law  
of Partial  
Pressure**

$$P_{total} = P_1 + P_2 + \dots + P_n$$

For non-reacting gas mixture.

$$P_i = x_i P_{total}$$

$$x_i = \frac{n_i}{n_{total}} \text{ (Mole fraction).}$$

**Graham's Law  
of Diffusion**

$$r \propto \frac{1}{\sqrt{M}} \implies \frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{\rho_2}{\rho_1}}$$

$r$ : Rate of diffusion/effusion.

Lighter gases diffuse faster.

At const  $P$  and  $T$ .

**Pressure of an  
Ideal Gas**

$$P = \frac{1}{3} \rho v_{rms}^2$$

$v_{rms}$ : Root Mean Square speed.

$$P = \frac{1}{3} \frac{mN}{V} v_{rms}^2$$

Pressure depends on density and square of RMS speed.

$$P = \frac{2}{3} E \text{ (where } E \text{ is K.E. per unit volume)}$$

$E$ : Total translational K.E. per unit volume.

**Root Mean  
Square Speed ( $v_{rms}$ )**

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_B T}{m_{molecule}}} = \sqrt{\frac{3P}{\rho}}$$

Speed effective in calculating kinetic energy.

**Note:**  $T$  must be in Kelvin,  $M$  in kg/mol.

For mixture:

$$v_{rms}(mix) = \sqrt{\frac{n_1 M_1 v_1^2 + n_2 M_2 v_2^2}{n_1 M_1 + n_2 M_2}}$$

**Average Speed  
( $v_{avg}$ )**

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8k_B T}{\pi m}}$$

Arithmetic mean of speeds.

$$v_{avg} \approx 0.92 v_{rms}$$

**Most Probable  
Speed ( $v_{mp}$ )**

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2k_B T}{m}}$$

Speed possessed by maximum fraction of molecules.

Ratio

$$v_{mp} : v_{avg} : v_{rms} = \sqrt{2} : \sqrt{8/\pi} : \sqrt{3}$$

<b>Kinetic Energy (Translational)</b>	Per molecule: $K.E. = \frac{3}{2}k_B T$	depends <b>ONLY</b> on Temperature ( $T$ ).
	Per mole: $K.E. = \frac{3}{2}RT$	Independent of nature of gas.
<b>Degrees of Freedom (<math>f</math>)</b>	Monoatomic: $f = 3$ (3 trans)	Used to calculate internal energy and $C_v$ .
	Diatomic (rigid): $f = 5$ (3 trans + 2 rot)	<b>Note:</b> For NEET/JEE, unless specified "high temp", take Diatomic $f = 5$ .
	Diatomic (vib at high T): $f = 7$	
	Triatomic (linear): $f = 5$ (rigid) / 7 (vib)	
	Triatomic (non-linear): $f = 6$	
<b>Law of Equipartition of Energy</b>	Energy associated with each D.O.F = $\frac{1}{2}k_B T$ (per molecule)	Total Internal Energy ( $U$ ) depends on $f$ .
<b>Internal Energy (<math>U</math>)</b>	$U = \frac{f}{2}nRT$	For an ideal gas, $U$ is a function of $T$ only.
		Change: $\Delta U = \frac{f}{2}nR\Delta T = nC_v\Delta T$
<b>Mean Free Path (<math>\lambda</math>)</b>	$\lambda = \frac{1}{\sqrt{2}\pi d^2 n_v}$	$d$ : diameter of molecule, $n_v$ : number density ( $N/V$ ).
		$\lambda \propto \frac{T}{P}$ (since $n_v = P/k_B T$ ).
<b>Mixture of Gases</b>	$M_{mix} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$	Used when non-reacting gases are mixed.
	$C_{v(mix)} = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}$	
	$\gamma_{mix} = \frac{C_{p(mix)}}{C_{v(mix)}}$	

## 2. Thermodynamics

Formula Name / Topic	Formula(e)	Conditions / specific usage
<b>First Law of Thermodynamics (FLOT)</b>	$dQ = dU + dW$	<b>Conservation of Energy.</b>
	$Q = \Delta U + W$	Sign Convention (Chemistry opposite for W):
		Heat added to system: $Q > 0$
		Work done BY gas (expansion): $W > 0$
		Internal Energy increases: $\Delta U > 0$
<b>Work Done (General)</b>	$W = \int_{V_1}^{V_2} P dV$	Area under P-V curve on Volume axis gives Work Done.
<b>Mayer's Relation</b>	$C_P - C_V = R$	Valid for Ideal Gas (per mole).
		$C_P > C_V$ because work is done in isobaric expansion.
<b>Specific Heat Ratio (<math>\gamma</math>)</b>	$\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$	Monoatomic $\gamma = 5/3 = 1.67$
		Diatomic $\gamma = 7/5 = 1.4$
		Triatomic $\gamma = 4/3 = 1.33$
<b>Bulk Modulus of Gas (<math>B</math>)</b>	<b>General:</b> $B = -V \frac{dP}{dV}$	Resistance to compression.
	<b>Isothermal (<math>B_T</math>):</b> $B_T = P$	Adiabatic elasticity is $\gamma$ times Isothermal elasticity ( $B_S = \gamma B_T$ ).
	<b>Adiabatic (<math>B_S</math>):</b> $B_S = \gamma P$	
		Isobaric ( $P=\text{const}$ ): $B = 0$ .

		Isochoric ( $V=\text{const}$ ): $B = \infty$ .
<b>Isochoric Process (<math>V</math> const)</b>	$W = 0$	Volume constant ( $\Delta V = 0$ ).
	$Q = \Delta U = nC_V \Delta T$	Gay-Lussac's Law holds.
	$\frac{P_1}{T_1} = \frac{P_2}{T_2}$	
<b>Isobaric Process (<math>P</math> const)</b>	$W = P(V_2 - V_1) = nR(T_2 - T_1)$	Pressure constant.
	$Q = nC_P \Delta T$	Charles's Law holds.
	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$	Fraction of heat into internal energy: $1/\gamma$ .
		Fraction of heat into work: $1 - 1/\gamma$ .
<b>Isothermal Process (<math>T</math> const)</b>	$W = nRT \ln(\frac{V_2}{V_1}) = 2.303nRT \log(\frac{V_2}{V_1})$	Temperature constant ( $\Delta T = 0$ ).
	$W = nRT \ln(\frac{P_1}{P_2})$	Internal energy change is zero for ideal gas.
	$\Delta U = 0 \implies Q = W$	Boyle's Law ( $P_1 V_1 = P_2 V_2$ ).
<b>Adiabatic Process (<math>Q = 0</math>)</b>	Equation: $PV^\gamma = \text{const}$	No heat exchange ( $dQ = 0$ ).
	$TV^{\gamma-1} = \text{const}$	Occurs suddenly/quickly or in insulated containers.
	$P^{1-\gamma} T^\gamma = \text{const}$	
<b>Work in Adiabatic Process</b>	$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{nR(T_1 - T_2)}{\gamma - 1}$	Since $Q = 0$ , $W = -\Delta U$ .

Expansion causes cooling ( $T_2 < T_1$ ).

Compression causes heating.

**Slope of P-V Graph**

Isothermal Slope:  $\frac{dP}{dV} = -\frac{P}{V}$

Adiabatic curve is steeper than Isothermal by a factor of  $\gamma$ .

Adiabatic Slope:  $\frac{dP}{dV} = -\gamma \frac{P}{V}$

**Polytropic Process**

$PV^x = \text{const}$

General process.

Molar Heat Capacity:

$$C = C_V + \frac{R}{1-x}$$

Work:  $W = \frac{nR(T_1 - T_2)}{x-1}$

**Cyclic Process**

$$\Delta U_{net} = 0$$

Work = Area enclosed by the loop.

$$Q_{net} = W_{net}$$

Clockwise = +ve Work (Engine).

Anticlockwise = -ve Work (Refrigerator).

**Efficiency of Heat Engine ( $\eta$ )**

$$\eta = \frac{\text{Work Output}}{\text{Heat Input}} = \frac{W}{Q_{in}}$$

$Q_{in}$ : Heat absorbed from source.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}}$$

$Q_{out}$ : Heat rejected to sink.

**Carnot Engine**

$$\eta = 1 - \frac{T_{sink}}{T_{source}}$$

**Maximum theoretical efficiency.**

$T$  must be in Kelvin.

Valid for reversible cycle only.

**Refrigerator (COP -  $\beta$ )**

$$\beta = \frac{\text{Heat Extracted}}{\text{Work Input}} = \frac{Q_{\text{cold}}}{W}$$

Coefficient of Performance.

$$\beta = \frac{Q_{\text{cold}}}{Q_{\text{hot}} - Q_{\text{cold}}} = \frac{T_{\text{cold}}}{T_{\text{hot}} - T_{\text{cold}}}$$

Relationship with efficiency:  
 $\beta = \frac{1-\eta}{\eta}$

**Entropy ( $\Delta S$ )**

$$\Delta S = \int \frac{dQ_{\text{rev}}}{T}$$

Measure of disorder.

Ideal Gas:

$$\Delta S = nC_v \ln\left(\frac{T_2}{T_1}\right) + nR \ln\left(\frac{V_2}{V_1}\right)$$

Adiabatic Reversible:  $\Delta S = 0$   
(Isentropic).

$$\text{Phase Change: } \Delta S = \frac{mL}{T}$$

**Free Expansion**

$$W = 0, Q = 0, \Delta U = 0, \Delta T = 0$$

Expansion against vacuum ( $P_{\text{ext}} = 0$ ).

Neither isothermal nor  
adiabatic in strict sense, but  
 $\Delta T = 0$  for ideal gas.

**Note from AP Sir:**

- Always check units: Pressure in Pa ( $N/m^2$ ), Volume in  $m^3$ , Temperature in Kelvin ( $K$ ).
- $R = 8.314 J/(mol \cdot K)$  when using SI units.
- $R = 0.0821 (L \cdot atm)/(mol \cdot K)$  when Pressure is atm and Volume is Liters.