Rotational Motion

BY AP Sir, Sakaar Classes

Formula / Topic Name	Formula(e)	Conditions / When to use
1. KINEMATICS OF ROTATION		
Angular Displacement	$\theta = \frac{l}{r}$	Angle in radians, l = arc length.
Angular Velocity	$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$	Rate of change of angular position.
	$\omega_{inst} = \frac{d\theta}{dt}$	
Angular Acceleration	$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$	Rate of change of angular velocity.
	$\alpha_{inst} = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$	
Equations of Kinematics	1. $\omega = \omega_0 + \alpha t$	**Strictly valid only when angular acceleration (α) is CONSTANT.**
	2. $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	
	3. $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$	
	4. $\theta_{nth} = \omega_0 + \frac{\alpha}{2}(2n - 1)$	
Linear vs Angular Variables	$v = \omega r$	v is tangential velocity.
	$a_t = \alpha r$ (Tangential acc.)	a_t changes speed.
	$a_c = \omega^2 r$ (Centripetal acc.)	a_c changes direction (always exists if $\omega \neq 0$).

$$a_{net} = \sqrt{a_t^2 + a_c^2}$$

2. MOMENT OF INERTIA (MOI)		
Discrete System	$I = \sum m_i r_i^2$	r_i is perpendicular distance from the axis of rotation.
Continuous Bodies	$I = \int r^2 dm$	Requires integration.
Radius of Gyration (k)	$I = Mk^2 \Rightarrow k = \sqrt{\frac{I}{M}}$	Distance where total mass is theoretically concentrated to give same I.
Perpendicular Axis Theorem	$I_z = I_x + I_y$	**Valid ONLY for planar (2D) bodies** (laminar objects). Axes x, y must be in the plane; z perpendicular to plane.
Parallel Axis Theorem	$I_{axis} = I_{CM} + Md^2$	Valid for **any** 3D or 2D body.
		d = perpendicular distance between parallel axes.
		One axis MUST pass through Center of Mass (CM).
3. MOI OF STANDARD BODIES	**(Axis through Center, unless specified)**	
Ring / Hollow Cylinder	$I = MR^2$	Axis perpendicular to plane (Ring) or along geometrical axis (Cylinder).
Disc / Solid Cylinder	$I = \frac{MR^2}{2}$	Axis perpendicular to plane (Disc) or along geometrical axis (Cylinder).

Thin Rod	$I = \frac{ML^2}{12}$ (Center)	Axis perpendicular to length.
	$I = \frac{ML^2}{3}$ (End)	
Solid Sphere	$I = \frac{2}{5}MR^2$	Axis along diameter.
Hollow Sphere (Shell)	$I = \frac{2}{3}MR^2$	Axis along diameter.
Rectangular Plate	$I = \frac{M(a^2+b^2)}{12}$	Axis perpendicular to plate, through center.
4. TORQUE (τ)		
Vector Definition	$\vec{\tau} = \vec{r} \times \vec{F}$	\tau
	$	
Newton's 2nd Law (Rotation)	$\vec{\tau}_{net} = I\vec{\alpha}$	Valid for rigid bodies rotating about a fixed axis or about CM.
	$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$	
Couple	$\tau = F \times d$	Two equal and opposite forces. Torque is independent of the choice of origin.
Rotational Equilibrium	$\sum \vec{F}_{ext} = 0$ AND $\sum \vec{\tau}_{ext} = 0$	Body is neither accelerating translationally nor rotationally.
5. ANGULAR MOMENTUM (L)		
Point Mass	$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$	About a specific origin.
	$L = mvr_\perp$	

Rigid Body (Fixed Axis)	$L = I\omega$	Axis must be fixed or passing through CM.
Combined Motion (Rolling)	$\vec{L} = \vec{L}_{CM} + \vec{L}_{aboutCM} = M(\vec{r}_{cm} \times \vec{v}_{cm}) + I_{cm}\vec{\omega}$	General formula for a body moving and rotating.
Conservation of Angular Momentum	$I_1\omega_1 = I_2\omega_2$	**Condition:** Net external torque on the system is ZERO ($\tau_{ext} = 0$).
6. WORK, POWER, ENERGY		
Rotational Kinetic Energy	$K_{rot} = \frac{1}{2}I\omega^2$	Pure rotation about an axis.
Work Done	$W = \int \tau d\theta = \vec{\tau} \cdot \Delta\vec{\theta}$	Analogous to $W = \vec{F} \cdot \vec{d}$.
Power	$P = \vec{\tau} \cdot \vec{\omega}$	Instantaneous power delivered by torque.
Work-Energy Theorem	$W_{ext} = \Delta K = K_f - K_i$	Work done by all torques equals change in Rotational KE.
7. ROLLING MOTION		
Condition for Pure Rolling	$v_{cm} = \omega R$	Velocity of the bottom-most point (contact point) is zero relative to ground.
	$a_{cm} = \alpha R$	
Total Kinetic Energy	$K_{total} = K_{trans} + K_{rot}$	Useful factor $\beta = (1 + \frac{k^2}{R^2})$.
	$K_{total} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$	Ring: $\beta = 2$, Disc: $\beta = 1.5$, Solid Sphere: $\beta = 1.4$.
	$K_{total} = \frac{1}{2}Mv_{cm}^2(1 + \frac{k^2}{R^2})$	
8. ROLLING ON INCLINED PLANE	**(Specific Question Formulae)**	**Body rolling down from rest without slipping.**

Acceleration	$a = \frac{g \sin\theta}{1 + \frac{I}{MR^2}} = \frac{g \sin\theta}{\beta}$	Standard NEET/JEE result. Solid sphere accelerates fastest (lowest β).
Velocity at bottom	$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$	Depends only on height h and shape factor $\frac{k^2}{R^2}$, not Mass or Radius.
Time to reach bottom	$t = \sqrt{\frac{2L(1 + \frac{k^2}{R^2})}{g \sin\theta}}$	L = length of incline. Ring takes max time, Solid sphere takes min time.
Min Friction Coefficient	$\mu_{min} = \frac{\tan\theta}{1 + \frac{MR^2}{I}}$	Condition to prevent slipping while rolling down.
9. COLLISION & TOPPLING		
Angular Impulse (J)	$J = \int \tau dt = \Delta L$	Change in Angular Momentum.
Rod hit by particle	$L_i = L_f$ (about hinge/pivot) $mvx = (\frac{ML^2}{3} + mx^2)\omega$	Use Conservation of Angular Momentum about the pivot point to find ω after impact.